Hypersphere Factoring Analysis of Daum's Most Precise Charged Pion Mass Determination

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The most precise mass determination of the charged pion listed by Particle Data Group is the one by Daum. Hypersphere factoring of that mass yields a theoretical mass that is only 0.00006 MeV from the experimentally determined value and well within the .00014 MeV experimental error. The most amazing thing about this analysis is that the best hypersphere surface volume factoring of the pion's mass was done with the surface volume of a unit radius 28-sphere, suggesting that the charged pion is a 28 dimensional object. (It appears 3-dimensional to us because it is attached to the Higgs field – which is 3 dimensional - and so moves with only 3 degrees of dimensional freedom as one would expect of a 3-dimensional particle.) Is the pion 28 dimensional? Examine the factoring and decide for yourself.

Introduction

Many hadrons factor convincingly with a unit radius hypersphere surface volume formulae times 'h'. (See Appendix A for the derivation of the unit radius hypersphere surface volume factoring formula, **Snh**. Also see Appendix B for examples of hypersphere factorings of some other hadron masses.)

Analysis

This is the most precise mass determinaton of the charged pion as listed by Particel Data Group in its 2024 listings.

π^+ MASS

VALUE (MeV)	*+/-*	*DOCUMENT ID*		*TECN*
139.57021	**.00014**	DAUM	19	SPEC

Dividing that mass by the surface volume of a unit radius 28-sphere times 'h' yields this:

$$\frac{139.57021}{\mathbf{S28h}} = 7189.336402$$

7189 is close to the number 7168, which is the sum of three consecutive powers of two, 4096, 2048, and 1024.

$$7168 = 4096 + 2048 + 1024$$

Subtracting 7168 from 7189.336402 yields 21.336402 which suggests the number 21.333… which is 64/3.
This suggests that the number should be 7189.333… At this point the factoring can be written as:

$$\begin{array}{rl} 4096 & = 2^{12} \\ 2048 & = 2^{11} \\ 1024 & = 2^{10} \\ + \quad 21.333 & = 64/3 \\ \hline 7189.333 & \end{array}$$

This factoring result multiplied by **S28h** yields a theoretical mass for the charged pion of: 139.5701504 MeV

$$7189.333\ \mathbf{S28h} = 139.5701504 \text{ MeV}$$

It would add credibility to this factoring if the neutral pion also factored with S28h and it is found that it does..

π^0 MASS

VALUE (MeV) +/-

134.9768 .0005 PDG FIT

Dividing the neutral pion's mass by the surface volume of a unit radius 28-sphere times 'h' yields this:

$$\frac{\mathbf{134.9768}}{\mathbf{S28h}} = 6952.727389$$

The series of numbers 7273 suggests multiplication by 11. (7272 would be better but 7273 is close.) Multiplication by 11 yields 76480.00128. The .00128 can be ignored, so the result is 76480. But since we want to compare the π^+ factoring to the π^0 factoring we should multiply also by 3 which yields 229440. The two pion factorings and their mass difference factoring are compared below.

π^0	π^+	$(\pi^+ - \pi^0)$
$\frac{229440}{33}$ **S28h** = 134.9767978	$\frac{237248}{33}$ **S28h** = 139.5701504	$\frac{7808}{33}$ **S28h** = 4.593352671
131072 = 2^{17}	131072 = 2^{17}	-----
65536 = 2^{16}	65536 = 2^{16}	-----
32768 = 2^{15}	32768 = 2^{15}	-----
-----	4096 = 2^{12}	4096 = 2^{12}
-----	2048 = 2^{11}	2048 = 2^{11}
-----	1024 = 2^{10}	1024 = 2^{10}
-----	512 = 2^{9}	512 = 2^{9}
-----	128 = 2^{7}	128 = 2^{7}
+ 64 = 2^{6}	+ 64 = 2^{6}	+ ---
229440	237248	7808

Comparisons

	π^0	π^+	$(\pi^+ - \pi^0)$
Theoretical Mass	134.9767978	139.5701504	4.593352671
Experimental Mass	- 134.9768	- 139.57021	- 4.5936
(TM-EM)	.0000022	.0000596	.000247329
ExpError	.0005	.00014	.0005
(TM-EM)/ExpError	0.44%	42%	49%

Conclusion

This discovery was made possible by the excellent high accuracy mass determinations of the experimentaists. What do the factorings mean? Is the pion 28 dimensional? It could be. What further information can be gleaned from the factorings? Well, the frequency of a photon with energy equal to the energy of the pion can be determined from Planck's Energy Frequency Law, E=hf, and thus its wavelength might coincide with the pion's circumference if the pion was 3d, but the factorings say it is probably 28d. I found a value of 0.64 fm in the literature for the pion's radius, but that would be the pion's radius of intersection with the Higgs field, which may not intersect the pion at its equator. At this time, no theoretical guidance is available for determining its radius in 28d space.

References

S. Navasetal. (ParticleDataGroup), Phys. Rev. D110, 030001 (2024) and 2025 update.

APPENDIX A

Derivation of the *Hypersphere Surface Volume* Factoring Formula

$$\mathbf{m}_{\mathrm{MeV}} = \mathbf{h}_{\mathrm{MeV}}(\mathbf{xSn})$$

The HSSV factoring formula, **m = h** (**xSn**), which is used to discover hadron dimensions and exact masses, can be derived from Planck's Energy-Frequency Relation: **E = hf.** The key to the derivation is associating a frequency with a unit of hypervolume. A main benefit of the derivation is that it explains how the 10^{-34} factor was removed from **h**, and its units changed from J-s to MeV.

If **m = h (xSn**) is correct, (and the factorings of hundreds of hadrons says it is) then a frequency of ($1.602176634 \times 10^{21}$ Hz) is associated with each unit of hypervolume of a hadron, no matter the dimension. Any valid factoring can be used to find the frequency per unit hypervolume using Planck's radiation law **E=hf**. A simple factoring to use is **Ds**'s which is 10.00 **S9h** = 1967.053624 MeV. Now convert 1967.053 to Joules, plug that into **E=hf**, and solve for **f**. You should get $4.75631288 \times 10^{23}$ Hz. Divide that by **Ds**'s hypervolume, which is **10.000 S9 =** 1967.053/**h** = 296.8657 hypervolume units and you should get $1.602176634 \times 10^{21}$ Hz. Multiplying 296.8657 – **Ds**'s hypervolume - by ($1.602176634 \times 10^{21}$ Hz/vol) - the frequency per unit hypervolume constant - will give you a frequency of $4.75631288 \times 10^{23}$ Hz as the frequency associated with the entire particle, which is correct. (Putting that frequency in Planck's energy-frequency law (**E=hf**) will give you the particle's mass in Joules.) So in terms of particle *hypervolume*, Planck's energy-frequency law can be rewritten as:

$$\mathbf{E}_{\mathrm{J}} = \mathbf{h}_{\mathrm{J\text{-}s}}\ (\mathbf{xSn}_{\mathrm{vol}})\ (\mathbf{1.602176634 \times 10^{21}}\ \mathrm{Hz/vol}) \qquad (\text{here } \mathbf{h} = 6.62607015 \times 10^{-34}\ \mathrm{J\text{-}s})$$

Which says a frequency (and therefore energy) is associated with a volume. To convert **h** to units of MeV divide the right hand side by $1.602176634 \times 10^{-13}$ Joules/MeV (the Joules to MeV conversion factor). The result is **h** in units of MeV and a factor of (1×10^{34}) times **h(xSn)** on the right . (**E** on the left hand side of the equation then has units of MeV by default.) When that factor, (1×10^{34}), is multiplied by Planck's constant, ($6.62607015 \times 10^{-34}$ MeV), you are left with just Planck's constant's coefficient (6.62607015 MeV) for **h**. The result is:

$$\mathbf{m}_{\mathrm{MeV}} = \mathbf{h}_{\mathrm{MeV}}\ (\mathbf{xSn}) \qquad (\text{So, here } \mathbf{h} = 6.62607015\ \mathrm{MeV}, \textbf{ not } 6.62607015 \times 10^{-34}\ \mathrm{J\text{-}s.})$$

Where **m** is in units of MeV, **h** = 6.62607015 MeV, and **Sn** is the hypervolume calculated from the surface volume formula for an n-sphere using a radius of one (a unit radius). (**Snh** values are given in an appendix for all **n** from dimensions 2 to 21.) That formula seems to work on any dimension of hadron, *which implies that the mass density of the hypervolume of hadrons remains the same over all dimensions*. What is the density of the hypervolume of any hadron? It is 6.62607015 MeV per unit hypervolume. That's what the formula says if it is rearranged.

$$\mathbf{h}_{\mathrm{MeV}} = \mathbf{m}_{\mathrm{MeV}}\ /\ (\mathbf{xSn})$$

So, if m=h(xSn) is valid, it means that if a correct factoring can be found for a hadron then, a dimension and a precise mass can be assigned to it.

APPENDIX B

Examples of n-Sphere Surface Volume Factorings of Some Hadron Masses

Subatomic Particle	ExpMass	Error	HSSV Factoring	ThrMass	Formation Quarks	
ρ(770)	775.02	0.35	**4.44444 S5h** =	775.071	dd	
η	547.865	0.031	**2.66666 S6h** =	547.8660	ds	
Δ(1232)	1232.9	1.2	**6.00000 S6h** =	1232.698	ddu	
K(1430)	1438	8/4	**7.00000 S6h** =	1438.148	ds	
Δ(1700)	1643	6/3	**8.00000 S6h** =	1643.598	ddu	
Xi^0	1314.86	0.20	**6.00000 S7h** =	1314.878	ddd	
Xi$^-$	1321.71	0.07	**6.03125 S7h** =	1321.727	ddd	
a2(1700)	1721	11/44	**8.00000 S8h** =	1721.172	cs	
Ds	1967.0	1.0/1.0	**64/7 S8h** =	1967.053	cs	
Ds(2460)	2458.9	1.5	**80/7 S8h** =	2458.817	cs	
B2(5747)	5737.2	0.7	**26.66666 S8h =**	5737.239	bd	
Ds	1967.0	1.0/1.0	**10.00000 S9h** =	1967.053	cc	
Ds(2460)	2458.9	1.5	**12.50000 S9h** =	2458.817	cc	
Ds(2700)	2688	4	**13.66666 S9h** =	2688.307	cc	
Ds(2700)	2710	2	**13.77777 S9h** =	2710.163	cc	
Bj(5732)	5704	4/10	**29.00000 S9h** =	5704.455	cc	
Ds(2212)	2112.2	0.4	**12.5000 S10h** =	2112.195	bc	
Ω(2250)	2253	13	**13.3333 S10h** =	2253.008	dcs	
Ds1(2536)	2534.6	0.3/0.7	**15.0000 S10h** =	2534.634	bc	
Ds2(2572)	2572.2	0.3/1.0	**15.2222 S10h** =	2572.185	bc	
Ds0(2590)	2591	13	**15.3333 S10h** =	2590.960	bc	
Pc(4337)	4337	7/4	**25.6666 S10h** =	4337.041	ddddu	
Pc(4457)	4449.8	1.7/2.5	**26.3333 S10h** =	4449.692	ddddu	
Y(4500)	4506	11	**26.6666 S10h** =	4506.017	ddddu	
b1(1235)	1236	16	**9.0000 S11h** =	1235.936	ddddd	
X(2175)	2197.4	4.4	**16.0000 S11h** =	2197.219	ddddd	
Z(3985)	3982.5	1.8	**29.0000 S11h** =	3982.461	ddddd	
X(4660)	4669	21/3	**34.0000 S11h** =	4669.092	ddddd	
Ds(2860)	2866.6(avg)		**27.0000 S12h** =	2866.605	bt	
D(3000)^0	2971.8	8.7	**28.0000 S12h** =	2972.775	bt	
D(3000)^0	3008.1	4.0	**28.3333 S12h** =	3008.165	bt	
Dsj(3040)	3044	8	**28.6666 S12h** =	3043.555	bt	
Λ	1115.59	0.08	**14.2222 S13h** =	1115.599	ccc	128/9 = 14.2222
Ω	1673.4	1.7	**21.3333 S13h** =	1673.398	ccc	64/3 = 21.3333
Xi(1950)	1952	11	**24.8888 S13h** =	1952.298	ccc	
Xi(2500)	2505	10	**31.9375 S13h** =	2505.195	ccc	
fj(2220)	2223.9	2.5	**40.0000 S14h** =	2223.630	vt	
Xc0(1P)	3415.5	0.4/0.4	**61.4400 S14h** =	3415.496	ccsd	
Xc2(1P)	3557.8	0.2/4	**64.0000 S14h** =	3557.808	ccsd	
ηb(1S)	9394.8	2.7/3.1	**169.0000 S14h** =	9394.839	vt	
f0(980)	977.3	0.9/3.7	**99.7500 S18h** =	977.298	cccb	
f0(980)	982.2	1.0/8.1	**100.2500 S18h** =	982.197	cccb	
f0(980)	984.7	0.4/2.4	**100.5000 S18h** =	984.646	cccb	

Source of ExpMass and Error data: P.A. Zyla et al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

APPENDIX C

Hypersphere Surface Volume Formulae

(Dimension 2 - Dimension 29)

Sphere Dimension	Sn	Surface Volume Formula	(π, r) Powers
2	S2 =	2 $\pi^1 r^1$	(1, 1)
3	S3 =	4 $\pi^1 r^2$	(1, 2)
4	S4 =	2 $\pi^2 r^3$	(2, 3)
5	S5 =	8/3 $\pi^2 r^4$	(2, 4)
6	S6 =	$\pi^3 r^5$	(3, 5)
7	S7 =	16/15 $\pi^3 r^6$	(3, 6)
8	S8 =	1/3 $\pi^4 r^7$	(4, 7)
9	S9 =	32/105 $\pi^4 r^8$	(4, 8)
10	S10 =	1/12 $\pi^5 r^9$	(5, 9)
11	S11 =	64 / 945 $\pi^5 r^{10}$	(5, 10)
12	S12 =	1 / 60 $\pi^6 r^{11}$	(6, 11)
13	S13 =	128 / 10395 $\pi^6 r^{12}$	(6, 12)
14	S14 =	1 / 360 $\pi^7 r^{13}$	(7, 13)
15	S15 =	256 / 135135 $\pi^7 r^{14}$	(7, 14)
16	S16 =	1 / 2520 $\pi^8 r^{15}$	(8, 15)
17	S17 =	512 / 2027025 $\pi^8 r^{16}$	(8, 16)
18	S18 =	1 / 20160 $\pi^9 r^{17}$	(9, 17)
19	S19 =	1024 / 34459425 $\pi^9 r^{18}$	(9, 18)
20	S20 =	1 / 181440 $\pi^{10} r^{19}$	(10, 19)
21	S21 =	2048 / 654729075 $\pi^{10} r^{20}$	(10, 20)
22	S22 =	1 / 1814400 $\pi^{11} r^{21}$	(11, 21)
23	S23 =	4096 / 1.374931058e10 $\pi^{11} r^{22}$	(11, 22)
24	S24 =	1 / 19958400 $\pi^{12} r^{23}$	(12, 23)
25	S25 =	8192 / 3.162341432e11 $\pi^{12} r^{24}$	(12, 24)
26	S26 =	1 / 239500800 $\pi^{13} r^{25}$	(13, 25)
27	S27 =	16384 /7.905853581e12 $\pi^{13} r^{26}$	(13, 26)
28	S28 =	1 /3113510400 $\pi^{14} r^{27}$	(14, 27)
29	S29 =	32768 / 2.134580467e14 $\pi^{14} r^{28}$	(14, 28)

APPENDIX D

Values of Hypersphere Surface Volume
Units of Factorization

(Dimension 2 - Dimension 29)

Sphere Dimension	Unit of Factorization		Formula			Value (MeV/c^2)
2	S2h	=	2	$\pi^1 r^1 h$	=	41.63282661
3	S3h	=	4	$\pi^1 r^2 h$	=	83.26565322
4	S4h	=	2	$\pi^2 r^3 h$	=	130.7933822
5	S5h	=	8/3	$\pi^2 r^4 h$	=	174.3911763
6	S6h	=		$\pi^3 r^5 h$	=	205.4497644
7	S7h	=	16/15	$\pi^3 r^6 h$	=	219.1464153
8	S8h	=	1/3	$\pi^4 r^7 h$	=	215.1464901
9	S9h	=	32/105	$\pi^4 r^8 h$	=	196.7053624
10	S10h	=	1/12	$\pi^5 r^9 h$	=	168.9756582
11	S11h	=	64 / 945	$\pi^5 r^{10} h$	=	137.3262492
12	S12h	=	1 / 60	$\pi^6 r^{11} h$	=	106.1705373
13	S13h	=	128 / 10395	$\pi^6 r^{12} h$	=	78.44057013
14	S14h	=	1 / 360	$\pi^7 r^{13} h$	=	55.59076334
15	S15h	=	256 / 135135	$\pi^7 r^{14} h$	=	37.91204905
16	S16h	=	1 / 2520	$\pi^8 r^{15} h$	=	24.94907624
17	S17h	=	512 / 2027025	$\pi^8 r^{16} h$	=	15.88056197
18	S18h	=	1 / 20160	$\pi^9 r^{17} h$	=	9.797479330
19	S19h	=	1024 / 34459425	$\pi^9 r^{18} h$	=	5.869441980
20	S20h	=	1 / 181440	$\pi^{10} r^{19} h$	=	3.419965454
21	S21h	=	2048 / 654729075	$\pi^{10} r^{20} h$	=	1.940989032
22	S22	=	1 / 1814400	$\pi^{11} r^{21}$	=	1.074413835
23	S23	=	4096 / 1.374931058e10	$\pi^{11} r^{22}$	=	0.580742340
24	S24	=	1 / 19958400	$\pi^{12} r^{23}$	=	0.306851874
25	S25	=	8192 / 3.162341432e11	$\pi^{12} r^{24}$	=	0.158648397
26	S26	=	1 / 239500800	$\pi^{13} r^{25}$	=	8.033363266e-2
27	S27	=	16384 /7.905853581e12	$\pi^{13} r^{26}$	=	3.987269099e-2
28	S28	=	1 /3113510400	$\pi^{14} r^{27}$	=	1.941350386e-2
29	S29	=	32768 / 2.134580467e14	$\pi^{14} r^{28}$	=	9.278796524e-3