Primality Has a Yielding Structure

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ABSTRACT

A simple-yet-fulfilling formula is presented showcasing a linkage between the prime number's [shifted] *index* (ordinal rank) vs its *signature trace*, or power sum over the characteristic 2-basis. A loose analogy of minimum action or energy orbitals could be conceived as one way of rationalizing the representation from amongst alternate, likewise 2-basis, candidates. A most minimalist calculus is proposed accommodating such singular-basis travel.

Keywords: primality calculus

Resubmitted 26 December 2025

Formula 1-2-3

$$p_k = 1 + \sum_{i=1}^{m} 2^{k_i} = \sum_{i=0}^{m} 2^{k_i}, \qquad k_0 = 0, \qquad \sum_{i=0}^{m} k_i = k \qquad (PC1)$$

$$p_k = p_0 + \sum_{i=1}^{l<m} 2^{k_i}, \qquad p_0 \geq 1 \qquad (PC2)$$

The conjecture (PC1) posits that, any prime number can be represented via a 2-basis powered-additive series, with the characteristic powers adding up to an equivalent of the implied prime *index*, or *rank*, as shifted by 1. This is because 2 is not [herewith] seen as a prime, rather as *basis* for, without being an odd number unlike, primes. While at it, please note that the initial *unit* is seen as 1=2^0 even as *any* other initial/interim prime smaller than the would-be resultant could be drawn upon to obtain one (PC2). In the latter case, it could be surmised their *index differential* would amount to that of their *signature traces*, or power sums (PC3). Notably, P0 could refer to *any* [lesser] starting prime kernel while most naturally denoting P0=1 globally (m=0), or outside a particular generating setup. In the meantime, the *m scope* is *open-ended a priori* and could vary extensively per any given number under alternate representations. One of the conjectures (PC4) embarks on it as one candidate way of settling for the core conjecture, i.e. index-signature equivalence.

[1] To Elya, her being my long-time & painstaking Muse

$$\forall p_k, p_s : k - l \equiv \Delta k = \Delta \sum_{i=0}^{m} k_i \equiv \sum k_i - \sum l_j \qquad (PC3)$$

$$[m * maxk_i] \to \min \text{wrt } \sigma(\text{m}) \; (PC4)$$

Not only does the candidate sigma above denote a particular power signature, it also points to a particular [expansion/generation] *path* possibly uniquely mapping into a specific *m*-scope. Notably, *m* effectively contracts per *repetitive* power patterns, somewhat akin to matrix ranks in case of degeneracy, or the *rad* operator over power composites.

Jet Calculus

The repetitive signature patterns can either be presented in the expanded form as-is or in a more compact fashion (PC5-6). In this light, (PC1) is augmentable to (PC1a):

$$p_k = 1 + \sum_{i=1}^{m} n_i 2^{k_i}, \qquad \sum n_i \geq m, \;\; k(m) = \sum_{i=1}^{m} n_i k_i \qquad (PC1a)$$

$$\sigma(m) \equiv (k_1 k_1 \dots k_m), \qquad k(m) = tr[\sigma(m)] \qquad (PC1b)$$

The null-kernel P0=1 enters but once, and its zero power may or may not appear as part of the sigma vector while adding nothing to its trace anyway. The one & possibly sole way of traveling across the *m*-paths (dub *m-pathetic* unless you shun awkward puns) of alternative representations per the otherwise exact same prime would be to embark on perhaps the simplest possible calculus yet (PC5):

$$kk \sim k + 1 \qquad (PC5)$$

$$kkk \dots k \equiv nk \;\; \to \;\; 2k \sim k + 1 \quad (PC6)$$

Any locally-repetitive power pattern could be collapsed to a higher power level, and the other way around—while preserving the prime value by varying the signature so as to ensure parsimony as in (PC4), likely aligned to the index/trace identity. *All*-units (*rep*units) versus *neg*unit are disallowed/allowed as match-enabling/trivial, respectively (Appendix). This will be elaborated on in the concluding section.

Rehashing & Re-Spanning

Primes appear to be powered-*additive* over the *2-basis* (singular, hence most economical), just like *composites* are known to be powered-*multiplicative* over the *prime basis* (potentially infinite albeit increasingly rare *a priori* yet effectively finite *ad hoc*).

The *index-trace equivalence* surprisingly resembles the interchangeability of upper versus lower indices as part of my ***L-gebra*** (forthcoming), while linking addition and multiplication & suggesting insight into proving conjectures as diverse as, FLP, ABC, & RH.

In hindsight, it remains to be seen whether the repetitive patterns in the sigma may have any bearing on the algebra of *repdigits* (forthcoming). In particular, if the (PC5) calculus has anything to do with a dual of sorts (PC7):

$$1kk1 = 11 * 1[k-1]1 \quad (PC7)$$

(For instance, 1881=171*11)

Finally, to illustrate the core case in point (cf. Appendix), consider e.g. 17=p(6)=1+2*2^3, which corresponds to the core *k-sigma* 33 (trace 33=3+3=k), equivalent to 4, 223, 1123, etc while securing the *tr=k* match amid minimizing the 'storage potential' requirement of *maxk***m*=3*1=3=*min*(3*1, 3*2, 3*3…).

Please note[2] the caveat qualifying the previous section whereby it is suggested that *repunits* in the signature be ruled out as trivial singularity. Indeed, whilst any signature could have an equivalent of 111..1 via the (PC5-6) transform, this collapses the prime to but a generic representation $p = n * 2^1 = 2n \pm 1$—admittedly still fully consistent with the core specification (PC1-2) and $p = 24\tau + 1$ alike, let alone the long-known $p = 4n \pm 3 = 4n' \pm 1$, as motivated in Shevenyonov (2025). In the meantime, the remark on allowing for a *negative* unit on par with a positive one points to how -1 may be implied dually while faring fully in line with the core approach and the pragmatic purposes alike, parsimony of representation not least: $-1 = n = nk = -1 * 1 = n * 2^k + 1 = -1 * 2^1 + 1 \equiv \Delta p$. Consider: $23 = p_8 = 3 * 2^3 - 1 = p_{3*3-1}, \sigma = -1333$. Whilst it remains to be seen if -1 acts to expand *or* contract the effective scope (basis size, or sigma rank), i.e. *m+1 vs m-1,* the overall effect (dub it 'energy savings' or 'storage economy,' if only to hint at an energy/information equivalence analogy) is clearly there: $m \leq 1 + 1 = 2,\ k_{max} * m \leq 3 * 2 = 6$.

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[2] Whereas the present paper was originally submitted on 22 December 2025, this paragraph is added on now so as to shed light on some of the more intricate issues that may have seemed obvious or rightfully deemed as spawned from the core exposition.

References

Shevenyonov, Arthur V. (2025). Another ‘Naive’ Study Worth of ‘(H)Eureka’ Results: Powers Aiding Differences. *vixra paper 2512.0109v1*

APPENDIX

p	**3**	**5**	**7**	**11**	**13**	**17**	**19**	**23**	**29**
index k	1	2	3	4	5	6	7	8	9
k-sigma	***1***	***2***	***12***	***13***	***23***	***33***	***133***	***-1333***	***234***
alt sigmas		11	-13	-123	113	223	1223	1114	2333
		-1111	-122	122	222	1123	-124	-134	1134
			111	-1222	1122	4			