Another 'Naive' Study Worth of '(H)Eureka' Results: Powers Aiding Differences

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ABSTRACT[1]

A 'naïve' look into simple & augmented natural power-difference forms (nPDF) unleashes implications & patterns in areas as diverse as, the RH, primality formulae, and structure parallels, to name but a few.

Keywords: Riemann Hypothesis, primality, characteristic differential, nPDF/pPDF

Motivation

Suffice it to zoom in on conventional *natural power-differences* of the first and higher order, for a host of patterns to obtain readily. Empirically, some of the PDs reveal the larger prime numbers as gaps, whilst a dual enterprise—a prime powered-differences (pPDF)—suggest a strict linear pattern. Based on the above, I have conjectured:

$$\Delta^k N^k = k! \quad k, N \in \boldsymbol{N} \qquad (C1)$$

$$\Delta^l N^x = \Delta^{l-x}\Gamma(1+x), \quad l, x \in \boldsymbol{R} \qquad (C2)$$

Whilst (C1) builds on an 'empirical' induction (confirming the trivial derivatives analogy), (C2) generalizes it all the way up to a conceptual extension, with implications being plenty.

[Pre-]RH

It straightforward to manipulate the above, among other things, as follows:

$$\Delta N^{1/k} = \Delta^{1-\frac{1}{k}}\Gamma\left(1+\frac{1}{k}\right) = \Delta^{1-\frac{1}{k}}\left[\Gamma\left(\frac{1}{k}\right) * \frac{1}{k}\right] \qquad (C2a)$$

$$N^{1/k} - (N-1)^{1/k} = \Delta^{1-\frac{1}{k}}\left[\Gamma\left(\frac{1}{k}\right) * \frac{1}{k}\right] \leftrightarrow kN^{1/k} = \Delta^{-\frac{1}{k}}\Gamma\left(\frac{1}{k}\right),$$
$$\Delta^{-1}\Delta^{1} = \Delta^{0} = \Delta^{1}\Delta^{-1} \sim 1 \quad (C2b)$$

Irrespective of how well the particular convention as above works or needs further polishing & calibrating (e.g. around a particular N if need be, as further proposed), the possibility seems fairly

[1]*In memoriam* Arina & children fallen victim to conflicts, kept hostage to evil designs

intriguing. More so given that, e.g. per *k=2*, this RHS above (C1a-b) showcases how *pi*, a *transcendental* value, could be inferred from, or lend itself with, a simple-yet-generalized *arithmetic* form. In the meantime, please note how a *negative* difference power may pertain to *summation*, as zero difference orders to levels (C2b). A similar convention is adopted as underpinning my ***L-gebra*** (forthcoming).

[Near-]Primality

Based on the selfsame study of [first-order] pPDs, a working prime-generating formula can be discerned (PG1):

$$\boldsymbol{p} = \sqrt{\boldsymbol{1 + 24\tau}} \qquad (\boldsymbol{PG1})$$

This builds on filtering the *tau* magnifier-values until after an *integer square root* has resulted. One will most likely/frequently prove prime; it is the relatively rare value that will turn out to be composite/singular -power, *modulo* 3 ruled out. Please note that 3 vs 2 stand alone as *basis builders*: *2=sqrt(1+3)*, *3=sqrt(1+2^3)*, which split contrasts with *24=2^3*3*, i.e. capturing both the subbases. (Alternatively, the respective *tau* values could be seen as the rational *1/2^3* vs *1/3*.) In the interim, select *tau* values may garner primality *without* an integer square root applying (PG2)—a weaker Boolean filter thus implied.

$$p' = 1 + 24\tau', \quad \tau' \in T = \left\{\sqrt{1 + 24\tau'}\, not \ \in \boldsymbol{N}\right\} \qquad (PG2)$$

At any rate, I have not been able to discern much ontology (beyond PG3-6) from the *tau* distribution other than a sufficient cut-off/search criterion serving as a necessary prerequisite for primality with adequate constructive sufficiency.

The other primality formulas churned out as auxiliary are featured below whilst ushering in *beta-like* or *quadratic* oscillatory patterns on the prime gaps amid motivating (PG1) beyond phenomenological regularity.

$$2t = \Delta n(p - \Delta n)\,, \qquad t = 3\tau \quad (PG3)$$

$$p = \frac{2t}{\Delta n} + \Delta n\,, \qquad p_0 = \frac{2t}{\Delta n} - \Delta n \quad (PG4)$$

$$p + p_0 = \frac{4t}{\Delta n}, \qquad p - p_0 = 2\Delta n \quad (PG5)$$

$$(p + p_0)(p - p_0) = p^2 - p_0^2 = 8t = 24\tau \qquad (PG6)$$

RH: Back to, Around, Beyond

The core PDF (C2) can now be rethought with an eye on Riemann's zeta (C3).

$$\zeta(s) \equiv \sum_{N=1}^{\infty} N^{-s} = \sum \Delta^s \Gamma(1-s) = \Delta^{s-1}\Gamma(1-s) \quad (C3)$$

Which re-construes the RH in terms of,

$$\Delta^{s-1}\Gamma(1-s) = 0 \qquad (C3a)$$

The special precondition might suggest that each & every term be tantamount to 0 (C3b):

$$\Delta^{s}\Gamma(1-s) = 0 \qquad (C3b)$$

In fact, it appears peculiar and fascinating just how this representation is structurally reminiscent of the *functional-equation* rendering of the zeta & RH alike in Shevenyonov (2025), and of *Xi* function for that matter. This may hold around *Re(s)=1/2* whereby the zeta is fixed at 0 and pi-powered terms canceled out by collapsing to finite-absolute-valued complexity: *pi^(1/2-1/2+2it)/2=pi^it*.

$$\Phi(s) = \Phi(1-s) \;\leftrightarrow\; \Delta^{s}\boldsymbol{\Phi}(\mathbf{1}-\boldsymbol{s}) = \mathbf{0} \quad (C3c)$$

One alternate approach would minimize zeta's absolute value wrt *s*, such that:

$$FOC: \Delta\zeta^2(s) \equiv 0 = 2\zeta * \Delta\zeta \;\; \rightarrow \;\; \boldsymbol{\zeta} = \Delta\boldsymbol{\zeta} \;\; \leftrightarrow \Delta^{s-1}\boldsymbol{\Gamma}(\mathbf{1}-\boldsymbol{s}) = \Delta^{s}\boldsymbol{\Gamma}(\mathbf{1}-\boldsymbol{s}) \quad (C4)$$

This, too, appears consistent with the two distant/disparate approaches above, albeit without having scrutinized the *complex*-case *S*OC (outside 'naïve' analysis).

'Naïvete' as per (C4) could, in fact, end up taken to greater depths & breadths along unaided RH lines (C5).

$$\Delta\boldsymbol{x} = \boldsymbol{x} = 0 \quad (C5)$$

$$x = \begin{cases} x_0 - x \;\rightarrow x = \dfrac{x_0}{2} \\ x - x_0 \rightarrow x = \ldots x \pm nx_0\,, \qquad ix = i(x - x_0)\;\; \forall n \end{cases} \qquad (C5a)$$

$$x = \frac{1}{2} * x_0 + i(x \pm nx_0), \qquad \frac{\boldsymbol{x}(\mathbf{1}-\boldsymbol{i})}{\boldsymbol{x_0}} \equiv \boldsymbol{s} = \frac{\mathbf{1}}{\mathbf{2}} \pm \boldsymbol{in} \quad (C5b)$$

The generic notation (C5) befits a wide variety of setups, RH included—from many a stance so. One need not tap into *variational/functional-analytic* premises to appreciate that less formally, if mnemonically. In particular, (C5b) depict just how this unwinds into what amounts to an RH structure, where *indefinite* iterations may straddle or bypass either *prior zero* or *posterior infinity*, complex *inter alia*, as degenerate corner-cases while informing a generalized solution—notably one consistent with the critical-line observation: *Re(s)=max Im(s), Im(s)=min Re(s)*. This, for one, is perhaps due to a *harmonic* representation of complexity, and could be reworked outside *fixed* or *saddle* points, by deploying hypothetic delta operators (C6a-c).

$$\exists\Delta\!: \sigma = \max t \sim \frac{t}{\Delta}, t = \min \sigma \sim \Delta\sigma \quad (C6a)$$

$$\sigma = maxmin\, \sigma \sim \Delta^{-1}\Delta\sigma = \sigma, \qquad t = minmax\, t \sim \Delta\Delta^{-1}t = t \quad (C6b)$$

$$cos\theta = \Delta sin\theta \qquad (C6c)$$

Whilst applying to *s* rather than *zeta*, this spans both under RH, if only ordinally/weakly so, as a reasonably *monotonous transform* of each other within an *s* band (C7) falling in between (-1, 2) for *zeta*, phase *theta*, and *s* (even though, ironically, RH seems to postulate theta-*in*variance save for mere presence thereof, specific & constructive as its nature might prove, e.g. given C5b).

$$\zeta(s) = \zeta\left[\sigma\big(1 + iTAN(\theta)\big)\right], \quad \zeta \sim s \in (-1, 2) \qquad (C7a)$$

For that matter, were one to embark on a Taylor-like expansion around the initial-value core, the whole of the residual, or its first terms, would amount to the [negative] latter (C7b) if RH is to hold:

$$\zeta(\sigma) \approx -[\zeta'(\sigma)it + \zeta''(\sigma)\frac{(it)^2}{2}] \qquad (C7b)$$

However, if any slight deviations around the specific value of *Re(s)*—whatever it is—ushers in sensitivity, that might collapse (C7b) to a dually simplified reduction (C7c):

$$\zeta''(\sigma)\frac{(it)^2}{2} = 0, \qquad \zeta(\sigma) \approx -\zeta'(\sigma)it = \frac{2const}{it} = \frac{2const}{t^2}\sigma + const_2 \qquad (C7c)$$

RHS of (C7c) does appear to second/pose strong local monotonicity all the way up to affinity, or linearity.

$$const \sim \zeta'(0) * (it)^2 = t^2 \sum logN, \;\; const_2 \sim \zeta(0) = -\frac{1}{2} \;\rightarrow\; \zeta(\sigma) \approx 2\sigma \sum logN - \frac{1}{2} \;\; (C8a)$$

On second thought, (C7b-c) looks in line with [an augmented, locally qualified]: $\zeta = \Delta\zeta \sim \frac{\partial\zeta}{\partial s}\Delta s$, $\Delta s = it$. Now, while dismissing the temptation of appreciating how sigma could take on 1/2, we obtain a complete affine approximation as in (C8b):

$$\zeta(s) \approx \; 2\sigma \sum logN - \frac{1}{2} + 2it * \sum logN = 2s * \sum logN - \frac{1}{2} \qquad (C8b)$$

Wherein, local sensitivity of zeta wrt *Re(s)* has nothing to do with *Im(s)*. Furthermore, though not fully legitimate (as *N*'s are not originally part & parcel of the powers, they could be so rendered *a posteriori* by applying on par with *s* exogenized, or linearized), a trick could be re-employed as in Shevenyonov (2025) whereby $\sum logN \sim \infty \sim 1/(2i\pi n)$, such that (C8c-d) holds tentatively without confirming much:

$$\zeta(s) \approx \frac{2s}{2i\pi n} - \frac{1}{2} = 0 \ \rightarrow \frac{\sigma - i\pi n}{-i\pi n} = \frac{1}{2} \rightarrow \frac{\sigma}{i\pi n} = 1/2 \qquad (C8c)$$

Alternatively, $\sum logN \sim \sum\sum \frac{1}{N} \sim \gamma * (1 + 1 + \ldots) = -\frac{\gamma}{2} \rightarrow s = -\frac{1}{2\gamma} \quad (C8d)$

What this suggests at best (and worst) is a *meta*-implication indulging the method attempted: No deviation from a 'naïve' study adds much structurally (for all the largely *phenomenological* nature of RH I have long allowed for) or pays off explanatorily, nor is devoid of dependence on precarious conventions. In contrast, not only does simple scrutiny of power-differences induce a potent primality formula (PG1), it posits a host of analogies largely irrespective of how well premises in (C2) or (C9a) hold.

Consider (C8f-g) for a simpler RH demonstration scheme which is McLaurin-type around *identical zeroes* while improving upon a (C8a-b) linearization (higher-order deviations around whatever match presumed zero).

$$\zeta(\bar{s}) \equiv 0 = \zeta(\sigma - \bar{\sigma}) + \zeta'(\sigma - \bar{\sigma}) * (\bar{\sigma} + it) \equiv \zeta(0) + \zeta'(0) * \bar{s}$$

$$0 = -\frac{1}{2} + \frac{\sum \Delta N^0}{0-0} * \bar{s} = -\frac{1}{2} + \bar{s} * \frac{\sum 1 - \sum 1}{0-0} = -\frac{1}{2} + \bar{s} * \frac{-\frac{1}{2} - (-1/2)}{0-0} = -\frac{1}{2} + \bar{s}$$

$$\bar{s} = \frac{1}{2} + 0 \sim \boldsymbol{\frac{1}{2} \pm 2i\pi n} \equiv \boldsymbol{\frac{1}{2} + it} \quad \forall n \in \boldsymbol{N} \qquad (C8f)$$

Alternatively,

$$\frac{\zeta'(0)}{\zeta(0)} = log'\zeta(0) = -\frac{1}{\bar{s}} \quad \rightarrow \frac{1}{\zeta(0)} = -\frac{1}{\bar{s}} \ \rightarrow \bar{s} = \begin{cases} \frac{1}{2} + 0 = \boldsymbol{\frac{1}{2} + 2i\pi n} \\ \boldsymbol{\frac{1}{2} + s_0} = \frac{1}{2} + it \end{cases} \qquad (C8g)$$

The latter holds insofar as monotonous/ordinal mapping does, which has however been *embedded* in the local linearization yet does involve the vicious circle of [admittedly nontrivial-yet-inconclusive] tautology:

$$\frac{\zeta'(0)}{\zeta(0)} = \frac{dlog\zeta(0)}{d0} = \frac{1}{\zeta(0)} * \frac{\partial\zeta(0)}{\partial 0} = \frac{1}{\zeta(0)} * \frac{\partial\zeta(\bar{s})}{\partial\bar{s}} = \frac{1}{\zeta(0)} * \zeta'(0)$$

The safer bet would be a more direct (and about as transitive) one:

$$dlog\zeta(0) = -\frac{d0}{\bar{s}} \rightarrow log\zeta(0) = const - \frac{0}{\bar{s}} \rightarrow \bar{s} = \frac{0}{const - log\zeta(0)}$$

$$\frac{\bar{s}}{0} \sim \frac{\bar{s}}{(\bar{s}-\bar{\sigma}) * t/2\pi n} \sim \frac{0}{const - log\zeta(0)} \rightarrow const \sim \frac{t}{2\pi n}, \qquad log\zeta(0) \sim \frac{\bar{\sigma}}{\bar{s}} * \frac{t}{2\pi n}$$

$$\left(\frac{\bar{\sigma}}{\bar{s}} - 1\right) * \frac{t}{2\pi n} = \frac{\pm 2i\pi n}{\bar{s}} \rightarrow t = \begin{cases} 2\pi n \\ 2i\pi n \sim 0 \end{cases}$$

This pairwise matching does not amount to much other than reiterating whatever findings secured *non*-analytically thus far.

Afterguesses: [HO] Difference Forms

Consider a stylized inversion:

$$\Delta^{-l} k! = \frac{k!}{l!} N^l = \Delta^{-l}\Gamma(k+1) = \frac{\Gamma(k+1)}{\Gamma(l+1)} N^l \quad (C9a)$$

For aught arguable, (C9b-c) hold with certainty:

$$\Delta^{-1} k! = k!\left(N - \frac{k-1}{2}\right),$$

$$\Delta^l + \Delta_L^{l-1} \equiv \Delta_R^{l-1} \rightarrow \begin{cases} \Delta^0 k! + \Delta_L^{-1} k! \equiv \Delta_R^{-1}\, k! \\ \Delta^{-l}(N+1)^k + \Delta^{-l-1} N^k = \Delta^{-l-1}(N+1)^k \end{cases} (C9b)$$

$$k! + \ k!\left(N - \frac{k-1}{2}\right) \equiv k!\left(N + 1 - \frac{k-1}{2}\right) \qquad (C9b')$$

$$\sum_{l=0\ or\ 1}^{k} \Delta^l N^k = \begin{cases} (N+1)^k \equiv \Delta^0 \\ \Delta(N+1)^k \equiv \Delta^1 \end{cases}, \qquad N > k \qquad (\boldsymbol{C9c})$$

What this points to is a surefire, identically finite *difference* counterpart of a Taylor expansion spanning every-order difference up to the factorial, which sum totals the next powered value. In essence, (C9b) would fare as but a special (or interior, local, adjacency) case of (C9c), or indeed *vice versa* (which phenomenon I dub *psi-orduale*, or automorphous), the same holding anywhere in the interim (C9d):

$$\sum_{l>1}^{k} \Delta^l N^k = \Delta^l (N+1)^k, \quad N > k \qquad (\boldsymbol{C9d})$$

For the same token, a *vertical-horizontal symmetry* or isotropy obtains, likewise carrying over into the interim (C9e):

$$\Delta^l N^k - \Delta^l (N-x)^k = \sum_{y=N-x+1}^{N} \Delta^{l+1} y^k \quad (\boldsymbol{C9e})$$

The above shortcut should suffice for most ‘traveling’ purposes without settling for awkward reduced-form pathways. Notably, these apply across *primes*, with $x \equiv 2\Delta n$:

$$\Delta^l p^k - \Delta^l {p_0}^k = \sum_{y=p-2\Delta n+1}^{p} \Delta^{l+1} y^k \ , \qquad \Delta n \geq 1 \quad (\boldsymbol{C9f})$$

$$\Delta(p^2-1) = \sum_{y=1}^{p} \Delta^2 y^2 = 4n = 24 * \Delta\tau \quad (\boldsymbol{C9g})$$

$$p = \begin{cases} \sqrt{4n + p_0} \\ 4n + p_0 \end{cases} \qquad (\boldsymbol{C9h})$$

(C9h) clearly underperforming (PG1) albeit somewhat qualifying & motivating the long-known formula.

Primality, RH [Likely] Having a [Naïve] Structure

The host of seemingly disjoint ‘naïve,’ or parsimoniously unaided, approaches do seem to be worth the consideration it takes. Not only do they suggest ‘constructive’ objects such as prime-generating formulas & RH analogies as spawned from otherwise mutually orthogonal premises, they also ensure the performance where advanced apparatus fails while leaving the jaded-yet-aided eye wondering about the hidden ontological structures vs. the very phenomenological regularities they fail to match/rationalize, conceptually explain, or predict with reasonable adequacy.

References

Shevenyonov, A. (2025). Riemann Hypothesis [Proving] Obfuscatedly Easy (forthcoming)