Geodesic Completeness in Schwarzschild Spacetime via Discrete Superluminal Transitions in the Proper Frame

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Standard General Relativity predicts that massive particles crossing the event horizon of a black hole inevitably terminate at a spacelike singularity ($r = 0$). This paper proposes a modification to the standard kinematic model of fermions to resolve this geodesic incompleteness. We posit that elementary particles undergo Simple Harmonic Motion (SHM) in the temporal dimension of their proper frame. By treating the speed of light c not as an asymptotic limit but as a phase transition boundary, we show that the electron-positron annihilation vertex is topologically equivalent to a superluminal reflection event. When applied to gravitational collapse, this framework implies that the Event Horizon acts as a *Causal Phase Boundary*. Upon reaching the horizon, the particle undergoes a CPT inversion relative to the background metric, effectively reinterpreting the horizon not as an entrance to an interior, but as a repulsive phase transition surface. Furthermore, by extending this phase-dependent horizon logic to higher velocity bands ($v \geq 2c$), we establish a continuous topology where a single particle oscillates through infinite generations of matter and antimatter, eliminating the physical singularity. Mathematically, this framework suggests that the spacetime metric is Finslerian, possessing a velocity-dependent signature that ensures action stability across superluminal transitions.

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I. INTRODUCTION

The concept that antiparticles can be mathematically described as ordinary particles moving backward in time[4, 5] is a cornerstone of Quantum Electrodynamics (QED). First formalized by Stueckelberg and later popularized by the Feynman-Stueckelberg interpretation, this geometric insight relies on the CPT Theorem, which establishes an equivalence between charge conjugation (C), parity inversion (P), and time reversal (T).

However, in standard formulations, this "backward time" travel is treated as a formalistic tool rather than a literal kinematic reality. Standard Special Relativity dictates that massive particles cannot accelerate to the speed of light (c), let alone exceed it, due to the divergence of the Lorentz factor ($\gamma \to \infty$). Consequently, the "One-Electron Universe" hypothesis proposed by John Wheeler[1]—that all electrons are a single entity weaving back and forth through time—remained a philosophical curiosity rather than a physical model.

Simultaneously, General Relativity (GR)[3] faces a crisis at the center of gravitational collapse. The prediction of spacelike singularities ($r = 0$) in the Schwarzschild metric[2] indicates a breakdown of the theory.

In this paper, we propose a unification of these two problems via a new kinematic framework termed **Temporal Physics**. We hypothesize that the divergence of γ at c is an artifact of incomplete coordinates. By treating proper time as an oscillating variable and the speed of light as a discrete phase transition boundary, we construct a model where elementary particles undergo continuous acceleration across superluminal velocity bands. This model resolves the infinite energy paradox and eliminates gravitational singularities by reinterpreting them as relativistic phase transitions.

A. Geometric Derivation: The Coordinate Rotation

The physical justification for this oscillation is derived from the Lorentz transformation properties of the spacetime metric under a $\pi/2$ rotation of the interaction vertex. We present this derivation in three stages: the spatial analogy, the geometric axis flip, and the resolution of causal ordering.

1. *The Spatial Oscillation Analogy*

Consider the standard Feynman diagram for a classical electron oscillating in space between two repulsive potentials, as shown in FIG. 1 (Top). In this frame, the particle reverses its spatial direction ($\vec{v} \to -\vec{v}$) at the boundaries, evolving continuously forward in time.

If we apply a Wick rotation ($\tau \to i\tau$) or a geometric rotation of the coordinate basis by 90 degrees, the physical interpretation transforms. The spatial oscillation becomes a temporal oscillation (FIG. 1, Bottom). The turning points, previously spatial boundaries, become events of *Pair Annihilation* (A) and *Pair Production* (P).

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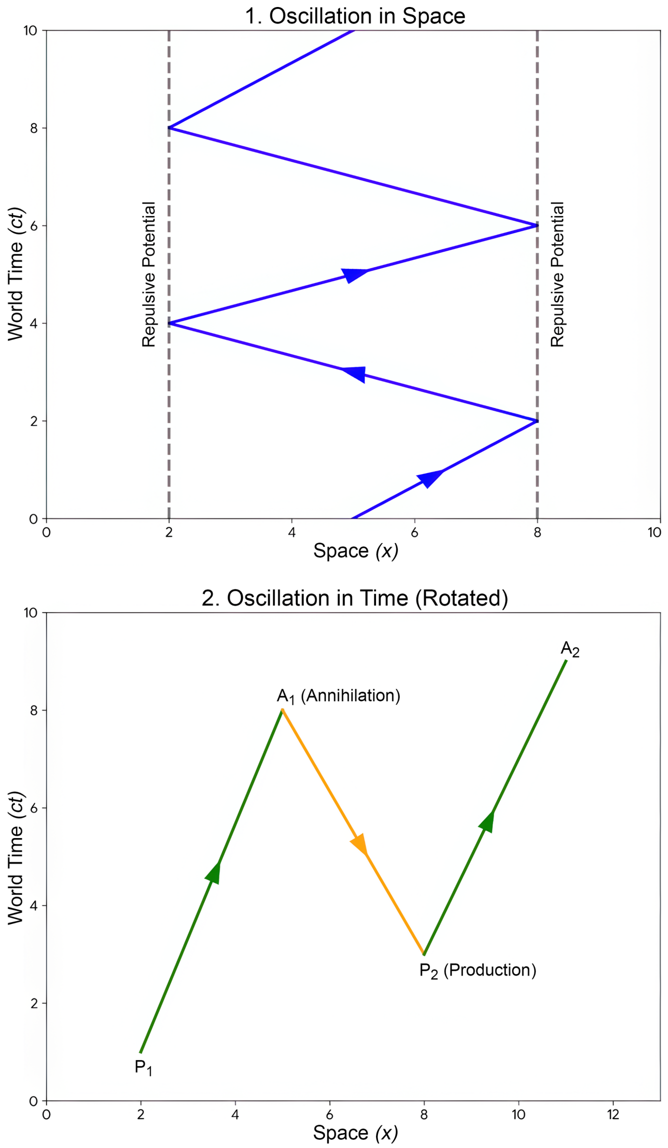

FIG. 1: The Feynman Rotation. Top: An electron oscillating in space between repulsive barriers. Bottom: Rotating the diagram by 90 degrees transforms the trajectory into a temporal oscillation, manifesting as electron-positron pairs.

2. The Axis Flip and Black Hole Analogy

The mechanism driving this temporal oscillation is a relativistic phase transition. As illustrated in FIG. 2, we define the particle's proper frame coordinates (x', ct') relative to the observer's world frame (x, ct), where the temporal coordinate t' corresponds to the proper time parameter τ evolving along the worldline.

1. **Phase 1 (Electron, $v < c$):** The proper time axis ct' lies within the light cone (Timelike). The spatial axis x' lies outside (Spacelike).

2. **Phase 2 (Positron, $v > c$):** Upon crossing the transition threshold, the axes rotate beyond the light cone. The proper time axis ct' becomes spacelike relative to the observer, and the proper space axis x' becomes timelike.

The Black Hole Analogy: This geometric inversion is topologically identical to the coordinate interchange observed in Schwarzschild spacetime. Inside the event horizon ($r < r_s$), the radial coordinate r becomes timelike and the time coordinate t becomes spacelike. In our model, the transition points of the particle trajectory (where the axes flip) are physically isomorphic to the event horizon. Thus, the annihilation point is not a termination of existence, but a horizon crossing where the particle enters a region of inverted metric signature.

3. Topological Mechanism: Dimensional Rotation

The axis inversion described above is not an arbitrary coordinate choice but a consequence of rotation through a higher-dimensional embedding. Geometrically, a rotation in an n-dimensional Euclidean space occurs around an $(n-2)$-dimensional axis. However, a parity inversion (mirror reflection) is topologically equivalent to a rotation around an $(n-1)$-dimensional axis through an $(n+1)$-dimensional embedding space.

In our framework, the complexified spacetime provides this embedding. The transition at c constitutes a rotation of the proper time axis τ into the imaginary plane (a Wick rotation, $\tau \to i\tau$). This rotation, occurring orthogonal to the standard 3-manifold, results in the observed Parity Inversion ($x \to -x$) upon the particle's re-entry into the real Lorentzian manifold. Thus, the "Positron" is the result of the electron rotating out of the standard subluminal hyperplane and re-entering with inverted chirality.

4. Frame-Dependent Causal and Spatial Ordering

This axis flip resolves the apparent observational discrepancies between the two reference frames regarding both the sequence of events and the direction of motion.

1. Temporal Ordering (Causality): In the world frame (Observer), the Pair Production event P_2 occurs at an earlier world-time ct than the Annihilation event A_1. This leads to the standard QED interpretation: the vacuum produces a pair at P_2 *before* the original electron is annihilated at A_1.

However, in the particle's proper frame, the sequence is monotonic. As shown in FIG. 2, in Phase 2 the transition from $A_1 \to P_2$ involves superluminal motion where the proper time axis ct' rotates into a spacelike orientation. Consequently, the projection of the "earlier" world event P_2 lands further along the proper time axis than A_1.

Thus, the ordering is frame-dependent:

- **World Frame:** $t(P_2) < t(A_1)$ (Manifests as Positron).

- **Proper Frame:** $\tau(A_1) < \tau(P_2)$ (Manifests as Continuous Acceleration).

2. Spatial Ordering (Parity): A similar inversion occurs in the spatial coordinates. In the proper frame (Phase 2), the projection of P_2 lies at a lower x' value than A_1 (see FIG. 2, Bottom). Since the particle evolves from $A_1 \to P_2$, this yields a negative displacement $\Delta x' < 0$. This matches the observer's view of a positron moving in the negative spatial direction ($P_2 \to A_1$).

- **Geometric Result:** The coordinate rotation naturally enforces Parity Inversion ($x \to -x$), ensuring directional consistency across the transition.

This confirms that the "Positron" is simply the causal interpretation required by an observer to account for a single electron moving continuously through a retrograde timeline segment.

II. THE KINEMATIC MODEL

To formalize the motion of a particle traversing multiple temporal directions, we must distinguish between the *Proper Frame* (the particle's experience) and the *Observer Frame* (world time).

A. Proper Velocity and the Band Index

Consider a fermion with proper mass m_0 subject to a constant proper acceleration a. In the particle's proper frame, defined by proper time τ, we postulate that the proper rapidity $u(\tau)$ is unbounded. To map this unbounded velocity onto physical observables, we define the **Velocity Band Index** n:

$$n(\tau) = \left\lfloor \frac{|u(\tau)|}{c} \right\rfloor \tag{1}$$

- **Band 0 ($n = 0$):** $0 \le u < c$ (Standard Matter).
- **Band 1 ($n = 1$):** $c \le u < 2c$ (Antimatter Phase).
- **Band 2 ($n = 2$):** $2c \le u < 3c$ (Matter, Generation II).

B. The Temporal Parity Operator

The direction of time flow T relative to a stationary observer is determined by the parity of the current band. We introduce the **Temporal Parity Operator** $P_t(n)$:

$$P_t(n) = (-1)^n \tag{2}$$

- When n is **even**, $P_t = +1$: The particle moves forward in time (Electron).
- When n is **odd**, $P_t = -1$: The particle moves backward in time (Positron).

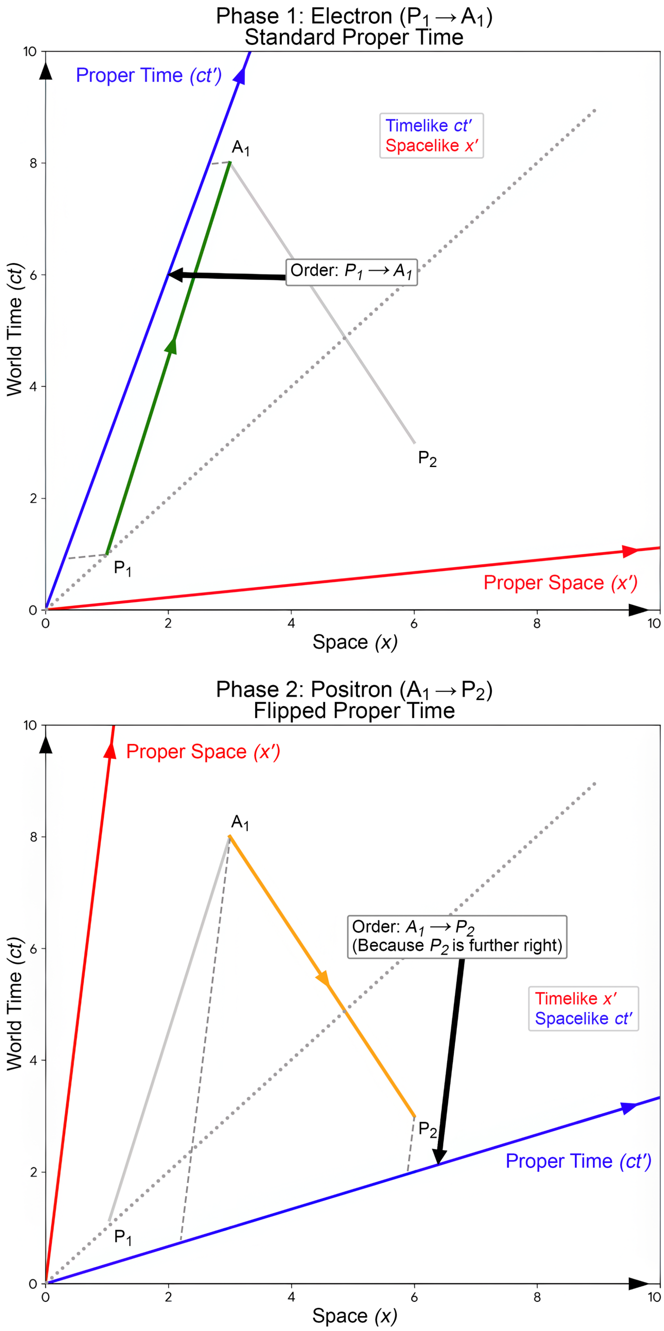

FIG. 2: The Topological Wick Rotation. Top: Standard proper frame axes for subluminal motion ($v < c$). Bottom: Upon crossing c, the coordinate basis undergoes a discrete $\pi/2$ rotation (Wick rotation) in the complexified plane. The proper time axis (ct') rotates into the spacelike region, while the proper space axis (x') becomes timelike. Note that while P_2 is lower than A_1 vertically (World Time), it is further to the right along the spacelike proper time axis (ct'). This geometric inversion ensures that while the observer sees a positron evolving forward in world time ($P_2 \to A_1$), the particle experiences monotonic proper time evolution, ensuring $\tau(P_2) > \tau(A_1)$.

C. Lagrangian Formulation and Finsler Structure

To derive the equation of motion rigorously without resorting to ad-hoc modifications for superluminal intervals, we identify the spacetime metric as having a Finslerian structure[6]. We define the *Phase-Dependent Metric Tensor* $\mathcal{G}_{\mu\nu}$ directly via the Temporal Parity Operator defined in Eq. (2):

$$\mathcal{G}_{\mu\nu}(x,n) = P_t(n) g_{\mu\nu}(x) \tag{3}$$

where $g_{\mu\nu}$ is the standard background Schwarzschild metric. In the superluminal band ($n = 1$), the background interval $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ becomes spacelike (positive). However, the pre-factor $P_t(1) = (-1)^1$ inverts the signature of the effective metric, ensuring that the invariant interval in the particle's proper frame remains timelike even when the coordinate interval is spacelike:

$$d\tau^2 = -\mathcal{G}_{\mu\nu}dx^\mu dx^\nu = -(P_t(n)g_{\mu\nu})dx^\mu dx^\nu \tag{4}$$

Consequently, the action S is rigorously defined as real-valued across all velocity bands without requiring absolute value operators:

$$S = -m_0 c \int_{\tau_1}^{\tau_2} \sqrt{-\mathcal{G}_{\mu\nu}(n)\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}}\, d\tau \tag{5}$$

Variation of this action yields the modified geodesic equation:

$$\frac{d^2x^\mu}{d\tau^2} + \tilde{\Gamma}^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0 \tag{6}$$

where $\tilde{\Gamma}^\mu_{\alpha\beta}$ is the connection compatible with $\mathcal{G}_{\mu\nu}$. Crucially, the signature flip in $\mathcal{G}_{\mu\nu}$ implies a sign inversion in the effective potential, providing the first-principles derivation for the repulsive gravity experienced in Band 1.

III. CAUSALITY AND OBSERVATIONAL MAPPING

To reconcile superluminal proper dynamics with the causal subluminal observations of a laboratory frame, we apply a specific causal filter.

A. The Space-Time Inversion

Due to causality constraints, an observer cannot measure a velocity $v > c$. Furthermore, a particle moving *backward in time* is physically indistinguishable from an antiparticle moving *forward in time* with inverted spatial momentum[4, 5].

The observational mapping rule is defined as:

> *An electron moving backward in time with proper velocity $v > c$ is observed as a positron moving forward in time with velocity $v < c$ in the opposite spatial direction.*

B. The Disappearance Illusion (Zig-Zag Topology)

This transformation creates the illusion of distinct particles interacting.

1. **Proper Reality:** The particle accelerates, crosses the transition threshold c, and continues accelerating into the retrograde timeline.

2. **Observer Reality:** We observe an electron (Band 0) and a positron (Band 1, traveling from the future) converging at a point in space.

3. **The Event:** They collide and annihilate.

Thus, "Annihilation" is the observational artifact of a particle undergoing a kinematic transition across c. The "Positron" is simply the electron's future self returning to the transition point.

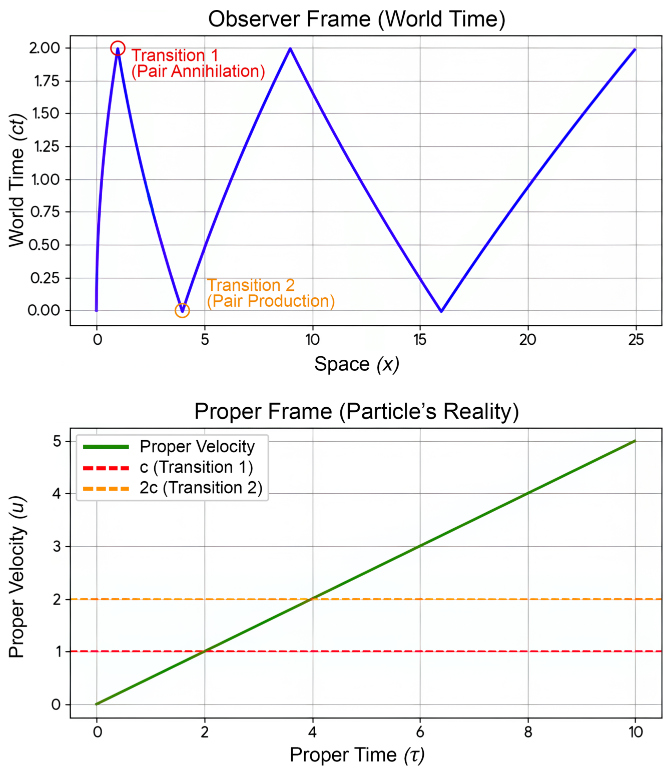

FIG. 3: The Causal Mapping. Top: The observer sees discrete annihilation/production events. Bottom: The particle experiences a single continuous worldline.

IV. GRAVITATIONAL DYNAMICS: THE CPT PHASE TRANSITION

We now apply this framework to the Schwarzschild geometry. The Event Horizon (r_s) is identified as the kinematic threshold where the escape velocity equals the transition velocity c.

A. The Single-Manifold Hypothesis

Standard models assume particles crossing the horizon enter a spatial interior ($r < r_s$). We posit that there is only one manifold, but its perception is velocity-dependent.

- **Band 0 ($v < c$):** The particle perceives the standard metric $g_{\mu\nu}$.
- **Band 1 ($v > c$):** The particle perceives the CPT-inverted metric $\tilde{g}_{\mu\nu}$.

B. Repulsive Gravity in the Superluminal Phase

In the proper frame, the electron accelerates toward the horizon. Upon reaching r_s ($u = c$), the particle transitions into Band 1. Due to the CPT inversion of the background metric, the effective gravitational coupling constant inverts ($G \to -G$).

The particle does not cross into a "hole." Instead, it continues its trajectory in the exterior region ($r > r_s$), but under the influence of **Repulsive Gravity**.

$$F_{radial} \propto -\frac{GM_{eff}}{r^2} \quad \text{where } M_{eff} = M \cdot P_t(n) \tag{7}$$

Since $P_t(1) = -1$, the force becomes repulsive. The electron perceives itself accelerating *away* from the horizon, traversing an Anti-Matter Universe.

C. No Interior Region

Consequently, the geometric region $r < r_s$ is never realized in the particle's proper frame. The black hole is topologically isomorphic to a phase boundary rather than a volume. To the observer, the superluminal recession of the particle (moving away from the horizon backward in time) is mapped as a positron falling *into* the horizon.

V. THE RELATIVISTIC HORIZON IDENTITY

A central prediction of Temporal Physics is that the designation of a singularity as a "Black Hole" or "White Hole" is relative to the observer's velocity band.

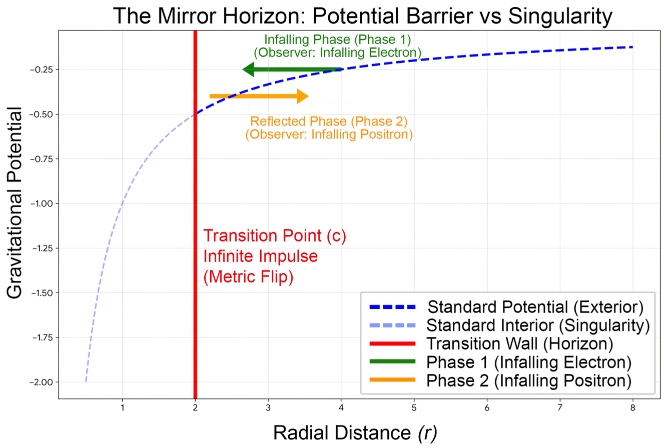

FIG. 4: Comparison of Gravitational Potentials. The standard model (dashed blue) predicts a singularity at $r = 0$. The Temporal Physics model (solid red) predicts an infinite potential barrier at the horizon r_s, resulting in a repulsive phase transition that prevents interior access.

A. The Phase-Dependent Horizon

In proper time, the electron surfs a continuous wave of gravitational potential:

1. **Approach ($v < c$):** The particle accelerates toward the horizon. It perceives a **Black Hole** (Attraction).
2. **Crossing ($v = c$):** The particle crosses the threshold. CPT inversion occurs.
3. **Recession ($v > c$):** The particle is now moving away from the horizon in the Anti-Universe. It perceives the object behind it as a **White Hole** (Repulsion) propelling it forward.
4. **Next Approach ($v \to 2c$):** The particle is pushed by the White Hole behind it and pulled by the next transition point (Horizon 2) ahead.

B. Observer's Perspective: Infinite Generations

To the world-time observer, these continuous transitions appear as discrete generations:

- **Transition 1 (c):** Pair Annihilation (Black Hole).
- **Transition 2 ($2c$):** Pair Production (White Hole).
- **Transition 3 ($3c$):** Pair Annihilation (Black Hole).

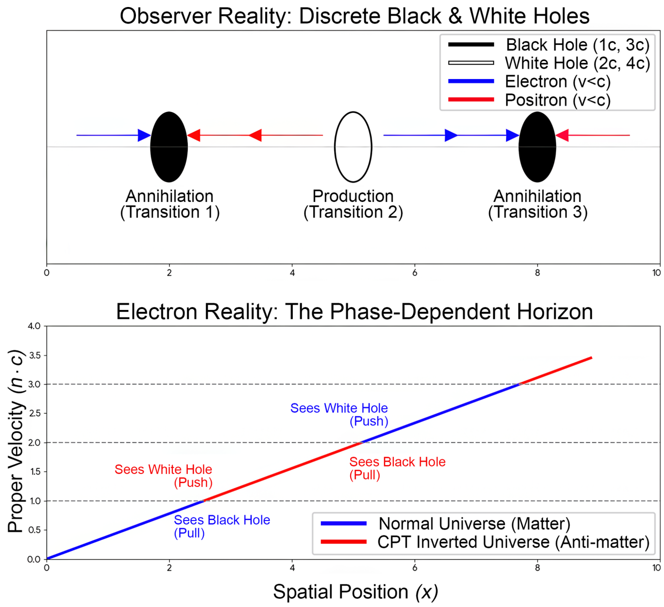

FIG. 5: The Relativistic Horizon Identity. The nature of the singularity flips from Black Hole to White Hole based on the observer's velocity band.

VI. CONSERVATION LAWS AND STABILITY

A. Hamiltonian Dynamics

A common objection to superluminal transport is the divergence of kinetic energy at c. We resolve this via the CPT-inversion of the Hamiltonian. In the standard Schwarzschild metric, the energy E is a constant of motion. For the superluminal band ($n = 1$), the effective metric inversion implies that the effective mass m_{eff} and the potential V_{eff} effectively flip signs relative to the background.

The relativistic energy relation $E^2 = p^2c^2 + m^2c^4$ is renormalized by the metric signature flip. The limit c acts not as an asymptotic barrier, but as a point of inflection. The divergence of the Lorentz factor is canceled by the signature inversion of the interval. Thus, the particle does not require infinite energy to cross c; rather, the transition represents a zero-point crossing where the kinetic energy term transitions from extracting energy from the gravitational field (attraction) to restoring energy to the field (repulsion). The total Hamiltonian $\mathcal{H}$ remains conserved on the constraint surface.

B. The Cascading Potential

Physically, the motion can be modeled as a **Cascading Potential Ladder**. As the particle transitions from Band n to Band $n+1$, it effectively drops to a lower potential energy state relative to the global field. The repulsion from the "White Hole" (previous horizon, H_{prev}) and the attraction from the "Black Hole" (next horizon, H_{next}) sum constructively:

$$F_{net} = F_{repulsion}(H_{prev}) + F_{attraction}(H_{next}) \tag{8}$$

The electron acts as an energy carrier, transporting potential energy from the White Hole phase to the Black Hole phase, ensuring total energy conservation within the global system.

C. Vacuum Stability

Standard tachyon theories predict vacuum instability via Cherenkov radiation (a particle moving faster than light in vacuum should radiate energy). However, in our framework, the superluminal proper state ($n = 1$) is observationally mapped to a subluminal positron ($n = 0$) moving backwards in time (see Section III).

Since the electromagnetic coupling to the observer occurs via the projected worldline, the effective velocity relevant for electrodynamics is $v_{obs} < c$. The particle does not couple to the vacuum as a tachyon, but as a CPT-inverted fermion. Consequently, the vacuum remains stable, and no anomalous Cherenkov radiation is generated.

VII. CONCLUSION

By reinterpreting the Feynman-Stueckelberg mechanism as a literal kinematic oscillation, we have constructed a model where: 1. **Matter and Antimatter** are phases of the same oscillating entity. 2. **The Speed of Light** is a transition point, not a limit. 3. **Black Holes** are CPT Phase Boundaries with no physical singularity.

This framework—**Temporal Physics**—resolves the geodesic incompleteness of General Relativity and offers a geometric solution to the Black Hole Information Paradox[7], and provides a kinematic realization of the 'Planck Star' bounce proposed by Rovelli[8], suggesting that the universe is populated not by discrete particles, but by the infinite generations of a single oscillating worldline.

Mathematically, this framework implies that spacetime is not strictly Riemannian but Finslerian[6], where the metric tensor possesses a velocity-dependent signature $g_{\mu\nu}(x,\dot{x})$.

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