# The Expansion of the Universe is a Transformation of Energy into Momentum

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## Abstract

**Any body existing in a gravitational field that changes with time due to the expansion of the universe or for any other reason remains subject to a conservation law for energy and momentum, but with a new concept of conservation, which is the conservation of the sum of the different components of the quantity that represents energy and momentum, not the conservation of each component separately. With this concept, we find that the expansion of the universe is a continuous transformation of energy into momentum, there is no loss or gain in the total sum of them.**


## Introduction

Physicists' views have differed greatly regarding the laws of conservation of energy and momentum in the presence of a gravitational field and the expansion of the universe. Concerning the gravitational field, there are now two main directions. The first can be attributed to the discoverer of general relativity, Einstein, and it says that energy and momentum are conserved if we use pseudotensors to represent the energy and momentum of the gravitational field **[1]**. This approach has been widely criticized because of the lack of covariance and localization. The second direction, which is dominant now and can be attributed to Einstein's partner in writing the field equation of general relativity, Hilbert, says that energy is conserved in the case of a static gravitational field such as the Schwarzschild field, as well as angular momentum, but linear momentum is not conserved due to the lack of symmetry in the three spatial axes to which the momentum components are related **[2]**.

As for the non-stationary of the gravitational field in time, there is almost a consensus among physicists that neither energy nor momentum is conserved.

In this paper, I show that the failure to obtain a complete and comprehensive law for the conservation of energy and momentum is the result of our use of an incomplete definition of the concept of conservation and of the conserved quantities themselves. Through purely mathematical analysis, without any reliance on physical assumptions other than the principles of general relativity, it can be shown that a unified and extended definition that includes energy and momentum for a body moving in a gravitational field in a continuously expanding universe satisfies a local and global law for conservation of energy and momentum.

## The Laws of Conservation of Energy and Momentum in Gravitational Field

### (1) Basic Assumption about the Carrier of Energy-Momentum:

A gravitational field has no energy or momentum by itself, it only modifies the definition of energy and momentum of a body moving within it.

**(2) Gravitational Field:**

Let us first define the quantity $\boldsymbol{U^p}$ which is a vector quantity whose magnitude is given by:

$$\boldsymbol{U} = \sqrt{\frac{2GM}{r}}$$

(*We note that this quantity resembles the scape velocity or Schwarzschild parameter, but here, we only want to use it simply as a calculation variable representing the gravitational field to some extent without linking it to any physical meaning, whether Newtonian or even relativistic*.)

This quantity is purely radial; therefore its components in spherical coordinate system in curved spacetime are given by:

$$\boldsymbol{U^t = 0} \quad , \quad \boldsymbol{U^r} = \frac{\boldsymbol{U}}{\sqrt{g_{\mathbf{rr}}}} \quad , \quad \boldsymbol{U^\theta = 0} \quad , \quad \boldsymbol{U^\varnothing = 0}$$

We have the relation between the magnitude and the components: $\boldsymbol{U^2} = \sum_{i.j=1,2,3} g_{\mathbf{ij}} \boldsymbol{U^i U^j}$

**(3) Motion of the Body:**

$\boldsymbol{V^p}$ are the components of the velocity of the t body as measured by an static observer. Therefore, $\boldsymbol{v^0} = \mathbf{c}$ , also we have; $\boldsymbol{V^2} = \sum_{i.j=1,2,3} g_{\mathbf{ij}} \boldsymbol{v^i v^j}$

**(4) Definition of Energy-Momentum in Gravitational Field:**

Let us define the matrix $\mathbf{B}_{\boldsymbol{pq}}$:

$$\mathbf{B}_{\boldsymbol{pq}} = \boldsymbol{m^2 \gamma}^2 ( g_{pq} \boldsymbol{V}^p \boldsymbol{V}^q - g_{pq} \boldsymbol{V}^p \boldsymbol{V}^q )$$

The time-time component $( \mathbf{B_{00}} )$ of this matrix is the square of the energy of the body in the gravitational field:

$$\mathbf{E}_{\boldsymbol{g}}{}^{\mathbf{2}} = \mathbf{B_{00}}$$

The sum of space-space components ( $\sum_{i.j=1,2,3} \mathbf{B}_{\boldsymbol{ij}}$ ) of this matrix is the square of the momentum of the body in the gravitational field:

$$\mathbf{p}_{\boldsymbol{g}}{}^{\mathbf{2}} = \sum_{i.j=1,2,3} \mathbf{B}_{\boldsymbol{ij}}$$

**(5) The Conservation laws of Energy-Momentum in Gravitational Field:**

It can be proved that:

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In static Gravitational Field both in geodetic motion of the body:

$\mathbf{E}_g$ is conserved.

$\mathbf{p}_g$ is conserved.

However, in variable gravitational field, such as expanding universe, there is no conservation of energy and momentum individually but the sum of their squares is conserved:

$$\mathbf{B}_{00} + \sum_{i,j=1,2,3} \mathbf{B}_{ij} = m^2 c^2$$

In any gravitational Field: ${\mathbf{E}_g}^2 + {\mathbf{p}_g}^2 = m^2 c^2$

This is the general energy-momentum relation which can be shown to reproduce the classical relation: $E^2 = p^2 c^2 + m^2 c^4$ in the absence of gravitational field.

**(6) Application of the New Definition in Schwarzschild Space-time to Verify the New Laws and Relations:**

In Schwarzschild with the signature $(+,-,-,-)$, we have: $g_{tt} = 1 - U^2/c^2$,

$$\mathbf{B}_{00} = m^2\gamma^2((1 - U^2/c^2)c^2 - 0) = m^2\gamma^2(c^2 - U^2)$$

$${\mathbf{E}_g}^2 = m^2\gamma^2(c^2 - U^2)$$

This (and, of course, its square root) is a conserved quantity in static gravitational filed..

$$\sum_{i,j=1,2,3} \mathbf{B}_{ij} = m^2\gamma^2\left(\sum_{i,j=1,2,3} g_{ij}V^iV^j - \sum_{i,j=1,2,3} g_{ij}U^iU^j\right)$$
$$= m^2\gamma^2(U^2 - V^2)$$

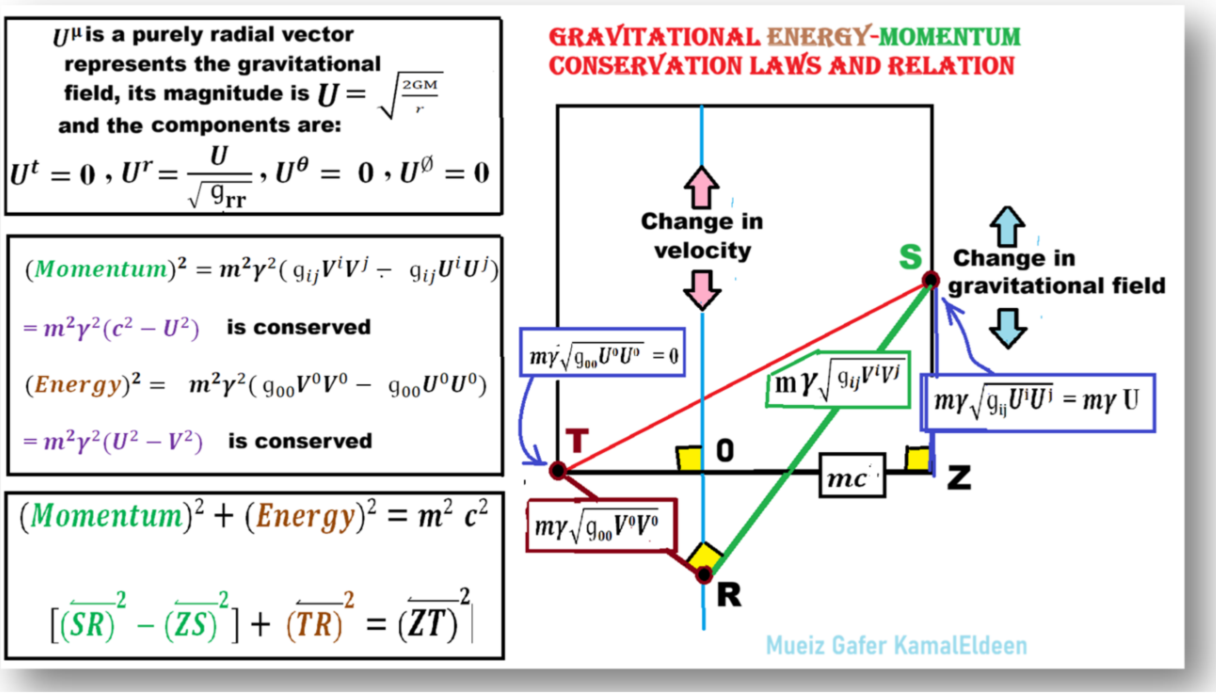
$${\mathbf{p}_g}^2 = m^2\gamma^2(U^2 - V^2)$$

Now combine:

$${\mathbf{E}_g}^2 + {\mathbf{p}_g}^2 = m^2\gamma^2\left(c^2 - U^2\right) + m^2\gamma^2(U^2 - V^2) = m^2c^2$$

This confirms the constancy of $\mathbf{p}_g{}^{2}$ in the case of a static gravitational field and confirms the validity of the relation between the square of the energy and the square of the momentum even when the gravitational field changes, because U disappears when energy and momentum are combined, regardless of whether it is constant or varying.

This new relationship between momentum and energy can be represented by two right angles constructed on a common hypotenuse as shown in the figure below. Where the side TZ represents the mass and is constant, while the side ZS, which represents the gravitational field, changes as S moves vertically upward. The position of point R, representing the motion of the body, also changes with the upward vertical motion. The location of point O along the horizontal line TZ represents the reference frame. This diagram, with these constraints, provides a complete picture of the relationship between momentum and energy.




**(7) Recovering Classical Conservation laws and Relation in the absence of Gravitational Field:**

We can see that the classical conservation laws and relation between the square of the energy and the square of the momentum can be recovered in the case of a vanishing gravitational field if we express the relation between the new definition of energy and momentum and the old definitions and substitute the value of flat metric in it.

**(8) Conservation Laws and Energy-Momentum Relation in Terms of Traditional Definitions of Momentum and Energy**

It may be more useful to write the conservation laws mentioned above in terms of the traditional definitions of energy and momentum:

## The law of Conservation of Energy of a Body in Gravitational Field:

$$E^2 - 2m^2\gamma^2c^2\ GM/r \text{ is conserved}$$

**The law of Conservation of Momentum of a Body in Gravitational Field:**

$$P^2c^2 - 2m^2\gamma^2c^2\ GM/r \text{ is conserved}$$

**This gives the energy-momentum relation in gravitational field:**

$$E^2 - P^2c^2 = m^2c^4$$

**The energy-momentum relation in gravitational field remains the same as it was in the absence of gravity.**

**And the more important relation::**

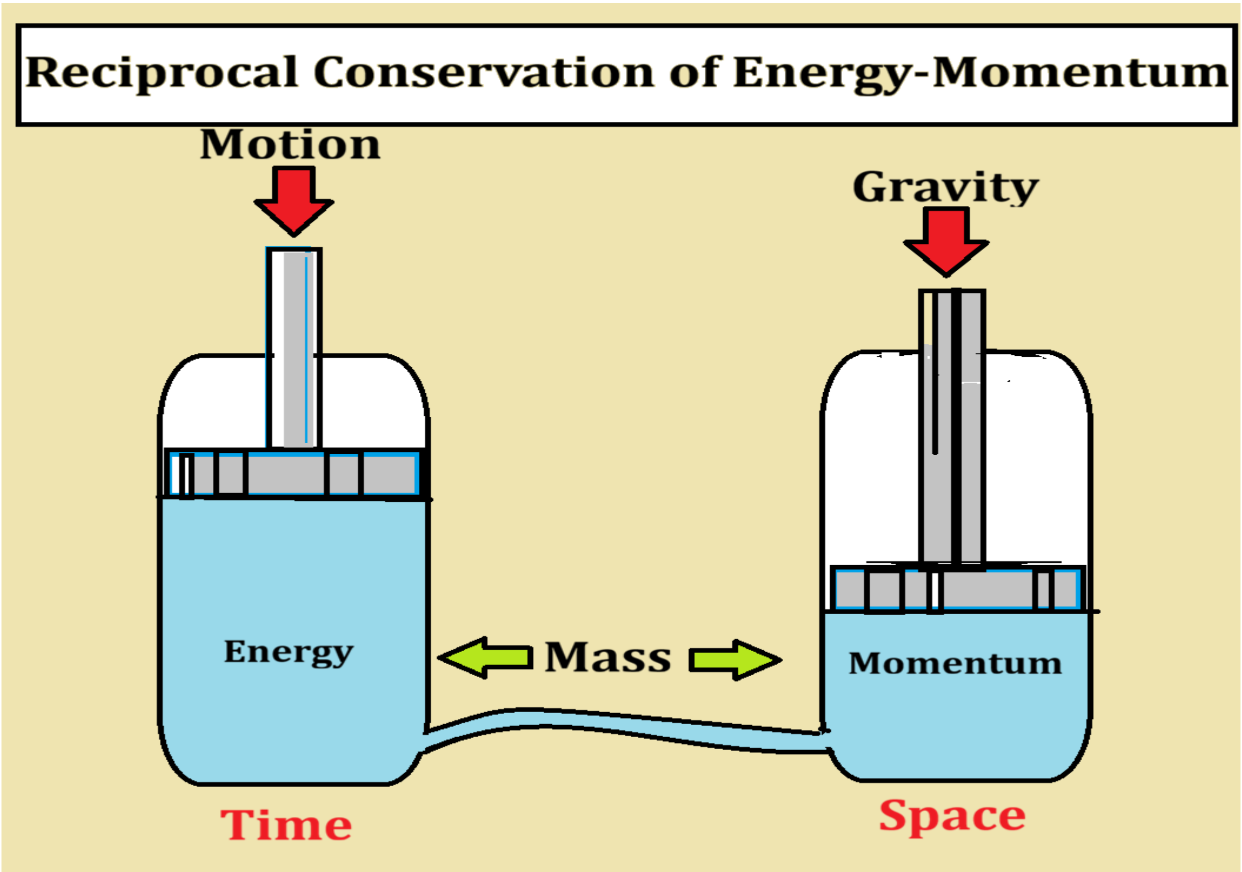
$$P^2c^2 + E^2 - 4m^2\gamma^2c^2\ GM/r \text{ is conserved}$$

$$= {P_0}^2c^2 + {E_0}^2 - 4m^2{\gamma_0}^2c^2\ GM/r$$

**(9) Some Conceptual Results of These New Laws and Relations:**

This formula represents a solution to the problem of the conservation of energy and momentum in general relativity. In the case of a static field, we have two separate laws: one for the conservation of the energy and one for the conservation of total momentum. The conservation of total momentum means that the components of the square momentum are not conserved individually, but the total sum is conserved. For example, a planet orbiting a star in a circular orbit does not have conservation for each component of its square momentum, but there is a conserved total sum of these components, which means that the components exchange momentum among themselves without any external intervention.

In a variable gravitational field, such as the gravitational field in an expanding universe, there is no law for the conservation of square energy and square of momentum individually; instead, there is a law for conserving the sum. This means that when the gravitational field changes, an exchange occurs between energy (exchange of energy or momentum, as they are of the same essence) and momentum, all happening within the body only. In this picture, the square of mass of the body represents a geometric constraint on this exchange, as it represents the total sum of the square of energy and the square of momentum. One can consider that the matter within the body, represented by its mass, is distributed into two vessels: a temporal vessel, in which the contained matter is called energy, and a spatial vessel, in which the contained matter is called momentum. The spatial vessel itself is further divided internally into sub-vessels representing the components of momentum. Here, the gravitational field does not intervene by taking or adding anything to the body; it only acts as an influencer in the internal exchange processes. In the absence of a gravitational field, each component of energy and momentum remains constant, this idea is illustrated in the figure below:




## Significant and Unexpected New Conservation Formula's Implications in Quantum Mechanics

Even without this new conservation formula, mass $m$ (or the rest energy $mc^2$) as a conserved quantity is very important in Newtonian physics and special relativity. However, from the perspective of quantum mechanics and general relativity, there are several reasons why "mass" alone is insufficient to define the *phase* under all conditions, for example:

1. Mass is not the sole "Generator" of motion .In quantum mechanics, the phase changes because the object moves or interacts. Using mass only will lead to phase that would depend solely on proper time, While this represents the "De Broglie phase" of a particle at rest, it fails to account for *interference* patterns caused by velocity (momentum) or changes in gravitational potential. To calculate a trajectory, we need a quantity that responds to changes in position and speed; rest mass remains indifferent to these factors.

2. Photons (light), for example, have zero rest mass yet they possess a phase that evolves and is affected by gravity and cosmic expansion (redshift). If the quantity used to calculate the phase were strictly mass, the phase of light would be indefinable or zero.

3. In the context of fundamental physics theories such as general relativity and quantum mechanics, mass is not a complete quantity but rather represents only the time-related component in a matrix that includes momentum in different directions.

Now, with the new conservation formula, we have overcome all those obstacles and barriers, because we have established the existence of an effective mass that does not change regardless of how much spacetime expands or curves, *while the distribution of its magnitude over the temporal and spatial vessels that carry it changes*, and this mass is not linked to time alone but results from the sum of components formed from all dimensions of space-time in a balanced way.

So, the invariant $\mathrm{m}^2\mathbf{c}^2$ naturally defines a space-time action:

$$S = \int \mathrm{m}^2\mathrm{c}^4 \, d\mathcal{T}$$

Where $\mathcal{T}$ is the proper time along the particle's worldline

And this allows us to associate a phase with this action:

$$\Delta\phi = \frac{1}{\hbar}\int_1^2 \mathrm{m}^2\mathrm{c}^4 \, d\mathcal{T}$$

Even if the mass is zero, as in the case of light, this does not mean phaselessness, because zero in this case is the sum of two quantities equal in magnitude but opposite in sign: energy and momentum. When we decompose the zero mass into these two parts, each part has a phase that changes with time.This phase is purely geometric and relativistic in origin. No quantum postulate has yet been introduced. This provides a unified and conceptually economical bridge between relativistic structure and quantum phenomena, Key structures of quantum mechanics –wave dynamics, uncertainty, interference and other structures can be derived using a single geometric invariant associated with space time redistribution.

## Gravitational Generalization of Energy-Momentum Relation and Its Reflection on Schrödinger Equation

Energy-Momentum Relations in the in the presence or absence of Gravity are:

$$E^2 - p^2c^2 = (mc^2)^2$$

$$E^2 + p^2c^2 - 2\left(m\gamma\, c\sqrt{2\frac{GM}{r}}\right)^2 = K^2$$

where:

$$K^2 = E_0^2 + P_0^2c^2 - 2\left(m\gamma_0\, c\sqrt{2\frac{GM}{r_0}}\right)^2$$

The first relation is usually used in the derivation of Schrödinger equation with another equation relating particle energy to the potential of the field taken from s classical theory to get a relation linking energy, momentum and a position-dependent function representing the field, This is done usually by applying a classical law that relates the particles energy to the potential of the field. This means that the accuracy of the resulting formula depends on the accuracy of this classical relation.

Applying this method with low gravity and low velocity approximations gives the *Gravitational Schrödinger* equation as:

$$\mathbf{i}\hbar\frac{\partial\psi}{\partial \mathbf{t}} = \left[-\frac{\hbar^2}{2\mathbf{m}}\nabla^2 - \frac{\mathbf{mU}^2}{2}\right]\psi$$

$$\mathbf{i}\hbar\frac{\partial\psi}{\partial \mathbf{t}} = \left[-\frac{\hbar^2}{2\mathbf{m}}\nabla^2 - \frac{\mathbf{GMm}}{\mathbf{r}}\right]\psi$$

With the second relation that relates energy, momentum and a function of position represents the gravitational field, we don't need to apply any classical theory of gravitation to derive Schrödinger equation because we have the precise relation;

$$E^2 + p^2c^2 - 2\left(m\gamma\, c\sqrt{2\frac{GM}{r}}\right)^2 = K^2$$

What is important about this formulation is that, the Schrödinger in a gravitational filed does not arise from introducing a potential or a geometric background, but from quantizing a redefined energy-momentum structure in which gravity manifests as a kinematical deformation rather than a dynamical entity.

## Conclusion

Instead of defining the energy and momentum of a body by the 4-momentum, we define them by a tensor quantity. The general conservation law of the tensor quantity that represents energy and momentum is for the sum of the components that represent energy and momentum in this extended definition, not for each component separately, and on this basis, the expansion of universe is a continuous transformation of the energy of all bodies into momentum. In this form, the conservation laws can be summarized by saying that a body moving in a gravitational field preserves the same value of the difference between the square of the magnitude of its velocity and the square of the magnitude of the quantity **U** representing the gravitational field. This formula has a significant impact on the formulation of quantum mechanics.

## References

**[1]**Einstein, A. (1916). Die Grundlage der allgemeinen Relativitätstheorie. Annalen der Physik, 49, 769–822.

**[2]** Hilbert, D. (1915). Die Grundlagen der Physik. Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse.