The Expansion of the Universe is a Transformation of Energy into Momentum

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Abstract

Any body existing in a gravitational field that changes with time due to the expansion of the universe or for any other reason remains subject to a conservation law for energy and momentum, but with a new concept of conservation, which is the conservation of the sum of the different components of the quantity that represents energy and momentum, not the conservation of each component separately. With this concept, we find that the expansion of the universe is a continuous transformation of energy into momentum, there is no loss or gain in the total sum of them.

Introduction

Physicists' views have differed greatly regarding the laws of conservation of energy and momentum in the presence of a gravitational field and the expansion of the universe. Concerning the gravitational field, there are now two main directions. The first can be attributed to the discoverer of general relativity, Einstein, and it says that energy and momentum are conserved if we use pseudotensors to represent the energy and momentum of the gravitational field **[1]**. This approach has been widely criticized because of the lack of covariance and localization. The second direction, which is dominant now and can be attributed to Einstein's partner in writing the field equation of general relativity, Hilbert, says that energy is conserved in the case of a static gravitational field such as the Schwarzschild field, as well as angular momentum, but linear momentum is not conserved due to the lack of symmetry in the three spatial axes to which the momentum components are related **[2]**.

As for the non-stationary of the gravitational field in time, there is almost a consensus among physicists that neither energy nor momentum is conserved.

In this paper, I show that the failure to obtain a complete and comprehensive law for the conservation of energy and momentum is the result of our use of an incomplete definition of the concept of conservation and of the conserved quantities themselves. Through purely mathematical analysis, without any reliance on physical assumptions other than the principles of general relativity, it can be shown that a unified and extended definition that includes energy and momentum for a body moving in a gravitational field in a continuously expanding universe includes a law for conservation of energy and momentum. In addition, this quantity is covariant and local.

The General Law of Conservation of Energy and Momentum

In a static gravitational field, we have a definition of the well-known conserved quantity:

$$\mathbf{m}\ \sqrt{\frac{c^2 - \boldsymbol{u}^2}{1 - \frac{\boldsymbol{v}^2}{c^2}}}$$

Where $\boldsymbol{u} = \sqrt{\frac{2GM}{r}}$

From this, we can deduce another conserved quantity by squaring this quantity and dividing it by the mass:

$\mathbf{m}\ \frac{c^2 - \boldsymbol{u}^2}{1 - \frac{\boldsymbol{v}^2}{c^2}}$, This quantity can be written in the form of quantities multiplied by the time-time component of the metric tensor:

$$\mathbf{m}\ \frac{\boldsymbol{u}^2 - c^2}{1 - \frac{\boldsymbol{v}^2}{c^2}} = \mathbf{m}\ \frac{g_{tt}\ \boldsymbol{v}^t\boldsymbol{v}^t - g_{tt}\ \boldsymbol{u}^t\boldsymbol{u}^t}{1 - \frac{\boldsymbol{v}^2}{c^2}} = \mathbf{T}_{tt}$$

That is because $\boldsymbol{v}^t\boldsymbol{v}^t = 1,\ g_{tt} = \boldsymbol{u}^2 - c^2$ and $\boldsymbol{u}^t\boldsymbol{u}^t = 0.$

This constant quantity is the energy $\mathbf{T}_{tt}$, and it is clear that it will lead to the well-known definition of energy in the absence of a gravitational field, because in this case, if $\mathbf{m}\ \frac{-c^2}{1 - \frac{\boldsymbol{v}^2}{c^2}}$ is conserved then $\mathbf{m}\ \frac{c^2}{\sqrt{1 - \frac{\boldsymbol{v}^2}{c^2}}}$ is also conserved.

Now, we have the quantity resulting from: $-\ \mathbf{m}c^2 -\ \mathbf{T}_{tt}$, which equals $\mathbf{m}\ \frac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{1 - \frac{\boldsymbol{v}^2}{c^2}}$, and it is also a constant quantity.

This quantity $\mathbf{m}\ \frac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{1 - \frac{\boldsymbol{v}^2}{c^2}}$ can be used as a definition of momentum, and it is also clear that it reduces to the well-known definition in the absence of a gravitational field, after using the law of conservation of angular momentum, which leads to the conservation of the direction of motion,

which leads to constant velocities, and the constancy of these velocities leads to the constancy of momenta in their traditional definition, as we explained in the case of energy.
The quantity $\boldsymbol{u}^2$ is the square of the magnitude of the scape velocity which is directed radially from the source of gravity; therefore, it can be written as a sum of components:

$$\boldsymbol{u}^2 = \sum_{i,j=1,2,3} g_{ij}\boldsymbol{u}^i\boldsymbol{u}^j$$

Also, the quantity $\boldsymbol{v}^2$ is the square of the magnitude of the velocity of the body therefore, it can be written as a sum of components:

$$\boldsymbol{v}^2 = \sum_{i,j=1,2,3} g_{ij}\boldsymbol{v}^i\boldsymbol{v}^j$$

Now the quantity $\mathbf{m}\,\frac{\boldsymbol{v}^2-\boldsymbol{u}^2}{1-\frac{\boldsymbol{v}^2}{c^2}}$ can be written as:

$$\mathbf{m}\,\frac{\sum_{i,j=1,2,3} g_{ij}\boldsymbol{v}^i\boldsymbol{v}^j - \sum_{i,j=1,2,3} g_{ij}\boldsymbol{u}^i\boldsymbol{u}^j}{1-\frac{\boldsymbol{v}^2}{c^2}}$$

$$= \sum_{i,j=1,2,3} \mathbf{m}\,\frac{g_{ij}\boldsymbol{v}^i\boldsymbol{v}^j - g_{ij}\boldsymbol{u}^i\boldsymbol{u}^j}{1-\frac{\boldsymbol{v}^2}{c^2}}$$

This allows us to define the momentum associated with the space coordinates i and j as; $\mathbf{P}_{ij} = \mathbf{m}\,\frac{g_{ij}\boldsymbol{v}^i\boldsymbol{v}^j - g_{ij}\boldsymbol{u}^i\boldsymbol{u}^j}{1-\frac{\boldsymbol{v}^2}{c^2}}$

And thus, we have the conservation law of momentum in static gravitational field in the form: $\sum_{i,j=1.2.3} \mathbf{T}_{ij}$ = constant, or more clearly: $\mathbf{P} = \mathbf{T}_{11}+\mathbf{T}_{12}+\mathbf{T}_{13}+\mathbf{T}_{21}+\mathbf{T}_{22}+\mathbf{T}_{23}+\mathbf{T}_{31}+\mathbf{T}_{32}+\mathbf{T}_{33}$ is conserved.

The conservation of the quantity $\mathbf{P}_{ij}$ means the conservation of the sum of the components, and this is the new concept of the law of conservation of linear momentum. There is no conservation law for each of the three spatial directions separately, but rather a conservation law for them collectively. A body that moves freely in a static gravitational field does not lose momentum, does not gain momentum, and does not exchange anything with the gravitational field or other bodies, but only the components of momentum exchange momentum among themselves. This is the law of conservation of energy in a static gravitational field.

The energy of the body $\mathbf{T}_{\mathrm{tt}}$ has the same form as energy because momentum can be written as:

$$\mathbf{T}_{\mathrm{tt}} = \mathbf{m}\,\frac{\boldsymbol{u}^2 - \mathrm{c}^2}{1-\frac{\boldsymbol{v}^2}{\mathrm{c}^2}} = \mathbf{m}\,\frac{\mathrm{g}_{\mathrm{tt}}\,\boldsymbol{v}^t\boldsymbol{v}^t - \mathrm{g}_{\mathrm{tt}}\,\boldsymbol{u}^t\boldsymbol{u}^t}{1-\frac{\boldsymbol{v}^2}{\mathrm{c}^2}}$$

This allows us to define a unified quantity T_{pq} that includes both energy and momentum:

$$\mathrm{T}_{pq} = \mathrm{m}\,\frac{\mathrm{g}_{pq}\boldsymbol{v}^p\boldsymbol{v}^q - \mathrm{g}_{pq}\boldsymbol{u}^p\boldsymbol{u}^q}{1-\frac{\boldsymbol{v}^2}{\mathrm{c}^2}}$$

$\boldsymbol{u} = \sqrt{\frac{2GM}{r}}$ ***in radial direction***, $\boldsymbol{u}^0 = 0$

$p, q = 0,1,2,3$

This unified definition of energy and momentum includes beside energy (T_{tt}) in one component and momentum ($\mathbf{T}_{11}, \mathbf{T}_{12}, \mathbf{T}_{13}, \mathbf{T}_{21}, \mathbf{T}_{22}, \mathbf{T}_{23}, \mathbf{T}_{31}, \mathbf{T}_{32}$) in nine components, other six components ($\mathbf{T}_{01}, \mathbf{T}_{02}, \mathbf{T}_{03}, \mathbf{T}_{10}, \mathbf{T}_{20}, \mathbf{T}_{30},$) that are neither energy nor momentum, but rather complements of the same type, we can prove that their sum equals zero. But we are now concerned with the components that represent linear momentum and energy.

In the case of the expansion of the universe and cases of changing gravitational fields in general, we find that what changes is the quantity $\boldsymbol{u}$, which leads to changes in momentum and energy, but if we take their total sum, we find that it is constant because it does not depend on the gravitational field it is the mass in minus sign:

$$\mathbf{m}\,\frac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{1-\frac{\boldsymbol{v}^2}{\mathrm{c}^2}} + \mathbf{m}\,\frac{\boldsymbol{u}^2 - \mathrm{c}^2}{1-\frac{\boldsymbol{v}^2}{\mathrm{c}^2}} = -\mathrm{mc}^2$$

If we change the metric signature, the sign changes from negative to positive and we obtain the following very interesting result:

Energy + Sum of the Components of Momentum = Mass

This means that in the case of a gravitational field changing with time, the body does not lose or gain anything from the outside, but what happens is that part of its energy is transformed into momentum or vice versa, so the total remains constant, while the mass remains constant. This is the general law of conservation of energy and momentum that includes all cases.

Significant and Unexpected New Conservation Formula's Implications in Quantum Mechanics

Even without this new conservation formula, mass m (or the rest energy mc^2) as a conserved quantity is very important in Newtonian physics and special relativity. However, from the perspective of quantum mechanics and general relativity, there are several reasons why "mass" alone is insufficient to define the *phase* under all conditions, for example:

1. Mass is not the sole "Generator" of motion .In quantum mechanics, the phase changes because the object moves or interacts. Using mass only will lead to phase that would depend solely on proper time, While this represents the "De Broglie phase" of a particle at rest, it fails to account for *interference* patterns caused by velocity (momentum) or changes in gravitational potential. To calculate a trajectory, we need a quantity that responds to changes in position and speed; rest mass remains indifferent to these factors.

2. Photons (light), for example, have zero rest mass yet they possess a phase that evolves and is affected by gravity and cosmic expansion (redshift). If the quantity used to calculate the phase were strictly mass, the phase of light would be indefinable or zero.

3. In the context of fundamental physics theories such as general relativity and quantum mechanics, mass is not a complete quantity but rather represents only the time-related component in a matrix that includes momentum in different directions.

Now, with the new conservation formula, we have overcome all those obstacles and barriers, because we have established the existence of an effective mass that does not change regardless of how much spacetime expands or curves, *while the distribution of its magnitude over the temporal and spatial vessels that carry it changes*, and this mass is not linked to time alone but results from the sum of components formed from all dimensions of space-time in a balanced way.

So, the invariant $\mathbf{mc^2}$ naturally defines a space-time action:

$$S = \int \mathrm{mc}^2 \, d\mathcal{T}$$

Where $\mathcal{T}$ is the proper time along the particle's worldline

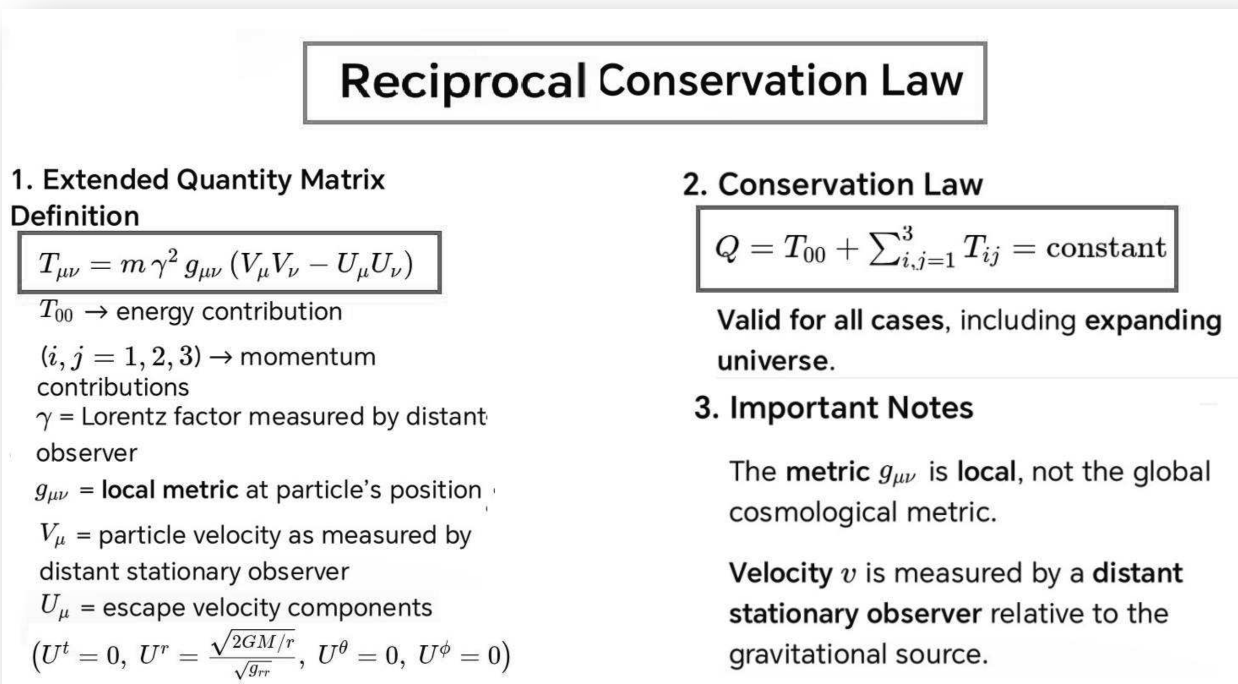
And this allows us to associate a phase with this action:

$$\Delta\phi = \frac{1}{\hbar}\int_{1}^{2} \mathrm{mc}^2 \, d\mathcal{T}$$

Even if the mass is zero, as in the case of light, this does not mean phaselessness, because zero in this case is the sum of two quantities equal in magnitude but opposite in sign: energy and momentum. When we decompose the zero mass into these two parts, each part has a phase that changes with time.This phase is purely geometric and relativistic in origin. No quantum postulate has yet been introduced. This provides a unified and conceptually economical bridge between relativistic structure and quantum phenomena, Key structures of quantum mechanics –wave dynamics, uncertainty, interference and other structures can be derived using a single geometric invariant associated with space time redistribution.

Conclusion

Instead of defining the energy and momentum of a body by the 4-momentum, we define them by a tensor quantity. The general conservation law of the tensor quantity that represents energy and momentum is for the sum of the components that represent energy and momentum in this extended definition, not for each component separately, and on this basis, the expansion of the universe is a continuous transformation of the energy of all bodies into momentum. This formula has a significant impact on the formulation of quantum mechanics.



Example Derivation of the Uncertainty Principle from the Reciprocal Conservation Law

Start from the Reciprocal Conservation Law:

$$Q = \gamma^2 \left(\|V\|_g^2 \quad U^2\right) = \text{constant}$$

where Q is conserved for a particle moving in a gravitational field.

Promote velocities to operators (quantization):

$$V_i \;\rightarrow\; \hat{V}_i = -i\hbar \frac{\partial}{\partial x_i}$$

and consider the spatial component contribution.

Express the conserved quantity in terms of position and momentum operators:

$$\hat{Q}_{\text{spatial}} \sim \sum_i \hat{V}_i^2$$

Compute the variance of position and momentum:

For any state $|\psi\rangle$:

$$(\Delta x_i)^2 = \langle\psi|\hat{x}_i^2|\psi\rangle - \langle\psi|\hat{x}_i|\psi\rangle^2$$

$$(\Delta p_i)^2 = \langle\psi|\hat{p}_i^2|\psi\rangle - \langle\psi|\hat{p}_i|\psi\rangle^2$$

Apply the Cauchy-Schwarz inequality:

$$(\Delta x_i)^2(\Delta p_i)^2 \geq \frac{1}{4}|\langle[\hat{x}_i, \hat{p}_i]\rangle|^2$$

$$[\hat{x}_i, \hat{p}_i] = i\hbar$$

Conclude the Uncertainty Principle:

$$\Delta x_i \, \Delta p_i \geq \frac{\hbar}{2}$$

Notes:

The derivation uses **the reciprocal conservation law** as the starting point.

No explicit energy operator is needed; uncertainty emerges from the **geometric relation between position and reciprocal momentum** enforced by the conservation.

This is concise, fully consistent with standard QM results, but rooted in the **geometric conservation framework.**

References

[1]Einstein, A. (1916). Die Grundlage der allgemeinen Relativitätstheorie. Annalen der Physik, 49, 769–822.

[2] Hilbert, D. (1915). Die Grundlagen der Physik. Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse.