The Expansion of the Universe is a Transformation of Energy into Momentum

Mueiz Gafer KamalEldeen

Abstract

Any body existing in a gravitational field that changes with time due to the expansion of the universe or for any other reason remains subject to a conservation law for energy and momentum, but with a new concept of conservation, which is the conservation of the sum of the different components of the quantity that represents energy and momentum, not the conservation of each component separately. With this concept, we find that the expansion of the universe is a continuous transformation of energy into momentum, there is no loss or gain in the total sum of them.

Introduction

Physicists' views have differed greatly regarding the laws of conservation of energy and momentum in the presence of a gravitational field and the expansion of the universe. Concerning the gravitational field, there are now two main directions. The first can be attributed to the discoverer of general relativity, Einstein, and it says that energy and momentum are conserved if we use pseudotensors to represent the energy and momentum of the gravitational field **[1]**. This approach has been widely criticized because of the lack of covariance and localization. The second direction, which is dominant now and can be attributed to Einstein's partner in writing the field equation of general relativity, Hilbert, says that energy is conserved in the case of a static gravitational field such as the Schwarzschild field, as well as angular momentum, but linear momentum is not conserved due to the lack of symmetry in the three spatial axes to which the momentum components are related **[2]**.

As for the non-stationary of the gravitational field in time, there is almost a consensus among physicists that neither energy nor momentum is conserved.

In this paper, I show that the failure to obtain a complete and comprehensive law for the conservation of energy and momentum is the result of our use of an incomplete definition of the concept of conservation and of the conserved quantities themselves. Through purely mathematical analysis, without any reliance on physical assumptions other than the principles of general relativity, it can be shown that a unified and extended definition that includes energy and momentum for a body moving in a gravitational field in a continuously expanding universe includes a law for conservation of energy and momentum. In addition, this quantity is covariant and local.

The General Law of Conservation of Energy and Momentum

In a static gravitational field, we have a definition of the well-known conserved quantity:

$$\mathbf{m}\sqrt{\frac{\mathrm{c}^2-\boldsymbol{u}^2}{1-\frac{\boldsymbol{v}^2}{\mathrm{c}^2}}}$$

Where $\boldsymbol{u} = \sqrt{\frac{2\mathrm{GM}}{r}}$

From this, we can deduce another conserved quantity by squaring this quantity and dividing it by the mass:

$\mathbf{m}\,\frac{\mathrm{c}^2-\boldsymbol{u}^2}{1-\frac{\boldsymbol{v}^2}{\mathrm{c}^2}}$, This quantity can be written in the form of quantities multiplied by the time-time component of the metric tensor:

$$\mathbf{m}\,\frac{\boldsymbol{u}^2-\mathrm{c}^2}{1-\frac{\boldsymbol{v}^2}{\mathrm{c}^2}} = \mathbf{m}\,\frac{\mathrm{g}_{\mathrm{tt}}\,\boldsymbol{v}^t\boldsymbol{v}^t-\mathrm{g}_{\mathrm{tt}}\,\boldsymbol{u}^t\boldsymbol{u}^t}{1-\frac{\boldsymbol{v}^2}{\mathrm{c}^2}} = \mathbf{T}_{\mathrm{tt}}$$

That is because $\boldsymbol{v}^t\boldsymbol{v}^t = 1,\ \mathrm{g}_{\mathrm{tt}} = \boldsymbol{u}^2-\mathrm{c}^2$ and $\boldsymbol{u}^t\boldsymbol{u}^t = 0.$

This constant quantity is the energy $\mathbf{T}_{\mathrm{tt}}$, and it is clear that it will lead to the well-known definition of energy in the absence of a gravitational field, because in this case, if $\mathbf{m}\,\frac{-\mathrm{c}^2}{1-\frac{\boldsymbol{v}^2}{\mathrm{c}^2}}$ is conserved then $\mathbf{m}\,\frac{\mathrm{c}^2}{\sqrt{1-\frac{\boldsymbol{v}^2}{\mathrm{c}^2}}}$ is also conserved.

Now, we have the quantity resulting from: $-\,\mathbf{m}\mathrm{c}^2-\ \mathbf{T}_{\mathrm{tt}}$, which equals $\mathbf{m}\,\frac{\boldsymbol{v}^2-\boldsymbol{u}^2}{1-\frac{\boldsymbol{v}^2}{\mathrm{c}^2}}$, and it is also a constant quantity.

This quantity $\mathbf{m}\,\frac{\boldsymbol{v}^2-\boldsymbol{u}^2}{1-\frac{\boldsymbol{v}^2}{\mathrm{c}^2}}$ can be used as a definition of momentum, and it is also clear that it reduces to the well-known definition in the absence of a gravitational field, after using the law of conservation of angular momentum, which leads to the conservation of the direction of motion, which leads to constant velocities, and the constancy of these velocities leads to the constancy of momenta in their traditional definition, as we explained in the case of energy.

The quantity $\boldsymbol{u}^2$ is the square of the magnitude of the scape velocity which is directed radially from the source of gravity; therefore, it can be written as a sum of components:

$$\boldsymbol{u}^2 = \sum_{i.j=1,2,3} \mathrm{g}_{ij}\boldsymbol{u}^i\boldsymbol{u}^j$$

Also, the quantity $\boldsymbol{v}^2$ is the square of the magnitude of the velocity of the body therefore, it can be written as a sum of components:

$$\boldsymbol{v}^2 = \sum_{i.j=1,2,3} \mathrm{g}_{ij}\boldsymbol{v}^i\boldsymbol{v}^j$$

Now the quantity $\mathbf{m}\ \frac{\boldsymbol{v}^2-\boldsymbol{u}^2}{1-\frac{\boldsymbol{v}^2}{\mathrm{c}^2}}$ can be written as:

$$\mathbf{m}\ \frac{\sum_{i.j=1,2,3} \mathrm{g}_{ij}\boldsymbol{v}^i\boldsymbol{v}^j - \sum_{i.j=1,2,3} \mathrm{g}_{ij}\boldsymbol{u}^i\boldsymbol{u}^j}{1-\frac{\boldsymbol{v}^2}{\mathrm{c}^2}}$$

$$= \sum_{i.j=1,2,3} \mathbf{m}\ \frac{\mathrm{g}_{ij}\boldsymbol{v}^2 - \mathrm{g}_{ij}\boldsymbol{u}^2}{1-\frac{\boldsymbol{v}^2}{\mathrm{c}^2}}$$

This allows us to define the momentum associated with the space coordinates i and j as; $\mathbf{P}_{ij} = \mathbf{m}\ \frac{\mathrm{g}_{ij}\boldsymbol{v}^2-\mathrm{g}_{ij}\boldsymbol{u}^2}{1-\frac{\boldsymbol{v}^2}{\mathrm{c}^2}}$

And thus, we have the conservation law of momentum in static gravitational field in the form: $\sum_{i.j=1.2.3} \mathbf{T}_{ij}$ = constant, or more clearly: $\mathbf{P} = \mathbf{T}_{11}+\mathbf{T}_{12}+\mathbf{T}_{13}+\mathbf{T}_{21}+\mathbf{T}_{22}+\mathbf{T}_{23}+\mathbf{T}_{31}+\mathbf{T}_{32}+\mathbf{T}_{33}$ is conserved.

The conservation of the quantity $\mathbf{P}_{ij}$ means the conservation of the sum of the components, and this is the new concept of the law of conservation of linear momentum. There is no conservation law for each of the three spatial directions separately, but rather a conservation law for them collectively. A body that moves freely in a static gravitational field does not lose momentum, does not gain momentum, and does not exchange anything with the gravitational field or other bodies, but only the components of momentum exchange momentum among themselves. This is the law of conservation of energy in a static gravitational field.

The energy of the body $\mathbf{T}_{tt}$ has the same form as energy because momentum can be written as:

$$\mathbf{T}_{tt} = \mathbf{m} \frac{\boldsymbol{u}^2 - c^2}{1 - \frac{\boldsymbol{v}^2}{c^2}} = \mathbf{m} \frac{g_{tt}\,\boldsymbol{v}^t \boldsymbol{v}^t - g_{tt}\,\boldsymbol{u}^t \boldsymbol{u}^t}{1 - \frac{\boldsymbol{v}^2}{c^2}}$$

This allows us to define a unified quantity T_{pq} that includes both energy and momentum:

$$T_{pq} = m \frac{g_{pq}\boldsymbol{v}^p \boldsymbol{v}^q - g_{pq}\boldsymbol{u}^p \boldsymbol{u}^q}{1 - \frac{\boldsymbol{v}^2}{c^2}}$$

$$\boldsymbol{u} = \sqrt{\frac{2GM}{r}}$$

$$p, q = 0,1,2,3$$

This unified definition of energy and momentum includes beside energy (T_{tt}) in one component and momentum ($\mathbf{T}_{11}, \mathbf{T}_{12}, \mathbf{T}_{13}, \mathbf{T}_{21}, \mathbf{T}_{22}, \mathbf{T}_{23}, \mathbf{T}_{31}, \mathbf{T}_{32}$) in nine components, other six components ($\mathbf{T}_{01}, \mathbf{T}_{02}, \mathbf{T}_{03}, \mathbf{T}_{10}, \mathbf{T}_{20}, \mathbf{T}_{30},$) that are neither energy nor momentum, but rather complements of the same type, we can prove that their sum equals zero. But we are now concerned with the components that represent linear momentum and energy.

In the case of the expansion of the universe and cases of changing gravitational fields in general, we find that what changes is the quantity $\boldsymbol{u}$, which leads to changes in momentum and energy, but if we take their total sum, we find that it is constant because it does not depend on the gravitational field it is the mass in minus sign:

$\mathbf{m} \frac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{1 - \frac{\boldsymbol{v}^2}{c^2}} + \mathbf{m} \frac{\boldsymbol{u}^2 - c^2}{1 - \frac{\boldsymbol{v}^2}{c^2}} = -mc^2$, this gives the following very interesting result:

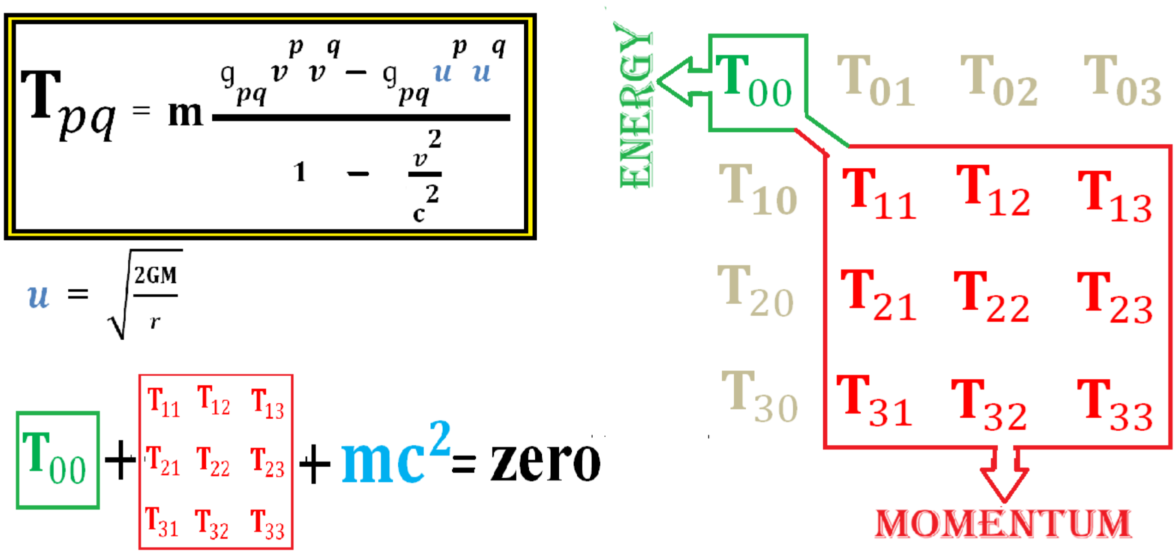
Energy + Sum of the Components of Momentum + Mass = Zero

This means that in the case of a gravitational field changing with time, the body does not lose or gain anything from the outside, but what happens is that part of its energy is transformed into momentum or vice versa, so the total remains constant, while the mass remains constant. This is the general law of conservation of energy and momentum that includes all cases.

Conclusion

Instead of defining the energy and momentum of a body by the 4-momentum, we define them by a tensor quantity. The general conservation law of the tensor quantity that represents energy and momentum is for the sum of the components that represent energy and momentum in this extended definition, not for each component separately, and on this basis, the expansion of the universe is a continuous transformation of the energy of all bodies into momentum.

GENERAL LAW OF ENERGY-MOMENTUM CONSERVATION



References

[1]Einstein, A. (1916). Die Grundlage der allgemeinen Relativitätstheorie. Annalen der Physik, 49, 769–822.

[2] Hilbert, D. (1915). Die Grundlagen der Physik. Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse.