The Prime Numbers Machine :
Basic Arithmetic Procedures to Validate and Predict
Primes and Twin Primes Sequences

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Summary

In this article, we intend to present two procedures of elementary simplicity, one for predicting the complete sequence of prime numbers, the second being more specifically dedicated to the identification of pairs of twin primes, with the primary motivation of creating the most concise method of achieving this end. To detect prime numbers in the continuum of natural numbers, our study is inspired by an algorithm well known to mathematicians working in the field, namely the formula $p^2 = 24n + 1$, to which we grafted two additional sieving steps aimed at discriminating false positives. The procedure for predicting the specific sequence of twin primes can be conceived independently or operationally grafted onto the previous one. We also submit to your attention a set of eleven postulates concerning twin primes, as well as a sample of eleven additional hypothesis of the order of conjecture.

1. Predicting the sequence of prime numbers using a simple and foolproof procedure

The search for a formal theoretical model revealing the existence of a coherent structure in the ordering of prime numbers within our numeral system has always aroused great fascination in scientific circles, as well as among the general public. There are methods for detecting these numbers, some of which are based on fairly basic formulas of algebraic arithmetic, with results of varying satisfaction. In the present study, we intend to understand the limitations of one of these formulas and to make the necessary improvements in the form of a procedure, as minimalist as possible, for locating and validating the complete sequence of prime numbers, without error or omission, and without the need for additional data input.

1.1 The starting algorithm

Without further ado, let's introduce the said algorithm and perform some calculations as a demonstration. The algebraic equation can be written according to two different formulations, namely variant A, which is the one we know, and variant B, which we propose here. These two variants correspond to the two main angles of attack available to us to identify or validate the prime number *p*.

Variant A:

p = Verified or searched prime.
n = Integer acting as the multiplicative factor of the number 24.

$$\boldsymbol{p = \sqrt{24n + 1}} \qquad \text{or} \qquad \boldsymbol{p^2 = 24n + 1}$$

Variant B:

p = Verified or searched prime.
n = Any positive integer resulting from a manipulation of *p*.

$$\boldsymbol{n = \frac{p^2 - 1}{24}}$$

In both cases, the variable *p* is considered a potential prime if and only if *n* is an integer. In other words, *p* will never be prime if *n* is not a whole number, but the affirmative does not constitute validation. We can translate these algorithms into a statement or a heuristic rule.

General rule: The square minus 1 of all prime numbers is divisible by 24.

The algorithm has a 100% success rate in spotting prime numbers, not omitting any of them. In other words, we always manage to get a correspondence between the square of prime numbers and the number 24, as a multiplicative factor or divisor. The difficulty comes rather from the fact that the algorithm also retains composite numbers in its nets, when it should have thrown them back into the water. We consider these composite numbers to be false positives.

In the present study, we have challenged ourselves to take logic further, to the point of establishing a complete procedure for detecting prime numbers that eliminates false positives and gives a glimpse of the arithmetic entanglement that determines the distribution of prime numbers.

An adequate detection procedure of prime numbers should include a triage function to identify these false positives, so as to reject them. It is this problem that we have chosen to solve. To achieve this, it is important to understand the arithmetic logic that causes the misselection of composite numbers. It is this understanding that will allow us to adjust our procedure accordingly by making the appropriate tuning.

However, as we will show later, it is easy to make the necessary correction operations. But let's first look at this correspondence between the square of primes and the multiples of the number 24.

1.2 The Pyramid Blocks

The following table highlights the close relationship between prime numbers and the number 24.

Table 1: Relationship between the squares of prime numbers and number 24

Prime number	Its square	Multiples of 24	Multiples of 24 + 1
5	25	24 (1 x 24)	25
7	49	48 (2 x 24)	49
11	121	120 (5 x 24)	121
13	169	168 (7 x 24)	169
17	289	288 (12 x 24)	289
19	361	360 (15 x 24)	361
23	529	528 (22 x 24)	529
29	841	840 (35 x 24)	841
31	961	960 (40 x 24)	961
37	1,369	1,368 (57 x 24)	1,369
41	1,681	1,680 (70 x 24)	1,681
43	1,849	1,848 (77 x 24)	1,849
47	2,209	2,208 (92 x 24)	2,209

Such data testify to the well-known correspondence between the sequence of prime numbers and the multiples of the number 24, as if the prime numbers were articulated and distributed according to it. Of course, the numbers 2 and 3, which are also prime, are excluded from our table since their squares are less than 24, which makes dividing by 24 impossible. However, we can apply the same kind of procedure to them by dividing their square by lower numbers arithmetically related to the number 24, i.e. its factors, considering that the number 24 can be interpreted as the product of two numbers.

24 = **3** x **8**

22 = 4 4 – 1 = 3 3 ÷ **3** = 1 (integer)

32 = 9 9 – 1 = 8 8 ÷ **8** = 1 (integer)

As well as:

52 = 25 25 – 1 = 24 24 ÷ 24 = 1 (or 24 ÷ 8 = 3 and 3 ÷ 3 = 1)

Thus, the prime numbers 2 and 3 are indeed an integral part of the same arithmetic construction as the set of prime numbers. We get a fun result by continuing this little game of dividing by 2, 3, 8, 12, 24 or 48 on the squares of the increasing sequence of prime numbers.

5^2 – 1 = 24 24 ÷ 24 = **1**

7^2 – 1 = 48 48 ÷ 24 = **2** (or 48 ÷ 3 = 16 16 ÷ 8 = 2)

11^2 – 1 = 120 120 ÷ 24 = **5** (or 120 ÷ 12 = 10 10 ÷ 2 = 5)
(or 120 ÷ 8 = 15 15 ÷ 3 = 5)

13^2 – 1 = 168 168 ÷ 24 = **7** (or 168 ÷ 12 = 14 14 ÷ 2 = 7)
(or 168 ÷ 8 = 21 21 ÷ 3 = 7)

The sequence of prime numbers from 2 to 7 is almost complete, as only the number 3 is missing. We still manage to obtain it by means of various operations that call on the number 24 and its factors (2, 3, 4, 6, 8, 12):

7^2 – 1 = 48	48 ÷ 8 = 6	6 ÷ 2 =	**3**
48 ÷ 24 = 2	8 x 2 = 16	48 ÷ 16 =	**3**
48 ÷ 12 = 4	4^2 = 16	48 ÷ 16 =	**3**

In the logic of this arithmetic sequence, which aims to obtain the increasing list of prime numbers as a result, it must also be understood that the more the number increases, the more the divisor must be increased to maintain the sequence (i.e. go from divisor 24, to 48, to 72, etc.). Thus, for prime numbers that follow 7 in the sequence, i.e. 11, 13, 17, 19, 23, 29, etc., it is still possible to obtain them by using the same technique, which consists of squaring a number, subtracting 1 from it and dividing it by a multiple of 24.

$23^2 - 1 = 528$	$528 \div 48 =$	**11**	(or $528 \div 24 = 22$	$22 \div 2 = 11$)
			(or $528 \div 12 = 44$	$44 \div 4 = 11$)
$25^2 - 1 = 624$	$624 \div 48 =$	**13**	(or $624 \div 24 = 26$	$26 \div 2 = 13$)
$35^2 - 1 = 1224$	$1224 \div 72 =$	**17**	(or $1224 \div 24 = 51$	$51 \div 3 = 17$)

In short, the sequence of prime numbers appears to us following these simple operations of multiplication and division articulated around the number 24 or its accomplices (its factors and multiples), testimony to the adequacy of this numeral assembly. It is true that number 24 has certain distinctive attributes that propel it to the forefront and lead it to play a particular role in some arithmetic contexts, including the study of prime numbers, encouraging us to reflect on its particular properties, but our study has different objectives.

1.3 Requirements for Recognition of the Integrity and Validity of Our Procedure

With the algorithm $p = \sqrt{24n + 1}$ as a starting point, the next step is to develop a procedure that translates this arithmetic structure into a sequence of steps with predictive value. Ideally, this procedure should adequately meet the following requirements or conditions:

1) It should enable us to validate the numbers as being primes or not and to predict the sequence of prime numbers in ascending or descending order, having as a single starting point an integer, prime or not.

2) It should not omit any element of the sequence (the sequence of prime numbers). In short, not to miss a single target.

3) It must not identify a number that is not prime as prime, i.e. produce false positive results. It must be equipped with simple and effective validation tests to identify intruders and eliminate them from the list, exposing the rationale behind this apparent anomaly by means of a minimum number of arithmetic operations.

4) It has to be self-sufficient, in the sense that it is not necessary to graft a chain of additional operations onto it, nor to inject it with additional information not initially accepted to obtain the expected result. In other words, the procedure must generate by itself the data it will need to fulfill its validation/prediction task.

1.4 False positives: justification and solution

The numbers pointed out by the algorithm are sometimes false positives, i.e. identified by our formula as prime numbers when they should not have been. The more the order of magnitude of the numbers increases, the more false positives multiply. In order to get a more precise idea of this, we tried to know the ratio that exists between the number of false positives (*fp*) – i.e. the numbers wrongly retained by the algorithm – and the number of potential false positives (*pfp*) – i.e. the set of numbers that could be wrongly identified by our algorithm as prime, or the set of odd nonprime numbers (excluding those ending in the number 5, which we know are not prime). The percentages represent the retention rate of *fp* across all *pfp*. They gradually increase according to the size of the numbers, until they slightly exceed the 50% mark.

Table 2: Percentage of false positives by increasing increments

Number samples	**Percentage of false positives** (numerator = *fp*, denominator = *pfp*)
1 to 100	18,75% (3/16) (these are 49, 77 et 91)
101 to 200	36,84% (7/19)
201 to 300	40% (10/25)
301 to 400	41,66% (10/24)
Between 841(29^2) and 961 (31^2)	48,14% (13/27)
Between 1,681 (41^2) and 1,849 (43^2)	50% (21/42)
Between 3,481 (59^2) and 3,721 (61^2)	49,2% (31/63)
Between 5,041 (71^2) and 5,329 (73^2)	56,09% (46/84)
Between 10,201 (101^2) and 10,609 (103^2)	55% (66/120)

We needed to address and solve this problem. We therefore focused on the analysis of the properties of false positive primes, which allowed us to discover common characteristics that make them easy to identify and discard from the outset. As the following examples show, their detection by the algorithm is always the result of one or the other of these two reasons:

A) They are a power of a prime number.

$5^2 = 25$ $25 – 1 = 24$ $24 \div 24 = 1$ (5 is a prime number)

$25^2 = 625$	625 – 1 = 624	624 ÷ 24 = 26	(25 is a *fp* because $5^2 = 25$)
$7^2 = 49$	49 – 1 = 48	48 ÷ 24 = 2	(7 is a prime number)
$49^2 = 2{,}401$	2,401 – 1 = 2,400	2,400 ÷ 24 = 100	(49 is a *fp* because $7^2 = 49$)
$11^2 = 121$	121 – 1 = 120	120 ÷ 24 = 5	(11 is a prime number)
$121^2 = 14{,}641$	14,641 – 1 = 14,640	14,640 ÷ 24 = 610	(121 is a *fp* because $11^2 = 121$)
$7^3 = 343$	3432 = 117,649	117,649 – 1 = 117,648	117,648 ÷ 24 = 4,902 (343 is an *fp* because $7^3 = 343$)
$7^4 = 2{,}401$	$2{,}401^2 = 5{,}764{,}801$	5,764,801 – 1 = 5,764,800	5,764,800 ÷ 24 = 240,200 (2,401 is a *fp* because $7^4 = 2{,}401$)

B) They have a pair of prime numbers as their only factors.

13 x 17 = 221	$221^2 = 48{,}841$	48,841 – 1 = 48,840	48,840 ÷ 24 = 2,035

(221 is a false positive because 13 x 17 = 221)

11 x 23 = 253	$253^2 = 64{,}009$	64,009 – 1 = 64,008	64,008 ÷ 24 = 2,667

(253 is a false positive because 11 x 23 = 253)

It could be argued that only one test is necessary, since a prime number to the power of 2 is the equivalent of the product of two prime numbers, but this would be to ignore the fact that the exponent can be greater than 2. For example, the number 2401 (7^4) is a false positive. Most of the time, of course, false positives are the product of two prime numbers. These numbers are known as semiprimes. In other words, the vast majority of the false positives represent a subset of semiprimes.

Table 3: False positive numbers and their factorization

False positive numbers (*fp*)	fp^2	$(n^2 – 1) ÷ 24$	Factors of *fp*
49	2,401	100	7^2
77	5,929	247	7 x 11
91	8,281	345	7 x 13
119	14,161	590	7 x 17
121	14,641	610	11^2
133	19,019	737	17 x 19
143	20,449	852	11 x 13
161	25,921	1,080	7 x 23
169	28,561	1,190	13^2

187	34,969	1,457	11 x 17
203	41,209	1,717	7 x 29
209	43,681	1,820	11 x 19
217	47,089	1,962	7 x 31
221	48,841	2,035	13 x 17
247	61,009	2,542	13 x 19
253	64,009	2,667	11 x 23
259	67,081	2,795	7 x 37
287	82,369	3,432	7 x 41
289	83,521	3,480	17^2
299	89,401	3,725	13 x 23
301	90,601	3,775	7 x 43
319	101,761	4,240	11 x 29
323	104,329	4,347	17 x 19
329	108,241	4,510	7 x 47
341	116,281	4,845	11 x 31
343	117,649	4,902	7^3
361	130,321	5,430	19^2
371	137,641	5,735	7 x 53
377	142,129	5,922	13 x 29
391	152,881	6,370	17 x 23

This does not mean that all semiprime numbers – i.e. all those which are the product of a pair of prime numbers – will come out of our procedure as false positives. For example, semiprime numbers can be even, whereas our validation procedure will never identify even numbers as potential primes, rejecting them out of hand in the first step of the analysis process. Likewise, semiprime numbers ending in a 5 are irrelevant, because we already know that these numbers are not prime numbers and therefore should be discarded without further consideration.

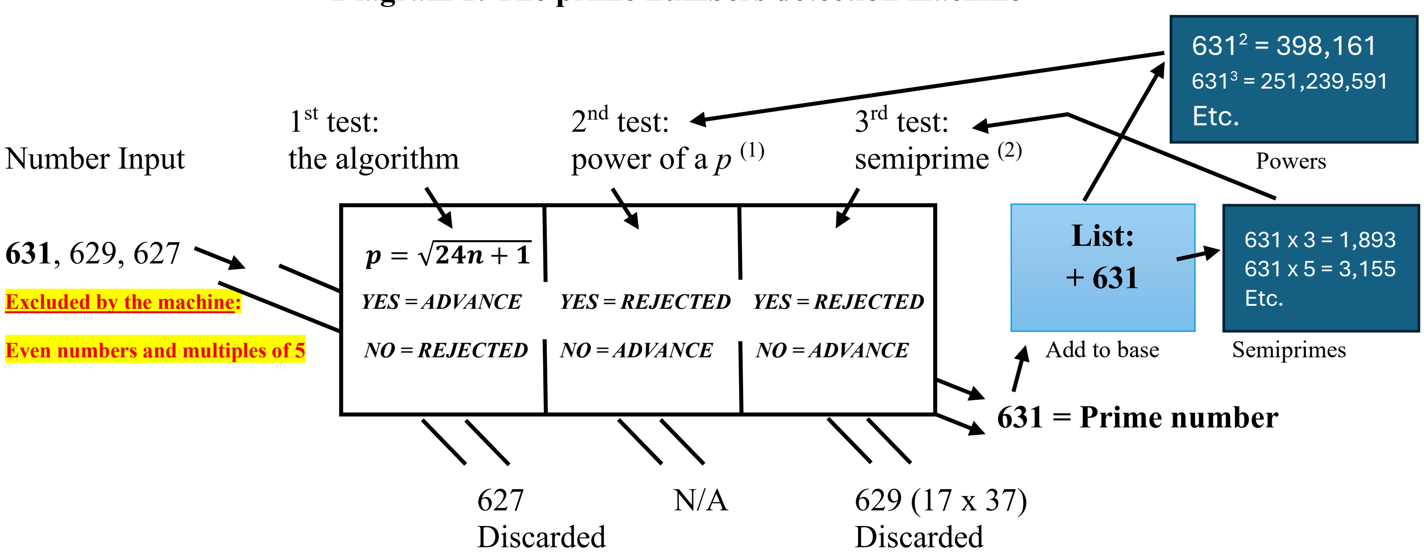
Rather than invalidating our model, these false positives only remind us that the numerical edifice is a structure crossed by a cleverly articulated framework and whose steel beams are the prime numbers, the very ones that ensure its integrity and act through it according to an infallibly rigorous principle of extension and redundancy that presides over their distribution.

This analysis led us directly to the addition of an additional step to our procedure for identifying prime numbers, which we call "sifting", like gold prospectors who strive to keep only the highly prized nuggets in their sieve. Our nuggets are obviously the constellation of prime numbers, but we must also emphasize the preponderant role of semiprime numbers in the validation procedure of our model and their potential role in the entire numerical architecture. We will see later that they are directly involved in the sequencing of twin prime pairs, which, in our opinion, gives them very special, if not essential, organizational properties in our numeral system, as well as a valuable predictive power.

1.5 Operationalization of the algorithm using the sieving procedure

Graphically, our model could be represented as a "machine" performing quality control or conformity testing on a production line, as shown in the following illustration. It should be noted that the same assembly, presented visually for the purposes of our presentation, can easily be translated into the form of a computer program.

Diagram 1: The prime numbers detection machine



(1) Check if the number tested is a power of a prime number (e.g. 7^3).
(2) Check if the number tested is the product of two prime numbers (e.g. 19 x 31).

Example:

What are the two prime numbers that follow 251?

Let's apply formula A, but first let's take the time to check if 251 is indeed a prime number.

$$p = \sqrt{24n + 1}$$

$$251 = \sqrt{24n + 1}$$

$$251^2 = 24n + 1$$

$$63\ 001 - 1 = 24n$$

$$63\ 000 = 24n$$

$$\frac{63\ 000}{24} = n = 2\ 625$$

The result of dividing by 24 the square minus 1 of 251 by 24 is an integer (2,625), 251 is not a power of a prime number and is not the product of two prime numbers, so it is a prime number. Now let's move on to the next step, which is to choose the next number to insert into our machine. It must not be an even number or a multiple of 5, which eliminates the numbers 252, 254, 255, 256, 258, 260, etc. So, we'll test the numbers 253, 257, 259, 261, 263, and so on in order.

$$p = \sqrt{24n + 1}$$

$$253 = \sqrt{24n + 1}$$

$$253^2 = 24n + 1$$

$$64\ 009 - 1 = 24n$$

$$64\ 008 = 24n$$

$$\frac{64008}{24} = 2\ 667$$

Square minus 1 of the number 253 is a multiple of 24, making it a potential prime number or "candidate." Now let's move on to the next step in our validation procedure, which is the double-filter sieving of number 253:

1st filter: Is it a power of a prime number?

Answer: NO → Switch to the second filter.

2nd filter: Does it have two prime numbers as only factors (apart from 1 and himself)?

Answer: YES (11 and 23) → Exclude, since it is a semiprime number.

The number 253 must therefore be rejected as a prime number, as it fails the sieving test of the 2nd filter. Let us pass on to the number 257.

$$p = \sqrt{24n + 1}$$

$$257 = \sqrt{24n + 1}$$

$$257^2 = 24n + 1$$

$$66\,049 - 1 = 24n$$

$$66\,048 = 24n$$

$$\frac{64008}{24} = 2\,752$$

Square minus 1 of the number 257 is a multiple of 24, making it a candidate prime number. After verification, we know that this number satisfies the two conditions of our compliance test. We are therefore in the presence of a prime number.

Despite the apparent simplicity of our procedure, we must admit that it works wonderfully. It meets the third and fourth validation conditions as described above, as it has a simple and effective two-tier verification component to detect and eliminate false positives. Knowing the specific attributes of the intruders drastically reduces the number of operations and computational time required for a computer program to perform validation (i.e. confirm whether a number should be excluded or retained), since it is no longer mandatory to look for divisors outside this class of numbers and the collection of factors already in the base. In addition, if we postulate that the program starts its search from scratch, it will have built its database from the prime numbers that it has previously detected and stored in a bank during the process. In other words, this means that it will already have, in a memory module, the list of primary factors to be used to perform its sieving task and will therefore not need to be fed by the addition of new data, the meshes of its double exclusion filter having been woven by the information collected during the very operationalization of the procedure. In doing so, the number of factors that our engine is called upon to cross to generate its list of semiprime numbers is as limited as the sample prime numbers already part of its collection.

2. The characterization of twin primes: postulates and hypothesis

2.1 Postulates

As we know, we are in the presence of a pair of twin primes when they are separated by a single number. However, we do not know why this is so. What a great challenge it is to better understand the properties of twin primes and to find an organizational rule governing the distribution of these numbers. It is reasonable to assume that a model capable of generating the sequence of prime numbers could be useful for any other task concerning numbers belonging to the same category, such as that of establishing correspondences or identifying regularities between twin primes, or even that of predicting their sequence. Such a result would testify to its versatility and would go in the direction of its validation by generalization.

Thus, mainly by association between the squares of the pairs of twin numbers, diagrams appeared to us, inviting us to formulate with confidence a number of eleven postulates, which we propose to you below, preceded by an example of an already known postulate of the same ilk.

Example of a postulate: The difference between the respective sums of two pairs of twin primes (successive or not) is always a multiple of 12. [1]

Postulate 1) The difference between the product of the numbers of two pairs of twin primes (consecutive or not and excluding the pair (3, 5)) is always a multiple of 12.

Postulate 2) The sum of the squares minus 2 of a pair of prime numbers (twin or not, successive or not, and excluding the numbers 2 and 3) is always a multiple of the number 24. This property is not unique to twin primes, since it is true for any pair of primes, but we think it is relevant to mention it here, since it is fundamental to our model of prime numbers as a whole. Be careful: it is not said here that any multiple of the number 24 corresponds to the sum of the squares of a pair of twin primes minus 2. It's quite the opposite.

Postulate 3) The difference between the squares of a pair of prime numbers (twin or not / successive or not) is always a multiple of 24 (except for the pair 3/5, whose sum of squares is less than 24). As before, this property is not exclusive to twin primes, since it is true for any pair of prime numbers, but it is of major importance in the development of our model, given its link with the number 24.

Postulate 4) The difference between the sum of the squares of two pairs (successive or not) of twin primes is always a multiple of the number 24 (except for the difference between the pairs 3/5 and 5/7).

Postulate 5) The gap between the difference of the squares of the numbers of two pairs of twin primes (successive or not) is always a multiple of the number 24 (except for the difference between the pairs 3/5 and 5/7).

Postulate 6) The difference between the squares of a pair of twin primes, when divided by 24, is always a divisor of the sum minus 2 of their squares, divided by 24. In other words, the quotient obtained from dividing by 24 the sum minus 2 of the squares of two twin primes is always a multiple of the quotient obtained from dividing by 24 the difference between these same squares.

Postulate 7) The sum of the two numbers of a pair of twin primes, when taken as a divisor and the dividend is the square minus 1 of either of these same two numbers, will always result in an integer, the order of magnitude of which increases in proportion to the starting number.

Postulate 8) The sum of the squares minus 2 of two twin primes is always divisible by the sum of them.

Postulate 9) The separator number (*sn*) is the number between the two numbers in a pair of twin primes (*tp*). Differentiate between the square of the separator number (*sn*) of a pair of twin primes and the square of a number of two units lower. Differentiate between the square of a number of two units greater than the *sn* and the square of the *sn*. The sum of these two results is equal to 8*ns* (eight times the *sn*). This is true for any pair of numbers separated by two units. The peculiarity with the *sn* of a pair of *tp* numbers is that the addition of these two results is a multiple of 24, which corresponds to $24(sn/3)$.

Postulate 10) The difference between the squares of a pair of *tp* numbers is equal to their sum multiplied by two. True for any pair of numbers whose difference is two units. The peculiarity of a pair of *tp* numbers is that the result is a multiple of 24 whose multiplicative factor corresponds to their sum divided by 12 or the separator number (*sn*) divided by 6.

Postulate 11) The multiples of the square of number 24 (i.e. the multiples of 576) sometimes correspond to the difference between the sum of the squares of two pairs of twin primes (successive or not). As we will see below, this last postulate requires further analysis and understanding on our part to correctly and more generally reflect the arithmetic principle it underpins.

For the purposes of our analysis, we will ignore the postulates that are universal mathematical truths, i.e. that concern all pairs of integers whose difference is 2, whether they are prime or not. In such cases, they are of little use in discerning the specifics of twin primes. Appendix A provides some examples.

In order to facilitate the understanding of the calculations that will be used to demonstrate of our postulates, we offer you the following three data tables, which aim to paint a comparative portrait between pairs of prime numbers, with the objective of identifying patterns (constants and correspondences). This exploratory approach is the one we adopt and generally recommend in this study. It's only a small sample, but it helps us better visualize the numbers we'll be juggling and can serve as a starting point for those interested in analyzing twin primes through the lens of our postulates and model guidelines. The majority of our postulates are inspired by it.

Table 4: Twin primes and their squares (1)

Pairs of primes	Square of these numbers	Sum of squares	Difference between these sums	Diff. between squares	Gap between these differences
3 / 5	9 / 25	34	Nil	16 (2 x 8)	Nil
5 / 7	25 / 49	74 (3x24+2)	+40 (5 x 8)	24 (1 x 24)	+8 (1 x 8)
11 / 13	121 / 169	290 (12x24+2)	+216 (9 x 24)	48 (2 x 24)	+24 (1 x 24)
17 / 19	289 / 361	650 (27x24+2)	+360 (15 x 24)	72 (3 x 24)	+24 (1 x 24)
29 / 31	841 / 961	1,802 (75x24+2)	+1,152 (2 x 242)	120 (5 x 24)	+48 (2 x 24)
41 / 43	1,681 / 1,849	3,530 (147x24+2)	+1,728 (3 x 242)	168 (7 x 24)	+48 (2 x 24)
59 / 61	3,481 / 3,721	7,202 (300x24+2)	+3,672 (153 x 24)	240 (10 x 24)	+72 (3 x 24)
71 / 73	5,041 / 5,329	10,370 (432x24+2)	+3,168 (132 x 24)	288 (12 x 24)	+48 (2 x 24)
101 / 103	10,201 / 10,609	20,810 (867x24+2)	+10,440 (435 x 24)	408 (17 x 24)	+120 (5 x 24)
107 / 109	11,449 / 11,881	23,330 (972x24+2)	+2,520 (105 x 24)	432 (18 x 24)	+24 (1 x 24)
137 / 139	18,769 / 19,321	38,090 (1587x24+2)	+14,760 (615 x 24)	552 (23 x 24)	+120 (5 x 24)
149 / 151	22,201 / 22,801	45,002 (1875x24+2)	+6,912 (288 x 24)	600 (25 x 24)	+48 (2 x 24)

Table 5: Twin primes and their squares (2)

A Pairs of primes	B Sum of squares -2 ÷ 24	C Difference between squares ÷ 24	D C x *n* = B	E *n* = C x 3
3 / 5	32 ÷ 8 = 4	16	Nil	Nil
5 / 7	72 ÷ 24 = 3	24 ÷ 24 = 1	1 x 3 = 3	3 = 1 x 3
11 / 13	288 ÷ 24 = 12	48 ÷ 24 = 2	2 x 6 = 12	6 = 2 x 3
17 / 19	648 ÷ 24 = 27	72 ÷ 24 = 3	3 x 9 = 27	9 = 3 x 3
29 / 31	1,800 ÷ 24 = 75	120 ÷ 24 = 5	5 x 15 = 75	15 = 5 x 3
41 / 43	3,528 ÷ 24 = 147	168 ÷ 24 = 7	7 x 21 = 147	21 = 7 x 3
59 / 61	7,200 ÷ 24 = 300	240 ÷ 24 = 10	10 x 30 = 300	30 = 10 x 3
71 / 73	10,368 ÷ 24 = 432	288 ÷ 24 = 12	12 x 36 = 432	36 = 12 x 3
101 / 103	20,808 ÷ 24 = 867	408 ÷ 24 = 17	17 x 51 = 867	51 = 17 x 3
107 / 109	23,328 ÷ 24 = 972	432 ÷ 24 = 18	18 x 54 = 972	54 = 18 x 3
137 / 139	38,088 ÷ 24 = 1 587	552 ÷ 24 = 23	23 x 69 = 1,587	69 = 23 x 3
149 / 151	45,000 ÷ 24 = 1 875	600 ÷ 24 = 25	25 x 75 = 1875	75 = 25 x 3

Table 6: Twin primes and their squares (3)

A Pairs of primes	B Sum of these two numbers	C Squares of these two numbers – 1	D C ÷ B	E B ÷ 12 or 24	F Sum of C ÷ B
3 / 5	3 + 5 = 8	8 / 24	8 ÷ 8 = 1 24 ÷ 8 = 3	Nile	4 (32 ÷ 8)
5 / 7	5 + 7 = 12	24 / 48	24 ÷ 12 = 2 48 ÷ 12 = 4	12 ÷ 12 = 1	6
11 / 13	11 + 13 = 24	120 / 168	120 ÷ 24 = 5 168 ÷ 24 = 7	24 ÷ 12 = 2 24 ÷ 24 = 1	12
17 / 19	17 + 19 = 36	288 / 360	288 ÷ 36 = 8 360 ÷ 36 = 10	36 ÷ 12 = 3 36 ÷ 24 = 1.5	18
29 / 31	29 + 31 = 60	840 / 960	840 ÷ 60 = 14 960 ÷ 60 = 16	60 ÷ 12 = 5 60 ÷ 24 = 2.5	30
41 / 43	41 + 43 = 84	1,680 / 1,848	1,680 ÷ 84 = 20 1,848 ÷ 84 = 22	84 ÷ 12 = 7 84 ÷ 24 = 3.5	42

59 / 61	59 + 61 = 120	3,480 / 3,720	3,480 ÷ 120 = 29 3,720 ÷ 120 = 31	120 ÷ 12 = 10 120 ÷ 24 = 5	60
71 / 73	71 + 73 = 144	5,040 / 5,328	5,040 ÷ 144 = 35 5,328 ÷ 144 = 37	144 ÷ 12 = 12 144 ÷ 24 = 6	72
101 / 103	101 + 103 = 204	10,200 / 10,608	10,200 ÷,204 = 50 10,608 ÷,204 = 52	204 ÷ 12 = 17 204 ÷ 24 = 8.5	102
107 / 109	107 + 109 = 216	11,448 / 11,880	11,448 ÷,216 = 53 11,880 ÷ 216 = 55	216 ÷ 12 = 18 216 ÷ 24 = 9	108
137 / 139	137 + 139 = 276	18,768 / 19,320	18,768 ÷,276 = 68 19,320 ÷,276 = 70	276 ÷ 12 = 23 276 ÷ 24 = 11.5	138
149 / 151	149 + 151 = 300	22,200 / 22,800	22,200 ÷,300 = 74 22,800 ÷ 300 = 76	300 ÷ 12 = 25 300 ÷ 24 = 12.5	300

A few observations:

We have identified various correspondences between the numbers in our different tables, the arithmetic reason for which we do not know. Here are a few.

The number 288 is:
- The square minus 1 of the number 17.
- The difference between the squares of the twin numbers (71, 72).
- The sum of the squares minus 2 divided by 24 of the pair of twin numbers (11, 13).

The number 360 is:
- The square minus 1 of the number 19.
- The difference between the sum of the squares of the pairs of twin numbers (11, 13) and (17, 19).

The number 432 is:
- The difference between the squares of the pair of twin numbers (107, 109).
- The sum of the squares minus 2 divided by 24 of the pair of twin numbers (71, 73).

The number 1,680 is:
- The square minus 1 of the number 41.
- The difference between the squares of the pair of twin numbers (419, 421).

Here is now a detailed illustration of our list of ten postulates, with supporting formulas, preceded by an already known example of the same type.

Postulate as an example) The difference between the respective sums of two pairs of twin primes is always a multiple of 12.

Detailed example:

Pair (11, 13) Pair (29, 31)

11 + 13 = 24 29 + 31 = 60

60 – 24 = 46

46 ÷ 12 = 3

Formula:

(*tpa*, *tpb*) = A pair of twin primes > (*tpc*, *tpd*).

(*tpc*, *tpd*) = A pair of twin primes < (*tpa*, *tpb*).

n = Integer.

$$n = \frac{pja + pjb - pjc - pjd}{12}$$

Application of the formula:

(*tpa*, *tpb*) = (29, 31)

(*tpc*, *tpd*) = (11, 13)

$$n = \frac{29 + 31 - 11 - 13}{12}$$

$$n = \frac{60 - 24}{12}$$

$$n = \frac{36}{12}$$

$$n = 3$$

Postulate 1) The difference between the product of the numbers of two pairs of twin primes (consecutive or not and excluding the pair (3,5)) is always a multiple of 12.

Detailed examples:

Pair (11, 13)	Pair (5, 7)	Pair (29, 31)	Pair (17, 19)
11 x 13 = 143	5 x 7 = 35	29 x 31 = 899	17 x 19 = 323
143 - 35 = 108,108	÷ 12 = 9,899	– 323 = 576,576	÷ 12 = 48

Formulas:

(*tpa*, *tpb*) = A pair of twin primes > (*tpc*, *tpd*).
(*tpc*, *tpd*) = A pair of twin primes < (*tpa*, *tpb*).
n = Integer.

A) $n = \frac{tpa \times tpb - tpc \times tpd}{12}$

B) $12n = tpa \times tpb - tpc \times tpd$

Application of Formula A:

(*tpa*, *tpb*) = (29, 31)
(*tpc*, *tpd*) = (11, 13)

$$n = \frac{29 \times 31 - 11 \times 13}{12}$$

$$n = \frac{899 - 143}{12}$$

$$n = \frac{756}{12}$$

$$n = 63$$

Postulate 2) The sum of the squares minus 2 of a pair of prime numbers (twin or not, successive or not, and excluding the numbers 2 and 3) is always a multiple of the number 24.

Detailed examples:

Pair (11, 13)		Pair (23, 37)	
$11^2 = 121$	$13^2 = 169$	$23^2 = 529$	$37^2 = 1,369$
121 + 169 = 290		529 + 1,369 = 1,898	
290 – 2 = 288		1,898 – 2 = 1,896	
288 ÷ 24 = 12		1,896 ÷ 24 = 79	

The numbers 12 and 79 are integers, as expected.

Formulas:

tpa = Twin prime > *tpb*.

tpb = Twin prime < *tpa*.

n = Integer.

A) $n = \frac{tpa^2+tpb^2-2}{24}$ B) $24n + 2 = tpa^2 + tpb^2$

Application of our two formulas:

tpa = 107

tpb = 109

$$n = \frac{107^2 + 109^2 - 2}{24}$$

$$n = \frac{11\ 449 + 11\ 881 - 2}{24}$$

$$n = \frac{23\ 330 - 2}{24}$$

$$n = \frac{23\ 328}{24}$$

$$n = 972$$

* * *

$$24n + 2 = 107^2 + 109^2$$

$$24n + 2 = 11\ 449 + 11\ 881$$

$$24n + 2 = 23\ 330$$

$$24n = 23\ 330 - 2$$

$$24n = 23\ 328$$

$$n = \frac{23\ 328}{24}$$

$$\mathrm{n} = 972$$

The formulas show that dividing by 24 does indeed result in an integer.

Postulate 3) The difference between the squares of two prime numbers is always a multiple of 24.

Detailed example:

Pair (29, 31)

$29^2 = 841$ $31^2 = 961$

$961 – 841 = 120$

$120 ÷ 24 = 5$

Formulas:

(*tpa*, *tpb*) = A pair of twin primes.

n = Integer.

A) $n = \frac{tpb^2 - tpa^2}{24}$ B) $24n = tpb^2 - tpa^2$

Application of our two formulas:

tpa = 7

tpb = 5

$$n = \frac{7^2 - 5^2}{24}$$

$$n = \frac{49 - 25}{24}$$

$$n = \frac{24}{24} \qquad n = 1$$

* * *

$$24n = 7^2 - 5^2$$

$$24n = 49 - 25$$

$$24n = 24$$

$$n = \frac{24}{24} \qquad n = 1$$

Postulate 4) The difference between the sum of the squares of the numbers of two pairs of twin primes is always a multiple of the number 24.

Detailed examples:

Pair (17, 19)

172 = 289 192 = 361

289 + 361 = 650

Pair (41, 43)

412 = 1,681 432 = 1,849

1,849 + 1,681 = 3,530

3,530 – 650 = 2,880

2,880 ÷ 24 = 120

Formulas:

(*tpa*, *tpb*) = A pair of twin primes > (*tpc* , *tpd*).

(*tpc* , *tpd*) = A pair of twin primes < (*tpa*, *tpb*).

n = Integer.

B) $n = \frac{tpa^2+tpb^2-tpc^2-tpd^2}{24}$

B) $24n = tpa^2 + tpb^2 - tpc^2 - tpd^2$

Application of Formula A:

(*tpa*, *tpb*) = (29, 31)

(*tpc*, *tpd*) = (11, 13)

$$n = \frac{29^2 + 31^2 - 11^2 - 13^2}{24}$$

$$n = \frac{841 + 961 - 121 - 169}{24}$$

$$n = \frac{1\ 802 - 290}{24}$$

$$n = \frac{1\ 512}{24}$$

$$n = 63$$

The formula shows that dividing by 24 does indeed result in an integer.

Postulate 5) The gap between the difference of the squares of the numbers of two pairs of twin primes is always a multiple of the number 24.

Detailed examples:

Pair (59, 61)		Pair (101, 103)	
$59^2 = 3{,}481$	$61^2 = 3{,}721$	$101^2 = 10{,}201$	$103^2 = 10{,}609$
$3{,}721 - 3{,}481 = 240$		$10{,}609 - 10{,}201 = 408$	

$$408 - 240 = 168$$

$$168 \div 24 = 7$$

Formulas:

(*tpa*, *tpb*) = A pair of twin primes > (*tpc*, *tpd*).

(*tpc*, *tpd*) = A pair of twin primes < (*tpa*, *tpb*).

n = Integer.

A) $n = \frac{tpb^2 - tpa^2 - tpc^2 - tpd^2}{24}$ B) $24n = tpb^2 - tpa^2 - tpc^2 - tpd^2$

Application of formula A:

(*tpa*, *tpb*) = (29, 31)

(*tpc, tpd*) = (11, 13)

$$n = \frac{31^2 - 29^2 - 11^2 - 13^2}{24}$$

$$n = \frac{1\,849 - 1\,681 - 841 - 961}{24}$$

$$n = \frac{48}{24}$$

$$n = 2$$

The formula shows that dividing by 24 does indeed result in an integer.

Postulate 6) The difference between the squares of the numbers of a pair of twin primes, when divided by 24, is always a divisor of the sum minus 2 of their squares, divided by 24.

Detailed example:

Pair (41, 43)

$41^2 = 1{,}681$ $43^2 = 1{,}849$	$41^2 = 1{,}681$ $43^2 = 1{,}849$
1,681 + 1,849 = 3,530	1,849 – 1,681 = 168
3,530 – 2 = 3,528	168 ÷ 24 = 7
3,528 ÷ 24 = 147	

147 ÷ 7 = 21

Formula:

(*tpa*, *tpb*) = A pair of twin primes.

n = Integer.

$$n = \frac{(tpa^2 + tpb^2 - 2) \div 24}{tpb^2 - tpa^2 \div 24}$$

Application of the formula:

(*tpa*, *tpb*) = (41, 43)

$$n = \frac{(41^2 + 43^2 - 2) \div 24}{43^2 - 41^2 \div 24}$$

$$n = \frac{(1\ 681 + 1\ 849 - 2) \div 24}{1\ 849 - 1\ 681 \div 24}$$

$$n = \frac{(3\ 530 - 2) \div 24}{168 \div 24}$$

$$n = \frac{3\ 528 \div 24}{7}$$

$$n = \frac{147}{7}$$

$$n = 21$$

Postulate 7) The sum of the two numbers of a pair of twin primes, when taken as a divisor and the dividend is the square minus 1 of either of these two numbers, will always result in an integer, the order of magnitude of which increases in proportion to the starting number.

Detailed example:

Pair (29, 31)

$29 + 31 = 60$ | $29^2 = 841$ | $31^2 = 961$

$841 - 1 = 840$ | $961 - 1 = 960$

$840 \div 60 = 14$ | $960 \div 60 = 16$

Formula:

(*tpa*, *tpb*) = A pair of twin primes.

n = Integer.

$$n = \frac{tpa^2-1}{tpa+tpb} \qquad n = \frac{tpb^2-1}{tpa+tpb}$$

Application of the formula:

(*tpa*, *tpb*) = (29, 31)

$$n = \frac{29^2-1}{29+31} \qquad n = \frac{31^2-1}{29+31}$$

$$n = \frac{841-1}{60} \qquad n = \frac{961-1}{60}$$

$$n = \frac{840}{60} \qquad n = \frac{960}{60}$$

$$n = 14 \qquad n = 16$$

Postulate 8) The sum of the squares minus 2 of two twin primes is always divisible by the sum of them.

Detailed example:

Pair (29, 31)

$29 + 31 = 60$ $\qquad$ $29^2 = 841$ $\qquad$ $31^2 = 961$

$841 + 961 = 1{,}802$

$1{,}802 – 2 = 1{,}800$

$1{,}800 \div 60 = 30$

Formula:

(*tpa*, *tpb*) = A pair of twin primes.

n = Integer.

$$n = \frac{tpa^2 + tpb^2 - 2}{tpa + tpb}$$

Application of the formula:

(*tpa*, *tpb*) = (71, 73)

$$n = \frac{71^2 + 73^2 - 2}{71 + 73}$$

$$n = \frac{5\,041 + 5\,329 - 2}{144}$$

$$n = \frac{10\,370 - 2}{144}$$

$$n = \frac{10\,368}{144}$$

$$n = 72$$

Postulate 9) Calculate the difference between the square of the separator number (*sn*) of a pair of twin primes and the square of a number of two units lower. Calculate the difference between the square of a number of two units greater than the *sn* and the square of the *sn*. The addition of these two results is equal to 8*sn* and is a multiple of 24, which corresponds to 24(*sn*/3).

Detailed examples:

Pair (5, 7) *sn* = 6

$6^2 - 4^2 = 20$ $\quad$ $8^2 - 6^2 = 28$ $\quad$ $20 + 28 = 48$

$(48 = 8 \times 6 = 24 \times \frac{6}{3})$

Pair (11, 13) *sn* = 12

$12^2 - 10^2 = 44$ $\quad$ $14^2 - 12^2 = 52$ $\quad$ $44 + 52 = 96$

$(96 = 8 \times 12 = 24 \times \frac{12}{3})$

Formulas: (the procedure requires three formulas)

sn = Separator number of a pair of twin primes.

x = *sn* – 2.

y = *sn* + 2.

c = Multiple of 24 equal to 8*sn* and also to $24\,\frac{ns}{3}$.

$sn^2 - x^2 = a$ $\quad$ $y^2 - sn^2 = b$ $\quad$ $a + b = c$

Pair (17, 19)

$18^2 - 16^2 = 68$ $\quad$ $20^2 - 18^2 = 76$ $\quad$ $68 + 76 = 144$

$144 = 8 \times 18$ $\quad$ $144 = 24 \text{ x } \frac{18}{3}$

Postulate 10) The difference between the squares of a pair of *tp* numbers is equal to their sum multiplied by two. True for any pair of numbers whose difference is two units. The peculiarity of a pair of *tp* numbers is that the result is a multiple of 24 whose multiplicative factor corresponds to their sum divided by 12 or the separator number (*sn*) divided by 6.

Detailed examples:

Pair (4, 6)

4 + 6 = 10 / 2 x 10 = 20

4^2 = 16 / 6^2 = 36 / 36 – 16 = 20

Pair (17, 19)

17 + 19 = 36 / 2 x 36 = 72

17^2 = 289 / 19^2 = 361 / 361 – 289 = 72

(36 ÷ 12 = 18 ÷ 6 = 3 / 24 x 3 = 72)

Pair (29, 31)

29 + 31 = 60 / 2 x 60 = 120

29^2 = 841 / 31^2 = 961 / 961 – 841 = 120

(60 ÷ 12 = 30 ÷ 6 = 5 / 24 x 5 = 120)

Formulas: (the procedure requires several operations)

x = Twin prime = $y - 2$.

y = Twin prime = $x + 2$.

sn = Separator number of the twin prime pair.

$x + y = a \quad 2a = b \quad x^2 = c \quad y^2 = d \quad d - c = b$

$\frac{a}{12} = \frac{sn}{6} = e \quad 24e = b$

Thus, the sum of two numbers in a pair of *tp* multiplied by 2 is equal to the difference between their squares, which is 24 times one-twelfth of their sum or 24 times one-sixth of the *sn*.

Postulate 11) The multiples of the square of number 24 (i.e. the multiples of 576) sometimes correspond to the difference between the sum of the squares of two pairs of twin primes (successive or not).

We found three such cases:

1) 1 x $24^2 = 17^2 + 19^2 - 5^2 - 7^2$
2) 2 x $24^2 = 29^2 + 31^2 - 17^2 - 19^2$
3) 3 x $24^2 = 41^2 + 43^2 - 29^2 - 31^2$

Detailed examples:

Pair (29, 31)		Pair (41, 43)	
29^2 = 841	31^2 = 961	41^2 = 1,681	43^2 = 1,849
841 + 961 = 1,802		1,681 + 1,849 = 3,530	

3,530 – 1,802 = 1,728

1,728 ÷ 576 = 3

Formulas :

(*tpa*, *tpb*) = A pair of twin primes > (*tpc*, *tpd*).

(*tpc*, *tpd*) = A pair of twin primes < (*tpa*, *tpb*).

n = Integer.

A) $n = \frac{tpa^2+tpb^2-tpc^2-tpd^2}{24^2}$ B) $24^2n = tpa^2 + tpb^2 - tpc^2 - tpd^2$

Application of formula B :

(*tpa, tpb*) = (41, 43)

(*tpc, tpd*) = (17, 19)

$$24^2n = 41^2 + 43^2 - 17^2 - 19^2$$

$$576n = 1{,}681 + 1{,}849 - 289 - 361$$

$$576n = 3{,}530 - 650$$

$$576n = 2{,}880$$

$$n = \frac{2{,}880}{576}$$

$$n = 5$$

This last postulate opens many other arithmetic possibilities based on the same principle, leaving us to assume that we have not fully grasped and defined the mathematical phenomenon we are trying to describe. For example, simple calculations show that in some cases we can get the same kind of equality by subtracting the square from an additional prime.

For example:

$4 \times 24^2 = 71^2 + 73^2 - 59^2 - 61^2 - \mathbf{29^2}$

We see that, to get 2,304 (i.e. 4 x 576), a fifth prime number must join the party, adding its square to the other two numbers to be subtracted. This means that many other combinations are possible and that it would be enough to perform a few calculations to find some new ones. Therefore, one of the questions that comes to mind is whether there is always a combination of numbers – no matter how many terms there are – that allows us to obtain each of the multiples of 24 by a series of subtraction operations – or a combination of subtractions and additions – these numbers being the squares of twin primes, or if not the squares of other non-twin primes.

For example:

$5 \times 24^2 = 109^2 - 73^2 - 61^2 + \mathbf{7^2}$

Here, we accept to be more flexible by permitting the use of addition, but the more the conditions are loosened, the more the possible number of operations increases, which at the same time makes the postulate less interesting and the task would then be to subdivide it into a series of more specific postulates. It seems likely that we can obtain all multiples of 24 if the constraints are insufficient, which would encourage us to tighten them. In any case, the problem posed is fascinating and deserves further investigation.

To conclude this section on the postulates about twin prime pairs, let's say that they pave the way for more advanced explorations. It would be desirable to obtain some form of validation or confirmation of our postulates through the production of several thousand cases without error (i.e. without exception or omission), but we consider them to be mathematical evidences.

2.2 Assumptions about arithmetic relationships between twin primes, primes, semiprimes, separator numbers, shoulder numbers, and false positives

As a complement to this section on twin primes (*tp*), we formulate eleven hypotheses that aim to examine a possible relationship between them and:

1. the set of prime numbers;
2. semiprimes;
3. the numbers in the middle of a pair of *tp*, which we call "separator numbers" (*sn*);
4. the numbers on each side, which we will call "epaulette numbers" (*en*);
5. False positive (*fp*) numbers from our algorithm, which essentially corresponds to a subset of semiprimes.
6. certain multiplicative factors associated with number 24.

Hypothesis 1:

The separator number (*sn*) is the number positioned between the two numbers of a pair of twin primes (*tp*). It can also be considered as the average of the two numbers of a *tp* pair. The corresponding sequence is listed in *The Online Encyclopedia of Integer Sequences* (OEIS) [2] under number A014574 (https://oeis.org/A014574). We know that the *sn* is always a multiple of 6. The only exception to this universal rule concerns the first of them, the number 4, which is less than 6 and therefore cannot be a multiple.

Our hypothesis expressed as a rule is as follows:

The *sn* of a *tp* pair is always equal to the sum of two other lower *sn* or equal to the same doubled *sn*. Below are some examples to illustrate this concept.

6 + 6 = 12   6 + 12 = 18   12 +18 = 30   12 + 30 = 42   30 + 30 = 60   12 + 60 = 72   30 + 72 = 102

6 + 102 = 108 (or 48 + 60 = 108)   30 + 108 = 138   42 + 108 = 150   30 + 150 = 180

12 + 180 = 192 (or 42 + 150)   18 + 180 = 198 (or 60 + 138)   30 + 198 = 228

42 + 198 = 240 (or 60 + 180 = 240)   30 + 240 = 270   12 + 270 = 282 (or 42 + 240 = 282)

30 + 282 = 312   108 + 240 = 348 (or 150 + 198 = 348)   ETC.

There are many other possible combinations, but the idea is always the same: the *sn* of all *tp* pairs, without exception, can always be obtained by adding two *sn* belonging to one or two lower pairs of *tp* numbers. Since all these numbers are multiples of 6, it is likely that this correspondence will be observed indefinitely. In the absence of evidence, it seems reasonable to us to consider this hypothesis as a new conjecture.

Hypothesis 2:

When we look at the sequence of twin prime pairs, the problem we face is the large amount of "false positives", i.e. all multiples of 6 that are not separator numbers (*sn*). We therefore focused on the analysis of the particular properties of numbers that frame a multiple of 6 that is not a separator of twin primes, in order to identify constants and extract a heuristic rule. We have come to the following conclusion, expressed in the form of a hypothesis:

All multiples of 6 that are not separator numbers of twin prime pairs are:

A) either numbers whose unit is 4 or 6 (therefore, it is impossible for them to find two prime numbers next to them, since multiples of 5 are never primes, which excludes them by default);

B) either enclosed by one or more of those numbers or a combination of them:

-a prime number (only one, before or after, otherwise it would be a pair of twin primes);

-a semiprime number;

-a power of 3 or more of a prime number.

The next table illustrates this phenomenon. It lists the first 23 multiples of 6 that are not *sn* (i.e. that are not framed by a pair of twin primes). Potential twin primes (*ptp*) appear in bold. These are those associated with a multiple of 6 whose unit digit is either 0, 2 or 8 and which are therefore candidates to be prime.

Table 7: Multiples of 6 from 0 to 222 that are not framed by a pair of twin primes

24 = 4 or 6 to units (23 = first) + (25 = semiprime)

36 = 4 or 6 to units (37 = prime) + (35 = semiprime)

48 = (47 = prime) + (49 = semiprime)

54 = 4 or 6 to units (53 = prime) + (55 = semiprime)

66 = 4 or 6 units (67 = prime) + (65 = semiprime)

78 = (79 = prime) + (77 = semiprime)

84 = 4 or 6 to units (83 = first) + (85 = semiprime)

90 = (89 = prime) + (91 = semiprime)

96 = 4 or 6 to units (97 = prime) + (95 = semiprime)

114 = 4 or 6 to units (113 = prime) + (115 = semiprime)

120 = (119 + 121 = semiprime)

126 = 4 or 6 to units (127 = prime) + (125 = 5^3)

132 = (131 = prime) + (133 = semiprime)

144 = 4 or 6 to units (143 + 145 = semiprime)

156 = 4 or 6 to units (157 = prime) + (155 = semiprime)

162 = (163 = prime) + (161 = semiprime)

168 = (167 = prime) + (169 = semiprime)

174 = 4 or 6 to units (173 = prime) + (175 = composite)

186 = 4 or 6 to units (185 + 187 = semiprime)

204 = 4 or 6 to units (203 + 205 = semiprimes)

210 = (211 = prime) + (209 = semiprime)

216 = (215 + 217 = semiprimes)

222 = (223 = prime) + (221 = semiprime)

As we can see, there is no composite number on either side of a multiple of 6 ending in 0, 2, or 8 in the set of pairs that are not pairs of twin primes, except in extremely rare and special cases, i.e. when the composite number is a power of a prime equal to or greater than 3. For example, for the pair of potential twin primes 341-343, located on either side of a multiple of 6 (342), we observe that 341 is a semiprime number and that 343 is the product of 7^3, which seems to reproduce the same pattern we observed earlier for the detection of prime numbers.

To summarize the matter in a kind of rule of thumb, we argue that:

All pairs of numbers that enclose a multiple of 6 (not ending in 4 or 6) are twin prime pairs, unless either number of these pairs is:

- a semiprime number;
-a power of a prime number.

Otherwise, both numbers are doomed to be prime numbers, and so they are twin primes.

We can therefore propose an expeditious heuristic method to obtain a list of twin prime pairs. It somewhat echoes the famous sieve of Eratosthenes.

Method:

1. List multiples of 6 from x to y.
2. Cross out numbers whose unit is 4 or 6. The remaining numbers are candidate separator numbers (*sn*) of pairs of twin primes (*tp*). In other words, it provides the list of potential twin prime pairs (*ptp*).
3. List the semiprime numbers and powers of prime numbers up to a maximum height of y, and then cross out the pairs of *ptp* with either of these numbers. Obviously, the semiprime numbers we are talking about here are never multiples or powers of the numbers 2 (since they are odd) or 5 (since no multiple of 5 can be prime). The remaining pairs are all "by default" pairs of twin primes, without exception.

As a demonstration, let's take the first 20 multiples of 6.

Step 1) List the first 20 multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108, 114, 120.

Step 2a) Identify multiples of 6 whose unit is a 4 or a 6, then eliminate the corresponding pairs (the number 6 is of course an exception, since the number 5 is prime).

Multiples of 6 to be subtracted: 24, 36, 54, 66, 84, 96, 114.

Pairs eliminated (7/20): (23, 25), (35, 37), (53, 55), (65, 67), (83, 85), (95, 97), (113, 115).

Step 2b) Form pairs with the numbers on either side of the conserved multiples of 6. So, we are left with 13 pairs of potential twin primes (*ptp*).

Multiples of 6 preserved: 6, 12, 18, 30, 42, 48, 60, 72, 78, 90, 102, 108, 120.

Preserved pairs (*ptp*) (13/20): (5, 7), (11, 13), (17, 19), (29, 31), (41, 43), (47, 49), (59, 61), (71, 73), (77, 79), (89, 91), (101, 103), (107, 109), (119, 121).

Step 3) List the semiprime numbers and powers of prime numbers up to 121:

p^2: $3^2 =$ **9** / $7^2 =$ **49** / $11^2 =$ **121** / $3^3 =$ **27** / $3^4 =$ **81**

p* x *p: 3 x 7 = **21** / 3 x 11 = **33** / 3 x 13 = **39** / 3 x 17 = **51** / 3 x 19 = **57** / 3 x 23 = **69** /
3 x 29 = **87** / 3 x 31 = **93** / 3 x 37 = **111** / 7 x 11 = **77** / 7 x 13 = **91** / 7 x 17 = **119**

Step 4) Cross out the pairs with either of these numbers. The remaining pairs are pairs of twin primes, every time!

Pairs eliminated (4/13): (47, **49**), (**77**, 79), (89, **91**), (**119**, **121**)

Retained Pairs (9/13 or 9/20 total): (5, 7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61), (71, 73), (101, 103) / (107, 109)

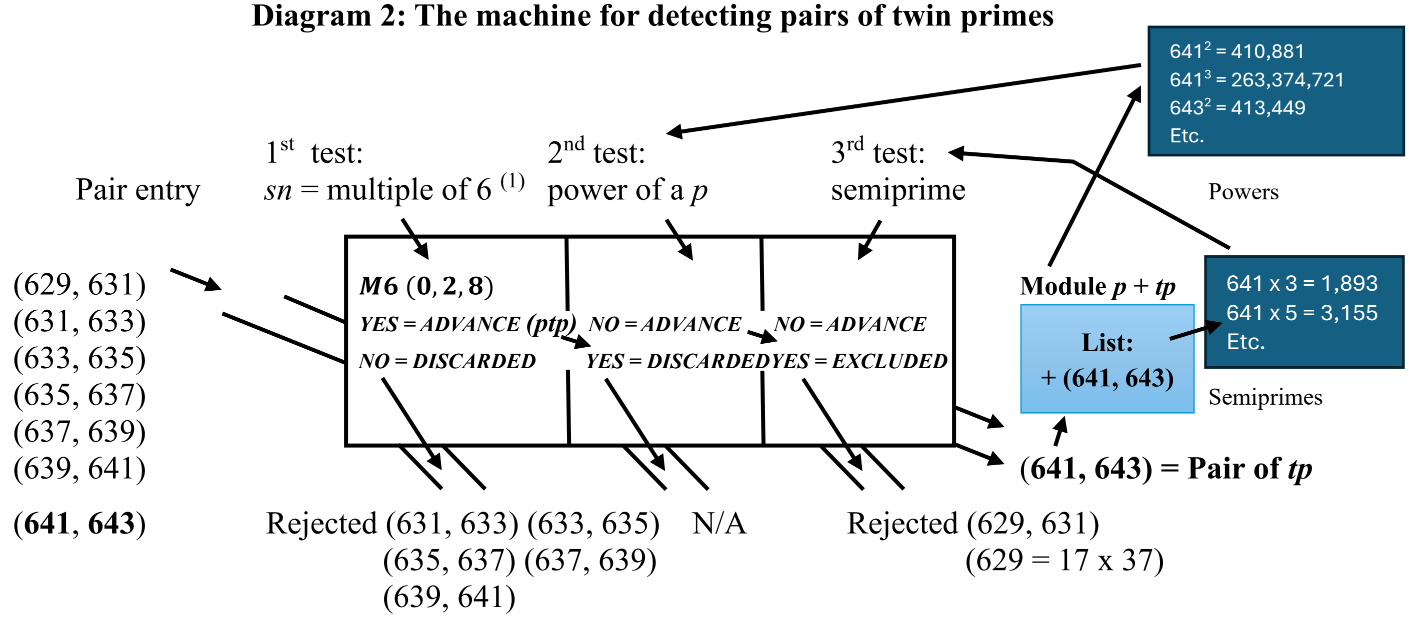
The temptation is strong to translate these statements into an algebraic formula whose function would be to find or identify pairs of twin primes, which could look like this:

n = Multiple of 6 having a 0, a 2 or an 8 at the position of the units.
p = Prime number.
sp = Semiprime number.
tp = Twin prime.
x = Superscript.

Formula: If $n - 1$ or $n + 1 \neq p^x$ or sp Then $n + 1$ and $n - 1 = tp$

Graphically, our model could be similar to our prime numbers detection machine (see Diagram 1) and even be functionally and operationally grafted onto it via the memory module, as shown in the following diagram.

Diagram 2: The machine for detecting pairs of twin primes



(1) Pairs must have a multiple of 6 in the middle, the units of which are 0, 2 or 8.

The memory module, whose initial function was to serve as a database for the storage of prime numbers following the identification procedure, proves its usefulness a second time by contributing to the validation of pairs of twin primes. Better still, we do not need to perform any verification or calculation on the remaining numbers to validate their status as twin primes, since they are arithmetically condemned to be twins.

The mechanism we use here to validate pairs of twin primes is based on the essential properties of our numeral system, which is articulated according to the multiplication of prime numbers and their "derivatives" in the form of semiprimes or powers of prime numbers. This principle of construction is responsible for the whole architecture and governs the distribution of pairs of twin primes. This brings us back to the universal arithmetic principle that prime numbers are the *building blocks* of our numeral system, moreover since they are the ones that spread semiprime numbers and prime powers along the continuum of natural numbers, thus producing the invisible matrix at the origin of our model.

Hypothesis 3:

In the continuum of positive integers, the first term of a pair of twin primes is always preceded by an integer that is less than 1 and the second term is always followed by an integer that is greater than 1. We are talking here about "epaulette numbers" (*en*).

We hypothesize that there are infinitely many pairs of twin primes enclosed by *nes*, which are semiprime numbers. Moreover, these *semiprime sn* seem to be systematically the result of the doubling of each of the numbers belonging to another pair of *tp*, which represents a spectacular singularity. We choose to call these pairs of arithmetically related twin primes – i.e. whose *en* of one are twice the prime numbers of the other – the pairs of "Sophia twin primes" (*stp*).

When we noticed the presence of semiprime numbers on either side of certain *tp* pairs, we tried to identify their origin. We therefore analyzed their divisors, which allowed us to find that they were the double of prime numbers belonging to another smaller pair of *tp*. We assume that this spectacular and unexpected arithmetic phenomenon is associated with the very nature of the construction of the numerical web, with semiprime numbers playing a more fundamental role than anticipated.

Here are a few examples. The epaulettes numbers (*en*) are in bold. The "donor pair" corresponds to the number multiplied by two and the "recipient pair" to the one framed by the epaulette numbers. Note that, apart from the pair (11, 13), the donor and recipient pairs must necessarily have the digits 9 and 1 as their respective units, in this precise order, for this doubling operation to be possible, since no other combination of *tp* pairs offers this compatibility. This criterion of rigor has the advantage of making it easier to locate them.

Donor pair:	(5, 7)	5 x 2 = **10**	7 x 2 = **14**
Recipient pair:	(11, 13)	**10**, 11, 12, 13, **14**	
Donor pair:	(29, 31)	29 x 2 = **58**	31 x 2 = **62**
Receiving pair:	(59, 61)	**58**, 59, 60, 61, **62**	
Donor pair:	(659, 661)	659 x 2 = **1,318**	661 x 2 = **1,322**
Recipient pair:	(1,319, 1,321)	**1,318**, 1,319, 1,320, 1,321, **1,322**	
Donor pair:	(809, 811)	809 x 2 = **1,618**	811 x 2 = **1,622**
Recipient pair:	(1,619, 1,621)	**1,618**, 1,619, 1,620, 1,621, **1,622**	
Donor pair:	(2,129, 2,131)	2,129 x 2 = **4,258**	2,131 x 2 = **4,262**
Recipient pair:	(4,259, 4,261)	**4,258**, , 4,259, 4,260, 4,261, **4,262**	
Donor pair:	(2,549, 2,551)	2,549 x 2 = **5,098**	2,551 x 2 = **5,102**
Recipient pair:	(5,099, 5,101)	**5,098**, 5,099, 5,100, 5,101, **5,102**	

Another way to represent the *stp* pairs is to target the separator number (*sn*) between the two pairs concerned and to double that of the lower pair, which is mathematically more simply expressed. The formulas used to define pairs of Sophia twin primes could therefore be as follows:

stpa = The two numbers of the "donor pair" of Sophia twin primes.

stpb = The two numbers of the "recipient pair" of Sophia twin primes.

sn = Separator number of the indicated pair.

S = Sum of the two numbers in the indicated pair.

<u>Formulas</u>:

$$Sstpa = \frac{Sstpb}{2}$$

or

$$2snstpa = \frac{snstpb}{2}$$

We can hypothesize that a third pair is grafted onto the other two, which would be twice the second pair, so that the *stpb* is the basis of the calculation to obtain a *stpc* larger than itself. We call "**triple** pairs of Sophia twin primes" the three pairs of *tp* numbers which, exceptionally, would be linked by this arithmetic relationship. The following equations reflect the situation where the three pairs of Sophia twin primes – i.e. *stpa*, *stpb,* and *stpc* –are triple *stp* pairs.

Formulas:

$$Sstpa = \frac{Sstpb}{2} = \frac{Sstpc}{4}$$

or

$$2nsstpa = \frac{nsstpb}{2} = \frac{nsstpc}{4}$$

Therefore, two questions must be asked of us:

1) Are there infinite pairs of *stp*?
2) Are there quadruple or even quintuple pairs of *stp* (etc.)?

We hypothesize that there are infinitely many pairs of *stp*. We also assume the existence of triple *stp*, although we have not yet identified any, as we do not have the necessary technological means at our disposal; this is more the job of professional mathematicians. We consider the enigmatic phenomenon of Sophia twin prime pairs to be mathematical evidence whose arithmetic mechanisms must be decoded and the meaning understood. All of these questions represent a conjecture requiring validation and we invite the community of researchers to take an interest in it.

Hypothesis 4:

We saw earlier that our initial algorithm – the formula $p = \sqrt{24n + 1}$ – accepts a number of false positives, which our procedure rejects as a result of a sieving. We investigated whether there could be a correlation between the number of false positive (*fp*) and twin primes (*tp*). We observed the gap between the squares of a pair of *tp* numbers and counted the number of *fp* within this interval, in order to identify some correspondence. The results obtained argue in favor of a more in-depth analysis than the one we have performed, because they lead us to assume the existence of a correlation between the number of *fp* contained in the range of squares of *tp* numbers – in the form of an increasing numerical sequence – and the distribution of the pairs of *tp*.

Our hypothesis is as follows: the number of false positive primes located between the squares of a pair of twin primes represents a term that is part of a logical sequence. For example, there are:

0 *fp* number between the squares of 3 (9) and 5 (25) not inclusive.

0 *fp* number between the squares of 5 (25) and 7 (49) not inclusive.

3 *fp* numbers between the squares of 11 (121) and 13 (169) not inclusive (i.e. 133, 143, 161).

7 *fp* numbers between squares of 17 (289) and 19 (361) not inclusive.

13 *fp* numbers between the squares of 29 (841) and 31 (961) not inclusive.

21 *fp* numbers between squares of 41 (1,681) and 43 (1,849) not inclusive.

31 *fp* numbers between squares of 59 (3,481) and 61 (3,721) not inclusive.

46 *fp* numbers between squares of 71 (5,041) and 73 (5,329) not inclusive.

66 *fp* numbers between the squares of 101 (10,201) and 103 (10,609) not inclusive.

Note (1) By "not inclusively", we mean that the squares of twin primes, which also are false positives, are excluded from the calculation. We are only counting the numbers situated in between their squares.

Note (2) This does not include even numbers or numbers that are multiples of 5 (such as 25), since our validation procedure does not accept them from the outset.

This gives us the following numerical sequence:

0, 3, 7, 13, 21, 31, 46, 66,…

The gaps (jumps) between the items can be used to generate another numerical sequence: +4, +6, +8, +10, +15, +20. The new sequence corresponds to certain sequences archived by the OEIS, including the following two:

A) Sequence number A211856 (https://oeis.org/A211856): Number of representations of *n* as the sum of products of distinct pairs of positive integers, considered equivalent when terms or factors are rearranged.

1, 1, 2, 3, **4**, **6**, **8**, **10**, **15**, **20**, 24, 34, 46, 58, 76, 97,…

B) Sequence number A238876 (https://oeis.org/A238876): Sub-diagonal growth partitions.

1, 1, 1, 2, 3, **4, 6, 8, 10, 15**, **20**, 25, 34, 44, 56, 74, 94,…

How can this help us? In our view, these progressions could serve as an indicator of the organizational logic of twin primes, if not as a method of predicting the next pair of twin primes.

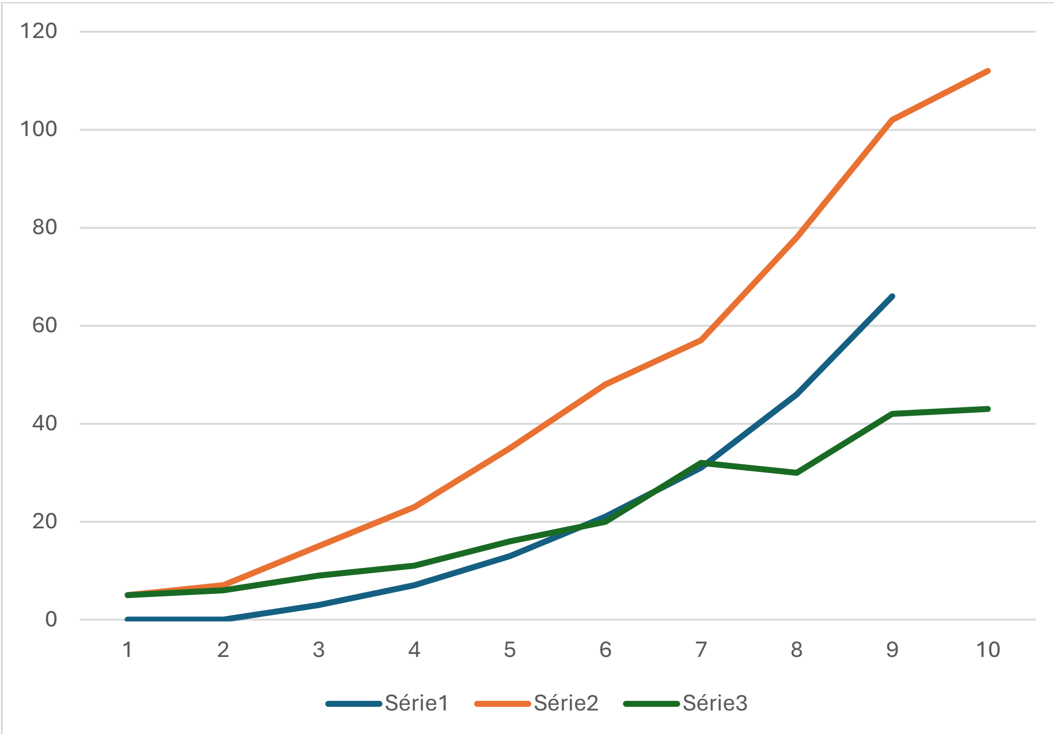
For example, given that there is a number of 31 *fp* between the squares of the numbers of the *tp* pair (59, 61), and that the next term in the sequence is 15 greater than that number, we can determine that the difference between the squares of the next pair of *tp* (71, 73) will be 46 *fp* (because 31 + 15 = 46). Thus, when our model detects the prime number following the pair (59, 61), it will be able to rule that it is a *tp* if and only if:

A) this prime number does not end with the number 3 (since the 2nd term of the pair cannot be a multiple of 5);
B) an exact number of 46 *fp* lies between its square and the square of the next prime number, which in such a case will be its twin.

If not, we move on to the next prime number and we satisfy the same two conditions. After rejecting 67, the next prime number is 71. It does not end in 3 and the number of false positives between its square and that of the next prime number (73) is 46. It is therefore a pair of twin primes (71, 73).

The observed correspondence is intriguing, to say the least: the number of false positives (*fp*) found in the interval between the squares of a pair of twin primes seems to follow an arithmetic progression akin to a logical sequence. From an arithmetic point of view, we could consider these numbers resulting from the operationalization of our algorithm as also being semiprime numbers, since they are practically a subset of them (except for numbers resulting from an exponent equal to or greater than 3 and which are therefore not *sp*), inviting us to wonder about a possible organizing function of semiprimes in the distribution of twin primes, acting as milestones or landmarks (bounds or reference points). But on closer examination of this correspondence, we discover that the number of *fp* and the set of *sp* do not give rise to the same progression curve, as shown in the following diagram:

Diagram 3: Comparison between the number of false positives, semiprimes and primes situated between the squares of a twin prime pair



1 = Between the squares of the 1st pair (3, 5) 2 = Between the squares of the 2nd pair (5, 7) Etc.

Series 1: False positive numbers
Series 2: Semiprime numbers (1)
Series 3: Prime Numbers (2)

(1) See sequence number A391619 of the OEIS (https://oeis.org/draft/A391619).
(2) See sequence number A137859 of the OEIS (https://oeis.org/A137859).

The curve corresponding to all semiprime numbers suggests relatively constant progression, while that of prime numbers is quite irregular. But the real surprise comes from the curve from the false positive numbers, which has a constant, almost perfectly regular pace. Unfortunately, this illusion dissipates when we push our analysis further. Indeed, we would have to obtain a number of 90 false positives between the squares of the next pair of twin primes, i.e. (107, 109), for the progression to be maintained, whereas there are only 72, which seems to invalidate our hypothesis, as shown in the following two diagrams.

Diagram 4: False positive curve with a number of 90 items (Series 1)

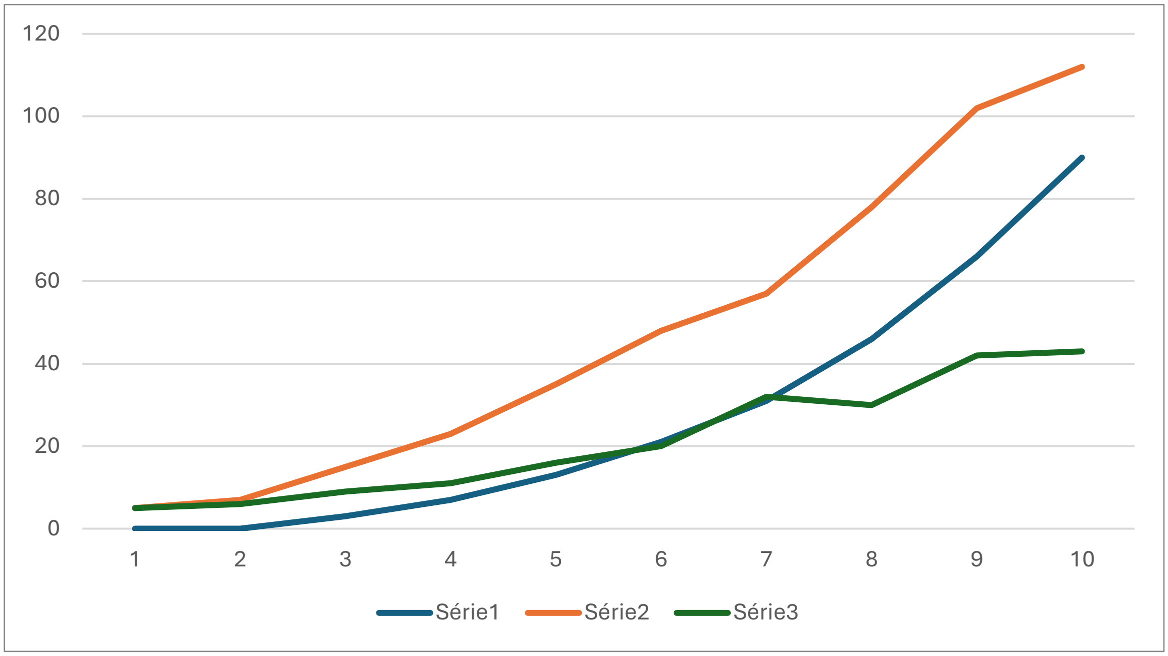
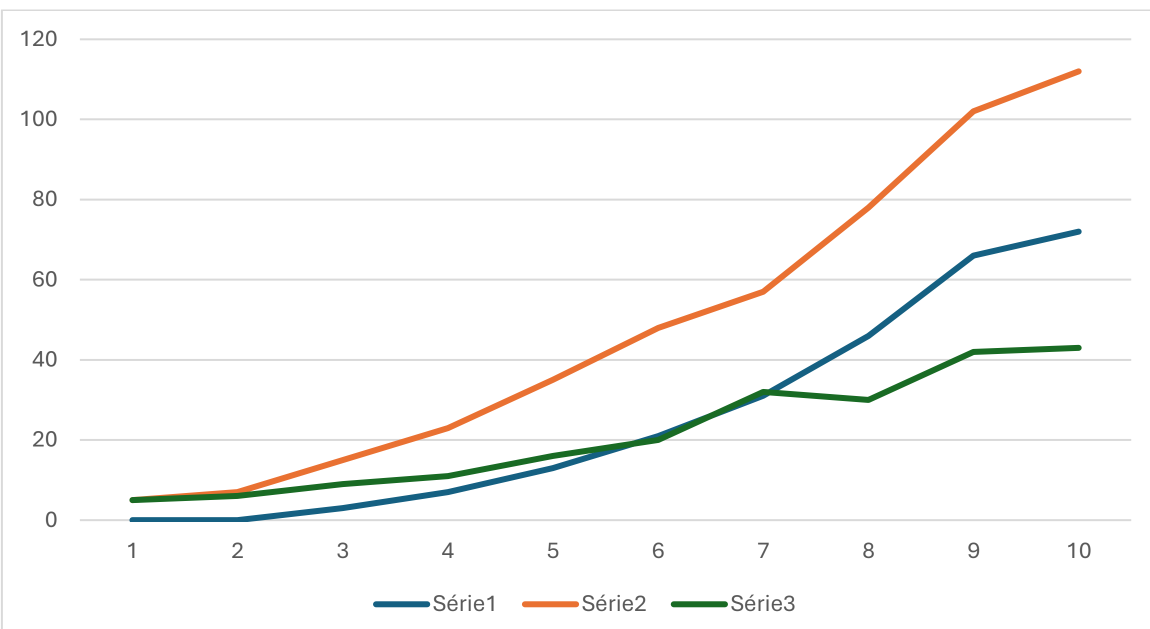


Diagram 5: False positive curve with a number of 72 items (Series 1)



It must be noted that the curve associating the false positive numbers with the squares of the twin primes does not maintain the expected regularity when we add to it the number of false positives that lie between the squares of the pair (107, 109), i.e. 72; it is rather nose-diving like the other two. This hypothesis therefore seems to be invalidated by these latest results.

Hypothesis 5:

The conjecture of the infinity of pairs of twin primes is a central research theme in the field of number theory. Our observations lead us to approach this question in these terms: if the semiprime numbers and the powers of the prime numbers are not enough to fill all the squares located on each side of the multiples of 6 (i.e. if this is arithmetically impossible), we must conclude that there are an infinite number of *tp* pairs, since there are infinitely many multiples of 6.

Considering the foregoing results and assuming that they are true, we must ask ourselves whether they represent mathematical proof for the conjecture concerning the infinity of pairs of twin primes or whether they at least allow us to envisage its elaboration. The question could be posed in the following terms: are there enough semiprimes and powers of prime numbers to fill all the squares of the pairs of potential twin primes? If this is arithmetically impossible, then we have proven that the *tp* pairs are infinite. Here is a table that can enlighten us on this matter.

Table 8: Semiprimes, multiples of 6 and twin primes per 100

sp = Semiprime numbers.
M6 = Multiples of 6.
ptp = Pairs of potential twin primes (i.e. multiples of 6 ending in a 0, a 2 or an 8 to units).
tp = Pairs of twin primes.

Sample	**Number of *sp***	**Number of odd *sp***	**Number of M6**	**Number of pairs = *ptp***	**Number of pairs = *tp***	**Number of pairs ≠ *tp***
1-100	34	19	16	9	8	1
101-200	28	18	17	11	7	4
201-300	32	22	17	10	4	6
301-400	32	22	16	9	2	7
401-500	27	20	17	11	3	8
501-600	33	24	17	10	3 (599-601)	7
601-700	26	18	16	9	3	6
701-800	30	22	17	11	0	11
801-900	28	20	17	10	5	5
901-1000	29	21	16	9	0	9
TOT	299	206	166	99	35	64

This table shows quite eloquently that, when we consider pairs of potential twin primes (*ptp*), the number of true pairs of *tp* numbers is systematically less than the number of pairs that are not pairs from the third hundred. Only the first two brackets (1-100 and 101-200) have a higher number of pairs of *tp* numbers than non-pairs (8 vs. 1 and 7 vs. 4). All other hundred increments show a lower ratio. This tells us, amongst all pairs of *ptp*, a scarcity in the rate of pairs of *tp* and an increase in the ratio of pairs that are not (i.e. that are not composed of two prime numbers).

It remains to be seen whether, at a certain point, the semiprime numbers and the powers of prime numbers occupy all the squares that could potentially belong to a pair of *tp* numbers. If not, we could have some sort of proof of the infinity of twin prime pairs. It is precisely this question that we will address in an upcoming article and to which we hope to provide a satisfactory answer, based on the well-established mathematical fact that the number of semiprimes gradually decreases – albeit irregularly – on the continuum of integers.

Hypothesis 6:

As we know, the separator number (*sn*) is the number between the two numbers in a pair of twin primes (*tp*). In our explorations, we observed that subtracting any prime number (*p*) greater than 3 from any *sn* equal to or greater than 6 always results in another *p*, excluding some well-defined special cases. In the form of a rule, our hypothesis could be read as follows:

> Subtracting from a ***sn*** equal to or greater than 6 any ***p*** equal to or greater than 5 always gives a ***p***, unless the result :
>
> - has the number 5 as its unit (excluding the number 5 itself);
> - is the power of a *p*;
> - is 1.

Increasing series of the first 9 consecutive *sn* (excluding the 4): 6, 12, 18, 30, 42, 60, 72, 102, 108,…

Subtraction operation between the 9 *sn* of our list and all the *p* that are numerically lower than them:

6-3=3 / 6-5=1 / 12-3=9 / 12-5=7 / 12-7=5 / 18-3=15 / 18-5=13 / 18-7=11 / 18-11=7 / 18-13=5 / 18-17=1 / 30-3=27 / 30-5=5 / 30-7=23 / 30-11=19 / 30-13=17 / 30-17=13 / 30-19=11 / 30-23=7 / 30-29=1 / 42-3=39 / 42-5=37 / 42-7=35 / etc.

The same demonstration using a pair of *tp* located several hundred years away:

(641, 643) / ns = 642 / 642-641=1 / 642-631=11 / 642-619=23 / 642-617=25 / 642-613=29 / 642-607=35 / 642-601=41 / 642-599=43 / 642-593=49 / 642-587=55 / Etc.

Formula:

p = Prime number.

tp = Twin prime.

$$\frac{tp + tp}{2} - p = p$$

Example:

If tp pair = (29, 31)

If sn = 23

$$p = \frac{29 + 31}{2} - 23$$

$$p = \frac{60}{2} - 23$$

$$p = 30 - 23$$

$$p = 7$$

Hypothesis 7:

It is always possible to obtain a prime number as a result by differentiating between a multiple of 24 and the square of a prime number that is less than it.

Here are a series of examples to explain this idea. Prime numbers resulting from subtracting between a multiple of 24 and the square of a prime number are in bold type. The first subtraction of a series is always made from the highest square (i.e. the one immediately below the multiple of 24).

Examples:

48 – 25 = **23**	72 – 49 = **23**	96 – 49 = **47**	120 – 49 = **71**
144 – 121 = **23**	168 – 121 = **47**	192 – 169 = **23**	216 – 169 = **47**
240 – 169 = **71**	264 – 169 = 95	288 – 169 = 119	312 – 289 = **23**
	(264 – 121 = 143)	(288 – 121 = **167**)	
	(264 – 49 = 215)		
	(264 – 25 = **239**)		
336 – 289 = **47**	360 – 289 = **71**	384 – 361 = **23**	408 – 361 = **47**
432 – 361 = **71**	456 – 361 = 95	480 – 361 = 119	504 – 361 = 143
	(456 – 289 = **167**)	(480 – 289 = **191**)	(504 – 289 = 215)
			(504 – 169 = 335)
			(504 – 121 = **383**)

528 – 361 = **167**	552 – 529 = **23**	576 – 529 = **47**	600 – 529 = **71**
624 – 529 = 95	648 – 529 = 119	672 – 529 = 143	696 – 529 = **167**
(624 – 361 = **263**)	(648 – 361 = 287)	(672 – 361 = **311**)	
	(648 – 289 = **359**)		
720 – 529 = **191**	744 – 529 = 215	768 – 529 = **239**	792 – 529 = **263**
	(744 – 361 = **383**)		
816 – 529 = 287	840 – 529 = **311**	864 – 841 = **23**	ETC.
(816 – 361 = 455)			
(816 – 289 = 527)			
(816 – 169 = **647**)			

As we can see, it is often the square of the prime number immediately below the multiple of 24 that allows us to obtain a prime number as the result of subtraction (25 times out of 35).

Another observation is that we never obtain a prime number when the digit at the units of the multiple of 24 is a 4 and the digit at the units of the square of the prime number is a 9, or when these digits are a 6 and a 1, since the result of their difference gives a 5 to the units. However, as our list of examples suggests, after a number of attempts with the squares of various prime numbers, we always manage to obtain a prime number as a result.

Hypothesis 8:

Our eighth hypothesis concerns a new concept: Rose's twin semiprime numbers (or "Rose numbers"). This concept of our invention, which characterizes a certain class of numbers, is defined as follows:

We are in the presence of Rose numbers (*rn*) when multiplying by 2 each of the numbers in a pair of twin primes (*tp*) results in semiprime numbers on either side of the numbers of another pair of twin primes that is greater than it. These twin semiprime numbers are therefore a special class of what we call the epaulette numbers (*en*). Here are some examples of pairs of *rn* (in bold).

Pair of *tp*	**Operations**	**Pair of *rn***	**Corresponding pair of *tp***
(5, 7)	5 x 2 = 10 and 7 x 2 = 14	(**10**, **14**)	**10**, (11, 13), **14**
(29, 31)	29 x 2 = 58 and 31 x 2 = 62	(**58**, **62**)	**58**, (59, 61), **62**
(809, 811)	809 x 2 = 1,618 and 811 x 2 = 1,622	(**1 618**, **1 622**)	1 618, (**1 619**, **1 621**), 1 622

Another way to make a similar association between numbers would be to simply multiply by 2 the separator number (*sn*), i.e. the number at the center of a pair of twin primes (*tp*), and check if the sum corresponds to the *sn* of another pair of *tp*, and then make sure that the epaulette numbers of this second pair are indeed semiprimes (see below).

Pair of *tp*	**Separator number (*sn*)**	**Operation**	**Corresponding pair of *tp***
(5, 7)	6	6 x 2 = 12	**10**, (11, 13), **14**
(29, 31)	30	30 x 2 = 60	**58**, (59, 61), **62**
(809, 811)	810	810 x 2 = 1,620	**1 618**, (1 619, 1 621), **1 622**

It seems quite conceivable to us that there are infinitely many pairs of numbers in this category. This supposition is conjecture and remains to be demonstrated. A challenge for research would be to verify whether there is an arithmetic logic behind the existence of these numbers or whether they are mere curiosities or “mathematical coincidences", if such a thing is possible.

<u>Hypothesis 9</u>:

In the same vein, we tried to find out what we got by multiplying by two the epaulette numbers (*en*) of a pair of twin primes (*tp*), i.e. those immediately located on each side. It seemed mathematically interesting to us to note that some of these *en*, when doubled, sometimes resulted in numbers that punctuated two other pairs of *tp*, the smaller sum being found immediately after the lowest pair of *tp* numbers, the other immediately before the next pair of *tp* numbers. We refer to the numbers resulting from this operation as pairs of "Alexy epaulette numbers" (*aen*):

In the following examples, the numbers in bold represent the epaulette numbers (*en*) which, when doubled, are positioned between two pairs – <u>always consecutive</u> – of *tp* numbers.

1st pair of *tp* numbers framed by its *en*	**Operations**		**2nd and 3rd pairs of *tp* numbers with their *aen***	
4, (5, 7), **8**	4 x 2 = 8	8 x 2 = 16	(5, 7), **8**…	**16**, (17, 19)
10, (11, 13), **14**	10 x 2 = 20	14 x 2 = 28	(17, 19), **20**…	**28**, (29, 31)
70, (71, 73), **74**	70 x 2 = 140	74 x 2 = 148	(137, 139), **140**…	**148**, (149, 151)
640, (641, 643), **644**	640 x 2 = 1,280	644 x 2 = 1,288	(1 277, 1 279), **1,280**…	**1,288**, (1 289, 1 291)

ETC.

Here are two characteristics of these *aen*:

- The doubled *en* are separated by 8 units and the pairs of *tp* numbers that border them are separated by 10 units.
- Either of these numbers can be semiprime, but it's not always the case.

Again, we assume that there are an infinite number of them, and we put this hypothesis on the table.

Hypothesis 10:

It seems relevant to us to pay particular attention to the succession of multiplicative factors of number 24 associated with prime numbers, which compose sequences directly related to the identification of prime numbers in the continuum of integers and can therefore inform us about the arithmetic logic of their distribution. Our hypothesis, which concerns the set of prime numbers including pairs of twin primes, is that the sequence of multiplicative factors of number 24 corresponding to the square of these numbers provides answers about their distribution. What we are looking at here is in fact the result of the formula we use (see Table 9).

Table 9: Multiplicative factors of 24 (*n*) and prime numbers

n = Multiplicative factor of 24.
Bold = Twin primes.
Underlined = False positive numbers.

n	**24n+1**	$\sqrt{24n+1}$	***n***	**24n+1**	$\sqrt{24n+1}$
1	25	**5**	187	4 489	67
2	49	**7**	210	5 041	**71**
5	121	**11**	222	5 329	**73**
7	169	**13**	247	5 929	77 (7 x 11)
12	289	**17**	260	6 240	79
15	361	**19**	287	6 889	83
22	529	23	330	7 921	89
35	841	**29**	345	8 281	91 (7 x 13)
40	961	**31**	392	9 409	97
57	1 369	37	425	10 201	**101**

70	1 681	**41**	442	10 609	**103**
77	1 849	**43**	477	11 449	**107**
92	2 209	47	495	11 889	**109**
100	2 401	49 (7 x 7)	532	12 769	113
117	2 809	53	590	14 161	119 (7 x 17)
145	3 481	**59**	672	16 129	127
155	3 721	**61**	715	17 160	131

Extending our list of numbers, we obtain the following sequences, which represent the multiplicative factors of 24 (i.e. *n*) for the squares of corresponding categories of numbers.

-Prime numbers:

1, **2**, **5**, **7**, **12**, **15**, 22, **35**, **40**, 57, **70**, **77**, 92, 117, **145**, **155**, 187, **210**, **222**, 260, 287, 330, 392, **425**, **442**, **477**, **495**, 532, 672, 715, **782**, **805**, **925**, **950**, 1027, 1107, 1162, 1247, **1335**, **1365**, **1520**, **1552**, **1617**, **1650**...

-Twin primes:

1, **2**, **5**, **7**, **12**, **15**, **35**, **40**, **70**, **77**, **145**, **155**, **210**, **222**, **425**, **442**, **477**, **495**, **782**, **805**, **925**, **950**, **1335**, **1365**, **1520**, **1552**, **1617**, **1650**...

-False positives:

100, 247, 345, 590, 610, 737, 852, 1080, 1190, 1457, 1717, 1820, 1962, 2035, 2542, 2667, 2795, 3432, 3480, 3725, 3775, 4240, 4347, 4510, 4845, 4902, 5430, 5735, 5922, 6370…

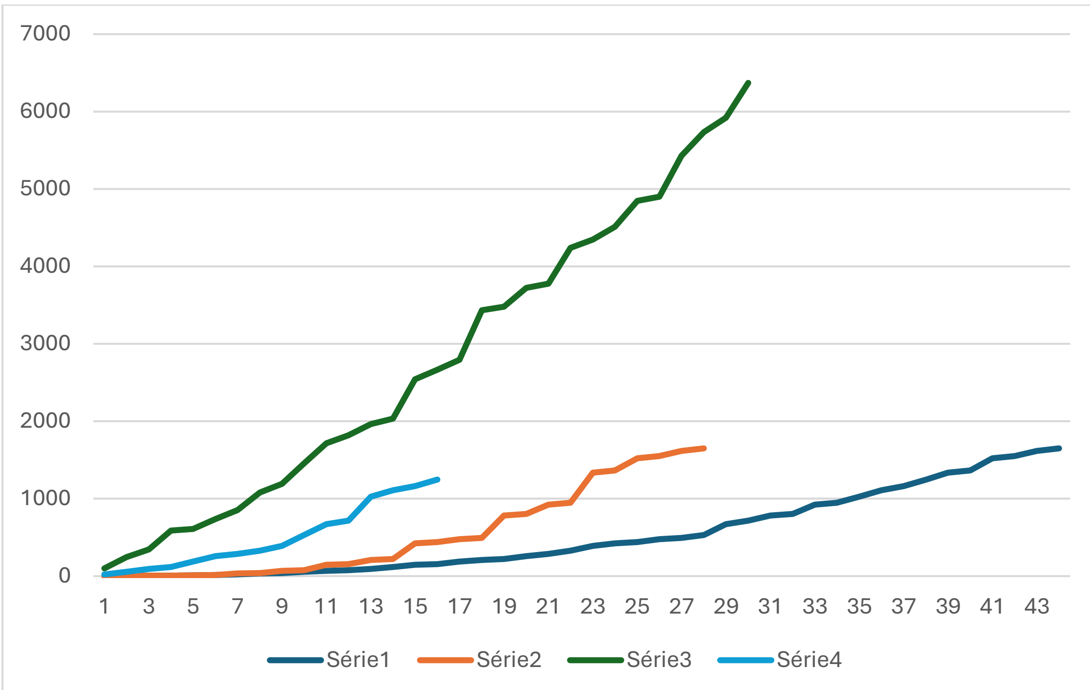
-Prime numbers without twin primes:

22, 57, **77**, 92, 117, 187, 260, 287, 330, 392, 532, 672, 715, 1027, 1107, 1162, 1247...

The sequence related to prime numbers was registered in the OEIS database by Clark Kimberling in 1991 under number A024702 (https://oeis.org/A024702), and corresponds to the application of our initial algorithm.

Since the sequences are of great interest for who is looking for hidden patterns in the distribution of primes, the question that arises here is whether it is possible to extract a rule from these sequences of numbers devoid of any apparent logic and, at best, to derive an algebraic formula from it. Here is a graphical portrait of these four progressions.

Diagram 6: Multiplicative factors of number 24 associated with the squares of primes, twin primes, and false positives



Series 1: All prime numbers Series 2: Twin primes

Series 3: False positive numbers Series 4: Prime numbers without *tp*

The multiplicative factors of the number 24, when coupled to the squares of prime numbers, twin primes and false positives, offer us curves of distinct appearance. The most regular of them seems to be the one that contains all the prime numbers (Series 1), inviting us to go further. For those who are ready to accomplish this task, they would first have to significantly increase the number of terms and see if a function can generate the curve of the resulting sequence.

<u>Hypothesis 11</u>:

Our eleventh and last hypothesis concerns the numerical sequence of the multiplicative factors of the number 24 associated with the difference between the squares of two twin prime pairs. We are wondering if, by any means, such a sequence could help us better understand the distribution of twin prime pairs.

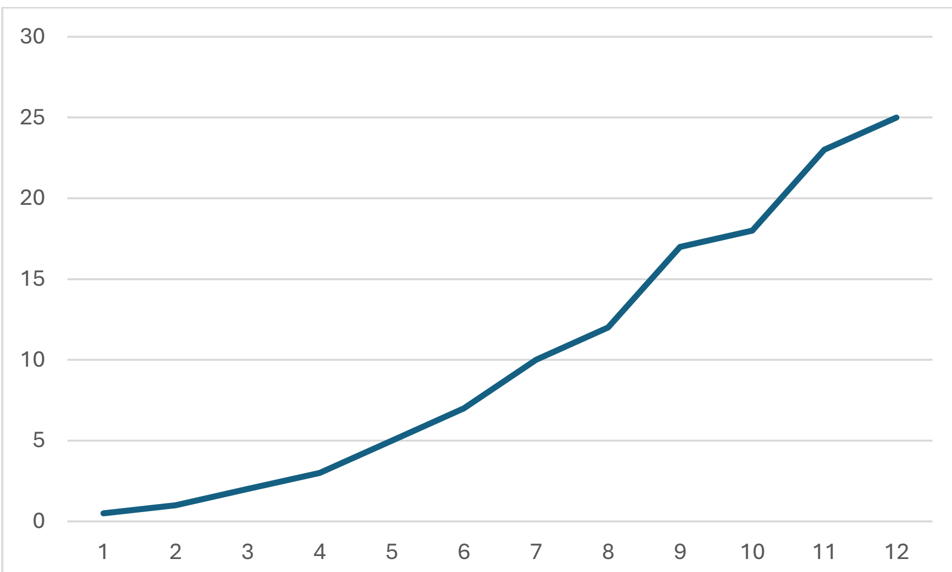
As stated in our postulate 5, the gap between the difference of the squares of two pairs of twin primes is systematically a multiple of the number 24 (excluding pair (3, 5)). Now, when we look at the ascending succession of quotients resulting from this operation (division by 24), this seems to produce a logical sequence, as we can observe in the column "Difference between squares" in Tables 4 and 5.

0.5, 1, 2, 3, 5, 7, 10, 12, 17, 18, 23, 25,…

This sequence of terms is sometimes referred to as the "twin ranks" sequence. It is recorded under number A002822 (https://oeis.org/A002822) in the OEIS. According to Lekraj Beedassy (2002), even terms correspond to the twin primes of the form (4k - 1, 4k + 1), odd terms to the twin primes of the form (4k + 1, 4k + 3).

Let's see how this translates in graphical form.

Diagram 7: Multiplicative factors of number 24 associated with the difference between the squares of two successive twin prime pairs



Suite : 0.5, 1, 2, 3, 5, 7, 10, 12, 17, 18, 23, 25,…

Interesting, but nothing very conclusive. We would have to produce a much larger set of terms to go further in our exploration of this question, namely whether these data can tell us more about the distribution of twin primes.

Overall, the results obtained with regard to twin primes are interesting and promising. If we haven't been able to discover the trick that guides us straight to writing an algorithm predicting their sequence, we certainly made some progress in that direction. The correspondences and regularities that we have observed suggest the existence of an infinity of twin prime pairs, if we agree that the model we have developed is solid and reliable. It is now up to the community of mathematicians to look at our results and assess their relevance, if they think our assumptions and hypothesis are worthwhile.

Discussion and conclusion: towards new theoretical horizons

As Cécile Dartyge, a researcher at the Élie Cartan Institute of the University of Lorraine says, if our numeral system can be conceived as a wall of which each number is a brick, then prime numbers are the elementary bricks that make up all numbers [3].

Our model of the arrangement of prime numbers suggests the existence of a structuring distribution rule associated with the number 24, which reflects the underlying organizing motif of our numeral system, the number 24 appearing as a major marker of the positional arrangement of prime numbers.

Inspired by this conception, our prime number detection "machine" successfully identifies 100% of prime numbers as far as we could test, without ever omitting a single one. False positives, initially retained in the identification sequence, can be systematically rejected by the application of a simple sieving technique. These false positives, far from casting doubt on the validity of our model, act in complete coherence with it. Our analyses highlight the possible role of these false positive numbers in the sequencing of pairs of twin primes, thus suggesting they could have a predictive value.

Knowing that Hardy considered the prime number theorem to be one of those statements whose depth made them accessible only through complex analysis, the great simplicity of our procedures for detecting prime numbers and pairs of twin primes is nothing short of astonishing. Our model as a whole is the result of an exploratory work which, we hope, will be of some use in algebraic arithmetic and analytic number theory, which is particularly interested in the distribution of integers within a series, by proposing to approach the problems concerning prime numbers in the light of their inseparable link with the number 24.

Appendix A

Mathematical formulas for pairs of integers, prime or not

1) The average of the numbers in a pair of *tp* (their sum divided by two) is equal to the square root of their product plus 1. True for any pair of numbers separated by two units.

Detailed examples:

Pair (29, 31)

$29 + 31 = 60$

$60 \div 2 = 30$

$29 \times 31 = 899$

$899 + 1 = 900$

$\sqrt{900} = 30$

$30 = 30$

Pair (14, 16)

$14 + 16 = 30$

$30 \div 2 = 15$

$14 \times 16 = 224$

$224 + 1 = 225$

$\sqrt{225} = 15$

$15 = 15$

Formula : $\frac{x + y}{2} = \sqrt{xy + 1}$

$$\frac{29 + 31}{2} = \sqrt{29 \times 31 + 1}$$

$$\frac{60}{2} = \sqrt{899 + 1}$$

$$30 = \sqrt{900}$$

$$30 = 30$$

2) The sum of the squares minus 2 of two *tp* is always the product of their sum multiplied by half of their sum. True for any pair of numbers of type $x - y = 2$, regardless of whether the numbers are prime or not.

Detailed examples: Pair (71, 73)

712 + 732 = 10,370

10,370 – 2 = 10,368

71 + 73 = 144

144 ÷ 2 = 72

144 x 72 = 10,368

10,368 = 10,368

Pair (6, 8)

62 + 82 = 100

100 – 2 = 98

6 + 8 = 14

14 ÷ 2 = 7

14 x 7 = 98

98 = 98

Formula : $x^2 + y^2 - 2 = (x + y)\frac{x+y}{2}$

$$71^2 + 73^2 - 2 = (71 + 73)\frac{71 + 73}{2}$$

$$5041 + 5329 - 2 = 144\frac{144}{2}$$

$$10370 - 2 = 144 \times 72$$

$$10368 = 10368$$

3) The sum of a pair of *tp* divided by 2 is equal to the square of these numbers minus 2 divided by their sum. True for all pairs of integers whose difference is two, prime or not.

Detailed examples: Pair (71, 73)

71 + 73 = 144

144 ÷ 2 = 72

712 + 732 = 10,370

10,370 – 2 = 10,368

10,368 ÷ 144 = 72

72 = 72

Pair (6, 8)

6 + 8 = 14

62 + 82 = 100

14 ÷ 2 = 7

100 – 2 = 98

98 ÷ 14 = 7

7 = 7

Formula: $\frac{x+y}{2} = \frac{x^2+y^2-2}{x+y}$

$$\frac{71+73}{2} = \frac{5041+5329-2}{71+73}$$

$$\frac{144}{2} = \frac{10370-2}{144}$$

$$72 = \frac{10368}{144}$$

$$72 = 72$$

4) The sum of the numbers in a pair of *tp* multiplied by 2 is equal to the difference between their squares. We propose two variants, one of which is that their sum is equal to the difference between their squares divided by 2. True for all pairs of integers whose difference is two, prime or not.

Detailed examples: Pair (17, 19)

192 – 172 = 361 – 289

361 – 289 = 72

17 + 19 = 36

2 x 36 = 72

72 = 72

Pair (14, 16)

162 – 142 = 256 – 196

256 – 196 = 60

14 + 16 = 30

2 x 30 = 60

60 = 60

Formulas: $x + y = \frac{x^2-y^2}{2}$ or $2(x+y) = x^2-y^2$ or $4\left(\frac{x+y}{2}\right) = y^2 - x^2$

$$13 + 11 = \frac{13^2-11^2}{2}$$

$$24 = \frac{169-121}{2}$$

$$24 = \frac{48}{2}$$

$$24 = 24$$

5) The difference between the squares of the numbers in a pair of *tp* is equal:

   Variant A) at twice the sum of the *tp* numbers;

   Variant B) to the greater of the two numbers minus 1 (or the smaller number plus 1) multiplied by 4.

   The formulas work with any pair of integers that have a difference of two, prime or not.

Note: Similarly, the difference between the squares of a pair of *tp* and its *sn* is a multiple of 12, with a difference of plus 1 with respect to the smallest square and minus 1 with respect to the largest square. The same sort of mathematical principle also applies to any triplet of consecutive numbers.

Detailed examples:

Variant A) Pair (11, 13)

$13^2 – 11^2 = 169 – 121$ $11 + 13 = 24$

$169 – 121 = 48$ $2 x 24 = 48$

$48 = 48$

Pair (10, 12)

$10^2 – 12^2 = 100 – 144$ $10 + 12 = 22$

$100 – 144 = 44$ $2 x 22 = 44$

$44 = 44$

Formula : $x^2 - y^2 = 2(x + y)$

$$13^2 - 11^2 = 2(13 + 11)$$

$$169 - 121 = 2 \times 24$$

$$48 = 48$$

Variant B)

Pair (11, 13)

132 – 112 = 169 – 121

169 – 121 = 48

13 – 1 = 12 (or 11 + 1 = 12)

4 x 12 = 48

48 = 48

Pair (10, 12)

102 – 122 = 100 – 144

100 – 144 = 44

12 – 1 = 11 (or 10 + 1 = 11)

4 x 11 = 44

44 = 44

<u>Formula</u>: $x^2 - y^2 = 4(x-1)$

$$13^2 - 11^2 = 4(13-1)$$

$$169 - 121 = 4 \times 12$$

$$48 = 48$$

6) Equation linking the *tp* numbers and the separator number, i.e. the one between the two numbers in the pair. Valid for any pair of integers diverging by two units.

<u>Formula</u>: $y + z = \frac{x^2 - y^2 + z^2 - x^2}{2}$

Pair (11, 13)

$$11 + 13 = \frac{12^2 - 11^2 + 13^2 - 12^2}{2}$$

$$24 = \frac{144 - 121 + 169 - 144}{2}$$

$$24 = \frac{23 + 25}{2}$$

$$24 = \frac{48}{2}$$

$$24 = 24$$

Pair (12, 14)

$$12 + 14 = \frac{13^2 - 12^2 + 14^2 - 13^2}{2}$$

$$26 = \frac{169 - 144 + 196 - 169}{2}$$

$$26 = \frac{169 - 144 + 196 - 169}{2}$$

$$26 = \frac{25 + 27}{2}$$

$$26 = \frac{52}{2}$$

$$26 = 26$$

Appendix B

Parallel findings: the number of factors of numbers resulting from the repeated exponentiation of a prime

We already know that, when a number is squared, the number of its factors increases due to the squaring operation creating new pairs of factors. A square number has an odd number of factors, and the number of factors is related to the prime factorization of the original number.

In our analyses, we observed something that we believe is worth pointing out to number theory researchers, in case it is not an already known fact. A prime number has two factors. When we square a prime number, the resulting number is a semiprime made up of three factors. For example, 32 = 9, whose factors are 1, 3, and 9. We were curious to know what the number of factors would result if we continued to square the following numbers obtained by such an iterative squaring procedure, starting with a prime number. The resulting sequence is: 2, 3, 5, 9, 17, 33, 65, 129, 257, 513, 1025,…

The first element of the sequence is the number of factors of a prime number. The next element is the number of factors of the number resulting from squaring that prime, and so on, squaring all subsequent numbers iteratively.

E.g. 2 (**2** factors), 2^2 = 4 (**3** factors), 4^2 = 16 (**5** factors), 16^2 = 256 (**9** factors), etc.
Ex: 5 (**2** factors), 5^2 = 25 (**3** factors), 25^2 = 625 (**5** factors), 625^2 = 390,625 (**9** factors), etc.

The increasing number of factors forms a sequence based on a very simple rule, which consists of doubling the number minus one (2n – 1). Another way to characterize the logic of this progression is to focus on the gap between the elements, as it doubles each time (+1, +2, +4, +8, +16, etc.). Interestingly, <u>this precise succession of numbers of factors appears regardless of the prime number chosen at the outset</u>.

This exact string of integers is recorded in the OEIS under the sequence number A000051 (https://oeis.org/A000051), but the wording of the sequence rule is different (a(n) = 3^n + 1 rather than 3n – 2). Moreover, the sequence does not seem to be related to what we are considering here, i.e. the increasing number of factors in the series of numbers resulting from this repetitive squaring operation, starting with any prime number.

The same phenomenon, that of an observable regularity at the level of the number of factors, occurs when a prime number is cubed (put to the power of 3), then this result is cubed in turn, and so on. The sequence is then different from the one from the squares, but as seen above, it remains the same regardless of the prime number chosen at the beginning, which corresponds to sequence no. A034472 (https://oeis.org/A034472) of the OEIS: 2, 4, 10, 28, 82,…

The first item in the sequence is the number of factors of a prime number. The next element is the number of factors in the cube of this number, and so on, putting all of the following resultant numbers to the power of 3 iteratively and adding to the sequence the number of factors of each of the resulting numbers.

Ex: 2 (**2** factors), 2^3 = 8 (**4** factors), 8^3 = 512 (**10** factors), 512^3 = 134217728 (**28** factors), 134217728^3 = 2417851639229258349412352 (**82** factors), etc.

Ex: 3 (**2** factors), 3^3 = 27 (**4** factors), 27^3 = 19.683 (**10** factors), 19.683^3 = 7,625,597,484,987 (**28** factors), $7,625,597,484,987^3$ = 443426488243037769948249630619149892803 (**82** factors), etc.

We are also of the opinion that the same arithmetic pattern (a regularity of the number of factors) will occur regardless of the prime number or exponent chosen at the outset. Thus, a sequence starting with any prime number set to the power of 4 will always produce the same sequence, the terms of which are the number of factors. We will then observe the rule a(n) = 4^n + 1 or 4n – 3, the items in the sequence always being the number of factors of the successive numbers obtained, which corresponds to sequence no. A052539 (https://oeis.org/A052539) of the OEIS: 2, 5, 17, 65,…

Ex: 2 (**2** factors), 2^4 = 16 (**5** factors), 16^4 = 65,536 (**17** factors), $65,536^4$ = 18,446,744,073,709,551,616 (**65** factors), etc.

Ex: 3 (**2** factors), 3^4 = 81 (**5** factors), 81^4 = 43,046,721 (**17** factors), etc.

To our knowledge, these regularities have not previously been associated with the growing number of factors of the numbers resulting from any prime number repeated exponentiation (being squared, cubed, or put at any other higher power). In such a case, it is a hypothesis that requires confirmation and we suggest considering it as a new conjecture related to prime numbers. If this is already a known fact, we apologize.

Tables and graphs:

References:

No scientific article published in a mathematics journal was consulted in connection with this research work on prime numbers. The entire content (tables, examples, calculations, algorithms, results, etc.) is solely the result of the author's personal investigations. All the postulates, hypothesis, data analyses and conclusions in this article are exclusively the result of the individual efforts of an independent researcher who explores the universe of numbers in a very free, intuitive and imaginative way. Not being a mathematician by profession, it is likely that some elements of our article are of little relevance or not entirely new. In such cases, we apologize and do not want to take credit for any ideas, formulas or research results that already exist. We welcome constructive feedback to improve our work at the following email address: les64cases@gmail.com.

[1] From the *Récéomath website* (http://www.recreomath.qc.ca/dict_jumeaux_np.htm).
[2] N.J.A. Sloane, *The Online Encyclopedia of Integer Sequences* (OEIS).
[3] Anaïs Culot: These numbers that draw the world, CNRS – Le Journal (2021).