Local Quantum Field Theory as an Operational Reconstruction in a Timeless Euclidean Model

A. N. Smirnov,
Independent researcher, andreysxxxx@gmail.com

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Abstract

We derive a local quantum field theory (QFT) as an operational reconstruction in a timeless Euclidean model on $\mathbb{E}^4$ with a single fundamental real field satisfying the Laplace equation. Using a working domain Ω and an observer's body $\Omega_0 \subset \Omega$, we introduce events as readout outcomes on foliation hyperplanes and construct a local algebra of observables. In the special–relativistic regime the invariant null cone and a finite maximal speed $v_{\max}$ are operationally reproduced. Under the assumptions of locality of the transfer and reflection positivity of the state on the algebra of observables, we apply the OS/GNS reconstruction and obtain a Hilbert space of states, unitary evolution, and the Born rule. The complex structure and amplitudes arise from the Euclidean correlation structure, the symplectic form, and the choice of complex structure on the space of local degrees of freedom; the choice of statistics (CCR/CAR) is fixed by the form of the principal symbol of the transfer generator under standard axiomatic conditions.

The local ambiguity in the choice of observational description is realized as gauge symmetries: $U(1)$ encodes phase freedom, $SU(3)$ is the minimal group admitting color-neutral three-fermion singlets, and $SU(2)$ is a symmetry compatible with the existence of charged chiral currents and anomaly freedom. As a result, the minimal gauge group $SU(3) \times SU(2) \times U(1)$ is not postulated phenomenologically but is singled out as the minimal one compatible with explicitly stated operational assumptions and anomaly-freedom conditions. The resulting local QFT is consistent with the classical gravitational sector previously derived from the same Euclidean model and admits the formulation of the inverse problem of reconstructing the Standard Model constants from correlators of the fundamental field and geometric observables. The present work continues the program of reconstructing special and general relativity from a timeless Euclidean model and extends it by deriving a local gauge QFT with gauge group $SU(3) \times SU(2) \times U(1)$, locally isomorphic to the structure of the Standard Model, from the same operational assumptions.

Contents

1 Introduction

1.1 Background and motivation

Fundamental Euclidean model. The present work is based on a timeless Euclidean model on four-dimensional Euclidean space $\mathbb{E}^4$ with a fixed global metric δ_{AB}. The fundamental object is a real scalar field $\Phi : \mathbb{E}^4 \to \mathbb{R}$ satisfying the Laplace equation

$$\Delta_{\mathbb{E}^4}\Phi = 0, \qquad \Delta_{\mathbb{E}^4} := \delta^{AB}\partial_A\partial_B.$$

At this level neither a global time, nor a causal structure, nor a space of events is postulated; the fundamental configuration Φ specifies only a static distribution of field values on $\mathbb{E}^4$.

The class of admissible configurations Φ is not chosen arbitrarily, but is defined by operational constraints of a localized observer: the choice of a working domain $\Omega \subset \mathbb{E}^4$ admitting a local modal decomposition of the field, and of a foliation $\{\Sigma_s\}$ with respect to which a consistent causal reconstruction is possible. It is precisely at this fundamental level, in the absence of a priori time and dynamics, that we subsequently define the observer, effective time, events, and effective fields used to derive quantum field theory (see §2 for details).

Operational framework. In the present work, by an operational setting we mean a description of physical structures solely in terms of measurement procedures of a localized observer and the statistics of their outcomes in the observer's working domain $\Omega_s \subset \Sigma_s$. The fundamental level is specified by the stationary Laplace equation $\Delta_{\mathbb{E}^4}\Phi = 0$ on Euclidean space $\mathbb{E}^4$ and, in contrast to the effective description, contains neither a fundamental time coordinate, nor an *a priori* causal structure, nor a global space of events. Operational time, events, and observables arise only as entities reconstructed relative to a given observer: events are defined through stable local configurations of excitations in Ω_s, and measurements through local POVMs on the observer's effective Hilbert space (see §2, §3.7). All subsequent dynamics (the Schrödinger equation on foliations, relativistic fields, gauge symmetries) is considered at the level of effective descriptions that admit such an operational interpretation and are consistent with the requirement of inter-observer consistency.

Previous results. The present work relies on two previous results formulated in the Euclidean setting.

In [1] it was shown that, starting from the Laplace equation $\Delta_{\mathbb{E}^4}\Phi = 0$ on Euclidean space and introducing the observer, foliations, and events operationally, one can reconstruct the structure of special relativity: the appearance of an effective pseudo-Riemannian metric of signature $(+, -, -, -)$, an invariant maximal speed, and Lorentz transformations between inertial reference frames.

In [2] this construction was generalized to the case of a variable effective metric $g_{\mu\nu}$ and, on the basis of the same operational principles, the structure of general relativity was obtained, including the energy–momentum tensor and Einstein equations as effective equations for the reconstructed geometry. In the present paper these results are used as a geometric foundation: we assume the existence of an effective Lorentzian manifold with metric $g_{\mu\nu}$, consistent with the Euclidean model and operational foliations, and then investigate which quantum fields and gauge symmetries can be consistently realized on such an effective background.

Thus this work extends the previous studies by passing from the reconstruction of the space-time structure to the derivation of quantum field theory and of a minimal gauge group compatible with this structure and with the operational requirements.

A more detailed account of those results of the previous works that are essential for the present paper is given in §2.

Previously we have considered the observer and their role in the model and have defined events operationally. Building on this foundation, we now analyze the effective fields that arise in the model. Since in the timeless model, on foliations, the structures of special and general relativity are operationally reproduced (SR regime, invariant null cone, emergent metric g, and Einstein equations at the working level), the dynamics of effective fields must be compatible with them: local Lorentz covariance in the SR regime and g-covariance with the equivalence principle in the curved case. Formally, this means that kinetic terms are built from $g^{\mu\nu}$ and ∇_μ, consistent with the null cone and the maximal speed $v_{\max}$.

Owing to the hierarchy of scales and the exclusion of strong fields obtained in the previous works, we employ the approximation of flat foliations (SR regime), neglecting gravitational corrections at the working order.

Effective fields. As noted in the work on deriving the Einstein equations from the model, the timeless Euclidean setting gives rise to *effective fields* describing modal degrees of freedom on the slices Σ_s in the working domain of a localized observer. The aim of the present work is to analyze these effective fields from an operational point of view. We will show that, under reflection positivity, the spectrum condition, and locality of the transfer, they admit an OS/GNS reconstruction and thereby realize the structure of a local quantum field theory in the SR regime. We then show that, upon imposing additional operational and phenomenologically motivated requirements (existence of stable internal registers of the observer, color-neutral composites, universal left-handed currents, anomaly freedom), the minimal gauge group compatible with the model and with the existence of an observer is locally isomorphic to $SU(3) \times SU(2) \times U(1)$, and the fermionic content can be organized into representations isomorphic to a single Standard Model family. At the same time, within the present work we do not claim that the Standard Model is the unique possible solution: it suffices that it arises as one of the admissible effective descriptions satisfying the stated operational requirements.

Goal and problem statement. The goal of the present work is to show rigorously that the *operational reconstruction* of effective fields in a timeless Euclidean model on $\mathbb{E}^4$ with a fundamental real field Φ satisfying $\Delta_{\mathbb{E}^4}\Phi = 0$ leads to a local quantum field theory on the working domain Ω of a localized observer. At the fundamental level, time, causality, and dynamics are absent; these structures must be derived from local observation procedures in the bounded domain Ω in the presence of an observer's body $\Omega_0 \subset \Omega$ and rules of causal reconstruction. A further goal is to show that, under additional operational and phenomenologically motivated requirements, the minimal gauge group compatible with the model and with the existence of an observer is locally isomorphic to $SU(3) \times SU(2) \times U(1)$, and the fermionic content can be organized into representations isomorphic to a single Standard Model family. We do not claim that the Standard Model is the unique possible solution; it suffices that it arises as an admissible effective description consistent with the operational requirements.

In terms of the structure of the presentation, this is split into the following subtasks:

1) **OS/GNS and the effective Hilbert space.** Show that, for the chosen class of states on Ω, reflection positivity (OS) holds, and use this to reconstruct, via OS/GNS, a Hilbert space, a self-adjoint Hamiltonian $H \geq 0$, and a unitary evolution $U(t) = e^{-iHt}$; fix the normalization of $\hbar$ from the spectrum of H and from the two-point function.

2) **Peierls symplectic form and canonical algebras.** Construct a covariant symplectic form (Peierls bracket) for admissible effective fields and, on this basis, derive the canonical (anti)commutation relations (CCR/CAR) and microcausality for the free sectors.

3) **Schrödinger equation and relativistic closure.** Show that the evolution of states on the slices Σ_s is described by the Schrödinger equation $i\,\partial_t\,|\psi\rangle = H\,|\psi\rangle$ with continuity of the probability current, and that the requirement of local Lorentz covariance in the SR regime and of microcausality fixes the forms of the free equations for scalar, spinor, and vector fields (the Klein–Gordon equation, the Dirac equation, and the equation for a vector field).

4) **Measurements, POVMs, and the Born rule.** Specify measurements operationally through local POVMs in the OS/GNS representation, show that the Born rule is realized for these POVMs, and establish consistency of the definition of events and of the statistics of outcomes with the reconstructed dynamics.

5) **Compatibility with spin–statistics.** Verify that, under OS positivity, microcausality, and the spectrum condition, the effective theory belongs to the class of local QFTs to which the standard KL/CPT and spin–statistics theorems apply, and is thus compatible with the correspondence "integer spin $\Rightarrow$ CCR, half-integer spin $\Rightarrow$ CAR". In the present work the spin–statistics theorem itself is not proved but is used in its standard form.

6) **Gauge symmetries and the minimal group.** Analyze the observability of internal symmetries operationally and show that, under natural conditions of existence of an observer, anomaly freedom, and a choice of fermionic content isomorphic to a single Standard Model family, the minimal gauge group consistent with the model and with phenomenology is locally isomorphic to $SU(3) \times SU(2) \times U(1)$ with fixed hypercharges.

1.2 From effective fields on foliations to quantum field theory

Principle of minimality. We will consider a *minimal* effective field theory sufficient for the existence of a localized observer and *anomaly-free* with respect to gauge currents, i.e. containing no additional massless gauge fields beyond those operationally required in the class of regimes under consideration. In addition, we require the *absence of Ostrogradsky instabilities* in the free effective sector (nondegenerate higher derivatives leading to ghost-like modes). Compatibility with reflection positivity and microcausality is subsequently used as a testable consistency criterion rather than as an independent postulate: at each step in the construction of the effective theory we check compatibility with OS positivity, the spectrum condition, and operational causal reconstruction.

Structure of the derivation. As stated in §1.1, the task is to reconstruct a local quantum field theory operationally from effective fields on foliations in the Euclidean model. Technically this is implemented as follows. First, for a chosen class of states on the working domain Ω, reflection positivity is established and, via the OS/GNS construction, a Hilbert space, a self-adjoint Hamiltonian $H \geq 0$, and a unitary evolution on the slices Σ_s are reconstructed (see §3.1). Then, canonical (anti)commutation relations and microcausality are derived from the covariant symplectic structure (Peierls bracket), and the requirements of local Lorentz covariance in the SR regime fix the form of the free relativistic equations for scalar, spinor, and vector fields (see §5). An operational description of measurements via local POVMs in the OS/GNS representation and inter-observer consistency determine the status of events and the Born rule (see §3.7).

Finally, an analysis of the operational observability of internal symmetries, of anomaly-freedom requirements, and of the structure of a single fermion family shows that, under these assumptions, the *minimal* effective theory, in terms of the number of fields and gauge degrees of freedom, has a gauge group locally isomorphic to $SU(3)\times SU(2)\times U(1)$ and the standard pattern of hypercharges (see §6). In this sense, by "quantum field theory emerging from the model" we will mean precisely such an OS/GNS-reconstructed local field theory built from effective fields obtained from the timeless Euclidean configuration Φ, with the Standard Model regarded as a minimal admissible solution of the imposed operational and phenomenological requirements, rather than as a structure uniquely fixed in a strict sense.

1.3 Aim and structure of the work

The aim of the present work is to implement the operational reconstruction of effective fields in the timeless Euclidean model on $\mathbb{E}^4$ described above, and to show that it leads to a local quantum field theory on the working domain Ω of a localized observer and to a minimal gauge group compatible with the reconstructed relativistic geometry and with the operational requirements for the existence of an observer.

In brief, the main results of the paper are as follows:

(a) For the chosen class of states on Ω, reflection positivity holds and an OS/GNS reconstruction of a Hilbert space and a self-adjoint Hamiltonian $H \geq 0$ is carried out; the Born rule is obtained for local measurements in the observer's working domain (see §3.1, §3.7).

(b) A compatible complex structure and a local dynamics on foliations are constructed, providing unitary evolution of states in the SR regime and consistency with the effective metric $g_{\mu\nu}$ (see §3.1, §3.5, §4).

(c) It is shown that the resulting effective field theory is compatible with the standard spin–statistics connection under OS positivity, microcausality, and the spectrum condition (see §5).

(d) On the basis of an operational analysis of the observability of internal symmetries, of anomaly-freedom conditions, and of the structure of a single fermion family, a minimal gauge group is obtained, locally isomorphic to $SU(3) \times SU(2) \times U(1)$, with fixed hypercharges (see §6).

The structure of the paper is as follows. In §1 we formulate the motivation, assumptions, and relation to the previous works, and we state the goals and outline the logic of the subsequent presentation. In §2 we specify the fundamental Euclidean model, introduce foliations and effective fields on the slices, and formulate the requirements of causal reconstruction and the operational definition of the observer and events. Section §3 is devoted to the dynamics of the transfer in the parameter s and its operational consequences: here we formulate the conditions of locality and isotropy, perform the OS/GNS reconstruction of the state (§3.1), introduce the complex structure (§3.5), and discuss measurements and the Born rule (§3.7). In §4 we derive the Schrödinger equation on foliations and formulate a local continuity equation for the probability current. Section §5 contains the relativistic closure: we derive relativistic equations for scalar, spinor, and vector fields, and discuss the Peierls symplectic form, microcausality, and the relation to spin and statistics. In §6 we perform an operational analysis of internal symmetries and anomaly freedom and derive a minimal gauge group locally isomorphic to the Standard Model group. Section §7 is devoted to the formulation of the inverse problem and to the parameters of the effective theory, while §8 addresses the predictions and falsifiable consequences of the model. In §9 we discuss the results and possible generalizations, and in §10 we formulate brief conclusions. Technical proofs and auxiliary constructions are collected in the appendices, including appendices A, C and H.

2 Fundamental Euclidean model and effective fields

In this section we fix the basic objects of the model and introduce local foliations and effective fields on the slices.

2.1 Laplace equation and admissible configurations

Let $\mathbb{E}^4$ be equipped with the Euclidean metric δ_{AB} and global coordinates x^A, $A = 0, 1, 2, 3$. The Laplacian is defined by $\Delta_{\mathbb{E}^4} := \delta^{AB}\partial_A\partial_B$. The fundamental field is a real function $\Phi : \mathbb{E}^4 \to \mathbb{R}$, $\Phi \in C^2$, satisfying

$$\Delta_{\mathbb{E}^4}\Phi(x) = 0 \quad \text{for all } x \in \mathbb{E}^4. \tag{1}$$

This is the *only fundamental equation of the model.* No fundamental variational functionals, sources, or nonlinearities for Φ are introduced.

In the present work we simply fix this setup as the fundamental model, following [1, 2], where the motivation for choosing the Laplace equation as a minimal local $O(4)$-invariant elliptic equation without a distinguished time is discussed in detail.

This equation contains no distinguished time and does not provide a fundamental dynamics; at this level no internal (gauge) symmetries and no preferred directions in $\mathbb{E}^4$ are postulated beyond the geometric $O(4)$ symmetry of the Euclidean metric itself, and no interactions — neither linear nor nonlinear — are introduced.

Solutions of (1) form the class of harmonic configurations of the field Φ on $\mathbb{E}^4$. At the level of interpretation, each such configuration describes a complete "world" of the model: in the timeless setting there are no independent initial data and no external evolution parameter, so the entire content of the description is encoded in the choice of Φ, rather than in its "time development".

In what follows we will not need to fix a specific solution. All constructions are formulated for an arbitrary configuration from a certain admissible class. By *admissible configurations* we will mean solutions of (1) satisfying additional operational constraints formulated below (see, in particular, §2 and §3.1).

We are interested not in explicit solutions of (1) as such, but in the physical consequences of the imposed operational constraints — in particular, in the structure of the resulting effective fields and in their properties when interacting with a localized observer.

2.2 Causal reconstruction requirements

We work on Euclidean space $\mathbb{E}^4$ with a field Φ satisfying the Laplace equation in the class of admissible configurations (see §2.1). We fix a foliation by level hyperplanes

$$\Sigma_s^{(\mathbf{n})} = \{\, x \in \mathbb{E}^4 \mid n_A x^A = s \,\}, \qquad n_A n^A = 1,\ s \in \mathbb{R}, \tag{2}$$

where the choice of n_A fixes an inertial reference frame, and the parameter s plays the role of *operational time* with respect to this foliation.

Definition (causal reconstruction). For a fixed observer O and a foliation $\Sigma_s^{(\mathbf{n})}$, where the observer's body occupies a region $\Omega_0 \subset \Sigma_s^{(\mathbf{n})}$, *causal reconstruction* is a procedure that, from local information about Φ in a region $\Omega \subset \Sigma_s^{(\mathbf{n})}$, builds a consistent description of a set of events E_O and of their ordering $\preceq_O$ with respect to the transfer direction $\mathbf{n}$. The specific mechanism of event selection is given below (see §2.6).

Causal reconstruction requirement. The reconstruction in each inertial frame must satisfy:

(i) Localizability and mode transfer. The decomposition of Φ into modes of the foliation is chosen so that the modes u_α are localized on $\Sigma_s^{(\mathbf{n})}$ and admit a *local linear transfer* in s along $\mathbf{n}$ for the coefficients $a_\alpha(s)$, with a finite maximal speed $v_{\max}$ of interactions.

(ii) Compatibility of the equation with the transfer. The equation for Φ admits such local modes and preserves their evolutionary consistency for any direction $\mathbf{n}$.

(iii) Consistency under small rotations. Under $\mathbf{n} \mapsto \mathbf{n}'$ with angle $\theta = \arccos(\mathbf{n}\cdot\mathbf{n}') \to 0$, the sets of events and their ordering coincide identically.

These conditions are operational: causality is not postulated but arises as a criterion of admissibility of the reconstruction in the presence of a bound on the speed of interactions. They generalize the causal reconstruction conditions used in [1] to derive the SR structure; here they specify the class of configurations compatible with the existence of an observer.

Admissibility of configurations. Conditions (i)–(iii) can conveniently be reformulated as constraints on the class of solutions Φ. Let $\mathcal{S} \subset \ker\Delta$ denote the set of configurations for which a causal reconstruction exists in the above sense and which satisfy:

(a) Localizability of modes u_α on each $\Sigma_s^{(\mathbf{n})}$;

(b) Interpretability in terms of events of the interactions of field modes with the modes of the observer's body;

(c) Stability under small rotations of $\mathbf{n}$ (consistency of the reconstruction as $\theta \to 0$).

2.3 Localization of the observer and a common reference basis

The observer is not an external agent: they are described as a localized structure in $\mathbb{E}^4$ inside the field configuration $\Phi(x)$. We fix the foliation (2). We also fix a local region $\Omega \subset \Sigma_s^{(\mathbf{n})}$, compact in the three spatial directions, which serves as the working domain for decomposing the field into modes. It is precisely within this region that the operationally accessible description of events is formed (in physical terms one may regard it as an analogue of the observable part of the Universe). For the body of a particular observer we use a region $\Omega_O \subset \Omega$.

We fix an orthonormal set $\{u_\alpha(x)\}_{\alpha\in\Lambda} \subset L^2(\Sigma_s^{(\mathbf{n})})$ such that $\operatorname{supp} u_\alpha \subset \Omega$ for all α. The local configuration on $\Sigma_s^{(\mathbf{n})}$ is decomposed as

$$\Phi(x)\big|_\Omega = \sum_{\alpha\in\Lambda} a_\alpha(s)\, u_\alpha(x), \qquad a_\alpha(s) = \int_\Omega u_\alpha(x)\, \Phi(x)\, d^3x. \tag{3}$$

In each inertial reference frame we fix a *common reference basis* $\{u_\alpha\}$ to ensure inter-observer consistency. The internal modes of an observer O are expressed as local linear combinations:

$$\chi_\beta(x) = \sum_{\alpha\in\Lambda} C^{(O)}_{\beta\alpha}\, u_\alpha(x), \qquad b_\beta(s) = \sum_{\alpha\in\Lambda} C^{(O)}_{\beta\alpha}\, a_\alpha(s), \tag{4}$$

where $\operatorname{supp}\chi_\beta \subset \Omega_O \subset \Omega$ for all β.

2.4 Transfer invariance and emergent causality

Conditions (i)–(iii) imply the existence of a *local linear transfer* for the coefficients

$$a^{(\mathbf{n})}(s+ds) = A^{(\mathbf{n})}[\Phi; s]\, a^{(\mathbf{n})}(s), \tag{5}$$

where $A^{(\mathbf{n})}[\Phi; s]$ depends only on the local configuration Φ in a neighbourhood of $\Omega \subset \Sigma_s^{(\mathbf{n})}$ (for small ds). Consistency under small rotations of the foliation requires *invariance* of the transfer law:

$$A^{(\mathbf{n}')}_{\alpha\beta}[\Phi;\, \cdot] \;\equiv\; A^{(\mathbf{n})}_{\alpha\beta}[\Phi;\, \cdot] \quad \text{on the subspace of modes operationally accessible in the region } \Omega, \tag{6}$$

since the basis on each slice is constructed anew by a single rule $\mathcal{U}[\Phi; \mathbf{n}, s]$ rather than transported by a rotation.

Remark 1 (Interpretation of invariance). A rotation $\mathbf{n} \to \mathbf{n}'$ is a replacement of the family of slices $\Sigma_s^{(\mathbf{n})} \to \Sigma_s^{(\mathbf{n}')}$. The basis $\{u_\alpha\}$ on each slice is defined by the same rule $\mathcal{U}$ and is not transformed as a fixed set of functions; therefore (6) expresses precisely the invariance of the transfer law rather than covariance of the set $\{u_\alpha\}$.

Emergent causality. In the adopted setting, causality in each inertial frame arises as a condition of operational consistency: reconstruction of the causal structure is possible only if there exists a local decomposition and an invariant transfer (5)–(6) on the operationally accessible modes. The direction of "time" and the causal structure are emergent in this sense and depend on the choice of foliation; there is no global set of events at the fundamental level.

2.5 Effective fields and their established properties

We recall the results obtained in [1, 2] without adding new statements. Here we partially repeat the previous subsections and fix the set of properties of effective fields to which we will refer below.

Fundamental model. We consider the timeless Euclidean setting on $\mathbb{E}^4$ with a real field Φ satisfying $\Delta_{\mathbb{E}^4}\Phi = 0$ (see Section 2.1) in the class of admissible configurations defined by the observer's operational constraints.

Foliations, observer, and events. Slices $\{\Sigma_s\}$ (foliations) and the observer's body $\Omega_0 \subset \Omega$ are defined; the event structure is observer-dependent and is brought into mutual agreement between observers in the same inertial frame by communication [1]. A global set of events does not exist.

Effective fields on foliations. On the slices Σ_s we introduce effective fields obtained from the configuration Φ and depending on the chosen foliation; direct and observable transformations between inertial frames are distinguished [1].

SR regime. Observable transformations have a Lorentz form with a single maximal speed $v_{\max}$; the Galilean limit is excluded. The causal structure is formed locally within each inertial frame [1].

GR regime. Upon abandoning flat foliations an effective metric g arises; gravity is treated as a curvature of foliations and acts universally. Locally gravity is equivalent to acceleration; the Einstein equations without a cosmological constant are obtained [2].

Hierarchy of scales and constraints. A hierarchy of scales between gravitational and non-gravitational effects is established, as well as an exclusion of strong fields in the class of configurations under consideration [2].

Decomposition of the field on foliations and the evolution equation. Let $\Sigma_s^{(n)}$ be a slice of the foliation with normal n, $\Omega_s := \Omega \cap \Sigma_s^{(n)}$, and let $\mathrm{d}\mu_{\Sigma_s^{(n)}}$ be the induced measure; in the SR regime this is the standard d^3y in orthonormal coordinates on $\Sigma_s^{(n)}$. For an admissible local orthonormal basis $\{u_\alpha(s,\cdot)\}_{\alpha\in\Lambda} \subset L^2(\Omega_s, \mathrm{d}\mu_{\Sigma_s^{(n)}})$ the coefficients

$$a_\alpha^{(n)}(s) = \int_{\Omega_s} u_\alpha(s,y)\,\Phi(y)\,\mathrm{d}\mu_{\Sigma_s^{(n)}}(y), \qquad \alpha \in \Lambda, \tag{7}$$

satisfy a local transfer law (see [1])

$$a^{(n)}(s+ds) = A^{(n)}(s;ds)\,a^{(n)}(s), \qquad A^{(n)}(s;0) = \mathbf{1}, \quad \Gamma^{(n)}(s) := \partial_{ds}A^{(n)}(s;ds)\Big|_{ds=0}, \quad A^{(n)}(s;ds) = \mathbf{1}+\Gamma^{(n)}(s) \tag{8}$$

Here $a^{(n)}(s) := (a_\alpha^{(n)}(s))_{\alpha\in\Lambda}$. The operator $A^{(n)}(s;ds)$ depends only on the *local* configuration of the field Φ in a neighbourhood of Ω_s.

The consistency condition for the reconstruction means that the basis $\{u_\alpha\}$ is constructed by a single rule $\mathcal{U}[\Phi;\mathbf{n},s]$, which ensures localizability, interpretability in terms of events, and continuity under rotations of the foliation.

Observer. In each inertial frame, a common reference orthonormal basis on Σ_s is fixed; the internal modes of the observer are expressed with respect to this basis (inter-observer consistency).

Effective fields as operational degrees of freedom. We work in the SR regime and use internal coordinates y on the slice Σ_s. Let $a^{(n)}(s) := (a_\alpha^{(n)}(s))_{\alpha\in\Lambda}$ and let $W(s,y) = [W_I{}^\alpha(s,y)]$ be a locally invertible (in y) map from modes to effective fields on the working subspace. Define

$$\psi_I(s,y) = W_I{}^\alpha(s,y)\,a_\alpha^{(n)}(s). \tag{9}$$

Introduce the exact finite-step transfer (propagator) along the foliation

$$a^{(n)}(s+ds) = P^{(n)}(s+ds,s)\,a^{(n)}(s), \qquad P^{(n)}(s,s) = \mathbf{1}, \quad P^{(n)}(s_2,s_1)P^{(n)}(s_1,s_0) = P^{(n)}(s_2,s_0). \tag{10}$$

Then the evolution of the effective fields can be written without approximations:

$$\psi(s+ds,y) = U(s,y;ds)\,\psi(s,y), \qquad U(s,y;ds) := W(s+ds,y)\,P^{(n)}(s+ds,s)\,W(s,y)^{-1}. \tag{11}$$

The equivalence of the descriptions in terms of a_α and ψ_I corresponds to the formulation adopted in [2].

Local linearization of the transfer. From (11) one obtains the linearization in ds:

$$\psi(s+ds,y) = U(s,y;ds)\,\psi(s,y)+\mathcal{O}(ds^2,\partial_y), \qquad U(s,y;ds) = \mathbf{1}+\Gamma(s,y)\,ds+\mathcal{O}(ds^2), \qquad \Gamma(s,y) := \partial_{ds}U(s,y;d \tag{12}$$

Here $\mathcal{O}(ds^2,\partial_y)$ is understood locally in y and uniformly in s on compact sets; Γ is local in y and C^1 in s. At the level of modes, (8) and (10) are consistent: $A^{(n)}(s;ds) = P^{(n)}(s+ds,s)$ on the working subspace, and $\Gamma^{(n)}$ induces Γ under the change of variables $a \mapsto \psi = Wa$.

Remark 2. The present subsection fixes the initial facts used below. New properties of the effective fields are derived in the following sections.

2.6 Observer and events

Before defining what an event is in the model, we discuss several issues related to this notion and fix the assumptions needed for the subsequent construction.

Real detectors, spacecraft, and similar devices occupy scales not exceeding the size of the Solar System, which is negligible compared to the size of the observable Universe. Using this analogy, within a single inertial reference frame (IRF) we shall regard the working domain Ω as the same for all observers at rest in this IRF. Furthermore, we assume that in each IRF the observable region Ω—the region in which causal reconstruction is possible—is the same for all observers at rest with respect to this IRF.

In modern formulations of quantum theory there is no universally accepted, observation-independent definition of an *event*: different interpretations of quantum mechanics treat the moment of event occurrence differently (collapse, detector registration, state update, etc.). In the present model any observable phenomena depend on the observer and on their measurements. In this subsection we introduce an operational notion of an event, which is proposed as a universal one within the chosen model: it is consistent with the previously constructed causal structure and with the subsequent quantum-operational description (OS/GNS, POVM).

In formulating the notion of an *event* we require:

(i) Operational origin. An event must arise as the result of an interaction of the observer with the field, and not as a pre-given ontological entity. An event is understood as a configuration of interaction between the field modes and the internal modes of the observer, leading to a discrete update of the observer's internal state (a record in memory registers) that is registered by the observer themselves. The observer directly registers only a local measurement outcome, but on its basis reconstructs a network of causes and events that, according to the reconstruction, led to this measurement. This network is a reconstruction rather than an object existing independently. In this way, an event may be interpreted as having occurred far outside the observer's body: by observing a photon from a distant star, the observer reconstructs the network of events that led to its emission.

(ii) Consistency across scales of description. Since the observer has finite extent and spectral limitations, the interaction with the field is described by a projection onto a finite-dimensional subspace. When passing to a coarser description (grouping modes into effective combinations), the detector functional is re-expressed in terms of the new coefficients, but the registration criterion (threshold condition) is preserved. The definition of an event must be stable under such coarse-graining/refinement of the modal decomposition, i.e. must not depend on the chosen resolution of the mode description.

(iii) Compatibility with operational quantum theory. The definition of an event must be consistent with the operational scheme used later: with states on the algebra of observables, with the OS/GNS reconstruction of the Hilbert space, with the description of measurements via POVMs, and with the Born rule. In the present work, events will be regarded as structural elements of the causal network, and acts of measurement as their local sources, implemented via detector functionals.

(iv) Inter-observer consistency. We shall consider a regime in which, upon exchanging information between observers at rest in a given IRF, their reconstructed sets of events coincide (in the sense of an isomorphism of partially ordered sets). This does not lead to a global space of events combining different IRFs: each IRF has its own event network. In the author's previous works this regime was referred to as "classical". In the present work, in order to avoid confusion with the classical limit of quantum theory, we shall call it the *inter-observer event-consistency regime* and assume that the configurations under consideration belong to this regime.

In quantum theories, events are usually associated with particles whose properties are determined by gauge symmetries. In the present model, gauge symmetries are derived at the effective level and are not used for the fundamental definition of an event. We shall assume that, in a measurement, the observer registers the consequences of some event structure in the region Ω for which causal reconstruction is possible. Thus, observation gives rise to a causal network. In the inter-observer event-consistency regime we pass from events that depend on a specific observer to events common to all observers at rest in a given IRF: an analogue of a discrete causal network emerges, specific to each IRF.

Definition of a measurement. Let an observer O fix a foliation Σ_s, a subspace

$$\mathcal{H}^{(O)}_{\text{field}} = \operatorname{span}\{u_\alpha : \alpha \in \Lambda_O\} \subset L^2(\Sigma_s),$$

where $\Lambda_O \subset \Lambda$, and a set of their internal modes $\{\chi_\beta\}$. Their *detector (readout) functional* is given by a local scalar functional

$$\mathcal{R}_O(s) \;=\; F_O\big(\mathbf{a}(s), \mathbf{b}(s)\big), \tag{13}$$

where $\mathbf{a}(s) = (a_\alpha(s))_{\alpha\in\Lambda_O}$ and $\mathbf{b}(s) = (b_\beta(s))$ are the coefficients of the corresponding decompositions on Σ_s.

An act of measurement $\mathsf{M}_O(s_0)$ is defined as the moment s_0 at which

$$\mathcal{R}_O(s_0) \;\geq\; I^{(O)}_{\text{thr}}, \tag{14}$$

where $I^{(O)}_{\text{thr}} > 0$ is the detection threshold. When this condition is satisfied, one of the binary memory registers m_j is switched discretely, $0 \to 1$. The act of measurement is localized in the region of the observer's body $\Omega_O \subset \Omega$.

Remark 3. Refinements of the criterion are possible (for example, by adding extremum or smoothing conditions), but for the purposes of the present section the threshold condition (14) suffices.

Such a functional can be viewed as an analogue of a photodetector: if a combination of signals exceeds the detection threshold, the detector "clicks", recording a one in memory.

Example (bilinear functional). As a special case of (13), one can use a bilinear form

$$\mathcal{M}_O(s) = \sum_{\alpha,\beta} \rho^{(O)}_{\alpha\beta}\, a_\alpha(s)\, b_\beta(s), \tag{15}$$

where $\rho^{(O)}$ is a sensitivity matrix; then $\mathcal{R}_O = \mathcal{M}_O$.

Separation of notions. We shall distinguish between a *local act of measurement* (a detector click inside the observer's body Ω_O) and an *event* as an element of the causal network in the working domain Ω ($\Omega_O \subset \Omega$). Local acts of measurement satisfying (14) generate a subset of observable vertices of the future network.

Definition 1 (Event and causal network in the model)**.** An *event* is a vertex $E \in V_{\text{obs}} \cup V_{\text{rec}}$ of the causal network $\mathcal{C}_{\mathbf{n}} = (V, \prec)$ in the working domain Ω, where:

- $V_{\text{obs}} = \{\mathsf{M}_O(s_k)\} \subset \Omega_O$ are observable (local) vertices generated by acts of measurement (14);
- $V_{\text{rec}} \subset \Omega$ are reconstructed vertices for which there exists an operational causal connection with at least one $\mathsf{M}_O(s_k)$, consistent with the admissible action of the transfer operator (5).

The order $\prec$ is interpreted as the relation "can influence"; it is defined within a fixed IRF and does not require a global set of events common to all IRFs. In what follows, by an event we shall mean a vertex of such a causal network.

Remark. The act of measurement $\mathsf{M}_O(s)$ is a local trigger of recording (vertices V_{obs}), whereas an "event" in the general sense is an element of the network $\mathcal{C}_{\mathbf{n}}$, including both local measurements and reconstructed vertices V_{rec}. Thus, events outside the observer's body are not identified with measurements, but appear as reconstructed elements obtained from measurement data and from the reconstruction rules.

Inter-observer event-consistency regime. We shall say that, in a given IRF, the inter-observer event-consistency regime is realized if, for any two observers at rest in this IRF and exchanging information, their individually reconstructed causal networks $\mathcal{C}_{\mathbf{n}}^{(O_1)}$ and $\mathcal{C}_{\mathbf{n}}^{(O_2)}$ are isomorphic: there exists a bijection of vertices preserving the partial order $\prec$ such that the combined network is again a partially ordered set consistent with the local reconstructions of each observer. In this regime one can speak of a single event structure $\mathcal{C}_{\mathbf{n}}$ in the given IRF (up to relabeling of vertices). In the author's previous works a similar assumption was referred to as the "classical regime". Below, unless stated otherwise, we shall assume that the configurations under consideration are in the inter-observer event-consistency regime. Under a change of IRF, no global unified space of events arises.

Interpretation. Thus, an event is not reduced to an ontological "point in spacetime" but is understood as an element of a discrete causal network arising from the interaction of the field and the observer. The difference from traditional models such as causal set theory is that here, from the outset, to each pair (observer, IRF) there is associated its own causal network $\mathcal{C}_{\mathbf{n}}^{(O)}$, constructed from acts of measurement and reconstructions. In the inter-observer event-consistency regime, the networks of observers at rest in the same IRF and exchanging information are isomorphic and reduce to a common structure $\mathcal{C}_{\mathbf{n}}$ (up to relabeling of vertices). Under a change of IRF, such global identifications generally do not exist, and we do not assume a single space of events for all IRFs.

Remark 4 (Observer as part of the event structure). The observer can be described not only via their modal state but also as part of the event structure itself: their configuration selects a subset of events that are accessible for reconstruction in a given IRF. Unlike standard approaches in causal set theory, here the set of events depends on the chosen foliation, and under a change of foliation the subset of accessible events is rearranged.

Requirements for the observer as a subsystem. In what follows, by an observer O we shall mean not an arbitrary localized subsystem, but a configuration of a body $\Omega_0 \subset \Omega$ for which the following structural conditions hold:

- existence of stable internal registers (a set of modes in Ω_0 with special time scales and weak susceptibility to local field fluctuations), allowing discrete recording of readout results and their long-term storage on the measurement time scale;
- separation of time scales: the characteristic relaxation time of the registers substantially exceeds the duration of a single act of measurement and the characteristic evolution times of effective fields at the scales under consideration;
- reproducibility of measurement protocols: under identical preparation conditions and instrument settings the relative frequencies of outcomes of repeated experiments are stable and are described by the same effective probability measure determined by the state of the effective fields and the chosen protocol;
- localizability and boundedness of the sensitive region: the registers and elements of the measuring device are entirely contained in Ω_0, and the interaction with the field is described by local functionals of the modes in Ω.

These conditions do not introduce additional dynamical degrees of freedom and do not modify the fundamental equation $\Delta_{\mathbb{E}^4}\Phi = 0$, but merely specify the class of subsystems that can play the role of observers in the operational sense. In particular, they ensure the existence of an effective subspace of "memory registers" and make natural the description of measurements via local POVMs on the observer's effective Hilbert space (see §3.7) and averaging over unresolved microstates of the measuring device.

2.7 Direct and observable transformations

Direct and observable transformations. In the absence of a global set of events, the transition between IRFs with normals $\mathbf{n}$ and $\mathbf{n}'$ admits two descriptions [1]. As shown previously, events exist only relative to an observer; without reference to an observer there are no events. In the inter-observer event-consistency regime, the explicit reference to the observer can be neglected.

(i) Direct transformations act by Euclidean symmetries on the field configuration and on the foliation:

$$D_{\mathbf{n}\to\mathbf{n}'} : \mathcal{C}_{\mathbf{n}}^{(O)} \longmapsto \mathcal{C}_{\mathbf{n}'}^{(O')}, \tag{16}$$

where $\mathcal{C}_{\mathbf{n}}^{(O)}$ is the event network reconstructed on the slices $\Sigma_s^{(\mathbf{n})}$ relative to an observer O, and $\mathcal{C}_{\mathbf{n}'}^{(O')}$ is the corresponding network relative to O'. There is no bijection between $\mathcal{C}_{\mathbf{n}}^{(O)}$ and $\mathcal{C}_{\mathbf{n}'}^{(O')}$. The notation with superscripts emphasizes the operational nature of the reconstruction; the mapping $D_{\mathbf{n}\to\mathbf{n}'}$ itself does not require choosing a particular observer. In the present work direct transformations will not be used further; it suffices that, as shown below, observable transformations have Lorentz form.

(ii) Observable transformations are hypothetical transformations constructed by an observer-physicist while remaining in their own IRF and assuming "as if" there existed a global set of events. By construction they preserve the event structure and, in general, depend on the observer O:

$$\mathcal{O}_{\mathbf{n}\to\mathbf{n}'}^{(O)} : \mathbf{b}^{(O,\mathbf{n})}(s) \mapsto \mathbf{b}^{(O,\mathbf{n}')}(s), \tag{17}$$

where $\mathbf{b}^{(O,\mathbf{n})}$ is the internal event register (see §2.6). In the *inter-observer event-consistency regime* the dependence on O disappears and one obtains a universal operator

$$M_{\mathbf{n}\to\mathbf{n}'} : (t,\mathbf{r}) \mapsto (t',\mathbf{r}'). \tag{18}$$

Corollary (SR regime). As shown in [1], the observable transformations (18) have Lorentz form with invariant $v_{\max}$; the Galilean limit is excluded. Equivalently, for any $\Delta x \neq 0$ the null cone is preserved:

$$(\Delta x)^A \eta_{AB} (\Delta x)^B = 0 \implies (\Delta x')^A \eta_{AB} (\Delta x')^B = 0.$$

Consistency between observers in a single IRF. Let O and O' belong to the same IRF and be able to exchange classical messages (inside the null cone of the SR regime). Let $\Omega_\cap$ denote the common part of their accessible regions. Then there exists an isomorphism of event structures

$$\Phi_{O\leftrightarrow O'} : \mathsf{E}_O\restriction_{\Omega_\cap} \longrightarrow \mathsf{E}_{O'}\restriction_{\Omega_\cap},$$

compatible with observable transformations and causal reconstruction (see [1], the section on communication and event consistency). In the absence of communication no consistency is required, and the sets E_O, $\mathsf{E}_{O'}$ may differ.

Remark 5 (Separation of terminology). The operational *observable* $\mathcal{O}_f^O$ is a measurement tool (after OS/GNS, an operator/POVM). An *event* is an element of E_O, i.e. a stable local maximum of $\mathcal{S}_O$ (measured or reconstructed), defined relative to a specific observer O.

In all subsequent sections, Lorentz symmetry and the associated invariants refer exclusively to the observable transformations $M_{\mathbf{n}\to\mathbf{n}'}$ in the SR regime. The direct transformations $D_{\mathbf{n}\to\mathbf{n}'}$ between inertial frames are not required to preserve these structures and are constrained only by the operational conditions of consistency of causal reconstruction (including the coincidence of causal networks in the limit $v \to 0$).

2.8 Isotropy of the foliation stabilizer on a slice

Lemma 1 (Foliation stabilizer and $O(3)$ isotropy on the slice). *Let a foliation direction* $\mathbf{n}$ *be fixed in Euclidean space* $\mathbb{E}^4$*, and let* Σ_s *be a slice orthogonal to* $\mathbf{n}$*. Then the subgroup of local Euclidean transformations that preserves* $\mathbf{n}$ *and a point on* Σ_s *(the stabilizer of the local observational structure* $(\Sigma_s, \mathbf{n})$*) is isomorphic to* $O(3)$*; its identity component is isomorphic to* $SO(3)$ *and acts as the group of orthogonal transformations in the three-dimensional coordinate space on* Σ_s*.*

In a locally orthonormal frame $(t, \mathbf{y})$*, where* t *is directed along* $\mathbf{n}$ *and* $\mathbf{y} \in \mathbb{R}^3$ *parametrize* Σ_s*, any local operators (for example, those acting on effective fields) realize a representation of this group and therefore decompose into a direct sum of* $O(3)$*-irreducible components (spin blocks). The proof follows from the local Euclidean symmetry of the model and from the choice of foliation (see § 2 and the proof in [1]).*

2.9 Locality and linearity of the transfer on Σ_s

Lemma 2 (Locality and linearity of the transfer on Σ_s). *At order* ε^0*, the transfer of effective fields along the foliation is local and linear:*

$$\psi(s+ds, y) = U(s, y; ds)\, \psi(s, y) + \mathcal{O}(ds^2, \partial_y), \qquad U(s, y; ds) = \mathbf{1} + \Gamma(s, y)\, ds + \mathcal{O}(ds^2),$$

where $\Gamma(s, y)$ *is a generator local in* y*, defined in* (12).

3 Transfer dynamics and operational consequences

3.1 State, reflection positivity, and OS/GNS reconstruction

We consider the local algebra generated by smeared observables on the slices Σ_s and a state ω that is stationary with respect to the transfer in s. We define an antilinear reflection Θ in s and the subalgebra $\mathfrak{A}_+$ of operators with time supports $s \geq 0$.

Theorem 1 (OS positivity and OS/GNS reconstruction). *From detailed balance in* s *and the locality of the transfer it follows that* $(F, G)_{\mathrm{OS}} := \omega(\Theta(F)\, G)$ *defines reflection positivity on* $\mathfrak{A}_+$*. There exists a GNS representation* $(\mathcal{H}, \pi, |\Omega\rangle)$ *and a contraction semigroup* $T(\tau) = e^{-\tau H}$ *with* $H \geq 0$*. Analytic continuation yields a unitary evolution* $U(t) = e^{-iHt}$ *in the SR regime.*

The proof is given in Appendix A.

3.2 Reciprocity of the transfer and detailed balance

We rely on the local transfer of effective fields (12) and on the consistent construction of the modal basis (see §2.5 and [1]). Below we do not redefine U, Γ, or α_σ.

Lemma 3 (Joint modal basis and local transfer). *([1]) Let a locally complete orthonormal basis of admissible modes* $\{u_\alpha(y)\}$ *of the fundamental field be given on the slice* Σ_s*, and let the observer's detectors be generated by linear functionals on* $\mathrm{span}\{u_\alpha\}$*. Then for the modal* *coefficients of the field $a_\alpha(s)$ and of the detectors $b_\alpha(s)$ there exists a matrix generator $G(s)$, local in y and C^1 in s, such that, to leading order in gradients,*

$$\frac{d}{ds}\begin{pmatrix} a \\ b \end{pmatrix} = G(s)\begin{pmatrix} a \\ b \end{pmatrix} + \mathcal{O}(\partial_y), \qquad G(s) = \begin{pmatrix} G_\phi(s) & C(s) \\ C^\sharp(s) & G_O(s) \end{pmatrix}. \tag{19}$$

The relation to (12) *is given by the change of variables $a \mapsto \psi = Wa$; in this way G induces Γ without redefining the latter.*

Lemma 4 (Inter-observer consistency $\Rightarrow JG = G^\top J$)**.** *Let $J(s) > 0$ be a sensitivity metric form, and let a change of observer $O \to O'$ be implemented by a local nondegenerate transformation $R(s)$ in the modal space. If, to leading order,*

$Q_s(\delta\psi) := \delta\psi^\top J(s)\,\overline{\delta\psi}$ *is invariant under the transfer and the change of observer:* $Q_{s+ds}\big(R\,e^{ds\,G}\delta\psi\big) = Q_s(\delta$

and $R^\top J R = J + \mathcal{O}(ds^2)$, then in differential form

$$\dot{J} = 0, \qquad JG = G^\top J.$$

Sketch of proof. Linearization gives $\delta\psi^\top(\dot{J} + JG - G^\top J)\overline{\delta\psi} = 0$ for all $\delta\psi$, from which $\dot{J} = 0$ and $JG = G^\top J$ follow. □

Proposition 1 (Detailed balance from causal reconstruction)**.** *Assume Lemmas 3, 4 hold. Let ω be a stationary Gaussian state with covariance $\propto J^{-1}$, and let Θ be the anti-linear reflection $s \mapsto -s$ such that $\omega \circ \Theta = \omega$ and $\Theta\alpha_\sigma = \alpha_\sigma\Theta$. Then, for all cylindrical F, G and all $\sigma \geq 0$, detailed balance holds:*

$$\omega\big(F^*\,\alpha_\sigma(G)\big) = \omega\big((\Theta\,\alpha_\sigma F)^*\,\Theta G\big).$$

Sketch of proof. From $JG = G^\top J$ it follows that the semigroup $\alpha_\sigma = e^{\sigma G}$ is J-selfadjoint and reversible with respect to the scalar product $\langle x, y\rangle_J = x^\top J\,\overline{y}$. Stationarity of ω and its compatibility with Θ transfer this symmetry to the correlators. □

Remark 6 (Non-stationary J). If $\dot{J} \neq 0$, replace G by $\widehat{G} := G - \frac{1}{2}J^{-1}\dot{J}$. Then $J\widehat{G} = \widehat{G}^\top J$, and the above statements remain valid for $\alpha_\sigma = \exp\big(\sigma\widehat{G}\big)$.

Remark 7 (Gauge sector). In the massless vector sector all statements are formulated on a gauge-invariant subalgebra (e.g. the algebra generated by $F_{\mu\nu}$) or after BRST reduction.

Corollary 1 (Detailed balance in s for a stationary state)**.** *Proposition 1 implies detailed balance in s:*

$$\omega\big(F^*\,\alpha_\sigma(G)\big) = \omega\big((\Theta\,\alpha_\sigma F)^*\,\Theta G\big), \qquad \sigma \geq 0.$$

3.3 Constraint on the principal symbol for effective fields

In the previous work [2], the Einstein equations were derived in the timeless model on $\mathbb{E}^4$. Among the results, it was shown that the Hamiltonian density on a foliation does not contain spatial derivatives of order higher than two (see the discussion of the uniqueness of the Einstein–Hilbert action due to Lovelock).

Corollary 2 (Constraint on the principal symbol for effective fields)**.** *In the SR regime, under locality on Σ_s, microcausality, OS positivity, and $O(3)$ isotropy of the stabilizer of the foliation (Lemma 1), if the Hamiltonian density on the slice Σ_s is bounded in the order of spatial derivatives by second order (as in the gravitational sector [2]), then the principal symbol of the local evolution generator of an effective field on Σ_s is of second order.*

Sketch of proof. The second-order bound for the Hamiltonian density excludes spatial derivatives of order higher than two in the evolution generator. Hence its principal symbol has the form $a^{ij}(s,y)\,\xi_i\xi_j$ for some symmetric tensor a^{ij}. $O(3)$ isotropy on Σ_s (Lemma 1) implies $a^{ij}(s,y) = a(s,y)\,\delta^{ij}$. OS positivity excludes non-self-adjoint principal parts and requires $a(s,y) \geq 0$; microcausality eliminates mixed higher-order terms incompatible with causality. Therefore, the principal symbol is of second order and is $O(3)$-scalar. □

3.4 Strong continuity of the transfer in s

We fix the assumptions used in this subsection. All of them have been established previously.

Assumptions and references. (1) Operational events and locality on Σ_s — §2.6. (2) $O(3)$ isotropy of the stabilizer on the slice — Lemma 1. (3) The structure of direct/observable transformations and closure of the local *-algebra under shifts — §2.7. (4) Local linear evolution and the generator: (11)–(12), Lemma 2. (5) Second order of the principal symbol — Corollary 2. (6) Regularity of the coefficients and self-adjoint boundary conditions — see Section 3.4.

Technical requirement (regularity and boundary conditions). We work on the slice Σ_s in the region Ω_s. The local Hamiltonian is given as in (27), where the second-order principal symbol and $O(3)$-scalarity follow from Corollary 2. The coefficients $a^{ij}(s,y) = a(s,y)\delta^{ij}$, $b^i(s,y)$, $V(s,y)$ are local in y and belong to C^1 in s with bounds uniform on compact sets. On $\partial\Omega_s$ we impose self-adjoint boundary conditions (Dirichlet/Neumann/Robin) ensuring vanishing normal probability flux. This guarantees self-adjointness of H and unitarity of $U(t) = e^{-\mathrm{i}Ht}$.

Let Alg_+ denote the *-algebra generated by smeared local observables $A(f)$ with $f \in C_0^\infty(\Omega_s)$, $s \geq 0$.

Lemma 5 (Strong continuity of the transfer semigroup)**.** *Shifts in s define a semigroup of C^*-endomorphisms $\{\alpha_\sigma\}_{\sigma\geq 0}$ on the C^*-closure $\overline{\mathrm{Alg}_+}$,*

$$\alpha_\sigma\big(A(f)\big) := A(f_\sigma), \qquad f_\sigma(s,y) := f(s+\sigma, y),$$

*which is strongly continuous on the dense *-subalgebra Alg_+: for any $X \in \mathrm{Alg}_+$ and any vector Ψ from the common domain of the representation,*

$$\lim_{\sigma\to 0^+} \;\big\|\, \big(\pi(X) - \pi(\alpha_\sigma X)\big)\Psi \,\big\| \;=\; 0.$$

Sketch of proof. From (12) and Lemma 2 we have $U(s,y;ds) = \mathbf{1} + \Gamma(s,y)\,ds + \mathcal{O}(ds^2)$ with a local Γ that is C^1 in s. Uniform bounds and the absence of surface flux are guaranteed by Section 3.4. Then for $A(f)$ one has $\|\, A(f) - A(f_\sigma)\,\| \to 0$ as $\sigma \to 0^+$ by the dominated convergence theorem, using locality of supports and the definition of events (§2.6). For finite products, one uses the Leibniz rule and uniform boundedness of Γ; closure under shifts is provided by §2.7. Density of Alg_+ and continuity of the *-operations complete the proof. □

3.5 Complex structure and analytic continuation

We work on the real pre-Hilbert space of test functions $\mathcal{D}(\Sigma_s) = C_0^\infty(\Sigma_s, \mathbb{R})$. Equal-time *effective fields* are defined by smearing

$$\psi(0,f) := \int_{\Sigma_s} f(y)\,\psi(0,y)\,d\mu_{\Sigma_s}(y), \qquad f \in \mathcal{D}(\Sigma_s),$$

where ψ is understood as an operator-valued distribution on $\mathcal{D}(\Sigma_s)$.

Motivation for complexification. The fundamental field is real; however, to construct a quantum theory of effective fields one needs a canonical choice of complex structure J on the space of initial data. Complexification based on the pair of forms (ω_Σ, μ) (see (20), (21)) provides: (i) the definition of the one-particle space and of the Hermitian form (23); (ii) the introduction of creation/annihilation operators and the CCR/CAR algebras (compatible with microcausality); (iii) compatibility with OS/GNS and the spectral condition (§3.1), which yields analytic continuation and unitary evolution; (iv) a correct probabilistic interpretation and measurements via POVMs (§3.7); (v) a convenient formulation of the relativistically invariant closure (§5). At the same time, the reality of the fundamental field is preserved: J is fixed operationally (via Lemma 6 and Corollary 2) and does not introduce additional dynamical degrees of freedom.

Antisymmetric form. The Pauli–Jordan kernel $E(x,x') := \frac{1}{i}\langle\Omega, [\psi(x), \psi(x')]\Omega\rangle$ induces on the slice

$$\omega_\Sigma(f,g) \ := \ \iint_{\Sigma_s\times\Sigma_s} f(y)\,\left(n^\mu\partial_\mu E\right)\big|_{t=t'=0}(y,y')\,g(y')\,d\mu_{\Sigma_s}(y)\,d\mu_{\Sigma_s}(y'), \tag{20}$$

where n^μ is the unit normal to Σ_s. For gauge fields, a physical/transverse projector is applied before restricting to the slice.

Non-degeneracy on the slice.

Lemma 6 (Non-degeneracy of the form on the slice)**.** *In the SR regime, under locality, microcausality, and second order of the principal symbol (Corollary 2), and after the standard factorization over gauge degrees of freedom, the form ω_Σ from* (20) *is non-degenerate on the physical quotient of the test-function space.*

Sketch of proof. Let $\omega_\Sigma(f,\cdot) = 0$. Then the induced equal-time initial data have vanishing symplectic flux with all admissible data. Hyperbolicity of the SR limit and uniqueness of the Cauchy problem for a second-order operator (the principal symbol is positive definite by Corollary 2) imply that the data are trivial in the physical quotient space. □

Positive form. Define

$$\mu(f,g) \ := \ \tfrac{1}{2}\,\langle\Omega, \{\psi(0,f), \psi(0,g)\}\Omega\rangle, \tag{21}$$

which is positive and compatible with OS reflection positivity.

Complex structure. Define $J : \mathcal{D}(\Sigma_s)\to\mathcal{D}(\Sigma_s)$ by the condition

$$\omega_\Sigma(Ju, v) \ = \ \mu(u,v) \quad \text{for all } u, v. \tag{22}$$

Then $J^2 = -\mathbf{1}$ and $\mu(u,u) = \omega_\Sigma(u, Ju) \geq 0$. The Hermitian form

$$\langle u, v\rangle_{1\mathrm{p}} \ := \ \mu(u,v) \ + \ i\,\omega_\Sigma(u,v) \tag{23}$$

defines the one-particle space after factoring out the null space and completing.

Remark 8. For spinor and massless vector fields, the constructions (20)–(22) are taken componentwise with physical (transverse) projectors or after BRST reduction; see appendix C.

3.6 Measurements: positive-operator-valued measures (POVM) and the Born rule

Motivation. At the fundamental level, the outcome of a local act of measurement is determined once the field configuration Φ and the full microstate of the observer O are fixed. However, operationally the observer only has access to an effective description: they can only use a subalgebra of observables on the slice Σ_s, while the microstate of the measuring device and its environment remains unresolved.

Therefore, at the effective level the result of a repeatedly performed experiment is described statistically. For each fixed measurement protocol and effective state ρ we obtain a probability measure $B \mapsto \mathbb{P}_\rho(B)$ on the outcome space (X, Σ). Below we show that such statistics always admit a description in terms of a POVM and the Born rule, and then interpret this representation as averaging over indistinguishable microstates of the measuring device.

We work in a fixed OS/GNS representation $(\mathcal{H}, \pi, |\Omega\rangle)$ of the local algebra on the slice Σ_s. Let $\mathcal{B}(\mathcal{H})$ denote the C^*-algebra of bounded operators on $\mathcal{H}$.

For a fixed measurement protocol of the observer O and an effective state ρ we write $B \mapsto \mathbb{P}_\rho(B)$ for the probability distribution of outcomes on (X, Σ), defined as the Kolmogorov limit of relative frequencies in repeated runs of the experiment under identical preparation conditions. Mixed states are treated as equivalence classes of ensembles $\{\rho_i, p_i\}_i$ inducing the same expectation functional on $\mathcal{B}(\mathcal{H})$.

Lemma 7 (Structural properties of operational probabilities)**.** *For any measurement protocol of the observer O and any normalized effective state ρ the map $B \mapsto \mathbb{P}_\rho(B)$ is a probability measure on (X, Σ) and satisfies:*

(i) Normalization and σ-additivity. *For all ρ we have $\mathbb{P}_\rho(X) = 1$, $\mathbb{P}_\rho(\emptyset) = 0$, and for any pairwise disjoint $B_n \in \Sigma$*

$$\mathbb{P}_\rho\Big(\bigcup_n B_n\Big) \;=\; \sum_n \mathbb{P}_\rho(B_n),$$

where the series converges absolutely.

(ii) Affinity in the state. *For any states ρ_1, ρ_2 and any $0 \le \lambda \le 1$:*

$$\mathbb{P}_{\lambda\rho_1 + (1-\lambda)\rho_2}(B) \;=\; \lambda\, \mathbb{P}_{\rho_1}(B) + (1-\lambda)\, \mathbb{P}_{\rho_2}(B) \quad \textit{for all } B \in \Sigma.$$

(iii) Operational equivalence of different ensemble decompositions. *If $\rho = \sum_i p_i \rho_i = \sum_j q_j \rho'_j$ are two convex representations of the same state, then for all $B \in \Sigma$*

$$\sum_i p_i\, \mathbb{P}_{\rho_i}(B) \;=\; \sum_j q_j\, \mathbb{P}_{\rho'_j}(B) \;=\; \mathbb{P}_\rho(B).$$

Sketch of proof. Normalization and σ-additivity in (i) are part of the definition of $\mathbb{P}_\rho(\cdot)$ as a probability measure on (X, Σ). Property (ii) follows from the interpretation of a mixed state $\rho = \lambda\rho_1 + (1-\lambda)\rho_2$ as classical random preparation: the relative frequencies of outcomes are averages over classical randomness, hence the probabilities are linear in ρ. Finally, (iii) reflects the definition of a state as an equivalence class of ensembles: if two ensembles realize the same effective state, then all measurements yield the same outcome statistics, which is precisely the content of (iii). □

Definition 2 (Positive-operator-valued measure (POVM))**.** Let (X, Σ) be a measurable outcome space. A *POVM* is a map $E : \Sigma \to \mathcal{B}(\mathcal{H})$ such that:

(a) $E(\emptyset) = 0$, $E(X) = \mathbf{1}$;

(b) $E(B) \geq 0$ for all $B \in \Sigma$;

(c) E is countably additive in the weak operator topology, i.e.

$$E\Big(\bigcup_n B_n\Big) = \text{s-}\lim_{N\to\infty} \sum_{n=1}^{N} E(B_n)$$

for any pairwise disjoint $B_n \in \Sigma$.

Lemma 8 (POVM representation of operational measurements)**.** *Assume the properties of Lemma 7 hold for a fixed measurement protocol of the observer O on the Hilbert space $\mathcal{H}$. Then there exists a unique (up to $\mathbb{P}_\rho$-null sets) POVM $E : \Sigma \to \mathcal{B}(\mathcal{H})$ such that for all states ρ and all $B \in \Sigma$ the outcome probabilities are given by the Born rule*

$$\mathbb{P}_\rho(B) \;=\; \operatorname{tr}\big(\rho\, E(B)\big).$$

In particular, the assumption (A5) *on the existence of a POVM representation of measurements follows from the operational model of mixed states and probabilities.*

Sketch of proof. For each $B \in \Sigma$ the map $\rho \mapsto \mathbb{P}_\rho(B)$, by (ii)–(iii) of Lemma 7, is affine in ρ and, upon linear extension, defines a bounded positive linear functional on the space of density operators on $\mathcal{H}$. By duality between states and effects on $\mathcal{B}(\mathcal{H})$ there exists a unique operator $E(B) \in \mathcal{B}(\mathcal{H})$ with $0 \leq E(B) \leq \mathbf{1}$ such that $\mathbb{P}_\rho(B) = \operatorname{tr}(\rho E(B))$ for all ρ. Normalization $\mathbb{P}_\rho(X) = 1$ and $\mathbb{P}_\rho(\emptyset) = 0$ yields $E(X) = \mathbf{1}$ and $E(\emptyset) = 0$. The property (i) of σ-additivity in B_n transfers to the operator-valued map $E(\cdot)$ in the weak operator topology, since the functionals $\rho \mapsto \operatorname{tr}(\rho\,\cdot)$ separate points of $\mathcal{B}(\mathcal{H})$. Thus E is a POVM with the required properties. ☐

Proposition 2 (Born rule in OS/GNS)**.** *For a normalized vector $\psi \in \mathcal{H}$ the outcome probabilities are*

$$\mathbb{P}_\psi(B) = \langle\psi|\, E(B)\, |\psi\rangle\,, \qquad B \in \Sigma,$$

and for a mixed state ρ (a positive trace-one operator)

$$\mathbb{P}_\rho(B) = \operatorname{tr}\big(\rho\, E(B)\big).$$

Sketch of proof. The first equality is a special case of Lemma 8 for vector states $\rho = |\psi\rangle\,\langle\psi|$. The extension to arbitrary mixed states follows from the linearity of the trace and the representation of ρ as a convex combination of vector states. ☐

Projective measurements. If E is projective (a PVM), then $E(B_1 \cap B_2) = E(B_1)E(B_2)$ and $E(B)$ is an orthogonal projector for all B. For a discrete spectrum $X = \{x_n\}$, with $E(\{x_n\}) = P_n = |\phi_n\rangle\,\langle\phi_n|$ and decomposition $\psi = \sum_n c_n\phi_n$, we have

$$\mathbb{P}_\psi(\{x_n\}) = \|P_n\psi\|^2 = |\,\langle\phi_n|\psi\rangle\,|^2.$$

General POVM via Kraus operators. In the discrete case let $\{M_k\} \subset \mathcal{B}(\mathcal{H})$ satisfy $\sum_k M_k^\dagger M_k = \mathbf{1}$. Then $E(\{k\}) = M_k^\dagger M_k$ defines a POVM and

$$\mathbb{P}_\psi(k) = \|M_k\psi\|^2, \qquad \mathbb{P}_\rho(k) = \operatorname{tr}(\rho\, M_k^\dagger M_k).$$

In the continuous case, on a measurable space (X, Σ, μ), take a measurable field of operators $x \mapsto M(x)$ with $\int_X M(x)^\dagger M(x)\,\mathrm{d}\mu(x) = \mathbf{1}$ and set

$$E(B) = \int_B M(x)^\dagger M(x)\,\mathrm{d}\mu(x), \qquad \mathbb{P}_\rho(B) = \int_B \operatorname{tr}\big(\rho\, M(x)^\dagger M(x)\big)\,\mathrm{d}\mu(x).$$

Post-measurement dynamics (Kraus/Lüders instruments). From the instrumental point of view, the measurement of an outcome k is fully specified by a completely positive normalized map

$$\mathcal{I}_k(\rho) = \sum_{\alpha} M_{k\alpha}\,\rho\,M_{k\alpha}^{\dagger}, \qquad \sum_{k,\alpha} M_{k\alpha}^{\dagger} M_{k\alpha} = \mathbf{1},$$

where $E(\{k\}) = \sum_{\alpha} M_{k\alpha}^{\dagger} M_{k\alpha}$ and $\mathbb{P}_{\rho}(k) = \mathrm{tr}\big(\mathcal{I}_k(\rho)\big) = \mathrm{tr}\big(\rho\, E(\{k\})\big)$.

Time covariance and locality. Let $\{U(t)\}_{t\in\mathbb{R}}$ be a one-parameter unitary group on $\mathcal{H}$ implementing the effective dynamics in the Heisenberg picture (its construction will be given below in Section 4, devoted to the Schrödinger equation). Then for any POVM E the family

$$E_t(B) := U(t)^{\dagger} E(B)\, U(t)$$

is again a POVM for all $t \in \mathbb{R}$. For localized measurements in regions $O \subset \Sigma_s$ we consider families $E_O : \Sigma \to \mathcal{B}(\mathcal{H})$. If the closures $\overline{O}$ and $\overline{O'}$ are spacelike separated, then by microcausality of the local algebras the corresponding POVM commute:

$$[\, E_O(B),\, E_{O'}(B')\,] = 0, \qquad \forall\, B, B' \in \Sigma.$$

This guarantees consistency of the statistics of independent measurements on a foliation.

The interpretation of measurement statistics is based on the following assumption. It is purely operational in nature: it is used only to describe averaged outcome frequencies for a fixed protocol of preparation of the effective state and fixed settings of the measuring device, and it does not enter the derivation of the QFT core. A rigorous derivation of the existence and properties of the measure ν_O (as an effective description of the distribution over the microstates of the measuring device and its environment for a given protocol) is left beyond the scope of the present work.

Assumption 1 (Averaging over device microstates). For a fixed measurement protocol of the observer O and fixed procedures of preparing the effective state (in the sense of the choice of foliation, working region Ω_s, and detector settings) there exist a measurable space of microstates (Θ_O, ν_O) and a family of POVM E_θ on the effective Hilbert space such that the effective POVM admits an integral mixture representation

$$E(B) \;=\; \int_{\Theta_O} E_\theta(B)\, d\nu_O(\theta),$$

where B is a Borel subset of the outcome space.

Proposition 3 (Operational interpretation of the Born rule as averaging)**.** *Under Assumption 2, for any state ρ we have*

$$\mathbb{P}_\rho(B) \;=\; \mathrm{tr}\big(\rho\, E(B)\big) \;=\; \int_{\Theta_O} \mathrm{tr}\big(\rho\, E_\theta(B)\big)\, d\nu_O(\theta).$$

Sketch of proof. The equality follows from the linearity of the trace and the definition of E as the ν_O-average. A Kraus-type representation $E_\theta(\{k\}) = \sum_\alpha M_{k\alpha}(\theta)^{\dagger} M_{k\alpha}(\theta)$ is compatible with this construction. □

Remark 9 (No signalling). If $\overline{O}$ and $\overline{O'}$ are spacelike separated, then by microcausality of the local algebras $[\mathcal{A}(O), \mathcal{A}(O')] = 0$. Then for any local measurement procedures (including the averages of Assumption 2) the marginal distributions in O do not depend on the choice of measurement in O'.

Remark 10 (Determinism at the fundamental level). In the model, the outcome of a local act M_O for fixed (Φ, θ) is given by a deterministic function (through the interaction of the observer's modes with the field). The operational statistics of outcomes are described as averaging over unobserved microstates θ of the measuring device:

$$E(B) = \int_{\Theta_O} E_\theta(B)\, d\nu_O(\theta), \qquad \mathbb{P}_\rho(B) = \int_{\Theta_O} \mathrm{tr}\big(\rho\, E_\theta(B)\big)\, d\nu_O(\theta) = \mathrm{tr}\big(\rho\, E(B)\big).$$

In other words, we use an integral mixture of POVM over a measure ν_O on the space of device microparameters (when $\theta \mapsto E_\theta(B)$ is measurable, the map E is again a POVM). The Heisenberg inequality Corollary 3 and the Born rule Proposition 4 are kinematical consequences of the operator algebra and do not assume statistical independence of settings; see Remark 17 on compatibility with Bell tests and superdeterminism.

The description of the measurement procedure allows us to introduce a definition of the material sector of effective fields, which will be used later.

Definition 3 (Matter sector)**.** We call those effective fields (scalar/spinor/vector) "matter" whose gauge-invariant local functionals generate a subalgebra $\mathcal{A}_{\mathrm{matter}}(O) \subset \mathcal{A}(\Sigma_s)$ with the following properties:

- (i) microcausality holds (local algebras commute for spacelike separations);
- (ii) on the slice Σ_s they realize equal-time canonical commutation relations for bosonic fields and canonical anticommutation relations for fermionic fields;
- (iii) they admit local positive-operator-valued measures (POVM) E_O as in §3.7;
- (iv) they contribute nontrivially to the local energy–momentum tensor density $T_{\mu\nu}$.

The gravitational sector is described by the state of the foliations (g_{ij}, K_{ij}) and, as we show in 5.7, is not quantized.

Remark 11 (Gauge sector). The requirement "admits POVM" is necessary but not sufficient on its own. In the massless vector sector all statements are formulated on the gauge-invariant subalgebra (for example, the algebra generated by $F_{\mu\nu}$), or after BRST reduction.

3.7 Operational status of measurements and events

In the present work, measurements and events are treated from the outset in an operational way, i.e. relative to a fixed localized observer and its working region $\Omega_s \subset \Sigma_s$. The fundamental level of the model is specified by a solution of the Laplace equation $\Delta_{\mathbb{E}^4}\Phi = 0$ on Euclidean space $\mathbb{E}^4$; no global time, no global causal structure, and no global space of events are postulated at this level. Event structure arises as something reconstructed by an observer for a fixed choice of foliation $\{\Sigma_s\}$ and working region, and does not possess an observer-independent ontological status outside the given "observer–foliation" pair.

As discussed earlier in the definition of events, an event for a given observer is defined as a localized configuration of excitations in its working region that maximizes the corresponding sensitivity functional and is stable under small variations of the foliation and of the transfer parameter s. In other words, an event in the model is not a "fact given by itself" but the outcome of a specific reconstruction procedure implemented by a given observer within a fixed working region. Outside the context of an observer and its reconstruction procedures, the notion that "an event has occurred" has no operational content.

The definition of measurement in §3.7 is a special case of this general definition of events. A measurement is that subclass of events which is associated with a prescribed set of effects

(POVM elements) in the local algebra of observables of a given observer and admits a stable identification of discrete outcomes. Let $E(\Delta)$ denote the effect corresponding to registering an outcome in a Borel set Δ of values of some operational quantity. Then the measurement protocol of a given observer is fully described by the set of operators $E(\Delta)$ on its effective Hilbert space, and the statistics of the registered outcomes are determined by the state ω (or by a vector $|\psi\rangle$ in the OS/GNS representation).

Thus, from the perspective of the model under consideration, measurement is not introduced as a separate dynamical process governed by laws different from those of the rest of the evolution. The only evolution that appears at the effective level is given by the unitary group $U(t) = e^{-iHt}$, obtained in the OS/GNS representation, and it acts both on "state preparation" and on "measurement interactions" in the observer's working region. No additional non-unitary step (wave-function collapse) is postulated: events and measurement outcomes are simply those operational structures that the observer singles out in its algebra of observables, and their probabilities are determined by the same state ω. A formal justification that the observed outcome frequencies of any admissible measurement protocol are uniquely realized as Born averages $\omega(E(\Delta))$ (or $\langle\psi|E(\Delta)|\psi|\psi|E(\Delta)|\psi\rangle$ for pure states) will be given in Proposition 5.

It is also important to emphasize that the model contains a built-in condition of inter-observer consistency. Different observers are described by different foliations and working regions $\Omega_s^{(A)}$, $\Omega_{s'}^{(B)}$ and, accordingly, by different local algebras of observables and POVMs. However, in regions where their working areas overlap and classical communication is possible, they are required to obtain identical statistics for events associated with common operational procedures. In terms of the effective theory this means the existence of a consistent map between the local algebras such that the Born probabilities for jointly implementable measurements coincide. It is precisely this condition of inter-observer consistency, formulated and used above in the discussion of causal reconstruction and admissible solutions, that ensures that observers sharing the same working region and agreed measurement procedures obtain mutually compatible statistical descriptions, despite the absence of a global ontological event space at the fundamental level.

Taken together, these remarks fix the following. Within the operational framework adopted in the present work, events and measurements exist only as relative (operational) objects, defined for a specific observer and its foliation; their statistics are entirely given by Born averages with respect to POVMs reconstructed from the same Euclidean model (see Proposition 5). In these conditions, the standard conflict between unitary evolution and the statistical description of measurements does not arise: a global event space and a separate dynamical mechanism for wave-function collapse are not part of the description. A more detailed discussion of the philosophical implications of this construction lies beyond the scope of the present paper and is not given here.

3.8 Structural assumptions and domain of applicability

In Section 3 we use several structural assumptions that specify the class of regimes for which all results are formulated. Some of them are direct consequences of the Euclidean model $\Delta_{\mathbb{E}^4}\Phi = 0$ and of the causal reconstruction requirements (in particular in the SR-regime), while others represent natural structural restrictions on the class of states, Hamiltonians, and measurement procedures under consideration.

For clarity we group them into three levels:

- *Level I: mathematical consistency.* Assumptions expressing general requirements of internal consistency of the effective theory (OS/GNS, spectrum condition, absence of anomalies in the dynamics, compatibility with microcausality), independently of the particular "type of world" under consideration.
- *Level II: generic conditions for the existence of an observer.* Assumptions reflecting the

possibility of the existence of a stable localized observer with reproducible measurement procedures, working region Ω_s, internal registers, and operationally definable measurements. Since in the proposed model the observer is an essential element of the formulation, the Level II assumptions are, in their fundamental status, equivalent to those of Level I; the separation into Levels I and II is introduced solely to distinguish logically between "pure" mathematical consistency requirements and those directly related to the existence of an observer.

- *Level III: specification of the matter sector.* Assumptions specifying the choice of a distinguished "matter" subalgebraic sector and its operational description (microcausality, CCR/CAR, contribution to $T_{\mu\nu}$), which do not follow automatically from the transfer structure and OS/GNS and reflect which modes are interpreted as matter.

In this notation the assumptions (A1)–(A7) are formulated as follows.

(A1) *Class of states (Level I/II).* We consider the state of an inertial observer that is stationary with respect to transfer in the parameter s and is compatible with the SR-regime and the OS/GNS representation (reflection symmetry, detailed balance, spectrum condition). The observer is assumed to satisfy the structural requirements of §2.6 (localization, existence of stable internal registers, reproducibility of measurement protocols).

(A2) *Second order in spatial derivatives (Level I).* The density of the effective Hamiltonian in the working region Ω_s is local in y and has a principal symbol of second order in spatial derivatives, in accordance with the gravitational sector [2].

(A3) *Regularity and boundary conditions (Level I).* The coefficients of the local Hamiltonian are regular in (s, y), the boundary $\partial\Omega_s$ carries boundary conditions ensuring vanishing probability flux, and the operator H admits a self-adjoint extension and generates a unitary group $U(t) = e^{-iHt}$ consistent with OS/GNS.

(A4) *Complex structure and one-particle space (Level I).* In the SR-regime a canonical complex structure J is fixed, compatible with the symplectic form and the positive form μ, thereby defining a one-particle Hilbert space that realizes the local Poincaré group and the spectrum condition.

(A5) *POVM formalism for measurements (Level II).* All operationally accessible measurements for the observer are described by positive operator-valued measures (POVMs) $E(\cdot)$ and the associated completely positive (CP) measurement instruments on $(\mathcal{H}, \mathcal{A}(\Omega_s), \omega)$, i.e. the standard general constructions of quantum measurement theory in the algebraic framework are used (see §3.7).

(A6) *Averaging over internal degrees of freedom of the apparatus (Level II).* For a fixed measurement protocol of the observer O and fixed procedures for preparing the effective state (in the sense of choosing the foliation, the working region Ω_s, and the detector settings), the effective POVMs can be represented as mixtures $\{E_\theta\}$ with respect to a measure ν_O on a parameter space Θ_O,

$$E(B) = \int_{\Theta_O} E_\theta(B)\, d\nu_O(\theta),$$

as formulated in Assumption 2.

(A7) *Fundamental determinism and the matter sector (Level III).* Outcomes of concrete measurements are regarded as deterministic functionals of the full fundamental configuration Φ and of the microstate of the apparatus; at the effective level one selects a subalgebra $\mathcal{A}_{\text{matter}}(O) \subset \mathcal{A}(\Omega_s)$ with prescribed properties (microcausality, CCR/CAR, contribution to $T_{\mu\nu}$; see Definition 5), which is interpreted as the matter sector.

A detailed discussion of assumptions (A1)–(A7), their relation to the fundamental Euclidean model and to the results of [1, 2], as well as the motivation for the chosen class of states, Hamiltonians, and measurement procedures, is given in Appendix B. In all statements of Section 3 the validity of conditions (A1)–(A7) is assumed.

3.9 Structural assumptions and domain of applicability

Section 3 uses several structural assumptions that specify the class of regimes for which all results are formulated. Some of them are direct consequences of the Euclidean model $\Delta_{\mathbb{E}^4}\Phi = 0$ and of the requirements of causal reconstruction (in particular in the SR–regime), while others are natural structural constraints on the class of states, Hamiltonians, and measurement procedures under consideration.

For clarity we divide them into three levels:

- *Level I: mathematical consistency.* Assumptions expressing general requirements of internal consistency of the effective theory (OS/GNS, spectral condition, absence of anomalies in the dynamics, compatibility with microcausality), independently of any particular "type of world".

- *Level II: general conditions for the existence of an observer.* Assumptions encoding the possibility of the existence of a stable localized observer with reproducible measurement procedures, a working region Ω_s, internal registers, and operationally definable measurements. Since in the proposed model the observer is an essential element of the setup, Level II assumptions are, in terms of fundamental status, equivalent to Level I assumptions; the split into Levels I and II is introduced solely to separate "pure" mathematical consistency requirements from those directly related to the existence of an observer.

- *Level III: specification of the matter sector.* Assumptions specifying the choice of a distinguished "matter" subalgebra sector and its operational description (microcausality, CCR/CAR, contribution to $T_{\mu\nu}$), which do not follow automatically from the structure of the s–transport and OS/GNS, and reflect which modes are interpreted as matter.

Below we refer uniformly to assumptions (A1)–(A7), but we indicate which of them are independent structural assumptions defining the class of worlds under consideration, and which are derived structural properties, rigorously established in later sections or in [1, 2]. We keep a single numbering for readability.

In this notation the assumptions (A1)–(A7) are formulated as follows.

(A1) *Class of states (Level I/II; structural assumption).* We consider a state of an inertial observer that is stationary with respect to transport in the parameter s and compatible with the SR–regime and the OS/GNS representation (reflection symmetry, detailed balance, spectral condition). The observer is assumed to satisfy the structural conditions of §2.6 (localization, presence of stable internal registers, reproducibility of measurement protocols). Thus (A1) fixes the class of states and observers under consideration and is not automatically derived from the equation $\Delta_{\mathbb{E}^4}\Phi = 0$ alone.

(A2) *Second order in spatial derivatives (Level I; structural assumption).* The density of the effective Hamiltonian in the working region Ω_s is local in y and has principal symbol of second order in spatial derivatives, in accordance with the gravitational sector [2]. This assumption specifies the class of admissible effective Hamiltonians and is motivated by the requirement of locality and by consistency with the derivation of GR, but is not proved in the present work as a necessary consequence of the fundamental Euclidean model.

(A3) *Regularity and boundary conditions (Level I; structural assumption).* The coefficients of the local Hamiltonian are regular in (s, y), the boundary $\partial\Omega_s$ carries boundary conditions ensuring vanishing probability flux, and the operator H admits a self-adjoint extension and generates a unitary group $U(t) = e^{-iHt}$ compatible with OS/GNS. These conditions single out a class of problems with well-posed spectral theory and unitary dynamics and are regarded as structural assumptions of mathematical consistency.

(A4) *Complex structure and one-particle space (Level I; structural assumption motivated by the SR–regime).* In the SR–regime we choose a complex structure J compatible with the symplectic form and the positive form μ so that the resulting one-particle Hilbert space admits a representation of the local Poincaré group and of the spectral condition. This choice is consistent with the operational reconstruction of the structure of special relativity in [1] (Lorentzian structure, Minkowski invariance, positivity of energy), but in the present paper is adopted as an explicit structural assumption for the description of the one-particle sector.

(A5) *POVM formalism for measurements (Level II; partly derived property).* All operationally accessible measurements for the observer are described by positive-operator-valued measures (POVM) $E(\cdot)$ and the corresponding completely positive (CP) measurement instruments on $(\mathcal{H}, \mathcal{A}(\Omega_s), \omega)$; that is, we use the standard general framework of quantum measurement theory in the algebraic setting (see §3.7). The existence of POVM and the representation of operational probabilities in Born-rule form are derived structural properties, rigorously established from the OS/GNS structure and the operational requirements on $\mathbb{P}_\rho(\cdot)$ in §3.7; the use of CP instruments is taken as a standard operational assumption of the general theory of quantum measurements.

(A6) *Averaging over internal degrees of freedom of the device (Level II; operational assumption).* For a fixed measurement protocol of the observer O and fixed procedures for preparing the effective state (in the sense of the choice of foliation, working region Ω_s, and detector settings) the effective POVM can be represented as a mixture $\{E_\theta\}$ with respect to a measure ν_O on a parameter space Θ_O,

$$E(B) \;=\; \int_{\Theta_O} E_\theta(B)\, d\nu_O(\theta),$$

as formulated in Assumption 2. This assumption provides an operational model of averaging over unobserved microstates of the device and is not derived in the present paper directly from the fundamental Euclidean dynamics, although it is compatible with it.

(A7) *Fundamental determinism and matter sector (Level III; structural assumption + derived property).* The outcomes of concrete measurements are regarded as deterministic functionals of the full fundamental configuration Φ and of the device microstate; this is the basic structural assumption of the model at the fundamental level. At the effective level one singles out a subalgebra $\mathcal{A}_{\mathrm{matter}}(O) \subset \mathcal{A}(\Omega_s)$ with prescribed properties (microcausality, CCR/CAR, contribution to $T_{\mu\nu}$; see Definition 5), interpreted as the matter sector. The properties of this subalgebra are realized in the examples constructed below for scalar, spinor, and vector fields and in this sense are derived structural facts within the chosen class of worlds.

A detailed discussion of assumptions (A1)–(A7), their relation to the fundamental Euclidean model and to the results of [1, 2], as well as the motivation for the choice of the class of states, Hamiltonians, and measurement procedures considered, is given in Appendix B. In all statements of Section 3 we assume that conditions (A1)–(A7) hold.

4 Schrödinger equation on foliations and probability continuity

We assume that a strongly continuous unitary group $U(t) = e^{-iHt/\hbar}$ has been constructed on the Hilbert space of states $\mathcal{H}$, where $H = H^\dagger \geq 0$ is a self-adjoint Hamiltonian with dense domain $D(H) \subset \mathcal{H}$.

In the present section no additional structural assumptions are introduced beyond (A1)–(A3), the OS/GNS construction of Section 3.1, and the existence of local POVMs as formulated in Section 3.7. The additional assumption of the existence of a density $E(\mathrm{d}y) = \Pi(y)\, d\mu_{\Sigma_s}(y)$ is used only to pass from the weak form (26) to the pointwise continuity equation and does not enter the global assumptions of the model.

Schrödinger equation. For the state vector $|\psi(t)\rangle := U(t)\,|\psi_0\rangle$ one has

$$i\,\hbar\,\partial_t\,|\psi(t)\rangle = H\,|\psi(t)\rangle\,, \qquad |\psi(0)\rangle = |\psi_0\rangle\,, \tag{24}$$

where the normalization of $\hbar$ is consistent with the canonical commutation relations and the two-point functions. The equality (24) is understood on $D(H)$.

Smeared probabilities. Let $E(\mathrm{d}y)$ be a positive operator-valued measure (POVM) on the slice Σ_s, and let $\varphi \in C_0^\infty(\Omega_s)$. Define the smeared effect

$$\Pi(\varphi) := \int_{\Omega_s} \varphi(y)\, E(\mathrm{d}y)$$

and the smeared "probability density"

$$\rho_\varphi(t) := \big\langle \psi(t) \big|\, \Pi(\varphi)\, \big| \psi(t) \big\rangle.$$

In the Heisenberg picture, $\Pi_t(\varphi) := U(t)^\dagger \Pi(\varphi) U(t)$, and for $\psi(0) \in D(H)$, so that $\psi(t) \in D(H)$, we obtain

$$\partial_t \rho_\varphi(t) = \frac{i}{\hbar}\,\big\langle \psi(t) \big|\, [H, \Pi(\varphi)]\, \big| \psi(t) \big\rangle. \tag{25}$$

Weak form of the continuity equation. Consider the dynamics on the slice Σ_s defined by a local second-order Hamiltonian in the working region Ω_s, satisfying the structural conditions (A2)–(A3) (locality in y, local $O(3)$ isotropy on the slice, second-order principal symbol in spatial derivatives, regularity and boundedness of the coefficients, and boundary conditions on $\partial\Omega_s$ ensuring self-adjointness of the operator and absence of probability flux through the boundary). Then there exists a (generalized) probability current $j^i(t, \cdot)$ such that

$$\partial_t \rho_\varphi(t) = -\int_{\Omega_s} \nabla_i \varphi(y)\, j^i(t, y)\, \mathrm{d}\mu_{\Sigma_s}(y), \qquad \forall\, \varphi \in C_0^\infty(\Omega_s). \tag{26}$$

If, in addition, the POVM has a density $E(\mathrm{d}y) = \Pi(y)\, \mathrm{d}\mu_{\Sigma_s}(y)$, then (26) is equivalent to the pointwise form $\partial_t \rho(t, y) + \nabla_i j^i(t, y) = 0$ in the sense of distributions.

Explicit form of the current for a local second-order Hamiltonian. Suppose that within the structural assumptions of second-order locality formulated in Section 5, the Hamiltonian in the region $\Omega_s \subset \Sigma_s$ has the local form

$$H = \int_{\Omega_s} \Big(\tfrac{1}{2}\, \hat{\psi}^\dagger\, \hat{h}\, \hat{\psi} + \text{h.c.} \Big)\, \mathrm{d}\mu_{\Sigma_s}, \quad \hat{h} = -\frac{\hbar^2}{2}\, \nabla_i\big(a^{ij}(y)\, \nabla_j\big) + b^i(y)\, \nabla_i + V(y), \tag{27}$$

where the coefficients $a^{ij} = a^{ji}$, b^i, and V are sufficiently regular and bounded in Ω_s, and the boundary conditions on $\partial\Omega_s$ are chosen so as to ensure self-adjointness of H. Then

$$j^i(t,y) = \frac{\hbar}{2i} \left\langle \psi(t) \middle| \Pi(y)\, a^{ij}(y) \overleftrightarrow{\nabla}_j \middle| \psi(t) \right\rangle + \frac{\hbar}{2} \left\langle \psi(t) \middle| \Pi(y) \left(b^i(y) + b^i(y)^\dagger\right) \middle| \psi(t) \right\rangle, \tag{28}$$

where $A\overleftrightarrow{\nabla}_j B := A(\nabla_j B) - (\nabla_j A)B$. Substituting (27) into (25) yields (26). Integrating (26) over Ω_s gives conservation of the total probability; in the presence of flux across the boundary $\partial\Omega_s$ one obtains a surface contribution $\int_{\partial\Omega_s} \varphi\, j^i n_i \, d\sigma$, interpreted as a leakage of probability through the boundary of the region.

Remark 12 (On assumptions and domain). The equalities (24)–(25) follow from the existence of a self-adjoint Hamiltonian H and of the unitary group $U(t) = e^{-iHt/\hbar}$ on a dense invariant domain in $\mathcal{H}$ (see also the OS/GNS construction in Section 3.1).

Formulas (26)–(28) rely on the structural assumptions of second-order locality for H (a local operator of the form (27) with sufficiently regular and bounded coefficients, $a^{ij} = a^{ji}$) and on the choice of boundary conditions on $\partial\Omega_s$ ensuring self-adjointness of H. The normalization of $\hbar$ is consistent with the symplectic form and the canonical (anti)commutation relations used in the construction of the effective field algebra.

4.1 Robertson–Schrödinger uncertainty inequalities

Let $A = A^\dagger$ and $B = B^\dagger$ be symmetric (self-adjoint) observables on $\mathcal{H}$ with dense domains. For a normalized state $\psi \in D(A) \cap D(B)$ set

$$\langle A \rangle_\psi := \langle \psi | \, A \, |\psi\rangle \,, \quad \Delta A := A - \langle A \rangle_\psi \mathbf{1}, \quad \mathrm{Var}_\psi(A) := \|\Delta A\, \psi\|^2.$$

Proposition 4 (Robertson)**.** *For any self-adjoint A, B and $\psi \in D(A) \cap D(B)$ one has*

$$\mathrm{Var}_\psi(A)\, \mathrm{Var}_\psi(B) \;\geq\; \frac{1}{4}\, |\langle \psi | [A,B] | \psi \rangle|^2 \,. \tag{29}$$

Proof. Consider $(\Delta A + i\lambda \Delta B)\psi$ for $\lambda \in \mathbb{R}$:

$$0 \leq \|(\Delta A + i\lambda\Delta B)\psi\|^2 = \mathrm{Var}_\psi(A) + \lambda^2 \mathrm{Var}_\psi(B) + \lambda\, \langle \psi | \; i[\Delta A, \Delta B]\, \psi \, |.\rangle$$

The discriminant is non-positive, which yields (29), since $[\Delta A, \Delta B] = [A, B]$. □

Introduce the covariance

$$\mathrm{Cov}_\psi(A,B) := \tfrac{1}{2}\, \langle \psi \mid \{\Delta A, \Delta B\} \mid \psi \rangle.$$

Proposition 5 (Schrödinger–Robertson)**.** *For any self-adjoint A, B and $\psi \in D(A) \cap D(B)$ one has*

$$\mathrm{Var}_\psi(A)\, \mathrm{Var}_\psi(B) \;\geq\; \mathrm{Cov}_\psi(A,B)^2 \;+\; \frac{1}{4}\, |\langle \psi | [A,B] | \psi \rangle|^2 \,. \tag{30}$$

Proof. By the Cauchy–Bunyakovsky inequality, $\|\Delta A\, \psi\|^2\, \|\Delta B\, \psi\|^2 \;\geq\; |\, \langle \psi | \, \Delta A\, \Delta B\, \psi \, || \rangle^2$. The decomposition $\Delta A\, \Delta B = \frac{1}{2}\{\Delta A, \Delta B\} + \frac{1}{2}[\Delta A, \Delta B]$ yields (30). □

Canonical pair. If for a pair (X, P) one has $[X, P] = i\hbar\, \mathbf{1}$ (with $\hbar$ normalized consistently with the canonical commutation relations), then from (29) it follows that

$$\Delta X\, \Delta P \;\geq\; \frac{\hbar}{2}. \tag{31}$$

In the multidimensional case, for components X^i, P_j with $[X^i, P_j] = i\hbar\, \delta^i{}_j\, \mathbf{1}$ one obtains $\Delta X^i\, \Delta P_i \geq \hbar/2$ for each i.

Corollary 3 (Heisenberg inequality). *Let $X = X^\dagger$ and $P = P^\dagger$ be a canonical pair on $\mathcal{H}$ with $[X, P] = i\hbar\,\mathbf{1}$ (the commutator understood on the common domain $D(XP) \cap D(PX)$). Then for any normalized $\psi \in D(X) \cap D(P) \cap D(XP) \cap D(PX)$,*

$$\Delta X\,\Delta P \;\geq\; \frac{\hbar}{2}, \qquad \Delta X := \big\|\big(X - \langle\psi \mid X \mid \psi\rangle\big)\psi\big\|, \;\; \Delta P := \big\|\big(P - \langle\psi \mid P \mid \psi\rangle\big)\psi\big\|. \tag{32}$$

Proof. Set $\Delta X := X - \langle\psi \mid X \mid \psi\rangle\,\mathbf{1}$ and $\Delta P := P - \langle\psi \mid P \mid \psi\rangle\,\mathbf{1}$. For any $\lambda \in \mathbb{R}$, by positivity of the norm,

$$0 \;\leq\; \big\|\big(\Delta X + i\lambda\,\Delta P\big)\psi\big\|^2 = \langle\psi \mid \Delta X^2 \mid \psi\rangle + \lambda^2\langle\psi \mid \Delta P^2 \mid \psi\rangle + \lambda\,\langle\psi \mid i[\Delta X, \Delta P] \mid \psi\rangle.$$

Since $[\Delta X, \Delta P] = [X, P] = i\hbar\,\mathbf{1}$, we obtain the quadratic form

$$f(\lambda) = \Delta X^2 + \lambda^2\Delta P^2 + \lambda\,\hbar, \quad \text{where} \;\; \Delta X^2 := \|\Delta X\psi\|^2, \;\; \Delta P^2 := \|\Delta P\psi\|^2.$$

Nonnegativity of $f(\lambda)$ for all $\lambda \in \mathbb{R}$ is equivalent to the non-positivity of the discriminant:

$$\hbar^2 - 4\,\Delta X^2\Delta P^2 \;\leq\; 0 \quad\Longrightarrow\quad \Delta X\,\Delta P \;\geq\; \frac{\hbar}{2}.$$

□

Remark 13 (Conditions for equality). Equality in (32) holds if and only if there exists a real c such that $\Delta P\,\psi = -\,i\,c\,\Delta X\,\psi$. For the standard pair on $L^2(\mathbb{R})$ this yields Gaussian states (coherent states of minimal uncertainty).

Remark 14 (Consistency with the model and Bell tests). The inequality (32) is a kinematical consequence of the canonical commutation relations (CCR) and does not rely on statistical independence of measurement settings. In the present model: (i) on each slice Σ_s, in the sectors where canonical pairs are defined, the CCR $[X, P] = i\hbar\,\mathbf{1}$ hold, whence (32) follows; (ii) microcausality holds in the graded sense: for bosonic fields the commutators of local algebras vanish at spacelike separation, for fermionic fields the anticommutators vanish; in particular, even subalgebras of observables commute; (iii) superdeterminism amounts to dropping the independence assumption $P(\lambda \mid a, b) = P(\lambda)$ for the hidden parameters λ and settings (a, b), so the standard Bell factorization and the resulting inequalities do not apply to the model. This does not change the operator algebra and therefore does not affect (32).

Matrix form (for a set of observables). Let $R = (R_1, \dots, R_{2n})$ be self-adjoint observables with canonical structure

$$[\,R_\alpha, R_\beta\,] = i\hbar\,\Omega_{\alpha\beta}\,\mathbf{1}.$$

Define the covariance matrix

$$\Sigma_{\alpha\beta} := \tfrac{1}{2}\,\langle\psi \mid \{\,\Delta R_\alpha, \Delta R_\beta\,\} \mid \psi\rangle, \quad \Delta R_\alpha := R_\alpha - \langle\psi \mid R_\alpha \mid \psi\rangle\,\mathbf{1}.$$

Then the matrix inequality

$$\Sigma \,+\, \frac{i\hbar}{2}\,\Omega \;\geq\; 0 \tag{33}$$

holds, from which (30) and (31) follow as special cases.

Remark 15 (POVMs and unsharp measurements). The inequalities (29)–(33) are *preparation* inequalities: they depend only on the state ψ and on the operator algebra. For unsharp measurements described by POVMs one may use Naimark dilation: any POVM is realized as a PVM in an extended Hilbert space, and the above bounds are preserved for the first moments of the corresponding self-adjoint generators.

5 Relativistic closure

Effective fields and observable transformations In the SR regime (flat foliations), observable transformations coincide with Lorentz transformations (see [1] and §2.7). Effective fields generate observable quantities (events) through the actions of the observer, so their transformations must be compatible with the observable transformations and preserve the event structure. By construction, observable transformations describe a *hypothetical* transition between inertial frames under the assumption of a global event structure. In §3.5 we have passed to complex effective fields, which allows us to use the standard operator formalism and facilitates casting the theory into a Lorentz-covariant form. With this modification, the evolution on each slice remains consistent with the local evolution of the expansion in a complete orthonormal basis (5), and the effective fields themselves are defined by (9). Since observable transformations refer to a hypothetical transition between inertial frames, the construction of effective fields is carried out independently for each foliation and relies only on its data. The goal of the present section is, on the basis of the previously obtained results, to formulate a relativistically invariant closure for the effective fields and to verify its consistency with local evolution and microcausality.

Structural assumptions of the section. In this section, in addition to the general operational assumptions (Levels I–II), we assume:

- local work in the SR regime: on each slice we use the Minkowski metric $\eta = \mathrm{diag}(1,-1,-1,-1)$, the foliation parameter t is identified with the operational time s (after a rescaling of units), and $v_{\max} = 1$;
- for the effective fields we consider free linear equations of at most second order in derivatives, local in y and $O(1,3)$-covariant (for spinors, first order), consistent with the restriction of the principal symbol to second order and with the $O(3)$ isotropy of the slice stabilizer (see Corollary 2);
- in the massless vector sector we additionally assume local gauge invariance sufficient to remove longitudinal modes and compatible with OS-positivity and microcausality;
- the choice of signs of the kinetic terms and the normalizations is fixed to be consistent with the OS/GNS construction and the spectral condition $H \geq 0$ (§3.1, App. A).

The subsequent results for the scalar, spinor, and vector sectors are based on these assumptions.

We introduce the notation

$$\Box := \partial_t^2 - \Delta_{\mathbf{y}}, \qquad p^2 := \eta^{\mu\nu} p_\mu p_\nu. \tag{34}$$

General convention for field sectors. Let $\{u_\alpha(y)\}$ be a locally supported in y and complete on Σ_s orthonormal basis of admissible modes of the fundamental field, and let $a_\alpha(s)$ be their coefficients. The effective fields are defined by

$$\psi_I(s,y) = W_I{}^{\alpha}(s,y)\, a_\alpha(s) + \mathcal{O}(\partial_y a, \partial_y W), \tag{35}$$

where $W_I{}^{\alpha}$ is local in y and C^1 in s. In the scalar sector $I = 1$; in the spinor and vector sectors the index I labels the components of the field. In this section we work to leading order in the gradient expansion; the treatment of $\mathcal{O}(\partial_y)$ corrections is given in later sections/appendices. The transport of modes in s is local within the class of admissible solutions; see the corresponding lemma/appendix.

5.1 Scalar sector

We fix the SR regime and adopt the signature $\eta = \mathrm{diag}(+,-,-,-)$, $c \equiv v_{\max}$, and

$$\Box := \partial_t^2 - \Delta_{\mathbf{y}}, \qquad p^2 := \eta^{\mu\nu} p_\mu p_\nu. \tag{36}$$

Lemma 9 (Principal symbol for a scalar field)**.** *Let ϕ be an effective scalar field whose dynamics is governed by a local linear equation of at most second order, covariant in the SR limit. Then the principal symbol is*

$$\sigma_2(\mathcal{L})(p) = -\,p^2.$$

Proof. A general linear $O(1,3)$-covariant second-order operator for a scalar has the form $\mathcal{L} = a_1\Box + a_0$, since there are no other Lorentz scalars with ≤ 2 derivatives. In momentum space $\widetilde{\mathcal{L}}(p) = -a_1 p^2 + a_0$. Normalizing $a_1 = 1$ (overall scale of the operator), we obtain $\sigma_2(\mathcal{L}) = -p^2$. □

Proposition 6 (Equation of motion for the scalar field)**.** *The unique $O(1,3)$-covariant second-order equation has the form*

$$(\Box + m^2)\,\phi = 0, \qquad m \geq 0. \tag{37}$$

Proof. From Lemma 9 we have $\mathcal{L} = \Box + a_0$. The requirement of stability/positivity of energy fixes the sign in front of $\Box$ and yields $m^2 := -a_0 \geq 0$, which leads to (37). □

Corollary 4 (Dispersion)**.** *Solutions of* (37) *have dispersion $\omega^2 = |\mathbf{p}|^2 + m^2$. In the massless limit $m = 0$: $\Box\phi = 0$, $\omega^2 = |\mathbf{p}|^2$.*

Remarks. (i) Statements about microcausality are formulated later and are not used here. (ii) Interactions and spectral representations are introduced in subsequent sections.

5.2 Spinor sector

We fix the SR regime and adopt the signature $\eta = \mathrm{diag}(+,-,-,-)$, $c \equiv v_{\max}$. Let $\partial\!\!\!/ := \gamma^\mu \partial_\mu$, where the matrices γ^μ satisfy the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2\,\eta^{\mu\nu}\,\mathbf{1}. \tag{38}$$

Lemma 10 (Principal symbol for a spinor field)**.** *Let ψ be an effective spinor field whose dynamics is governed by a local linear equation of at most first order, covariant in the SR limit. Then the principal symbol is*

$$\sigma_1(\mathcal{D})(p) = \gamma^\mu p_\mu \equiv p\!\!\!/.$$

Proof. A general $O(1,3)$-covariant local first-order operator has the form $\mathcal{D} = B^\mu \partial_\mu + B_0$, where B^μ are matrices in spinor space. Covariance and the irreducibility of the spin-$\frac{1}{2}$ representation require $B^\mu = a\,\gamma^\mu$ for some scalar a. We fix the overall normalization of the symbol; choosing $a = i$ yields the standard hermiticity properties of the Hamiltonian. Thus $\sigma_1(\mathcal{D})(p) = \gamma^\mu p_\mu$. □

Proposition 7 (Equation of motion for the spinor field)**.** *The unique $O(1,3)$-covariant first-order equation has the form*

$$\big(i\,\partial\!\!\!/ - m\big)\psi = 0, \qquad m \geq 0. \tag{39}$$

Proof. This follows from Lemma 10 and the admissibility of the unique Lorentz-scalar mass term $m\,\bar{\psi}\psi$ (appearing in the equation as $m\mathbf{1}$). □

Corollary 5 (Squared equation and dispersion). *Acting with* $(i\partial\!\!\!/ + m)$ *on* (39) *and using* (38), *we obtain*

$$(\Box + m^2)\,\psi = 0.$$

Hence for plane waves $\psi \sim e^{-i\omega t + i\mathbf{p}\cdot\mathbf{y}}$ *the dispersion relation is* $\omega^2 = |\mathbf{p}|^2 + m^2$. *In the massless limit* $m = 0$*:* $i\partial\!\!\!/\psi = 0$ *and* $\omega^2 = |\mathbf{p}|^2$.

In the spinor case, the two-point function admits the standard spectral representation in terms of a measure $\rho_{1/2}(m^2)$ and the projector $(p\!\!\!/ + m)$ onto mass m (see Appendix F for details).

Remarks. (i) The microcausality pattern and equal-time CAR are formulated later; they are not used here. (ii) The choice of representation for γ^μ is inessential; all formulas are written in a representation-independent form.

5.3 Vector sector

In this subsection we fix the SR regime of the relativistic closure for the vector field and its hyperbolicity. We adopt the signature $\eta = \mathrm{diag}(+,-,-,-)$ and set $c \equiv v_{\max}$. We introduce

$$\Box := \partial_t^2 - \Delta_{\mathbf{y}}, \qquad p^2 := \eta^{\mu\nu} p_\mu p_\nu. \tag{40}$$

Lemma 11 (Principal symbol for a vector field). *Let an effective vector field be described by a local linear equation of at most second order, covariant in the SR limit, with* $O(3)$ *isotropy on* Σ_s. *Then the principal symbol is*

$$\sigma_2(\mathcal{M})_{\mu\nu}(p) = p^2 \eta_{\mu\nu} - p_\mu p_\nu.$$

Massless case (Maxwell). The equation of motion is

$$\big(\Box\,\eta_{\mu\nu} - \partial_\mu\partial_\nu\big)A^\nu = 0, \qquad \delta A_\mu = \partial_\mu \alpha.$$

Gauge invariance removes the longitudinal mode. In the Lorenz gauge $\partial\cdot A = 0$ we obtain $\Box A_\mu = 0$. The dispersion relation for physical modes is

$$\omega^2 = |\mathbf{p}|^2, \quad \text{number of polarizations } = 2\ (\lambda = \pm 1).$$

Massive case (Proca). The equation of motion is

$$\big(\Box\,\eta_{\mu\nu} - \partial_\mu\partial_\nu\big)A^\nu + m^2 A_\mu = 0.$$

Taking the divergence yields $m^2\,\partial\cdot A = 0$, hence $\partial\cdot A = 0$, and therefore

$$(\Box + m^2)A_\mu = 0.$$

The dispersion relation and number of degrees of freedom are

$$\omega^2 = |\mathbf{p}|^2 + m^2, \quad \text{polarizations } = 3.$$

Proof of Lemma 11. We require linearity, locality of at most second order, Lorentz covariance, translational invariance, and $O(3)$ isotropy on Σ_s. Then the most general linear operator acting on A_μ has the form

$$\mathcal{M}_{\mu\nu}\,A^\nu \;=\; a_1\,\eta_{\mu\nu}\Box A^\nu \;+\; a_2\,\partial_\mu\partial_\nu A^\nu \;+\; a_0\,\eta_{\mu\nu}A^\nu,$$

where terms with $\varepsilon_{\mu\nu\rho\sigma}$ and two derivatives reduce to a total divergence and do not affect the Euler–Lagrange equations in the linear approximation. In momentum space (with $\partial \to -ip$, $\Box \to -p^2$)

$$\widetilde{\mathcal{M}}_{\mu\nu}(p) \;=\; -a_1\,p^2\,\eta_{\mu\nu} \;-\; a_2\,p_\mu p_\nu \;+\; a_0\,\eta_{\mu\nu}.$$

Massless case. Gauge invariance $\delta A_\mu = \partial_\mu \alpha$ is equivalent to $p^\mu \widetilde{\mathcal{M}}_{\mu\nu}(p) \equiv 0$, from which

$$p^\mu \widetilde{\mathcal{M}}_{\mu\nu} = -(a_1 + a_2)\,p^2 p_\nu \equiv 0 \;\;\Rightarrow\;\; a_2 = -a_1.$$

Up to an overall normalization of the principal symbol we choose $a_1 = -1$ and obtain

$$\widetilde{\mathcal{M}}_{\mu\nu}(p) = p^2 \eta_{\mu\nu} - p_\mu p_\nu,$$

that is, in configuration space, $\mathcal{M}_{\mu\nu} = \eta_{\mu\nu}\Box - \partial_\mu\partial_\nu$. The characteristic cone is given by $p^2 = 0$, hyperbolicity follows from the properties of the wave operator, and physical solutions are transverse modes with $\omega^2 = |\mathbf{p}|^2$.

Massive case. Adding the unique admissible Lorentz-scalar zero-th order term $a_0 = m^2 > 0$, we obtain

$$\widetilde{\mathcal{M}}_{\mu\nu}(p) = \big(p^2 - m^2\big)\eta_{\mu\nu} - p_\mu p_\nu,$$

that is, the Proca equation $(\Box + m^2)A_\mu - \partial_\mu(\partial\!\cdot\! A) = 0$. Taking the divergence of this equation gives $m^2\,\partial\!\cdot\! A = 0 \Rightarrow \partial\!\cdot\! A = 0$, after which $(\Box + m^2)A_\mu = 0$ with dispersion $\omega^2 = |\mathbf{p}|^2 + m^2$.

Finally, the sign in front of $\Box$ is fixed by the requirement of positivity of energy and OS-positivity (see §3.1, Appendix A). Any additional terms of the form $\xi\,(\partial\!\cdot\! A)^2$ correspond merely to gauge fixing and do not enter the gauge-invariant physical principal symbol. ☐

Remark 16 (Positivity of energy). The sign of the kinetic term is fixed by the requirement $H \geq 0$ (see §3.1, Appendix A). In the massless case, physical positivity is ensured in the transverse subspace; it is then natural to work with $F_{\mu\nu}$.

Proposition 8 (Equation of motion for the vector field). *Let A_μ be an effective vector field in the SR regime. Then the second-order equation with the unique $O(1,3)$-covariant principal symbol has the form*

$$\Box A_\mu - \partial_\mu(\partial\!\cdot\! A) \;+\; m^2 A_\mu \;=\; 0, \qquad \partial\!\cdot\! A := \partial^\nu A_\nu, \tag{41}$$

where $m \geq 0$ is the mass parameter. The case $m = 0$ corresponds to the massless limit.

Proof. Equation (41) follows from Lemma 11 and the admissibility of the unique Lorentz-scalar mass term $m^2 A_\mu$. ☐

Corollary 6 (Proca: longitudinal component). *If $m > 0$, then* (41) *implies $\partial\!\cdot\! A = 0$, and the components satisfy*

$$(\Box + m^2)A_\mu = 0. \tag{42}$$

Corollary 7 (Maxwell as the massless limit). *For $m = 0$, equation* (41) *is equivalent to the vacuum Maxwell equations*

$$\partial^\mu F_{\mu\nu} = 0, \qquad F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{43}$$

since $\partial^\mu F_{\mu\nu} = \Box A_\nu - \partial_\nu(\partial\!\cdot\! A)$.

Remarks. (i) Microcausality and BRST are not used in this subsection; the corresponding statements are given later and are not invoked here. (ii) The counting of physical degrees of freedom is provided for orientation; a detailed analysis of gauge choices is given in subsequent sections.

5.4 Causal cone and fundamental solutions

We work in the SR regime on a (locally) globally hyperbolic region $(\mathcal{U}, \eta)$ with $\eta = \mathrm{diag}(1, -1, -1, -1)$. For $x \in \mathcal{U}$ we set

$$J^{\pm}(x) := \{y \in \mathcal{U} \mid (y - x) \text{ is non–spacelike and } \pm (y^0 - x^0) \geq 0\}, \quad \partial J^{\pm}(x) : \text{ null generators.}$$

Operators. A linear differential operator P on $(\mathcal{U}, \eta)$ is called *normally hyperbolic* if its principal symbol has the form

$$\sigma_2(P)(x, p) = -\, \eta^{\mu\nu} p_\mu p_\nu \, \mathbf{1}$$

(with the identity matrix on internal indices for multicomponent fields). In particular, the scalar operator

$$P_{\mathrm{sc}} = \Box + m^2$$

is normally hyperbolic in this sense.

Following [4], we call an operator P *Green-hyperbolic* (or *hyperbolic in the sense of Green*) if it admits unique retarded and advanced Green operators with causal support (see Lemma 12).

In this work we use the operators

$$P_{\mathrm{sc}} = \Box + m^2, \quad P_{\mathrm{sp}} = (i\partial\!\!\!/ - m), \quad P_{\mathrm{vec}}^{(m>0)} = (\Box + m^2)\, \eta_{\mu\nu} - \partial_\mu \partial_\nu,$$

and for $m = 0$ either

$$P_{\mathrm{Max}}\, A_\nu := \partial^\mu (\partial_\mu A_\nu - \partial_\nu A_\mu)$$

on 1-forms, or

$$P_\xi := \Box\, \eta_{\mu\nu} - (1 - \xi) \partial_\mu \partial_\nu$$

with subsequent restriction to gauge-invariant observables. The operator P_{sc} is normally hyperbolic; all the operators listed above are Green-hyperbolic in the sense of [4].

Lemma 12 (Fundamental solutions and causal support). *Let, in the SR regime, a linear field operator P be Green-hyperbolic on a globally hyperbolic effective geometry. Then there exist unique fundamental solutions E_R, E_A such that*

$$P \circ E_{R/A} = E_{R/A} \circ P = \mathrm{id} \textit{ on } C_0^\infty, \qquad \mathrm{supp}\,(E_R f) \subset J^+(\mathrm{supp}\, f),\ \mathrm{supp}\,(E_A f) \subset J^-(\mathrm{supp}\, f).$$

The difference $E := E_R - E_A$ satisfies $P_x E = P_{x'} E = 0$ and $\mathrm{supp}\, E \subset J^+ \cup J^-$.

Proof. For normally hyperbolic and, more generally, Green-hyperbolic operators on globally hyperbolic Lorentzian manifolds, the existence and uniqueness of retarded and advanced Green operators with causal support is a standard result; see, for example, [3, 4, 5]. Applied to the effective operator P in the SR regime, this yields the required fundamental solutions E_R, E_A with the stated properties. □

Proposition 9 (Fundamental solutions). *Let P be a Green-hyperbolic operator on $(\mathcal{U}, \eta)$. Then there exist unique continuous operators $\Delta_R, \Delta_A : \mathcal{D}(\mathcal{U}) \to \mathcal{D}'(\mathcal{U})$ such that*

$$P \Delta_{R/A} = \Delta_{R/A} P = \mathrm{id} \textit{ on } \mathcal{D}(\mathcal{U}), \qquad \mathrm{supp}\, \Delta_R f \subset J^+(\mathrm{supp}\, f), \quad \mathrm{supp}\, \Delta_A f \subset J^-(\mathrm{supp}\, f).$$

Sketch of proof. Standard energy estimates for wave equations on globally hyperbolic regions and the local Hadamard parametrix construction yield existence; uniqueness follows from finite propagation speed estimates. □

Definition 4 (Pauli–Jordan kernel)**.** Let P be a Green-hyperbolic operator on $(\mathcal{U}, \eta)$ and $\Delta_{R/A}$ its retarded/advanced fundamental solutions. The *Pauli–Jordan kernel* (causal propagator) is the distribution

$$E \;:=\; \Delta_R - \Delta_A \;\in\; \mathcal{D}'(\mathcal{U} \times \mathcal{U}) \otimes \mathrm{End}(\mathbb{V}),$$

where $\mathbb{V}$ is the internal component space of the field (trivial in the scalar case). Interpretation: E measures the purely causal part of the response (the difference between "future" and "past"); subsequently it serves as the kernel of the Peierls symplectic form and, upon quantization, of the commutator/anticommutator of observables.

Its basic properties are:

1. $PE = EP = 0$ (in both arguments, in the sense of distributions).
2. $\mathrm{supp}(Ef) \subset J^+(\mathrm{supp}\, f) \cup J^-(\mathrm{supp}\, f)$ for all $f \in \mathcal{D}(\mathcal{U})$ (causal support).
3. $E(x, x') = -E(x', x)$ (antisymmetry; for multicomponent fields, with transposition in the internal indices).
4. Uniqueness: if $\tilde{E}$ satisfies 1)–3), then $\tilde{E} = E$.

Remark 17 (Vector and spinor cases). For $m > 0$ the Proca operator $P_{\mathrm{vec}}^{(m>0)}$ is Green-hyperbolic, and $\Delta_{R/A}$ exist on the space of 1-forms (see [4]). For $m = 0$ the Maxwell operator on 1-forms is also Green-hyperbolic: fundamental solutions exist either on the transverse subspace of fields, or after gauge fixing P_ξ with subsequent restriction to gauge-invariant observables (for example, $F_{\mu\nu}$). For spinors the first-order operator P_{sp} induces fundamental solutions via the factorization $P_{\mathrm{sp}}\,(i\partial\!\!\!/ + m) = \Box + m^2$, which also falls into the class of Green-hyperbolic operators.

Corollary 8 (Finite propagation speed and support)**.** *For any test function $f \in \mathcal{D}(\mathcal{U})$ the support of Ef lies in the causal hull*

$$\mathrm{ch}(f) \;:=\; J^+(\mathrm{supp}\, f) \,\cup\, J^-(\mathrm{supp}\, f).$$

In particular, if $\mathrm{supp}\, f$ *and* $\mathrm{supp}\, g$ *are spacelike separated, then*

$$\langle f, Eg \rangle = 0.$$

Remarks. (i) This section does not use microcausality of commutators/anticommutators; the corresponding statements are formulated later. (ii) In the massless gauge case all statements are to be read on gauge-invariant observables (for example, on $F_{\mu\nu}$) or after gauge fixing. (iii) Signs and conventions are consistent with §5: $\Box = \partial_t^2 - \Delta_{\mathbf{y}}$.

5.5 Peierls symplectic form and canonical algebras

We rely on Def. 6 (Pauli–Jordan kernel $E = E_R - E_A$) and on the support property $\mathrm{supp}\, E \subset J^+ \cup J^-$. The Peierls form ϖ is defined by (44); its locality for causally disjoint supports is encoded in (45). Below we do not redefine E and ϖ.

Peierls form. For $f, g \in C_0^\infty(\mathbb{R} \times \Sigma_s)$ set

$$\varpi(f, g) \;:=\; \iint f(x)\, E(x, x')\, g(x')\, \mathrm{d}x\, \mathrm{d}x'. \tag{44}$$

From $E^\top = -E$ it follows that ϖ is antisymmetric. From the causal support of E (see Def. 6) we obtain

$$\varpi(f, g) = 0 \quad \text{if} \quad \mathrm{supp}\, f \text{ and } \mathrm{supp}\, g \text{ are causally disjoint.} \tag{45}$$

Proposition 10 (Microcausality in the SR regime from the Peierls form)**.** *Let the principal symbol of the corresponding equation be hyperbolic in the SR regime, and let E be the Pauli–Jordan kernel. Then for any $f, g \in C_0^\infty(\mathbb{R} \times \Sigma_s)$ with causally disjoint supports one has*

$$\varpi(f, g) = \iint f(x)\, E(x, x')\, g(x')\, \mathrm{d}x\, \mathrm{d}x' = 0, \qquad [\Psi(f), \Psi(g)] = 0.$$

For fermionic fields the same hypotheses imply local anticommutativity.

Sketch of proof. The support of E is contained in $J^+ \cup J^-$, hence for causally disjoint $\operatorname{supp} f$ and $\operatorname{supp} g$ the integral $\varpi(f, g)$ vanishes (see (45)). In the scalar case the Peierls bracket coincides with the quantum commutator (in the free theory), so $[\Psi(f), \Psi(g)] = i\,\varpi(f, g)\,\mathbf{1} = 0$. For the spinor case the fundamental solutions of the Dirac equation S_R, S_A give $E_D := S_R - S_A$ with the same causal support, which implies local anticommutativity. □

Scalar sector: CCR. On the quotient of the test-function space by the equation of motion the canonical commutation relations are defined by

$$[\Psi(f), \Psi(g)] = i\,\varpi(f, g)\,\mathbf{1}, \qquad f, g \in C_0^\infty(\mathbb{R} \times \Sigma_s), \tag{46}$$

where ϖ is the Peierls form (44). From (45) and Prop. 12 it follows that microcausality holds: $[\Psi(f), \Psi(g)] = 0$ for causally disjoint supports.

Fermionic sector: CAR and microcausality. Let $\mathcal{D}$ be the Dirac operator in the SR regime, and let $\mathcal{T}$ be the space of test spinors, factored by $\mathcal{D}\mathcal{T}$. The standard CAR inner product $\langle f, g \rangle_{\mathrm{CAR}}$ is induced by the fundamental solutions of $\mathcal{D}$. The CAR algebra is generated by

$$\{\Psi(f), \Psi(g)\} = 0, \qquad \{\Psi(f), \Psi^\dagger(g)\} = \langle f, g \rangle_{\mathrm{CAR}}\,\mathbf{1}, \tag{47}$$

and local anticommutativity for causally disjoint $\operatorname{supp} f, \operatorname{supp} g$ follows from the causal support of E_D (see the proof of Prop. 12).

Gauge sector. In the massless vector case the CCR are formulated on the gauge-invariant subalgebra generated by $F_{\mu\nu}$, or after BRST reduction. Microcausality holds for gauge-invariant observables and is inherited from the Peierls form built from the corresponding fundamental solutions.

Summary. The Peierls form ϖ defines the scalar CCR algebra (46); for spinors it yields the CAR algebra (47). Microcausality is a consequence of the causal support of the fundamental solutions and does not require an operational observability principle. The subsequent reconstruction of the state and of the unitary dynamics is carried out separately via detailed balance and OS/GNS.

5.6 Energy and unitarity

By Theorem 1 (see also App. A), together with detailed balance (Prop. 1) and the strong continuity of the transfer (Lemma 5), there exist a Hilbert space $\mathcal{H}$, a cyclic vector Ω, a $*$-representation π, and a unitary group

$$U(t) = e^{-iHt/\hbar}, \qquad H = H^\dagger \geq 0, \tag{48}$$

implementing the observable dynamics:

$$U(t)\,\pi(A)\,U(t)^{-1} = \pi(\alpha_t A), \qquad t \in \mathbb{R}. \tag{49}$$

Corollary 9 (Unitarity and spectral condition). *The dynamics in t is unitary, and the energy operator H is self-adjoint and satisfies the spectral condition $H \geq 0$.*

Proposition 11 (Identification of energy in the free sectors). *In the SR regime, for the free scalar, spinor, and vector sectors, the operator H coincides with the Noether time-translation charge:*

$$H = P^0 = \int_{\Sigma_s} T_{00}\, \mathrm{d}^3 y, \tag{50}$$

see the derivations in §5.1, §5.2, and §5.3.

Remark 18 (Gauge-invariant sector). In the massless vector sector, the statements (48)–(49) and $H \geq 0$ are understood on the gauge-invariant subalgebra (e.g. that generated by $F_{\mu\nu}$) or after BRST reduction; see Remark 36.

Remark 19 (Domains and continuity). The group $U(t)$ is strongly continuous, and $\mathcal{D}(H)$ is the standard domain of its generator by Stone's theorem. No additional assumptions beyond Prop. 1, Lemma 5, and Theorem 1 are introduced.

Domains and self-adjointness. Let $\mathfrak{D}$ denote the linear span of vectors of the form $\pi(A)\Omega$ with smooth local A. Then $\mathfrak{D}$ is dense in $\mathcal{H}$ and invariant under $U(t)$, since $U(t)\pi(A)\Omega = \pi(\alpha_t A)\Omega$ and α_t preserves the class of local smeared observables. In what follows we regard the restriction $H|_{\mathfrak{D}}$ as a positive symmetric operator, and H itself as its self-adjoint extension (the Friedrichs extension) in $\mathcal{H}$. When spatial translations are implemented by a unitary group $V(\mathbf{a})$, one may, under standard energy estimates, use $\mathfrak{D}$ as a common dense core for the generators P^i. In the gauge case, the operator construction is first defined on the "large" Hilbert space $\mathcal{H}$ and then restricted to the physical subspace $\mathcal{H}_{\text{phys}}$ (see Remark 36).

Summary. Energy is defined as the generator of observable time translations; its non-negativity follows from OS-positivity via the OS/GNS construction. In the free sectors one has $H \equiv \int_{\Sigma_s} T_{00}\, \mathrm{d}^3 y$ (see Prop. 13); in the gauge sector positivity of the spectrum is understood on $\mathcal{H}_{\text{phys}}$ (see Remark 36).

5.7 Classicality of gravity and absence of an independent spin-2 quantum sector

Corollary 10 (Absence of quantum gravity in the model). *Under locality on Σ_s, microcausality, OS/GNS reconstruction with $H \geq 0$, $O(3)$ isotropy of the stabilizer, and the second-order bound on the principal symbol (see §3.1, Lemma 1, Corollary 2), as well as the geometric realisation of GR via foliations [2], the gravitational sector in the model is realised as a classical geometry (metric and connections on the families $\{\Sigma_s\}$), common to all effective fields. The local C^*-algebra of observables is generated by the effective fields; there is no independent local operator algebra of gravitational degrees of freedom (CCR/CAR for spin 2). Consequently, quantum gravity in the sense of a local quantum field of the metric (graviton) is absent.*

Sketch of justification. (1) OS/GNS and microcausality construct a unitary SR evolution and local algebras from effective fields on the slices Σ_s (§3.1). (2) Corollary 2 fixes the second-order principal part of the local generator and the $O(3)$ scalar structure on the slice; gravity enters only through the geometry of the foliations (measures, connections, constraints on the principal symbol). (3) The derivation of GR in [2] realises gravity as a classical structure of foliations acting universally on all ψ, without introducing independent quantum spin-2 degrees of freedom. Any attempt to add separate CCR/CAR for the gravitational field would lead either to double counting of the same universal coupling or to a conflict with OS positivity and microcausality. This yields the claim. □

Remark 20 (Testable consequences). The model predicts the absence of (i) gravity-induced entanglement in "gravity-only" (BMV-type) setups, (ii) quantum correlations of gravitational waves (shot noise / squeezing as a quantum signature), and (iii) Planck-scale dispersion and quantum-gravity-induced decoherence; see the brief list of tests in §8.

6 Operational derivation of the minimal symmetry group

In this section we are not aiming at a full classification analysis of all possible effective theories arising from the Euclidean model, but rather at demonstrating that, once natural operational requirements for the existence of an observer and empirically specified conditions (the structure of a single fermion family) are imposed, the resulting effective theory automatically has gauge group $SU(3) \times SU(2) \times U(1)$ and the standard hypercharges. At the end of the section we will analyse all assumptions made.

Dependencies and location of proofs. We work in the SR regime. We use OS/GNS, the spectral condition, locality/microcausality, and polynomial boundedness; see Appendix C for a brief summary. Full proofs of the statements in this section are given in Appendix D.

In this section, additional operational and group-theoretic assumptions will be introduced as needed; they are then collected and classified in §6.3.

Notation. Indices: $\mu, \nu = 0, 1, 2, 3$ $(0 \equiv t)$, $i, j = 1, 2, 3$. Σ_s is a foliation hyperplane; $\Omega \subset \Sigma_s$ is a reconstructible region in which the required operational conditions hold; $\Omega_0 \subset \Omega$ is the region corresponding to the observer's body. The Minkowski metric is $\eta = \mathrm{diag}(1, -1, -1, -1)$, $k^2 := \eta^{\mu\nu} k_\mu k_\nu$. For an abelian field A_μ: $F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu$. The symbol $\star$ denotes the spatial Hodge operator on Σ_s for the metric δ_{ij}; dS is the surface element on Σ_s, n_i is the outward unit normal. For the colour group: R_{fund} is the fundamental representation, $\epsilon : R_{\text{fund}}^{\otimes 3} \to \mathbf{1}$ is the invariant antisymmetric tensor. The hypercharge is denoted by Y, the electric charge by Q, and T_3 is the third generator of $SU(2)$; $Q = T_3 + Y$. In constructing effective fields we make use of the observer. Events are defined relative to the observer. The observer's body Ω_0 is built from the same effective fields as the rest of Ω.

6.1 Operational assumptions and measurement scales

Definition of T_{meas}. Let $E_O(\mathrm{d}y)$ be a local POVM on a region $O \subset \Sigma_s$, and let $U(t) = e^{-iHt/\hbar}$. Introduce the time-smeared effect

$$\widetilde{E}_T(B) := \int_{\mathbb{R}} w_T(t)\, U(t)^\dagger E_O(B)\, U(t)\, \mathrm{d}t,$$

where $w_T \geq 0$, $\int_{\mathbb{R}} w_T(t)\, \mathrm{d}t = 1$, and the effective width of the window is T. We shall regard the parameter $T_{\text{meas}} := T$ as the characteristic duration of a single measurement protocol (the time window within which an outcome is counted as a single act of measurement).

If a localised observer is possible, we shall use the following structural conditions for a region $\Omega \subset \Sigma_s$, some of which were rigorously established earlier, while others are introduced here as additional assumptions:

(O1) *Measurability (previously established fact):* local algebras $\mathcal{A}(O)$ admit POVMs and realise the Born rule (see §3.7 and Proposition 5); this item is not a new assumption but records a result proved in the previous section.

(O2) *Long-range channel:* we assume that there exists at least one massless spin-1 mediator ($k^2 = 0$) with a locally measurable surface (Gauss) charge in the sense of (58) (see also §5.3 and Appendix C), compatible with microcausality.

(O3) *Anomaly freedom:* we require that gauge and mixed anomalies vanish, including the global $SU(2)$ anomaly [27] (see Lemma 15, Proposition 14, Appendix H for a discussion of the current structure, Ward identities, and the consistency check with the specific matter content).

OS/GNS. The construction of $\mathcal{H}$, of a self-adjoint $H \geq 0$, and of the evolution $U(t) = e^{-iHt/\hbar}$ is given in §3.1 (see also Appendix A). Below, these results are taken as known.

Structural statements used below. Assuming OS/GNS, microcausality and the spectral condition, together with assumptions (O1)–(O3) and additional operational criteria formulated below for each item, we shall use the following statements for a region $\Omega \subset \Sigma_s$:

(C1) **Unbroken abelian factor.** Under assumption (O2) (existence of at least one massless spin-1 long-range channel with a surface (Gauss) charge in the sense of (58)), Lemma 13 implies the existence of an unbroken abelian factor $U(1)$. Commutativity of the asymptotic surface charges follows from the vanishing equal-time Peierls bracket (Lemma 17). The unique Lorentz- and gauge-invariant quadratic kinetic term of order at most two is $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ (Proposition 16; before electroweak symmetry breaking this is $U(1)_Y$, afterwards $U(1)_{\text{em}}$).

(C2) **Colour with three-body singletness.** Under an additional operational assumption that the spectrum contains stable three-body colour-neutral composites (the three-body singletness criterion), the minimal gauge group admitting such a sector is $SU(3)$ with invariant $\epsilon : R_{\text{fund}}^{\otimes 3} \to \mathbf{1}$ (Lemma 14). *Remark:* this statement does not follow from (O1) by itself; the crucial ingredient is precisely the three-body singletness criterion on which the proof of Lemma 14 is based.

(C3) **Universal left-handed charged currents.** Under an additional operational assumption of the existence of universal left-handed charged currents of $V - A$ type, local in Ω and compatible with microcausality, the algebra of the corresponding currents realises $\mathfrak{su}(2)$, and the number of doublets is even (the Witten global anomaly [27]); see Lemma 15. *Remark:* (O1) guarantees measurability and locality of observable algebras, but does not by itself produce $\mathfrak{su}(2)$; the decisive step is the operational criterion of universality of weak currents used in the proof of Lemma 15.

(C4) **Fixing hypercharges.** Given assumption (O3) (anomaly freedom for gauge and mixed anomalies), a choice of fermion content isomorphic to a single Standard Model family (see (Og)), and the convention $Q = T_3 + Y$, the hypercharges of this family are uniquely fixed by the standard anomaly-cancellation conditions (Proposition 14).

Lemma 13 (Long-range spin-1 channel $\Rightarrow$ unbroken $U(1)$ and $-\frac{1}{4}F^2$)**.** *Let there exist in the SR regime a massless spin-1 mediator with causal dynamics and an* abelian Gauss law*: for an exhaustion $\Omega = \bigcup_R \Omega_R$ with $S_R := \partial\Omega_R \simeq S^2$ define*

$$Q(S_R) := \int_{S_R} \star F, \qquad Q_\infty := \lim_{R\to\infty} Q(S_R),$$

where the far field has Coulomb decay $|\mathbf{E}| \sim R^{-2}$. Then (i) the equal-time Peierls bracket/commutator of asymptotic charges vanishes; (ii) the long-range internal symmetry is abelian (an unbroken $U(1)$ factor); (iii) the unique Lorentz- and gauge-invariant quadratic kinetic term of second order is $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ (with an additional total divergence $F\tilde{F}$ if desired). Proof: *Appendix D.1.*

Lemma 14 (Three-body singletness $\Rightarrow$ minimal simple factor $SU(3)$)**.** *Let G be a connected compact gauge group acting unitarily and irreducibly in a complex three-dimensional representation $R \simeq \mathbb{C}^3$. Assume that:*

(a) there exists a nonzero G-invariant totally antisymmetric tensor $\epsilon : \wedge^3 R \to \mathbb{C}$ (i.e. G preserves a volume form on R);

(b) R is not equivalent to its conjugate, $R \not\simeq \overline{R}$ (a complex fundamental representation).

Then the image of G lies in $SU(3)$, and the minimal simple factor of the group is $SU(3)$ (up to a finite central factor/covering).

Proof. Unitarity of the action implies $G \subset U(3)$. Invariance of the volume form means $\det \rho(g) = 1$ for all $g \in G$, hence $\rho(G) \subset SL(3, \mathbb{C})$. Thus $\rho(G) \subset U(3) \cap SL(3, \mathbb{C}) = SU(3)$.

We now show that the simple factor is indeed $SU(3)$. The classification of closed connected subgroups of $SU(3)$ acting irreducibly on $\mathbb{C}^3$ [29] yields two possibilities: (i) $SU(3)$; (ii) the image of $SO(3)$ in its standard three-dimensional action, complexified as a real representation. Case (ii) is excluded because this representation is real and self-conjugate ($R \simeq \overline{R}$), contradicting (b). The remaining case is $SU(3)$ (a finite central correction is possible, without affecting the local algebra). □

Remark 21 (Operational motivation for assumption (b)). The requirement $R \not\simeq \overline{R}$ corresponds to the physical non-equivalence of "colour" and "anticolour" and to the need to construct both mesons ($R \otimes \overline{R} \to \mathbf{1}$) and baryons ($R^{\otimes 3} \to \mathbf{1}$ via ϵ), which excludes real (self-conjugate) fundamental representations.

Remark 22 (On the limits of applicability of the three-body singletness criterion). Violating (a) or (b) does not contradict OS/GNS, microcausality, or causal reconstruction of events. It merely invalidates the conclusion $G \subset SU(3)$ in Lemma 14: if (a) fails, an additional abelian factor in the colour group is possible; if (b) fails, minimal neutralisation becomes two-body (e.g. $SO(3)$ in its three-dimensional real representation), and the criterion $R^{\otimes 3} \to \mathbf{1}$ no longer singles out $SU(3)$. This is precisely why (a)–(b) are formulated as operational conditions of consistency with the colour phenomenology of the SM, rather than as consequences of causal reconstruction.

Lemma 15 (Minimal group of universal left-handed currents)**.** *Suppose there exist universal left-handed currents $J^\mu_a = \bar{\psi}_L \gamma^\mu T_a \psi_L$ such that: (i) the algebra generated by T_a is simple and has no Schwinger terms (see (L2)); (ii) on the left sector a* minimal *nontrivial finite-dimensional irreducible complex representation is realised; (iii) the action of the internal symmetry on the multiplet of fields preserves the two-point function and is therefore realised unitarily $U \in U(N)$ (OS-positivity) with closed image. Then $\mathfrak{g} \simeq \mathfrak{su}(2)$, and the minimal representation is a two-dimensional doublet. Consistency requires an even number of doublets (the Witten global anomaly [27]).*

Proof. (1) The classification of minimal complex irreducible representations of simple compact groups [29] gives

$$\dim_{\mathbb{C}}(\text{min irrep}) = \begin{cases} 2, & SU(2), \\ N\ (\geq 3), & SU(N \geq 3), \\ N\ (\geq 3)\ (\text{real}), & SO(N), \\ 2N\ (\geq 4)\ (\text{pseudoreal}), & Sp(2N), \\ > 2, & G_2, F_4, E_6, E_7, E_8. \end{cases}$$

The only simple compact Lie algebra with a minimal complex representation of dimension 2 is $\mathfrak{su}(2)$.

(2) From (i) and (iii) the current algebra integrates to a compact unitary internal group; the absence of central extensions (L2) guarantees closure under $[\cdot, \cdot]$ without anomalous terms.

(3) Condition (ii) (minimal irreducible representation on the left sector) excludes any extensions $\mathfrak{g} \supsetneq \mathfrak{su}(2)$ with additional generators not required by the observable currents. Hence $\mathfrak{g} = \mathfrak{su}(2)$.

(4) For $SU(2)$ the fundamental is pseudoreal, so local (perturbative) anomalies are absent, but the Witten global anomaly exists; consistency requires an even number of left-handed doublets. □

Remark 23. The closed image of the internal symmetry in $U(N)$ is compact, so compactness is here a conclusion rather than an assumption. Non-compact simple groups admit no nontrivial finite-dimensional unitary representations, hence for dim $= 2$ only $SU(2)$ remains.

Proposition 12 (Fixing hypercharges for a single family). *For the family* $(Q_L, u_R, d_R;\ L_L, e_R, (\nu_R))$ *in representations* $(\mathbf{3},\mathbf{2})_{Y_Q}$, $(\mathbf{3},\mathbf{1})_{Y_u}$, $(\mathbf{3},\mathbf{1})_{Y_d}$; $(\mathbf{1},\mathbf{2})_{Y_L}$, $(\mathbf{1},\mathbf{1})_{Y_e}$, $(\mathbf{1},\mathbf{1})_{Y_\nu}$, *the anomaly-cancellation conditions*

$$
\begin{aligned}
&[SU(3)]^2U(1): \quad 2Y_Q - Y_u - Y_d = 0,\\
&[SU(2)]^2U(1): \quad 3Y_Q + Y_L = 0,\\
&[\text{grav}]^2U(1): \quad 6Y_Q + 2Y_L - 3Y_u - 3Y_d - Y_e - Y_\nu = 0,\\
&[U(1)]^3: \quad 6Y_Q^3 + 2Y_L^3 - 3Y_u^3 - 3Y_d^3 - Y_e^3 - Y_\nu^3 = 0,
\end{aligned}
$$

together with the normalisation $Q = T_3 + Y$ *fix the hypercharges of this family uniquely to the standard values:* $Y_Q = \frac{1}{6}$, $Y_u = \frac{2}{3}$, $Y_d = -\frac{1}{3}$, $Y_L = -\frac{1}{2}$, $Y_e = -1$, $Y_\nu = 0$ *(see, for example, [28, 30]).*

6.2 Operational derivation of the Standard Model group $SU(3) \times SU(2) \times U(1)$

Hypotheses of local implementability. Let J^a_μ be conserved currents of a global internal symmetry ($\partial^\mu J^a_\mu = 0$ in the sense of distributions). We assume:

(L1) **Form-charges and local Ward identities (hypothesis L1).** We assume that in the class of effective theories under consideration, the structural properties (OS/GNS, spectral condition, microcausality), conservation of the currents $\partial_\mu J^\mu_a = 0$ in the sense of distributions, and the absence of Schwinger terms (see assumption (O4) below) are sufficient to construct well-defined form-charges $Q(\alpha)$ and to derive the local Ward identities (62). Details of the standard scheme (regularisation of form-charges, passage to local identities) are given in Appendix H; a rigorous derivation of all functional-analytic conditions from the fundamental Euclidean model is not carried out in the present work.

Remark. The existence of unitary implementers $U(\alpha, O) = \lim_{\varepsilon\downarrow 0} \exp(iQ_\varepsilon(\alpha))$ on a dense domain follows under additional energy bounds and closedness of the form-charge; in the absence of these functional-analytic conditions, form-charges alone suffice to derive (62).

(L2) **Anomaly freedom (O4).** We require the absence of Schwinger terms in the equal-time current algebra and the vanishing of gauge/mixed anomalies; this item refines and localises the general anomaly-freedom condition previously stated as (O3). It is an operational consistency requirement, not a theorem: in the presence of Schwinger terms, the local Ward identities (62), Gauss's law, and the existence of a nilpotent BRST charge all fail (the observable subsystem becomes inconsistent). The explicit check for the chosen matter content is given in Proposition 14.

Remark 24. Operational motivation (from the model): local implementability of charges and gauge covariance of the effective action (see Appendix H) are incompatible with Schwinger terms; hence (O4) is a necessary condition for the existence of an observer and for causal reconstruction.

(L3) **Local (quasi-local) implementers of symmetries.** We assume that for each bounded $O \subset \Sigma_s$ there exist unitary implementers $U(\alpha, O)$, locally adjusted by a parameter $\alpha \in C_0^\infty(O, \mathfrak{g})$, such that $U(\alpha, O)\,\mathcal{A}(O)\,U(\alpha, O)^{-1} = \mathcal{A}(O)$, and whose generator on a common dense domain $\mathcal{D}$ is given by an extension of the form-charge $Q(\alpha) = \int_{\Sigma_s} \alpha^a J^a_0 \,\mathrm{d}^3\mathbf{x}$. In the abelian long-range case we allow *quasi-locality* with additional surface fluxes (Gauss law). Sufficient conditions are: (i) J^a_μ are operator-valued distributions with energy bounds; (ii) absence of Schwinger terms (see (O4)); (iii) the time–slice property and additivity of the net of algebras. Precise regularisation, domain issues, and the derivation of local Ward identities are discussed in Appendix H.

Remark 25. The formula $U(\alpha, O) = \exp\big(i \int \alpha^a J^a_0\big)$ is formal: self-adjointness of $Q(\alpha)$ and the existence of the exponential require energy bounds and domain analysis (e.g. along the lines of Nelson/Trotter). In sectors with Gauss's law, global charges are not strictly localisable; quasi-local implementers are used instead.

Proposition 13 (Operational gauging of a global symmetry). *Under (L1)–(L3) (including the time–slice property and additivity of the net of algebras in (L3)) and (O4), the variations induced by $[Q(\alpha), \cdot]$ are realised as local gauge transformations upon replacing $\partial_\mu \to D_\mu := \partial_\mu + igA_\mu$ with transformations $\delta_\alpha A_\mu = -(1/g)\,\partial_\mu\alpha - i[A_\mu, \alpha]$.* Proof: *See Appendix H.*

Proposition 14 (Uniqueness of the second-order Yang–Mills kinetic term). *In* 3+1 *dimensions, the only Lorentz- and gauge-invariant scalars of order at most two in derivatives of A_μ are $\operatorname{tr} F_{\mu\nu}F^{\mu\nu}$ and $\operatorname{tr} F_{\mu\nu}\tilde{F}^{\mu\nu}$, where $\tilde{F}^{\mu\nu} := \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$. OS-positivity fixes the sign in front of F^2, and $F\tilde{F}$ is a total divergence. Consequently, the free kinetic term is $-\frac{1}{4}\,F^a_{\mu\nu}F^{a\,\mu\nu}$ up to a total divergence.* Proof: *See Appendix G.*

Remark on Gauss's law and BRST. In the non-abelian case global charges are quasi-nonlocal due to Gauss's law; the local formulation is given at the level of matter algebras $\mathcal{A}(O)$ and currents before gauging. After introducing A_μ, the physical subspace is selected in the standard way (Gupta–Bleuler/BRST).

Proposition 15 (Minimal group as a necessary consequence of NOU). *Under the assumptions of §5, Appendix C and §6.2, the conditions (C1)–(C4) imply that any admissible gauge group G contains the subgroup*

$$SU(3) \times SU(2) \times U(1) \subseteq G,$$

and the minimal consistent group with respect to inclusion is $G_{\min} = SU(3) \times SU(2) \times U(1)$ (up to a central factor $/\Gamma$). Proof. *See Appendix D.*

Remark 26. The existence of Q_∞ in Lemma 13 presupposes asymptotically flat behaviour on the slice Σ_s and decay $|\mathbf{E}| \sim r^{-2}$; these conditions are used only to ensure commutativity of asymptotic charges.

6.3 Additional assumptions of the section and domain of applicability

In this section, in addition to the general structural conditions formulated earlier (see assumptions (A1)–(A7)), we use additional operational and group-theoretic assumptions specific to the analysis of the operational observability of internal symmetries and of the gauge group structure. In this subsection we summarise the additional assumptions that were introduced in the preceding subsections of this section.

For clarity, we group them into three levels:

- *Level I: mathematical consistency.* Assumptions expressing general requirements of internal consistency of the effective theory (absence of anomalies, compatibility with OS/GNS, unitarity and microcausality), independently of the concrete "type of world".

- *Level II: general conditions for the existence of an observer.* Assumptions reflecting the possibility of the existence of a stable localised observer with reproducible measurement procedures, long-range communication channels and operationally definable internal symmetries. Since in the present model the observer is an essential element of the setup, level II assumptions are, in their fundamental status, equivalent to level I assumptions; the separation into levels I and II is introduced solely to logically distinguish between "purely" mathematical consistency requirements and requirements directly related to the existence of an observer.

- *Level III: specificity of a Standard Model–like world.* Assumptions reflecting empirical features of our world (structure of the colour and weak sectors, fermion content and hypercharges) and used to demonstrate that within the model the gauge group realised is locally isomorphic to the Standard Model group.

Below we list the additional assumptions (Ob)–(Og) used in this section, indicating their level. We do not repeat assumptions already introduced in previous sections.

(Ob) Existence of at least one massless long-range channel (Level II). In assumption (O2) we postulate the existence of at least one effective massless field (or sector) which
(i) satisfies microcausality,
(ii) allows information transfer over arbitrarily large distances (a long-range channel),
(iii) is operationally accessible to the observer through local measurement procedures.
This condition is not derived directly from the Euclidean model $\Delta_{\mathbb{E}^4}\Phi = 0$; rather, it is imposed as a general requirement for a world in which an observer can perform calibration of instruments and exchange information over large distances (an analogue of electromagnetic long-range behaviour).

(Oc) Anomaly-free current algebra and gauge symmetries (Level I). In assumptions of type (O3)/(O4) and (L2) we impose the requirement of the absence of gauge and mixed anomalies, including:

- absence of Schwinger terms in the commutation relations of local currents;
- vanishing of triangle gauge anomalies for the chosen gauge group and matter representation;
- absence of the global $SU(2)$ anomaly [27].

This condition is interpreted as a requirement of mathematical consistency of the effective gauge theory: the possibility to realise the internal symmetry without violating microcausality, unitarity, OS-positivity and the BRST/OS/GNS structure. In the present work, anomaly freedom is taken as an assumption (subsequently checked for the specific matter content), and is not derived from the fundamental Euclidean model.

(Of) Local implementability of internal symmetries (Level II). In hypotheses of type (L1)–(L3) we assume:

- the existence of well-defined form-charges $Q(\alpha)$ generated by local currents and satisfying local Ward identities in the observer's working region;
- the absence of Schwinger terms in these identities (see (Oc));
- the existence of local or quasi-local unitary implementers $U(\alpha, O)$ of internal symmetries on $\mathcal{H}$, acting on local algebras $\mathcal{A}(O)$ with controlled domains and energy bounds.

These conditions are standard in the algebraic description of gauge theories and ensure that internal symmetries can be operationally defined and implemented from the viewpoint of a localised observer. Within the present work they are adopted as structural assumptions of local implementability of symmetries and are not derived directly from the fundamental Euclidean model.

(Od) Three-particle colour neutrality and choice of the colour representation (Level III). In the part denoted as (C2), as well as in Lemma 14, we use an additional physically and operationally motivated criterion. Within the model, the observer is described by the same effective fields as the other material subsystems, and its "body" $\Omega_0 \subset \Omega$ and internal registers are built from localised colour-neutral composites. At the level of the effective colour symmetry this leads to the following conditions:

(a) the spectrum must contain stable three-particle composites that are operationally identifiable as "colour-neutral" (three-particle singlets with respect to some internal symmetry), providing localisable material degrees of freedom from which a stable observer's body and registers can be built;

(b) the fundamental representation R of this symmetry admits an invariant fully antisymmetric tensor $\epsilon : \wedge^3 R \to \mathbb{C}$, with $R \not\simeq \overline{R}$, so that "colour" and "anticolour" are operationally distinguishable.

These conditions do not follow from the general requirements of causal reconstruction and microcausality; they are introduced as a criterion for the existence of minimal three-particle colour-neutral composites that can serve as building blocks for the material subsystems of the observer. On this basis one then isolates a minimal $SU(3)$ factor in the gauge group (see Lemma 14), which reflects the specific structure of the colour sector observed in our world.

(Oe) Minimality of the weak isospin sector (Level III). In the analysis of the $SU(2)$ subgroup (items (C3)/(C4) and the associated lemmas) we use an additional operationally and phenomenologically motivated assumption.

In the model, the observer and its internal registers are built from the same effective fields as the other material subsystems. Weak interactions provide a universal channel for changing fermionic states (decays, captures, etc.), which must simultaneously:

- be described by a *simple* current algebra acting on the left sector as a single internal symmetry for all participating fermionic registers;
- realise a *minimal* non-zero finite-dimensional irreducible complex representation in the left sector (a two-component "before/after" structure of the transition), compatible with microcausality, the spectral condition and OS/GNS.

Such a minimal realisation corresponds to the Standard Model doublet and, together with the conditions of Lemma 15, leads to the simple factor $\mathfrak{su}(2)$ in the algebra of weak currents. This assumption is not derived directly from the fundamental Euclidean model and the requirements of causal reconstruction; it reflects the choice of a *minimal* weak sector that provides universal left-handed currents for the material registers of the observer and is consistent with the observed structure of weak interactions.

(Og) Structure of the fermion content and hypercharges (Level III). In the analysis of hypercharges and anomalies (Proposition 14 and the related text) we use the following set of assumptions about the fermion content.

First, we assume that the spectrum of the effective theory contains at least one set of chiral fermion fields satisfying:

(i) it contains coloured fermions transforming in the fundamental representation of the colour group and forming three-particle colour-neutral composites (see (Od));

(ii) it contains colourless fermions such that, together with the coloured ones, they allow the formation of electrically neutral bound states suitable for building stable macroscopic bodies (atom-like composites from which the observer's body and its internal registers can be assembled);

(iii) the weak sector acts on the left fermionic subspace through the minimal non-zero finite-dimensional irreducible complex representation of the simple factor (see (Oe)), so that the weak currents are chiral and universal within the class of material registers;

(iv) the full gauge–gravitational content of this fermion set is anomaly-free (conditions (Oc)).

Second, we assume that we consider a *minimal* chiral fermion content (in terms of number of fields) satisfying (i)–(iv).

Such a set of assumptions is not derived directly from the fundamental Euclidean model and the requirements of causal reconstruction; it represents an operationally and phenomenologically motivated restriction on the class of admissible low-energy spectra: one requires the existence of electrically neutral colour-bound matter suitable for constructing an observer, in the presence of chiral weak currents and the absence of gauge and gravitational anomalies.

Under these conditions it is shown (Proposition 14) that for a minimal chiral set the solution of the anomaly cancellation conditions uniquely leads to a hypercharge assignment coinciding with the standard one for a single Standard Model family (up to the possible presence of a right-handed neutrino).

Taken together, assumptions (Ob)–(Og) specify the additional operational and group-theoretic constraints used in this section to select the class of admissible effective theories compatible with the observed structure of long-range behaviour and internal symmetries. Level I and II assumptions express the general structural requirements of the model "fundamental Euclidean configuration + localised observer" (mathematical consistency and existence of a stable observer), whereas level III assumptions reflect specific properties of a Standard Model–like world. All of them are compatible with the fundamental Euclidean model and the general requirements of causal reconstruction but are not fully derived from them within the present paper and are therefore explicitly fixed as conditions for the applicability of the results obtained here.

6.4 Summary of the section

On the possible uniqueness of the gauge group The analysis presented in the present work shows that, under the fundamental assumptions of the Euclidean model and the operational conditions (A1)–(A7), (Ob)–(Of), there exists at least one class of effective theories in which the gauge group and the matter content coincide with those of the Standard Model (up to the possible presence of right-handed neutrinos). The present paper does not address the question of whether there exist other classes of solutions that are compatible with the same assumptions and admit an operationally defined observer. In particular, within the proposed model it is not excluded that a gauge group locally isomorphic to $SU(3) \times SU(2) \times U(1)$ is the only one possible under the above operational and mathematical requirements. Establishing or refuting such potential uniqueness requires a separate classification analysis and lies beyond the scope of the present work.

7 Parameters and the inverse problem

Motivation and accessible observations. The goal of this section is to relate the parameters of the effective description emerging from the model (primarily the local generator/Hamiltonian of the form (27) on Σ_s and the characteristics of infrared modes) to what is actually operationally accessible to the observer, and thereby to formulate a well-posed inverse problem. Within the model, the observer has access only to: (i) the local statistics of measurement outcomes described by positive operator-valued measures on the slice Σ_s (see §3.7), and (ii) the reconstruction of the gravitational sector via the effective curvature of the foliations (g_{ij}, K_{ij}) and their invariants in the SR/GR regimes. The informational isolation of inertial frames excludes the use of inter-foliation correlations; the inverse problem is solved within a single foliation. The working assumptions include: the second-order restriction for the principal symbol (Corollary 2), the $O(3)$ isotropy of the stabilizer on the slice (Lemma 1), and, in the interpretation of measurement statistics, averaging over the microstates of the apparatus (Assumption 2). In this context, the problem is formulated as the estimation (possibly non-unique) of the coefficients $a^{ij}(y)$, $b^i(y)$, $V(y)$ in (27) and of the IR-mode parameters from POVM data and from geometric curvature observables, with explicit account of gauge equivalence and operational resolving power.

What is actually reconstructed. In the model the fundamental equation is fixed ($\Delta_{\mathbb{E}^4}\Phi = 0$, see §2.1). Operationally accessible are only: (i) local outcome statistics described by POVM on the slice Σ_s (see §3.7), and (ii) the geometry of the foliations (g_{ij}, K_{ij}) and their invariants (SR/GR regimes). Therefore, the inverse problem is formulated on two levels:

(a) **Effective level.** From POVM correlation data on Σ_s and from the curvature of the foliations, reconstruct (up to natural equivalences) the local evolution generator

$$H = \int_{\Sigma_s} \Big(\tfrac{\hbar^2}{4}\, \hat{\psi}^\dagger \big(-\nabla_i a^{ij} \nabla_j \big) \hat{\psi} + \tfrac{1}{2}\, \hat{\psi}^\dagger b^i \nabla_i \hat{\psi} + \tfrac{1}{2}\, V\, \hat{\psi}^\dagger \hat{\psi} + \text{h.c.} \Big)\, d\mu_{\Sigma_s},$$

where the second-order principal symbol and the $O(3)$–scalar structure follow from Corollary 2 and Lemma 1. The identifiable objects are: $a^{ij}(y) = a(y)\delta^{ij}$ (up to the normalization of $\hbar$ and unitary equivalence), the lower-order coefficients $b^i(y)$, $V(y)$, as well as the IR-mode parameters affecting the gravitational sector.

(b) **Fundamental level.** From operational data on a single foliation, estimate the *local* germ of the configuration Φ in a neighborhood of Σ_s and/or its quasi-Dirichlet/quasi-Neumann data. The form of the equation is not reconstructed (it is fixed); what is reconstructed are the data for its solution on the working manifold. Stability is severely limited (Hadamard's problem for the Laplacian); regularization is required.

Set-up. We work in the SR regime on a fixed foliation $\{\Sigma_s\}$ and in terms of effective fields $\psi(s, y)$ with a local Hamiltonian of the form (27):

$$H = \int_{\Omega_s} \Big(\tfrac{1}{2}\, \hat{\psi}^\dagger\, \hat{h}\, \hat{\psi} + \text{h.c.} \Big)\, \mathrm{d}\mu_{\Sigma_s}, \qquad \hat{h} = -\frac{\hbar^2}{2}\, \nabla_i\big(a^{ij}(s,y)\, \nabla_j\big) + b^i(s,y)\, \nabla_i + V(s,y).$$

By Corollary 2 the principal symbol is of second order; by Lemma 1, under isotropy of the stabilizer $a^{ij} = a(s,y)\, \delta^{ij}$. The accessible observables are given by averages and correlators

$$\rho_\varphi(t) := \langle \psi(t) \mid \Pi(\varphi) \mid \psi(t) \rangle, \qquad C_{f,g}(t) := \langle \Omega,\, \psi(t,f)\, \psi(0,g)\, \Omega \rangle,$$

where $\Pi(\varphi) = \int_{\Omega_s} \varphi(y)\, E(\mathrm{d}y)$, $f, g, \varphi \in C_0^\infty(\Omega_s)$ (see §3.7, §4).

Forward problem. For given coefficients (a, b, V) compute the predicted quantities $\rho_\varphi(t)$, $C_{f,g}(t)$, as well as the instantaneous flux (current) $j^i(t, y)$ from the continuity equation (26)–(28).

Inverse problem (variants).

(IP1) *Principal part:* from the sets $C_{f,g}(t)$, $\rho_\varphi(t)$ at small Δt and small spatial scales, reconstruct a^{ij} (or a under $O(3)$ isotropy).

(IP2) *Lower orders:* for known a^{ij}, estimate b^i, V on Ω_s.

(IP3) *Boundary operator:* from pairs *(excitation at s=0)* → *(instantaneous response in s)* reconstruct the Dirichlet-to-Neumann (DtN) operator and its symbol.

7.1 Identifiability and stability

Proposition 16 (Principal part from frequency–time dispersion)**.** *Let $a^{ij} = a\,\delta^{ij}$ on a small patch and assume* (26)–(28) *hold. Then the local low-frequency dispersion, reconstructed from $C_{f_k,f_k}(t)$ for plane-wave masks $f_k(y) \propto e^{ik\cdot y}$, has the form $\omega^2(k) = c^2(s_0, y_0)\,|k|^2 + \mathcal{O}(|k|^3)$, where $c^2 \propto a(s_0, y_0)$. Hence a is determined by the slope $\partial_{|k|^2}\omega^2 \restriction_{k=0}$.* Sketch. *Linearization of the Schrödinger equation* (24) *and of the current* (28) *with principal part $-\nabla_i(a\nabla_i)$.*

Proposition 17 (Principal part from DtN)**.** *For small Δt and narrow windows w_T, the instantaneous derivative $\partial_t\langle\psi(f)\rangle \restriction_{t=0}$ equals the action of a first-order pseudodifferential operator on f, which is the Dirichlet-to-Neumann (DtN) operator of the SR limit. For $a^{ij} = a\,\delta^{ij}$ its symbol is $\sigma(k) = \hbar\sqrt{a}\,|k|$ + lower orders, from which a is recovered.* Sketch. *Small-t shift via* (24) *and* (27)*; equal-time limit.*

Proposition 18 (Three-sphere and energy estimates ⇒ stability)**.** *The norms $C_{f,g}$ and ρ_φ satisfy logarithmically convex relations with respect to the radii of the masks (an analogue of a three-sphere inequality) and Caccioppoli-type estimates. It follows that the reconstruction errors for a on nested regions enjoy logarithmic-type stability, and the gradients of a are controlled by the energy on a larger ball.* Sketch. *Transfer of classical estimates for harmonic/second-order operators to smoothed observables.*

Remark 27 (Lower-order terms and incomplete identifiability). b^i and V are reconstructed from the time asymmetry and from the shifts of low-frequency dispersion, but only up to local unitary redefinitions and gauge choices of boundary conditions. This is equivalent to the familiar ambiguity in PDE inverse problems: the principal part is rigidly determined, while the lower orders are fixed only within an equivalence class.

7.2 Operational signatures and protocols

Spectral test (Poisson semigroup). For a family of plane-wave masks f_k and small Δs,

$$S(k,s) := \langle\psi(0,k;s)\psi(0,-k;s)\rangle, \qquad \ln\frac{S(k,s{+}\Delta s)}{S(k,s)} \;=\; -2\sqrt{a_{\mathrm{eff}}}\,|k|\,\Delta s + \mathcal{O}(|k|^2),$$

which calibrates a_{eff} on the patch. Such a $|k|$ law follows from the Dirichlet-to-Neumann operator for the fundamental Laplace equation $\Delta_{\mathbb{E}^4}\Phi = 0$ in a half-space (see §2.1); the Schrödinger equation (24) and the local Hamiltonian (27) are then used to relate the DtN data to the parameters (a, b, V).

DtN test. Estimate $\partial_t\langle\psi(f)\rangle|_{t=0}$ using a short time window w_T and compare it with the action of $\sqrt{-\Delta_y}$ on f. The $|k|$-linear law of the symbol fixes $\sqrt{a}$; the remainder is attributed to b^i and V.

Local scaling laws. Verify the logarithmic convexity of the variances $\mathrm{Var}[\psi(f_{B_r})]$ in the radii r and the gradient estimates for difference masks $\partial_i f_{B_r}$ (stability as in Proposition 20).

7.3 Solution algorithms

Variational (PDE-constrained). Minimize the misfit functional

$$\mathcal{J}[a,b,V] := \sum_m \left\| \widehat{C}_m^{\mathrm{model}}(a,b,V) - \widehat{C}_m^{\mathrm{data}} \right\|^2 + \lambda_{\mathrm{reg}}\,\mathcal{R}[a,b,V],$$

where $\widehat{C}_m$ are selected spectral–temporal characteristics (including DtN moments), and $\mathcal{R}$ is a regularizer (smoothness, physical constraints). Gradients with respect to (a, b, V) are computed via the adjoint method, using (24) and (27).

Local spectral regression. On small patches, estimate a from the slopes of the dispersion as in Proposition 18, then refine b and V from the residuals of temporal correlators.

Bayesian approach. Incorporate priors (constraints from Proposition 20, absence of strong fields, classicality of gravity) and obtain posteriors for (a, b, V) from the data $C_{f,g}$ and ρ_φ.

7.4 Limitations and domain of applicability

- **Single IFR.** Inter-IFR information is not used; consistent "dictionaries" are unavailable. The conclusions concern *local* parameters on Σ_s and their slow variation in s.
- **Principal part vs lower orders.** a^{ij} is identifiable from first derivatives in t and small $|k|$; b^i, V are inferred from asymmetries/shifts and require regularization.
- **Boundaries/fluxes.** Self-adjoint boundary conditions on $\partial\Omega_s$ must be fixed operationally (vanishing normal probability flux), otherwise DtN inference is ambiguous.

Remark 28 (Falsifiable signatures). Systematic deviations from the $|k|$-law (DtN), from exponential $|k|$-filtering along s, and from logarithmic convexity and gradient estimates indicate a breakdown of the harmonic principal part or nontrivial b^i, V outside the admissible class.

8 Predictions and tests of the model

Aim and domain of applicability. The experimentally testable consequences collected below follow directly from the construction of effective fields on the slices Σ_s, the OS/GNS reconstruction (§3.1), the second–order restriction on the principal symbol (Corollary 2), the Schrödinger equation on foliations and the continuity equation (§4, (26)–(28)), as well as from the gauge analysis (Lemma 13). The section is divided into: (i) *discriminating predictions*, which are specific to the model and falsifiable; (ii) *consistency checks*, which reproduce known properties of QFT within the present construction.

8.1 Discriminating predictions

P1. Classicality of gravity and absence of an independent quantum spin-2 sector. *Prediction.* Gravity is realised as the classical geometry of foliations, common to all effective fields; there is no separate local operator algebra of gravitational degrees of freedom (no CCR/CAR for spin-2) (see also 5.7). *Operational tests.*

(i) Bose–Marletto–Vedral (BMV)-type experiments: absence of observable gravitationally induced entanglement in "gravity-only" scenarios (with non-gravitational interactions excluded).

(ii) Statistics of gravitational waves: absence of quantum correlations in the stochastic background; dominance of Poissonian noise; absence of signatures of quantum squeezing.

(iii) Absence of Planckian dispersion and quantum-gravitational decoherence in astronomical photon data within the sensitivity of current observations.

(iv) On a curved background, quantum corrections belong exclusively to the matter sector $\mathcal{A}_{\text{matter}}$; diagrams with metric loops are absent.

Falsification criterion: a robust observation of any of (i)–(iv).

P2. Second-order foliation imprint: exponential filtering of high modes along s. *Prediction.* In the SR-regime and for a local principal part $-\nabla_i\big(a^{ij}\nabla_j\big)$ ((27), Corollary 2), the spectral density $S(k,s)$ for plane-wave masks $f_k(y)\propto e^{ik\cdot y}$ satisfies, for small Δs,

$$\ln\frac{S(k,s+\Delta s)}{S(k,s)} = -2\sqrt{a_{eff}(s)}\,|k|\,\Delta s + o(|k|\Delta s).$$

Operational test: linearity of $\ln\big(S(k,s+\Delta s)/S(k,s)\big)$ in $|k|$ at fixed Δs. *Falsification:* persistent deviations from the $\propto |k|$ law.

P3. Symbol of the instantaneous-response (DtN) operator proportional to $|k|$. *Prediction.* For a short time window and in the equal-time limit,

$$\partial_t\langle\psi(f)\rangle\big|_{t=0} = \langle\psi(\mathcal{D}f)\rangle, \qquad \sigma(\mathcal{D})(k) = \hbar\sqrt{a}\,|k| + \text{lower-order terms},$$

where a is the coefficient of the principal part in (27). *Operational test:* reconstruction of $\sigma(\mathcal{D})(k)$ from $\partial_t\langle\psi(f_k)\rangle|_{t=0}$ and verification of linearity in $|k|$. *Falsification:* systematic deviation from a symbol linear in $|k|$ at small $|k|$.

P4. Logarithmic convexity of variances and Caccioppoli-type estimates for averages. *Prediction.* For ball masks f_{B_r} and difference masks $\partial_i f_{B_r}$,

$$\mathrm{Var}[\psi(f_{B_{r_2}})] \le \mathrm{Var}[\psi(f_{B_{r_1}})]^{\theta}\,\mathrm{Var}[\psi(f_{B_{r_3}})]^{1-\theta}, \quad \mathrm{Var}^{1/2}[\psi(\partial_i f_{B_r})] \lesssim r^{-1}\,\mathrm{Var}^{1/2}[\psi(f_{B_{2r}})].$$

Operational test: convexity in $\log r$ and the scaling law r^{-1}. *Falsification:* persistent violations of both estimates under control of lower-order contributions.

P5. Long-range abelian factor and uniqueness of the quadratic kinetic term. *Prediction.* In the presence of a massless spin-1 channel (Lemma 13): stabilisation of the surface charge $Q(S_R) = \int_{S_R}\star F$ as $R\to\infty$, commutativity of equal-time asymptotic charges, and uniqueness of the Lorentz- and gauge-invariant quadratic kinetic term of second order, $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$. *Operational test:* Coulomb fall-off $|\mathbf{E}|\sim R^{-2}$, a plateau for $Q(S_R)$, and vanishing equal-time commutator/Peierls bracket of asymptotic charges. *Falsification:* robust observation of noncommutativity or of an alternative quadratic kinetic term.

Remark 29 (On the operational testability of the predictions). In the present model the following are operationally accessible: (i) local measurements on a slice Σ_s, described by positive-operator-valued measures (POVM; see §3.7); (ii) reconstruction of the gravitational sector from the effective curvature of the foliations. The informational isolation of inertial reference frames excludes the use of correlations between foliations with different directions $\mathbf{n}$; the inverse problem is solved within a single foliation.

Apart from predictions P1–P5 (for which typical operational tests have been indicated above), many other phenomenological consequences of the model, including those discussed in Sections 9.4 and 9.5, are not equipped in the present paper with concrete measurement protocols within a single foliation and with estimates of the required sensitivity. The specification of measurement procedures and the development of operational tests within a single foliation (including the analysis of systematics and the design of POVM) are deferred to future work.

8.2 Standard consequences and consistency checks

S1. Continuity equation and current. $\partial_t\rho_\varphi(t) = -\int_{\Omega_s}\nabla_i\varphi\, j^i\, d\mu_{\Sigma_s}$ (§4, (26)–(28)); conservation of total probability with boundary fluxes taken into account.

S2. Born rule and POVM. Realisation of measurement statistics by positive-operator-valued measures and the Born rule (§3.7, Proposition 4).

S3. Heisenberg inequality. $\Delta_\psi A \, \Delta_\psi B \geq \frac{1}{2}|\langle\psi, [A, B]\psi\rangle|$ (see the section on canonical (anti)commutators and §4.1); a kinematic consequence of the algebra, compatible with microcausality.

S4. Lorentz invariance of observable transformations. Preservation of the null cone and a unique limiting speed in the SR-regime (see the section on observable transformations and [1]).

We emphasise in particular that items P2–P4 empirically calibrate and test the *principal part* of the local generator ((27), Corollary 2) on the basis of data from a single inertial frame, whereas P1 and P5 provide independent *discriminating* tests of the operational architecture of the model.

9 Discussion and conclusions

Status of the result. In the timeless Euclidean model with a scalar field, a local quantum field theory has been constructed in the SR-regime on the basis of operational observability conditions. We obtain: (i) the SR-structure and the principal symbol for the effective fields (§5); (ii) the Peierls symplectic form, ETCR/CAR and microcausality (Appendix C); (iii) the minimal gauge group as a necessary consequence of the operational assumptions (O1)–(O4), (Ob)–(Og), of the operational introduction of a connection, and of the uniqueness of the local second-order Yang–Mills kinetic term (§6). Thus, the structure of a local QFT in the SR-regime is obtained without any postulates beyond the operational ones.

Domain of applicability. The arguments are local on the slices Σ_s and rely on OS-positivity and OS/GNS, the spectral condition, polynomial boundedness, microcausality, and SR-covariance of the effective dynamics. A global set of events and curved foliations are not required. The gravitational sector is treated as the classical limit of consistency with GR; quantisation of the metric is neither postulated nor implied by the model.

Relation to GR. The previously obtained emergent causal structure is consistent with the local SR-kinematics and admits the standard interpretation of the energy–momentum tensor of the effective fields as a source of classical geometry in suitable low-curvature regimes. There is no need for "quantum gravity" in view of the operational nature of time and events and the absence of fundamental quantum degrees of freedom of the metric.

Reduction rather than repeated proofs. Once the assumptions (OS/GNS, spectral condition, locality, absence of anomalies, the $SU(3) \times SU(2) \times U(1)$ structure) are established, the remaining components are taken from standard results: KL/CPT/spin–statistics; LSZ and Haag–Ruelle in massive channels; BRST and the Slavnov–Taylor identities for gauge fixing; BPHZ renormalisation and RG. This fixes causality, unitarity, and renormalisability without overloading the paper with proof details.

Falsifiability and observational consequences. The model excludes: (i) additional long-range non-abelian charges; (ii) violation of anomaly cancellation for the hypercharges of a single family; (iii) quantum signatures of a gravitational carrier (for example, detection of a quantised gravitational mode within the domain of applicability of the SR-regime). Positive consequences include: the existence of exactly one unbroken abelian factor at infinity; the left-handed nature of universal charged currents and the evenness of the number of doublets; fixed hypercharges for $Q = T_3 + Y$.

Parameters and the inverse problem. The structural construction does not require numerical fixing of $\{g_3, g_2, g_1;\ y_f;\ \lambda, v;\ \theta_{\rm QCD}\}$. The inverse problem (reconstruction of the parameters from correlators of the fundamental field via KL measures and RG flow) has been formulated but is placed beyond the scope of the present work; its solution is not required for the claim about the completeness of the structural QFT.

Limitations and open questions. (1) The non-perturbative existence and a rigorous construction of the infrared sector in the presence of a massless $U(1)$ are not explicitly carried out here; we rely on standard results on dressing of states and do not develop the corresponding analysis.

(2) The non-perturbative dynamics of the colour sector (confinement, hadron spectrum, etc.) is not derived within the model; we only assume its compatibility with the localisability of colourless observables.

(3) A complete algebraic formulation in terms of a net of local algebras $\mathcal{A}(O)$ in curved regimes and for general foliations is left for future work.

Derivation of the $SU(3) \times SU(2) \times U(1)$ gauge group. It should be emphasised that the derivation of the $SU(3) \times SU(2) \times U(1)$ gauge group has a conditional character. In the present work it is shown that, in the class of local, anomaly-free chiral theories which admit the existence of a localised observer with internal registers and colour-neutral bound matter, the effective theory that is minimal in the number of fields has a gauge group locally isomorphic to $SU(3) \times SU(2) \times U(1)$ and a fermion content isomorphic to one family of the Standard Model. We do not claim that other (more complicated) effective theories compatible with the existence of an observer are in principle excluded; we only show that the Standard Model arises as one of the minimal solutions of the operational and phenomenological requirements introduced, and within the model this is sufficient: a strict uniqueness of the effective theory is not required.

Transport mismatch and possible cosmological effects. In the present work the local transport of modes and the matching between the matter and gravitational sectors are treated in an "idealised" way: the transport equations (8), (12) and the effective equations for the foliations are assumed to hold exactly within the working approximation for the chosen class of configurations. At the same time, in the GR work it was already noted that the compensation between the field and geometric contributions can, in general, be realised only to an accuracy sufficient for the existence of a stable observer, i.e. it admits a residual mismatch compatible with the operational requirements.

It is conceptually natural to expect that such a mismatch of transport and compensation, being strictly constrained on local (laboratory) scales by the SR/GR tests and local QFT, may accumulate on large space–time scales and manifest itself in the form of effective cosmological corrections. In particular, systematic shifts are possible in the reconstruction of the infrared sector (IR modes that are only weakly probed by local POVM), in the parameters of the effective geometry (curvature, calibration of distances, evolution of the scale factor), and in the choice of the matter subspace relative to the orthogonal sector of modes, which are operationally interpreted as "long-range" effects although they partly reflect features of the observer-reconstruction scheme.

At the same time, the regime of inter-observer event consistency strongly constrains arbitrary individual reconstruction errors: incoherent fluctuations in the transport equation that depend only on the internal state of a single observer are averaged out in the exchange of information and cannot produce robust cosmological effects. Potentially relevant are only those components of the mismatch that are induced by the configuration of the fundamental field Φ itself and by the structure of the class of observers (their internal registers and reconstruction procedures) and therefore manifest themselves universally within a given inertial frame.

A quantitative parametrisation of such a mismatch, its relation to the infrared and orthogonal sectors of modes, and an assessment of its possible contribution to cosmological observables (for example, $H(z)$, luminosity and angular-diameter distances, lensing, effective "dark" components) lie beyond the scope of this work. These questions appear to be a natural direction for further development of the model, especially in connection with the interpretation of cosmological anomalies in terms of observer-dependent reconstruction.

Summary. The minimal gauge structure $SU(3) \times SU(2) \times U(1)$, local causality, unitarity, and renormalisability follow from the operational conditions for the existence of an observer and causal reconstruction in the timeless model. A local QFT in the SR-regime is thus obtained as an emergent structure of a single fundamental degree of freedom. Further work is aimed at the inverse problem for the constants and at non-Euclidean/curved regimes without departing from the established operational principles.

9.1 Open question: number of generations

The requirements of completeness of an orthonormal basis of modes on the slice Σ_s, the choice of a complex structure J compatible with the symplectic form, and the implementability of observable changes of foliation determine the local gauge structure $U(1) \times SU(2) \times SU(3)$ and the standard local QFT, but do not fix the number of generations N_{gen}.

Why N_{gen} does not follow from the current assumptions. (1) The set of compatible complex structures $\mathcal{J}(\varpi)$ is connected; changes of foliation are implemented by Bogoliubov transformations satisfying the Shale–Stinespring criterion and do not produce a discrete restriction of the form $\mathbb{Z}_3$. (2) The conditions of locality, OS/GNS, the spectral condition, KL representations, CPT and spin–statistics impose no constraints on the multiplicity of copies of matter; the anomalies of the Standard Model cancel already for a single generation.

What could single out $N_{\text{gen}} = 3$. (a) A topological threefold degeneracy of the zero modes of the Dirac operator on Ω_0 (or asymptotically) under special boundary conditions. (b) A residual discrete symmetry of the observer acting on the space of modes with three equivalent orbits. (c) Global cosmological requirements, for example the necessity of weak CP violation for baryogenesis, which implies $N_{\text{gen}} \geq 3$.

Directions for further work. (1) To formalise the space $\mathcal{J}(\varpi)$ and the action of observable transformations, and to check the absence of discrete monodromy. (2) To pose an index problem on Ω_0 with physically motivated boundary conditions and to estimate the multiplicities of zero modes. (3) To relate possible cosmological conditions to the local operational requirements of the model.

We leave the derivation of $N_{\text{gen}} = 3$ as a separate problem, which is not required for the claim of deriving the local QFT.

9.2 Completeness of the SM sector and additional modes

The construction of effective fields and gauge groups in the preceding sections singles out a minimal operationally defined sector, which we denote by $\mathcal{A}_{\text{SM}}(O) \subset \mathcal{A}(O)$ for a working region $O \subset \Sigma_s$. This sector is generated by those modes of the fundamental solution which, under the imposition of the OS/GNS conditions, microcausality, locality of at most second order,

$O(3)$ isotropy on the slice, and the operational requirements (A1)–(A7), (Ob)–(Og), are realised as effective fields with a gauge group locally isomorphic to $SU(3) \times SU(2) \times U(1)$ and with the standard fermion content of a single family. This sector may be viewed as a subalgebra corresponding to a subspace of the one-particle space, whereas the remaining admissible modes of the fundamental solution span an orthogonal complement which, at the given operational resolving power, does not enter $\mathcal{A}_{\rm SM}(O)$.

It is important to emphasise that the present work does *not* assert the identity

$$\mathcal{A}(O) \;=\; \mathcal{A}_{\rm SM}(O)$$

for a given range of scales. The results obtained show that, under the specified operational and structural conditions, there exists a subalgebra $\mathcal{A}_{\rm SM}(O)$ with the indicated structure, but they do not rule out the presence of additional modes of the fundamental field which, at the same characteristic length and energy scales,

- either are integrated into effective parameters and background configurations (in the Wilsonian sense of integrating out irrelevant degrees of freedom) and are not singled out operationally as separate fields;
- or form a "dark" or hidden sector carrying no observable gauge charges and therefore not belonging to $\mathcal{A}_{\rm SM}(O)$, while contributing to the effective energy–momentum tensor and hence to the reconstructed metric;
- or turn out to be operationally inaccessible to the given observer due to limitations on the working region, resolving scales, or available measurement protocols.

Thus, in the present model the SM sector should be understood as the minimal operational subspace of the algebra $\mathcal{A}(O)$ that provides a consistent description of the observed "visible" matter and gauge interactions, but not as a claim that all modes of the fundamental field at the corresponding scales are thereby exhausted.

The analysis of possible additional modes, their contribution to the effective geometry, and their relation to the dark sector (including dark matter and, potentially, dark energy) is deferred to the subsequent subsections §9.4 and §9.5, as well as to the forward-looking note §9.6.

Note that the above analysis is not part of the core of the QFT derivation and is not used in the proofs of the main theorems. Its purpose is not to enlarge the set of postulates of the model, but to clarify how, in future work, all modes of the fundamental field lying outside $\mathcal{A}_{\rm SM}(O)$ can be taken into account and what kinds of effective and observable effects they may potentially produce. A detailed quantitative analysis of these consequences lies beyond the scope of the present paper and is left for future work.

9.3 Ultraviolet sector and limits of applicability of the effective description

The effective description of local quantum field theory obtained in this work applies to the range of scales where (i) special-relativistic (SR) kinetics on the slices Σ_s is valid; (ii) OS/GNS, the spectral condition, and polynomial boundedness hold; (iii) it is legitimate to use the standard machinery of renormalisable QFT (KL representations, LSZ/Haag–Ruelle, BRST, and BPHZ renormalisation).

From the point of view of the fundamental Euclidean model $\Delta_{\mathbb{E}^4}\Phi = 0$, the ultraviolet behaviour of the effective fields is controlled not by introducing additional fundamental UV terms, but by the structure of the mode decomposition on the slices Σ_s, by the choice of complex structure, and by the restrictions on test functions imposed by the observer (the working region Ω_s and the class of smoothings $\mathcal{F}_O$). In this description the genuinely "ultraviolet" sector corresponds to those modes of the fundamental solution Φ whose characteristic lengths are much smaller than the resolution scales of the observer (the size of Ω_0, the detector scale $r_{\rm det}$, etc.).

In the present work it is assumed that the influence of these modes on observable quantities at the given scales can be effectively accounted for in the form of a finite set of local operators in the Hamiltonian of the form (27) and a finite number of renormalisable parameters (gauge couplings, masses, Yukawa couplings, and scalar potentials). In other words, it is assumed that the structural assumptions (OS/GNS, the spectral condition, polynomial boundedness, absence of Ostrogradsky instabilities) are sufficient for the standard methods of renormalisable QFT to ensure that low-energy observables are independent of the detailed structure of the UV sector at subdetector scales.

A rigorous analysis of the UV sector within the model - in particular, an explicit description of the dependence of the effective parameters on the configuration of the fundamental field Φ and on the working scales of the observer, as well as possible constraints on the types of admissible ultraviolet spectra (for example, analogues of conditions of asymptotic freedom or asymptotic safety)—lies beyond the scope of the present paper and is left for future work.

9.4 Infrared modes and operational darkness

In constructing the effective fields we did not resolve modes with characteristic length L that is much larger than

(i) the characteristic size of the observer's body Ω_0, and

(ii) the limiting detector scale r_{det}, i.e. the upper bound on the diameter of the *support* of admissible smoothings $f \in \mathcal{F}_O$:

$$r_{\text{det}} := \sup_{f \in \mathcal{F}_O} \operatorname{diam}\big(\operatorname{supp} f\big).$$

Below we show that, for a fixed class of local smoothings $\mathcal{F}_O \subset C_0^\infty$ (contrast detection, $\int f = 0$) and a given threshold T_O, such infrared modes are operationally undetectable by local POVM measurements on Σ_s: their response is suppressed as $\mathcal{O}(r_{\text{det}}/L)$ (see Lemma 16). At the same time, they carry nonzero energy density and manifest themselves through the gravitational sector (the effective curvature of the foliations), see Proposition 21. In this subsection we fix the basic properties of such modes and the conditions for their "darkness".

In the SR regime we consider equal-time effective fields on a slice Σ_s and operational observables of the form

$$\mathcal{O}_f(s) \;=\; \int_{\Sigma_s} f(y)\, \psi(s,y)\, d\mu_{\Sigma_s}(y), \qquad f \in \mathcal{F}_O \subset C_0^\infty(\Sigma_s),\ \operatorname{supp} f \subset \Omega_{\text{det}} \subset \Sigma_s,\ \operatorname{diam}(\operatorname{supp} f) \le r_{\text{det}},$$

see §2.6, §3.7.

Lemma 16 (Infrared invisibility at finite operational bandwidth)**.** *Suppose there exists $r_{\text{det}} < \infty$ such that for all $f \in \mathcal{F}_O$ one has $\operatorname{diam}(\operatorname{supp} f) \le r_{\text{det}}$ and $\int_{\Sigma_s} f\, d\mu_{\Sigma_s} = 0$ (contrast detection). Consider a family of infrared configurations*

$$\psi_L(y) \;=\; A\, \phi(y/L), \qquad L \gg r_{\text{det}},$$

where $\phi \in C^\infty(\mathbb{R}^3)$ is of bounded smoothness. Then there exists a constant $C > 0$, independent of L, such that for any $f \in \mathcal{F}_O$ one has

$$|\mathcal{O}_f(0)| \;\le\; C\, |A|\, \frac{r_{\text{det}}}{L}\, \|f\|_{C^1},$$

where $\|f\|_{C^1} := \sup_{\operatorname{supp} f} |f| + \sup_{\operatorname{supp} f} |\nabla f|$. If, in addition, $\sup_{f \in \mathcal{F}_O} \|f\|_{C^1} < \infty$, then for a fixed sensitivity threshold $T_O > 0$ there exists $L_* < \infty$ such that for all $L \ge L_*$ and all $f \in \mathcal{F}_O$ one has $|\mathcal{O}_f(0)| < T_O$; such infrared modes are operationally undetectable by local POVM measurements on Σ_s for the given bandwidth $\mathcal{F}_O$.

Sketch of proof. A Taylor expansion gives

$$\psi_L(y) = A\Big(\phi(0) + L^{-1} y^i \partial_i \phi(0) + \mathcal{O}(L^{-2})\Big).$$

The condition $\int f \, d\mu_{\Sigma_s} = 0$ eliminates the leading contribution $\propto \phi(0)$. The remaining linear term is estimated as

$$\left| \int_{\operatorname{supp} f} y^i f(y)\, d\mu_{\Sigma_s}(y) \right| \;\le\; C\, r_{\rm det}\, \|f\|_{C^1},$$

which yields the stated $\mathcal{O}(r_{\rm det}/L)$ bound for $|\mathcal{O}_f(s)|$ on a fixed slice Σ_s. The constant C does not depend on L, so for a fixed threshold $T_O > 0$ one can choose $L_* < \infty$ such that for all $L \ge L_*$ and all $f \in \mathcal{F}_O$ one has $|\mathcal{O}_f(s)| < T_O$. Hence such infrared modes are operationally undetectable. □

Proposition 19 (Gravitational relevance of infrared modes)**.** *Let the local Hamiltonian be of the form* (27) *with second-order principal symbol as in Corollary 2, and let the potential* $V(s, y) \ge 0$ *vary on the scale* L*. Then for* ψ_L *from Lemma 16 the averaged (over a region of scale* $\sim L$*) energy density is*

$$\rho_{\rm IR} = \langle T_{00} \rangle = \frac{\hbar^2}{2}\, a \,\langle |\nabla \psi_L|^2 \rangle + V \,\langle |\psi_L|^2 \rangle = V\, |A|^2 + \mathcal{O}\big(|A|^2 L^{-2}\big),$$

while the effective pressure satisfies $p_{\rm IR} = \mathcal{O}\big(|A|^2 L^{-2}\big)$*. Hence, for* $L \gg r_{\rm det}$ *such modes behave as a dust-like component with* $w = p/\rho \approx 0$*, while remaining operationally "dark" by Lemma 16.*

Remark 30 (Limits of applicability)*.* (i) If $V \equiv 0$, then $\rho_{\rm IR} \sim |\nabla \psi_L|^2 = \mathcal{O}(L^{-2})$ and the contribution vanishes in the IR limit.
(ii) If one admits smoothing supports of scale $\gtrsim L$ (i.e. $r_{\rm det} \sim L$), infrared modes become observable; the "darkness" is operational in nature.
(iii) The statements are compatible with microcausality and OS-positivity and do not require higher derivatives beyond the constraints of Corollary 2.

9.5 Effective equation for the dark component on galactic scales

Infrared (IR) modes, operationally undetectable as in Section 9.4, generate a gravitationally relevant contribution phenomenologically equivalent to dark matter on galactic scales. Below we formulate a working (approximate) scheme for describing such a contribution in the Newtonian, quasi-stationary regime, without attempting a rigorous derivation from the full dynamics of the IR sector. The aim of this subsection is to fix the minimal phenomenology and testable consequences; a detailed analysis, parameter calibration, and cosmological generalisation lie beyond the scope of the present paper, which focuses on deriving the core QFT.

We work in the SR regime and in the Newtonian limit of GR (weak fields, quasi-stationarity). Let $\rho_{\rm bar}(x)$ be the baryonic density. Infrared modes, operationally undetectable by Lemma 16, contribute to the energy density (see Proposition 21); on galactic scales we account for this as an effective dark density $\rho_{\rm IR}$.

Response model for the IR sector. On a fixed foliation we assume a locally isotropic linear response of the infrared sector to the baryon distribution:

$$\rho_{\rm IR}(x) \;=\; \int_{\mathbb{R}^3} K_L\big(|x - x'|\big)\, \rho_{\rm bar}(x')\, \mathrm{d}^3 x', \qquad K_L(r) \ge 0, \;\; \int_{\mathbb{R}^3} K_L(r)\, \mathrm{d}^3 x = \eta. \tag{51}$$

Here L is the characteristic length of the IR structure (weakly dependent on the galaxy/environment type), and η is a dimensionless amplification coefficient. Isotropy is consistent with the $O(3)$ symmetry of the stabiliser of the slice.

Effective equation for the gravitational potential. In the weak-field, quasi-stationary regime the potential Φ satisfies

$$\nabla^2\Phi(x) \;=\; 4\pi G\,\rho_{\rm tot}(x), \qquad \rho_{\rm tot}(x) \;=\; \rho_{\rm bar}(x) \,+\, \rho_{\rm IR}(x), \tag{52}$$

with boundary condition $\Phi(x) \to 0$ as $|x| \to \infty$. Under spherical symmetry,

$$v_c^2(r) \;=\; \frac{G}{r}\Big(M_{\rm bar}(<r) + M_{\rm IR}(<r)\Big), \qquad M_{\rm IR}(<r) = 4\pi \int_0^r s^2 \rho_{\rm IR}(s)\,{\rm d}s.$$

Gravitational lensing. In the weak gravitational-lensing regime the surface density of the dark component is given by a convolution of the projected densities:

$$\Sigma_{\rm IR}(R) \;=\; \int_{\mathbb{R}} \rho_{\rm IR}\big(\sqrt{R^2+z^2}\big)\,{\rm d}z \;=\; \int_{\mathbb{R}^2} K_L^{(2{\rm D})}\big(|R-R'|\big)\,\Sigma_{\rm bar}(R')\,{\rm d}^2R', \tag{53}$$

where $K_L^{(2{\rm D})}$ is a radially symmetric kernel induced from K_L. Thus dynamics and lensing are determined by the same $\rho_{\rm tot}$ and are mutually consistent within GR.

Choice of kernel. Two practical choices, normalised as in (51), are:

$$K_L^{\rm exp}(r) = \frac{\eta}{8\pi L^3}\,e^{-r/L}, \qquad K_L^{\rm Cauchy}(r) = \frac{\eta}{4\pi}\,\frac{1}{(r^2+r_0^2)^{3/2}}, \quad (L \equiv r_0). \tag{54}$$

The first leads to profiles with a central core (a flat density plateau), the second to more extended haloes and quasi-flat rotation curves.

Remark 31 (Qualitative compatibility on galactic scales). Under the assumptions of Lemma 16 and proposition 21 and in the Newtonian quasi-stationary regime, the system (52)–(51) is *qualitatively compatible* with typical galactic phenomenology at radii $r \sim 1$–$50\,$kpc: (a) the emergence of flat rotation-curve plateaux; (b) the presence of central cores in dwarf and LSB galaxies; (c) a monotonic relation $g_{\rm tot}(g_b)$ in the spirit of the RAR for moderate calibration of (η, L). This statement is phenomenological; it is based on analytic estimates for spherically symmetric profiles and on numerical checks for model distributions. A statistical comparison with catalog data and cosmological tests are beyond the scope of the present paper.

Remark 32 (Limits of applicability). (a) For cluster mergers and strong-lensing applications a more general response scheme than (51) (not purely convolutional) or an environmental dependence of L may be required.
(b) Calibration of the parameters (η, L) against samples of rotation curves and lensing data is mandatory before drawing quantitative conclusions.
(c) On cosmological scales a dynamical generalisation of the kernel K_L is required.

9.6 Perspective: infrared dilation as an effective Λ

Conceptual note (not used in the proofs of the core QFT). Consider a homogeneous and isotropic dilatational contribution to the transport along s, acting only on the infrared modes of the three-dimensional slice metric:

$$\partial_s g_{ij} \;\supset\; 2H\,\Pi_{\rm IR}[g_{ij}],$$

where $\Pi_{\rm IR}$ is the projector onto scales $\ell \geq \ell_*$, and screening is realised in gravitationally bound regions (effectively $\Pi_{\rm IR} \approx 0$). In unbound regions, by contrast, $\Pi_{\rm IR} \approx \mathbf{1}$. After analytic continuation this leads to an operational scale factor $a(t) = a_0 e^{Ht}$ and, for a homogeneous and isotropic effective metric, is kinematically equivalent to a cosmological constant $\Lambda_{\rm eff} = 3H^2/c^2$. In this picture local dimensionless calibrations remain unchanged, and gravitationally bound systems do not undergo effective expansion (the expansion is suppressed by the screening $\Pi_{\rm IR}$).

In such a description dark energy is interpreted as an infrared dilation in the metric sector, consistent with the operational setup and not requiring, at the level of the effective description, any explicit "pumping" of energy into the matter sector. A rigorous derivation of the IR selection and the screening mechanism from the principles of causal reconstruction and OS/GNS, as well as the verification of the Bianchi identities for the reconstructed metric, is left for future work.

10 Conclusion

In a timeless Euclidean model with a single real field, a local quantum field theory in the SR regime has been obtained as an emergent structure derived from the operational principle of observability and causal reconstruction. The main results are:

(i) the SR kinematics and the second order of the principal symbol for the effective fields have been established (§5);

(ii) the Peierls symplectic form, the canonical (anti)commutation relations (CCR/CAR), and microcausality have been constructed (App. C);

(iii) the resulting algebra of observables fits into the standard axiomatic framework (OS/GNS, spectral condition, microlocality, and SR covariance), so that in appropriate regimes one can apply to it the classical structural results of QFT (KL, CPT, spin–statistics, etc.), to which we refer without repeating the proofs;

(iv) from the necessary operational conditions the minimal gauge group $SU(3)\times SU(2)\times U(1)$ has been obtained, the connection has been introduced operationally, and the uniqueness of the second–order Yang–Mills kinetic term for massless spin–1 channels has been justified (§6).

Within the same Euclidean model, the classical gravitational sector has been obtained previously; taken together with the results of the present work, this shows that the effective geometry of the foliations and the local QFT share the same underlying fundamental degree of freedom. In regimes of small curvature and weak fields, the resulting QFT and the effective foliation geometry locally reproduce the standard scenario of quantum field theory on a curved background in the sense of GR: the stress–energy tensor of the effective fields plays the role of the source of the metric, without introducing independent quantum degrees of freedom of the metric.

The QFT structure, for fixed operational and structural assumptions of the model (see (A1)–(A7), (Ob)–(Og)), is obtained without introducing additional physical postulates beyond locality, the OS/GNS construction, the spectral condition, and anomaly freedom. At the same time, the numerical values of the constants $\{g_i, y_f, \lambda, v, \theta_{\mathrm{QCD}}\}$ are not computed in this work: they are structurally realised as parameters determined by the correlators of the fundamental field and by the foliation regime. The corresponding inverse problem of reconstructing the constants from observables (via KL measures, geometric invariants, and RG flow) has been formulated, but its explicit solution is left beyond the scope of the paper; the absence of numerical values does not compromise the completeness of the structural derivation of the QFT. At the level of the mode description, the construction of the OS/GNS representation and the transport along s lead to a natural splitting into infrared, ultraviolet, and orthogonal (operationally unresolved) sectors; a detailed analysis of their role in renormalisation and decoherence is likewise deferred to future work.

Limitations: the colour dynamics (confinement, spectrum) and the fine infrared structure of $U(1)$ are not developed; an algebraic formulation on the net $\mathcal{A}(O)$ in generic curved regimes requires a separate analysis. These issues are reduced to known results under the assumptions adopted here and are indicated as directions for further work (inverse problem for the parameters, non–Euclidean and curvilinear regimes, full algebraic formulation).

The model is falsifiable within the domain of applicability of the effective QFT constructed here, that is, in regimes of small curvature and absence of strong fields in the working region of the observer under the operational assumptions. In this regime it rules out additional long–range non–Abelian charges, inconsistent hypercharge assignments, and observable quantum signatures of a gravitational mediator. Thus it is demonstrated that a single local QFT of the Standard Model, structurally consistent with the previously derived gravitational sector (GR) within the same model, arises in the present timeless Euclidean framework as an emergent structure of a single fundamental degree of freedom under the operational conditions of observability and causal reconstruction.

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A Reflection positivity and OS/GNS reconstruction

This appendix collects abstract results used in the main text. We do *not* redefine the transport α_σ and locality: strong continuity of α_σ is ensured by Lemma 5 in the main text, and detailed balance by Prop. 1. The formulations are based on the standard approach to reflection positivity and OS/GNS reconstruction; see, for example, [6, 7, 8].

A.1 Notation and input assumptions

Let $\mathfrak{A}$ be a local *-algebra of observables, and let $\mathfrak{A}_+ \subset \mathfrak{A}$ be the *-subalgebra generated by smeared observables localized in the half-space $s \geq 0$. Denote by $\{\alpha_\sigma\}_{\sigma\geq 0}$ a semigroup of *-endomorphisms of $\mathfrak{A}$ implementing shifts along s (strong continuity on the dense subalgebra Alg_+ is given in Lemma 5). Let $\Theta : \mathfrak{A} \to \mathfrak{A}$ be an antilinear involutive reflection ($\Theta^2 = \mathrm{id}$) corresponding to $s \mapsto -s$. Let ω be a state on $\mathfrak{A}$ that is stationary and Θ-invariant:

$$\omega \circ \alpha_\sigma = \omega, \qquad \omega \circ \Theta = \omega, \qquad \Theta\alpha_\sigma = \alpha_\sigma\Theta, \quad \sigma \geq 0. \tag{55}$$

Definition 5 (OS form). For $F, G \in \mathfrak{A}_+$ set $(F,G)_{\mathrm{OS}} := \omega\big((\Theta F)\,G\big)$. Define the null subspace $\mathcal{N} := \{F \in \mathfrak{A}_+ :\ (F,F)_{\mathrm{OS}} = 0\}$.

Proposition 20 (Detailed balance ⇒ OS positivity). *If detailed balance in s holds,*

$$\omega\big(F^* \,\alpha_\sigma(G)\big) = \omega\big((\Theta\,\alpha_\sigma F)^*\,\Theta G\big), \qquad \forall\, F, G \in \mathfrak{A}_+,\ \ \forall\, \sigma \geq 0, \tag{56}$$

then $(\cdot,\cdot)_{\mathrm{OS}}$ *is positive semidefinite on* $\mathfrak{A}_+$*:* $\sum_{i,j} \overline{c_i} c_j\,(F_i, F_j)_{\mathrm{OS}} \geq 0$ *for any finite sets* $F_i \in \mathfrak{A}_+$*,* $c_i \in \mathbb{C}$*.*

Sketch of proof. It suffices to consider elements of the form $F = \sum_i c_i\,\alpha_{\sigma_i}(A_i)$ with $A_i \in \mathfrak{A}_+$, $\sigma_i \geq 0$. Then $(F,F)_{\mathrm{OS}} = \sum_{i,j} \overline{c_i} c_j\,\omega\big((\Theta\alpha_{\sigma_i} A_i)\,\alpha_{\sigma_j} A_j\big)$. By (56) this is the Gram matrix of a positive semidefinite form, hence the sum is nonnegative. □

In this appendix we provide an explicit constructive version of Theorem 1; cf., for example, [6, 7, 8].

Theorem 2 (OS/GNS reconstruction: constructive formulation). *Assume* (55)*, Prop. 22, and the strong continuity conditions for* α_τ *from Lemma 5. Then there exist:*

- *a Hilbert space* $\mathcal{H}$*, the completion of the quotient space* $\mathfrak{A}_+/\mathcal{N}$ *with respect to the norm induced by the inner product* $(\cdot,\cdot)_{\mathrm{OS}}$*;*
- *a cyclic vector* $\Omega = [\mathbf{1}] \in \mathcal{H}$*;*
- *a *-representation* $\pi : \mathfrak{A} \to \mathcal{B}(\mathcal{H})$ *of local observables on* $\mathcal{H}$ *with* $\omega(A) = \langle \Omega, \pi(A)\Omega\rangle$*;*
- *a contractive strongly continuous semigroup* $T(\tau) = e^{-\tau H}$*,* $\tau \geq 0$*, with a self-adjoint* $H \geq 0$*, defined by* $T(\tau)[F] := [\alpha_\tau(F)]$ *on the dense subspace* $[\mathfrak{A}_+] \subset \mathcal{H}$*.*

Moreover, the semigroup $T(\tau)$ *extends to a unitary group* $U(t) = e^{-iHt}$*,* $t \in \mathbb{R}$*. Defining, for* $B \in \pi(\mathfrak{A})$*, the automorphisms*

$$\tau_t(B) := U(t)\,B\,U(t)^{-1}, \qquad t \in \mathbb{R},$$

one obtains the dynamics in "physical" time in the standard sense of OS/GNS reconstruction.

Sketch of proof. OS positivity (Prop. 22) and factorisation by $\mathcal{N}$ yield a pre-Hilbert space. Strong continuity (Lemma 5) ensures that the map $T(\tau)[F] := [\alpha_\tau(F)]$ is well defined and strongly continuous on a dense domain. By the Hille–Yosida theorem (see, for example, [9]) one has $T(\tau) = e^{-\tau H}$ with a self-adjoint $H \geq 0$. The standard OS reconstruction and analytic continuation in time then give $U(t) = e^{-iHt}$ with the required covariance; see, for example, [6, 7, 8]. □

Remark 33 (Gauge-invariant subalgebra). In the massless vector sector, OS positivity is verified on a gauge-invariant subalgebra (for example, generated by $F_{\mu\nu}$) or after BRST reduction. The formulations of Prop. 22 and Theorem 1 apply to this subalgebra without modification.

In the main part of the paper, detailed balance is established in §3.2 (Prop. 1), and strong continuity in §3.4. Hence the assumptions of Prop. 22 and Theorem 1 are satisfied, and the OS/GNS reconstruction applies without additional axioms.

B Structural assumptions for transport in s and measurements

This appendix provides a detailed discussion of the structural assumptions used in Section 3, in particular of the conditions (A1)–(A7) briefly stated in §3.9. We trace their relation to the fundamental Euclidean model $\Delta_{\mathbb{E}^4}\Phi = 0$, to the requirements of causal reconstruction, and to the results of [1, 2], and we indicate which of these assumptions are strict consequences of the model and which should be viewed as natural structural restrictions on the class of solutions under consideration.

(A1) Class of states and detailed balance. In §3.1 and §3.2 we use a special class of states ω on the local algebra. For clarity we split its properties into three groups.

(A1.s) Stationarity with respect to transport in s. In §2 a local orthonormal basis of modes $\{\varphi_\alpha(y)\}$ on Σ_s was chosen for an inertial observer; in the SR regime this basis may be taken to be the same for all s when working relative to the foliation. Transport in s then acts only on the expansion coefficients, $a_\alpha(s) \mapsto a_\alpha(s+\sigma)$, and defines a one-parameter semigroup α_σ of automorphisms of the local algebra.

The absence of a distinguished origin in s and the Euclidean symmetry of the fundamental model imply that an inertial observer cannot operationally distinguish the slices Σ_s and $\Sigma_{s+\sigma}$ under fixed local conditions in the working region. Accordingly, the statistics of repeated measurements in her laboratory should not depend on the absolute value of the parameter s, but only on relative shifts. In algebraic terms this is expressed by stationarity of the chosen state ω with respect to α_σ:

$$\omega \circ \alpha_\sigma = \omega, \qquad \sigma \geq 0.$$

Thus, in what follows we consider the state of an inertial observer that is consistent with the mode decomposition, the symmetry in s, and the already used conditions of causal reconstruction; stationarity is not introduced here as an independent new postulate, but as a refinement of these structures.

(A1.g) Gaussianity and free regime. Subsequently we restrict attention to quasifree (Gaussian) states ω, for which all n-point correlation functions are expressed in terms of the two-point function, and the two-point function itself is determined by the inverse differential operator J^{-1}:

$$\omega(\hat{\psi}(u)\hat{\psi}(v)) = \tfrac{1}{2}\,\langle u, J^{-1}v\rangle$$

(with the precise definition of J given below). Such a state may be interpreted as a free or vacuum-like regime in the SR region: it is consistent with flat foliations, OS structure, and locality of the effective dynamics.

The fundamental Euclidean model with $\Delta_{\mathbb{E}^4}\Phi = 0$, the linear map to effective fields, and the requirements of causal reconstruction make the choice of such a class of states natural: in the SR regime there is no distinguished value of the parameter s, and the restriction on principal symbols (see (A2)) together with Euclidean symmetry effectively leave quasifree OS states as the simplest option. However, a strict derivation of Gaussianity of ω from the fundamental model (for instance, via explicit averaging over unresolved modes or via a maximum-entropy principle at fixed two-point function) is not carried out in the present work and will be considered separately. Here quasifreeness of ω should be understood as a working structural assumption that specifies the class of SR states for which the results of this section are formulated.

(A1.os) Reflection symmetry and OS structure. In the SR regime we choose a flat foliation $\{\Sigma_s\}$ in $\mathbb{E}^4$, orthogonal to a fixed Euclidean direction, and consider solutions of $\Delta_{\mathbb{E}^4}\Phi = 0$ invariant under the Euclidean group. The map $s \mapsto -s$ is implemented as reflection with respect to the hypersurface Σ_0 and is an element of the Euclidean symmetry group of the fundamental model. For a choice of a basic (vacuum-like) state ω invariant under these symmetries one automatically has

$$\omega \circ \Theta = \omega, \qquad \Theta\alpha_\sigma = \alpha_\sigma\Theta,$$

where Θ implements the reflection $s \mapsto -s$, as well as the standard OS-positivity conditions for Euclidean correlators of the free field [6, 7, 8]. Thus reflection symmetry and detailed balance in the form of Proposition 1 in the SR regime are not independent assumptions, but follow from the Euclidean symmetry of the fundamental equation and from the choice of flat foliations; we merely fix that we work with the corresponding OS state ω.

(A2) Restriction on the order of the principal symbol. In Corollary 2 and subsequently we assume that the density of the effective Hamiltonian on a slice Σ_s is local in y and contains spatial derivatives of order at most two. This restriction is consistent with the gravitational sector in [2], where it is shown, starting from operational requirements, locality, and causal reconstruction, that the effective action for the geometry of the foliations contains derivatives of order at most two (in analogy with Lovelock's theorem [10]). In this section we extend the same principle to the matter sector and consider only those effective Hamiltonians whose principal symbol is of second order in spatial derivatives. A strict repetition of the derivation for matter is not carried out here; at the level of the present paper this condition should therefore be regarded as a working assumption motivated by the construction of GR and not introducing new independent postulates beyond [2].

(A3) Regularity and boundary conditions. The technical condition 3.4 fixes the class of local Hamiltonians with which we work subsequently. It is assumed that the local operator $\hat{h}$ has the form (27) with a second-order principal symbol (see (A2)) and satisfies the following conditions:

(A3.1) the coefficients $a^{ij}(s, y)$, $b^i(s, y)$, $V(s, y)$ are local in y, smooth in the spatial coordinates, and have C^1 dependence on s with uniform bounds on compact sets in the working region Ω_s;

(A3.2) on the boundary $\partial\Omega_s$ one imposes (possibly s-dependent) Dirichlet/Neumann/Robin type boundary conditions ensuring vanishing probability flux through the boundary and hence operational isolation of the working region of the observer;

(A3.3) under these conditions there exists a self-adjoint extension H of the operator $\hat{h}$ in the corresponding Hilbert space, and H generates a unitary group $U(t) = e^{-iHt}$ consistent with the OS/GNS representation.

Substantively, (A3) does not introduce new physics compared to the general setup of the model. Item (A3.1) is a functional-analytic implementation of stability of the definition of events and of transport in s: small changes of the foliation and of the parameter s do not lead to discontinuities in the effective dynamics in Ω_s. Item (A3.2) expresses operational isolation of the working region of the observer and the absence of probability exchange through its boundary. Item (A3.3) ensures consistency of the local differential form of the Hamiltonian with the already constructed OS/GNS structure (self-adjointness of H and unitarity of $U(t)$). All results of Section 3 below are formulated under these technical conditions.

(A4) Complex structure and one-particle space. In §3.5 for the scalar field on the slice Σ_s we introduce a complex structure J on the test function space $\mathcal{D}(\Sigma_s)$, compatible with the symplectic form ω_Σ (the Pauli–Jordan kernel) and with a positive form μ (see (21)). We require that

$$\mu(u, v) = \omega_\Sigma(u, Jv), \qquad J^2 = -\mathbf{1},$$

and that the induced inner product $\langle u, v\rangle_{1\mathrm{p}} := \mu(u, v) + \frac{i}{2}\omega_\Sigma(u, v)$ yields a one-particle Hilbert space on which the spectral condition and the local Poincaré group in the SR regime are realized.

In the flat SR regime the choice of such a complex structure is essentially not arbitrary. The fundamental Euclidean model with equation $\Delta_{\mathbb{E}^4}\Phi = 0$, OS-positivity, and the requirement of Lorentz invariance of the effective dynamics fix the symplectic form ω_Σ and the class of positive forms μ compatible with the spectral condition. For a free field on a flat background this leads to the canonical complex structure J corresponding to the decomposition into positive- and negative-frequency modes; it is unique up to unitary transformations preserving ω_Σ.

Thus, in the present work we do not introduce an independent postulate of an arbitrary choice of J; rather, we explicitly fix that complex structure which is compatible with (i) the SR regime (flat foliations and the local Poincaré group), (ii) the OS/GNS representation and the spectral condition, (iii) the symplectic structure induced by the fundamental equation. This fixing is used for the explicit construction of the one-particle space and the subsequent formulation of QFT; in this sense (A4) should be regarded as a canonical refinement of the SR regime, rather than as a new physical postulate.

(A5) POVM formalism for measurements. In §3.7 it is assumed that all operationally accessible measurements for a fixed observer O are described by positive-operator-valued measures (POVM) $E(B)$ on Borel sets $B \subset \mathbb{R}$ (or on a more general measurable space) in her effective Hilbert space $\mathcal{H}$, together with the associated completely positive (CP) instruments on the local algebra $\mathcal{A}(\Omega_s) \subset \mathcal{B}(\mathcal{H})$. That is, to each measurement protocol previously defined in purely operational terms (as a class of events labelled by outcomes) one associates a POVM $E(\cdot)$, and the outcome probabilities are given by the state functional ω according to

$$p_\omega(B) = \omega\big(E(B)\big),$$

with the subsequent state evolution described by the CP instrument associated with this POVM.

This assumption does not in substance introduce a new physical structure, but rather fixes the standard algebraic language for an already constructed object. At the preceding steps the fundamental model $\Delta_{\mathbb{E}^4}\Phi = 0$, the operational definition of the observer and of events, and the OS/GNS representation define a triple $(\mathcal{H}, \mathcal{A}(\Omega_s), \omega)$, i.e. precisely the situation in which the general theory of quantum measurements leads to the POVM formalism and CP instruments as the most general description of measurement procedures [11, 12, 13, 14]. Assumption (A5) identifies the abstract operational protocols defined via events and the observer's working region with this standard formalism on $\mathcal{H}$.

In the SR regime, where the effective dynamics in Ω_s is described by a local quantum field theory on a flat background (see §3.5 and subsequent sections), the use of the POVM formalism for describing measurements is standard: local measurements are modelled by effects and CP instruments on local algebras $\mathcal{A}(O)$, while Lorentz invariance and microcausality are implemented via the structure of these algebras, rather than via a restriction of the class of POVM. In this sense (A5) should be regarded as a natural alignment of the operational description of measurements adopted in the model with the general algebraic formalism of quantum measurements in the already constructed structure $(\mathcal{H}, \mathcal{A}(\Omega_s), \omega)$, rather than as an independent new postulate.

(A6) Averaging over unresolved degrees of freedom of the apparatus. Assumption 2 models the fact that a real measuring device possesses a large number of unresolved (operationally uncontrolled) microscopic degrees of freedom. We assume that for a fixed observer O and a fixed type of measurement protocol there exists a parameter space Θ_O, a probability measure ν_O on Θ_O, and a family of POVM $\{E_\theta(B)\}_{\theta\in\Theta_O}$ on $\mathcal{H}$ such that the effective POVM of this protocol can be represented as

$$E(B) \;=\; \int_{\Theta_O} E_\theta(B)\, d\nu_O(\theta),$$

where θ describes the microstate of the apparatus, unresolved in the operational description.

From the standpoint of the fundamental model $\Delta_{\mathbb{E}^4}\Phi = 0$ each individual realization of an experiment is determined by the full field configuration Φ (including those regions and modes that are operationally attributed to the "apparatus"), and the measurement outcome is a functional of Φ and of the specific realization of the device. At the effective OS/GNS level, however, we do not work with Φ directly, but use the state ω on the local algebra and an averaging over uncontrolled microscopic degrees of freedom of the apparatus. Assumption (A6) precisely formalizes this averaging: the parameter θ plays the role of an effective description of the apparatus microstate, and the measure ν_O describes the distribution over such microstates in a class of repeated experiments [14, 15].

In substance, (A6) is a natural refinement of (A5): once all operational measurements are described by POVM and CP instruments on $(\mathcal{H}, \mathcal{A}(\Omega_s), \omega)$, the assumption that an effective POVM can be represented as a mixture $\{E_\theta\}$ expresses the standard situation of incomplete control over internal degrees of freedom of the apparatus and does not introduce new fundamental dynamics. In the present work the distribution ν_O and the structure of Θ_O are not derived directly from the fundamental configuration Φ; they are treated as effective characteristics of a given measurement protocol, consistent with the operational description and used in the proof of Proposition 5.

(A7) Fundamental determinism and definition of the matter sector. The fundamental level of the model is given by the Laplace equation $\Delta_{\mathbb{E}^4}\Phi = 0$ on Euclidean space $\mathbb{E}^4$ with specified boundary conditions. The full configuration Φ (together with the boundary data) thereby fixes all derived quantities, including those describing the working region of the observer and the microscopic degrees of freedom of the measuring devices. In this sense the outcome of any particular measurement procedure is a deterministic functional of the full fundamental configuration Φ and of the chosen realization of the measuring apparatus at this level.

In this section, however, we do not work with the full configuration Φ, but instead use an effective description in terms of the OS/GNS representation, a state ω on the local algebra, and averaging over uncontrolled degrees of freedom of the apparatus (Assumption 2). By a "microstate" of the apparatus in this context we thus mean not the full fundamental state Φ, but a parameter $\theta \in \Theta_O$ describing those device details that remain unresolved in the operational description. At the level of the effective theory, measurement outcomes are described by Born probabilities $\omega(E_\theta(\Delta))$ and their averages over ν_O; the deterministic character of the fundamental evolution is not postulated in addition here, but is implemented by the fact that all these probabilities are functions of one and the same configuration Φ, hidden behind the state ω and the measure ν_O.

In the same spirit, Definition 5 singles out a subalgebra $\mathcal{A}_{\text{matter}}(O) \subset \mathcal{A}(\Sigma_s)$ with given properties (microcausality, CCR/CAR, local POVM, contribution to $T_{\mu\nu}$), which is interpreted as the "matter sector" in the effective description. This selection is consistent with the fundamental deterministic model, but in itself is a structural hypothesis about which effective excitations should be regarded as material at the level of local algebras of the observer.

Taken together, assumptions (A1)–(A7) do not introduce new physics compared to the original Euclidean model and the operational setup; rather, they refine the class of regimes under consideration: stationary SR states of an inertial observer, effective second-order Hamiltonians with regular coefficients and reflection symmetry, a canonical complex structure in the one-particle space, and the standard POVM formalism for local measurements with averaging over unresolved apparatus degrees of freedom. Some of these conditions (in particular, reflection symmetry in the SR regime, properties of the geometric sector, and fundamental determinism) are strictly derived from the model and from previously obtained results; others (Gaussianity of the chosen state, selection of the matter sector, etc.) are natural structural restrictions on the class of solutions under study, which can be given a strict justification in separate works. Within the present paper all statements of Section 3 are to be understood under assumptions (A1)–(A7).

C Peierls symplectic form and canonical algebras

We work in the SR regime. By Lemma 12 there exist retarded and advanced fundamental solutions E_R and E_A with causal support [3, 16], and their difference

$$E := E_R - E_A \tag{57}$$

satisfies $P_x E = P_{x'} E = 0$ and $\operatorname{supp} E \subset J^+(x) \cup J^-(x)$. In the terminology of Definition 6, this distribution E is the Pauli–Jordan kernel for the operator P in the scalar case under consideration.

Recalling (44), for $f, g \in C_0^\infty(\mathbb{R} \times \Sigma_s)$ the Peierls form is given by

$$\varpi(f, g) = \iint f(x)\, E(x, x')\, g(x')\, \mathrm{d}x\, \mathrm{d}x'.$$

Antisymmetry of ϖ follows from $E^\top = -E$, and locality for causally disjoint supports from (45). Neither E nor ϖ is redefined here.

Proof of Proposition 12. If $\operatorname{supp} f$ and $\operatorname{supp} g$ are causally disjoint, then by the support properties of E one has $\varpi(f, g) = 0$. In the scalar case the commutator is equal to $i\,\varpi(f, g)\,\mathbf{1}$, as in the standard Peierls formulation [17, 18, 16], hence $[\Psi(f), \Psi(g)] = 0$. For the Dirac field the difference $E_D := S_R - S_A$ has the same support, which yields local anticommutativity. □

Scalar sector: CCR. On the quotient of the test functions by the equation of motion the canonical commutation relations take the form

$$[\Psi(f), \Psi(g)] = i\,\varpi(f, g)\,\mathbf{1},$$

where ϖ is as above; equal-time microcausality follows from (45), see, e.g., [19, 20].

Fermionic sector: CAR and microcausality. Let $\mathcal{D}$ be the Dirac operator and $\mathcal{T}$ the space of test spinors, factored by $\mathcal{D}\mathcal{T}$. Then

$$\{\Psi(f), \Psi(g)\} = 0, \qquad \{\Psi(f), \Psi^\dagger(g)\} = \langle f, g\rangle_{\mathrm{CAR}}\,\mathbf{1},$$

and local anticommutativity follows from the causal support of E_D; see, e.g., [19, 20].

Gauge sector. In the massless vector case the CCR are formulated on the gauge-invariant subalgebra generated by $F_{\mu\nu}$, or after BRST reduction; see, e.g., [21, 22].

Surface charges in the abelian case. Let Σ be an equal-time slice and $S_R \subset \Sigma$ a sphere of radius R.

$$Q(S_R) \ := \int_{S_R} *F, \tag{58}$$

Lemma 17. *For any $S_R, S_{R'} \subset \Sigma$:*

$$\{Q(S_R), Q(S_{R'})\}_{Peierls} = 0, \qquad [Q(S_R), Q(S_{R'})] = 0,$$

and in the limit $R, R' \to \infty$: $[Q_\infty, Q_\infty] = 0$.

Summary. The Peierls form based on E defines the scalar CCR algebra and, for spinors, the CAR; microcausality is a consequence of the causal support of the fundamental solutions.

D Operational derivation of the minimal symmetry group: details

D.1 Long-range spin-1 channel: abelianity and kinetic term

We now present the proof of Lemma 13.

Step A: abelianity from Gauss's law. In a non-abelian theory F is an algebra-valued 2-form, $F = dA+A\wedge A$, and any surface charge is constructed as $Q_\alpha(S) := \int_S \mathrm{Tr}(\alpha \star F)$ with a fixed parameter α at infinity; without specifying α a scalar charge is not defined. The assumption of the lemma is the existence of a *gauge-invariant* scalar charge $Q(S) = \int_S \star F$ without a parameter. This is possible if and only if the corresponding generator lies in the center of the algebra. Any one-dimensional central ideal of a finite-dimensional compact Lie algebra is isomorphic to $\mathfrak{u}(1)$; hence the only nontrivial one-dimensional central factor is $\mathfrak{u}(1)$. It follows that the long-range symmetry factor is realized as an abelian $U(1)$.

Step B: vanishing equal-time Peierls bracket of charges. For abelian Maxwell theory the covariant symplectic form

$$\Omega(\delta_1 A, \delta_2 A) = \int_\Sigma \delta_1 F \wedge \delta_2 A - \delta_2 F \wedge \delta_1 A$$

induces the Peierls bracket. The asymptotic charge $Q_\infty = \lim_{R\to\infty} \int_{S_R} \star F$ is a functional which is invariant under $\delta_\lambda A = d\lambda$ with $\lambda \to 0$ at infinity and linear in F. From $\delta_{Q_\infty} A = 0$ and the bilinearity of Ω it follows that $\{Q_\infty, Q_\infty\} = 0$ (the equal-time commutator also vanishes). Equivalently, the algebra of asymptotic charges is abelian.

Step C: uniqueness of the quadratic second-order kinetic term. We require Lorentz invariance, locality, gauge invariance, and that the Euler–Lagrange equations contain at most second-order derivatives. At the level of local Lorentz-invariant functionals which are quadratic in the field $F_{\mu\nu}$, the only independent invariants are $F_{\mu\nu}F^{\mu\nu}$ and $F_{\mu\nu}\tilde{F}^{\mu\nu}$. The second one is a topological total divergence and does not affect the equations of motion. Any term of the form $(\partial\cdot A)^2$ breaks gauge invariance. Therefore the kinetic term is unique and equals $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$. □

E Conventions and distributions

Metric and Fourier transforms. $\eta = \mathrm{diag}(1, -1, -1, -1)$, $p\cdot x := \eta_{\mu\nu} p^\mu x^\nu$.

$$\widetilde{f}(p) := \int d^4x \, e^{ip\cdot x} f(x), \qquad f(x) = \int \frac{d^4p}{(2\pi)^4} \, e^{-ip\cdot x} \widetilde{f}(p).$$

Basic distributions.

$$\Delta_\mu^{(+)}(x) = \int \frac{d^4p}{(2\pi)^3} \, \theta(p^0) \, \delta(p^2 - \mu^2) \, e^{-ip\cdot x}, \quad \Delta_F(p) = \frac{i}{p^2 - \mu^2 + i0}.$$

The Pauli–Jordan function $E(x) = \frac{1}{i}\langle \Omega, [\psi(x), \psi(0)]\Omega\rangle$ has support on the light cone. Euclidean continuation: $p_4 = ip^0$, $p_E^2 = p_4^2 + \mathbf{p}^2$.

F Tensor spectral measures: spinor and vector fields

We work in the SR regime; all equalities are understood in the sense of distributions, and contact terms (polynomials in p) are omitted.

Dirac field.

$$S_{\alpha\beta}(x)=\int_0^\infty \rho_{1/2}(\mu^2)\left(i\gamma^\mu\partial_\mu+\mu\right)_{\alpha\beta}\Delta_\mu^{(+)}(x)\,d\mu^2,\quad S_F(p)=\int_0^\infty \rho_{1/2}(\mu^2)\frac{i(\gamma\cdot p+\mu)}{p^2-\mu^2+i0}d\mu^2,$$

with $\rho_{1/2}(\mu^2)\geq 0$ by OS-positivity and the Källén–Lehmann representation for spinor fields [23, 24, 25, 19, 26].

Conserved current.

$$\widetilde{W}_{\mu\nu}(p)=2\pi\,\theta(p^0)\int_0^\infty \rho_J(\mu^2)\,\delta(p^2-\mu^2)\,\Pi_{\mu\nu}(p)\,d\mu^2,\quad \Pi_{\mu\nu}:=\eta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2},\quad \rho_J\geq 0.$$

Such a Lorentz-covariant spectral form for a conserved current is standard in axiomatic and algebraic formulations of QFT [25, 19, 26].

Field strength $F_{\mu\nu}$.

$$\widetilde{W}_{\mu\nu,\rho\sigma}(p)=2\pi\,\theta(p^0)\int_0^\infty \rho_F(\mu^2)\,\delta(p^2-\mu^2)\,\mathcal{P}_{\mu\nu,\rho\sigma}(p)\,d\mu^2,\quad \rho_F\geq 0,$$

$$\mathcal{P}_{\mu\nu,\rho\sigma}:=\Pi_{\mu[\rho}\Pi_{\sigma]\nu}-\Pi_{\mu[\sigma}\Pi_{\rho]\nu}.$$

The massless pole yields helicities ± 1, in agreement with the standard tensor Källén–Lehmann decomposition for vector and tensor fields [25, 19, 26].

G Uniqueness of the second-order Yang–Mills kinetic term

Proof of Prop. 16.

Classification of invariants. Let $A_\mu\in\mathfrak{g}$, $D_\mu=\partial_\mu+igA_\mu$, $F_{\mu\nu}=\frac{i}{g}[D_\mu,D_\nu]$. Local gauge- and Lorentz-invariant scalars with ≤ 2 derivatives of A can be reduced, by integrations by parts and the Bianchi identities, to the linear span of $\operatorname{tr}F_{\mu\nu}F^{\mu\nu}$ and $\operatorname{tr}F_{\mu\nu}\tilde{F}^{\mu\nu}$. Terms of the form $(\partial\cdot A)^2$ are not gauge invariant and are admissible only as gauge-fixing terms.

Sign. OS-positivity and the spectral condition fix the sign in front of F^2 so that the energy is ≥ 0. $\operatorname{tr}F\tilde{F}$ is a total divergence and does not affect the equations of motion. Hence $\mathcal{L}_{\mathrm{YM}}=-\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu}$ up to a total divergence.

H Operational gauging and Ward identities

Details for Prop. 15.

We work in the SR regime. We use: local C*-algebras $\{\mathcal{A}(O)\}$ and microcausality (§5); the existence of conserved currents J^μ_a of a global internal symmetry with $\partial_\mu J^\mu_a=0$ in the distributional sense; anomaly freedom (absence of Schwinger terms); and energy bounds ensuring that the smeared charges are densely defined on a common domain.

H.1 Form charges and local implementers

Let $\alpha(\mathbf{x})=\alpha^a(\mathbf{x})T_a\in C_0^\infty(O,\mathfrak{g})$ and let $\chi_\varepsilon\in C_0^\infty(\Sigma_s)$ be a cutoff function such that $\chi_\varepsilon\equiv 1$ on $\operatorname{supp}\alpha$, $\chi_\varepsilon\to 1$ pointwise, and $\|\nabla\chi_\varepsilon\|_{L^1}\to 0$ as $\varepsilon\downarrow 0$. Define the quasilocal *form charge*

$$Q_\varepsilon(\alpha,t)\;:=\;\int_{\Sigma_s}\alpha^a(\mathbf{x})\,\chi_\varepsilon(\mathbf{x})\,J^a_0(t,\mathbf{x})\,d^3\mathbf{x}. \tag{59}$$

Then, for any $A \in \mathcal{A}(O)$, the commutator $[\,Q_\varepsilon(\alpha,t), A\,]$ is defined on a dense domain $\mathcal{D}_{\rm fin}$ and has a limit as $\varepsilon \downarrow 0$, independent of t:

$$\delta_\alpha A \;:=\; i \lim_{\varepsilon\downarrow 0} [\,Q_\varepsilon(\alpha,t), A\,], \qquad \operatorname{supp}\alpha \subset O. \tag{60}$$

Independence of t follows from $\partial_\mu J^\mu_a = 0$ and the vanishing of the surface flux as $\varepsilon \downarrow 0$; locality follows from microcausality. The absence of central terms is guaranteed by anomaly freedom.

Define the local implementers $U_\varepsilon(\alpha, O) := \exp\big(iQ_\varepsilon(\alpha,t)\big)$. Then, in the limit $\varepsilon \downarrow 0$, there exist unitary automorphisms such that

$$U(\alpha,O)\, A\, U(\alpha,O)^{-1} \;=\; A \;+\; i\,\delta_\alpha A \;+\; \mathcal{O}(\alpha^2), \qquad A \in \mathcal{A}(O). \tag{61}$$

H.2 Ward identities

Let $\mathcal{O}_k(x_k)$ be local operators with pairwise distinct points x_k. Then, for time-ordered correlators, the local Ward identity holds:

$$\partial_\mu \big\langle T\, J^\mu_a(x)\, \mathcal{O}_1(x_1)\cdots \mathcal{O}_n(x_n)\big\rangle \;=\; -\,i \sum_{k=1}^{n} \delta^{(4)}(x-x_k)\, \big\langle T\, \mathcal{O}_1(x_1)\cdots (\delta_a \mathcal{O}_k)(x_k)\cdots \mathcal{O}_n(x_n)\big\rangle. \tag{62}$$

Here $\delta_a \mathcal{O} := \lim_{\varepsilon\downarrow 0}[\,Q_\varepsilon(\alpha_a), \mathcal{O}\,]$ for α_a locally constant in a neighbourhood of $\operatorname{supp}\mathcal{O}$, and the equality is understood in the distributional sense. The proof proceeds by integrating $\partial_\mu J^\mu_a = 0$ over a cylinder in t using (59)–(60), microcausality, and the limit $\varepsilon \downarrow 0$; the contact terms yield the right-hand side of (62).

An equivalent equal-time form (on the slice Σ_s) is

$$\partial_i \big\langle J^i_a(t,\mathbf{x})\, \mathcal{O}_1\cdots \mathcal{O}_n\big\rangle \;=\; i \sum_{k=1}^{n} \delta^{(3)}(\mathbf{x}-\mathbf{x}_k)\, \big\langle \mathcal{O}_1 \cdots [\,J^0_a(t,\mathbf{x}_k),\, \mathcal{O}_k\,]\cdots \mathcal{O}_n\big\rangle. \tag{63}$$

H.3 From Ward identities to gauge covariance

Consider the free matter kinetic term $\mathcal{L}_0 = \bar\psi\, i\gamma^\mu \partial_\mu \psi$ and the local variation $\delta_\alpha\psi = i\alpha\psi$, $\delta_\alpha\bar\psi = -i\bar\psi\alpha$. From (62) it follows that the additional term $\bar\psi\,\gamma^\mu(\partial_\mu\alpha)\psi$ is purely a contact source in the variation of correlators. This is compensated by introducing a connection A_μ and the replacement

$$D_\mu := \partial_\mu + ig\, A_\mu, \qquad \delta_\alpha A_\mu \;=\; -\frac{1}{g}\,\partial_\mu\alpha \;-\; i[\,A_\mu, \alpha\,], \tag{64}$$

so that $\delta_\alpha(\bar\psi\, i\gamma^\mu D_\mu\psi) = 0$. Hence, operational local implementability of the symmetry together with (62) implies gauge covariance of the effective action under the replacement $\partial \to D$.

Introducing the connection A_μ and the replacement

$$D_\mu := \partial_\mu + ig\, A_\mu, \qquad \delta_\alpha A_\mu = -\frac{1}{g}\,\partial_\mu\alpha - i[\,A_\mu, \alpha\,],$$

we obtain that the variation of the fermionic term $\bar\psi i\gamma^\mu D_\mu\psi$ compensates the contact contribution, and the action becomes invariant under local transformations with parameter $\alpha(x)$.

Thus, under conditions (L1)–(L3) and (O4), the variations induced by $[Q(\alpha), \cdot]$ are realised as local gauge transformations with the above D_μ and $\delta_\alpha A_\mu$, which completes the proof of Prop. 15.

Remarks. (i) All equalities are in the sense of distributions; the equations (62)–(63) are valid in the absence of Schwinger terms, as ensured by (O4). (ii) The constructions are local on Σ_s and do not use the global event structure. (iii) In the non-abelian case, (64) is understood in the Lie-algebra sense with the normalisation of generators adopted in the main text.