Variants for Demonstrating Aristotle's Wheel Paradox

Author: Viktor Strohm

Independent Researcher

Contacts: vfstrohm@zahoo.de

Abstract

This article discusses two variants for demonstrating Aristotle's Wheel Paradox, based on differing approaches to defining the "connection of the wheels" and the conditions of their rolling. In the first variant, the classic paradox is reformulated by changing the point of support for a rigid body. The second variant clarifies the concept of slippage of the smaller disk using a discrete "forcing mechanism."

Keywords: disk, disk motion, friction, slippage, forcing mechanism.

1. Introduction and Classical Formulation of the Paradox

The Wheel Paradox, historically attributed to Aristotle (first described in the treatise "Mechanics," 4th century BC), constitutes a fundamental problem in kinematics that sparked debate among prominent scientists, including Galileo, Descartes, and Fermat.

The essence of the paradox is as follows. Consider two disks, D_1 (radius r_1) and D_2 (radius r_2), which are rigidly fixed together and share a common center O, where $r_1 > r_2$ (see Fig. 1). When the outer, larger disk D_1 rolls along a flat surface **without slipping** and completes one full revolution, its center O travels a distance equal to the circumference of D_1: $L_1 = 2\pi r_1$.

Because both disks share a common axis of rotation, the inner disk D_2 travels the exact same distance L_1. A logical contradiction arises: if D_2 also completes one revolution, and the distance traveled by it is $2\pi r_1$, then its own circumference $2\pi r_2$ must be equal to $2\pi r_1$. This conclusion is erroneous since $r_1 > r_2$.

The resolution to this problem, first strictly formulated by Jean-Jacques Dortous de Mairan in 1715, is that the smaller disk D_2 **cannot perform pure rolling**. The motion of D_2 is combined, including a necessary **slippage** (or sliding) component, which allows its smaller circumference to cover the distance equivalent to the circumference of the larger disk.

Fig. 1

2. Variants of Demonstration

2.1. Variant 1: Rigid Body and Change of Support Point

In this variant, disks D_1 and D_2 are considered as a **single rigid body** (rigidly connected), which excludes the slippage of D_2 relative to D_1. Thus, the linear distance between any points on the disks remains constant.

Let O be the common center of the disks. Point O_1 on the circumference of D_1 is the support point (the point of contact with the plane). P_2 is a point on the circumference of the small disk D_2, and P_1 is a point on the circumference of the large disk D_1. All these points lie on the same vertical line O, O_1 in the initial position.

1. **Rolling on D_1 (support point O_1):** When the large disk D_1 completes a full revolution (Fig. 2a), the body travels a distance $L_1 = 2\pi r_1$. The line O, O_1 returns to its initial state, demonstrating a full revolution of the rigid body.
2. **Rolling on D_2 (support point P_2):** If the rolling condition is changed, and the point P_2 (on the circumference of D_2) is taken as the support point (the point of rolling without slipping), then upon completing a full revolution of the rigid body (Fig. 2b), the distance traveled will be $L' = 2\pi r_2$.

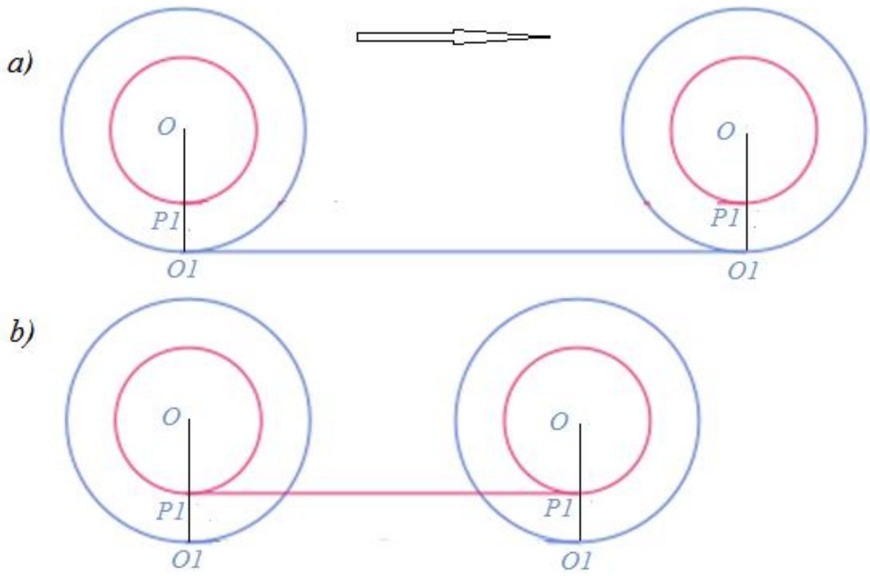

Fig. 2

This variant demonstrates that for a single rigid body, the distance traveled during one revolution **depends on the radius of rolling** (the support point). This resolves the paradox, as D_1 and D_2 travel the same distance **only** if D_1 is the support wheel. If D_2 were the support wheel, the distance would be $2\pi r_2$.

2.2. Variant 2: Demonstration of Slippage with a Forcing Mechanism

This variant aims to **clarify** the concept of slippage of the small disk D_2, which is the true resolution of the paradox (according to Mairan).

Consider a situation where disk D_2 **has no friction** with disk D_1. If D_1 rolls without slipping (support point O_1) and rotates D_2 only by means of the rigid connection of their centers (allowing $\omega_1 \neq \omega_2$), a **forcing action** is necessary for the rotation of disk D_2.

The author proposes to **discretize** the rolling process for a clear demonstration of the necessary slippage[21].

1. The period of a full revolution of the large disk D_1 is divided into N intervals.
2. In every **even** moment in time, a **forcing mechanism** is activated, compelling D_2 to rotate.
3. In every **odd** moment, the mechanism is switched off.

For example, with $N = 8$:

- At moment $N = 2$ (after 1/4 of the path of D_1 has been traversed), point P_1 will be displaced, but point P_2 on D_2 may remain in the initial position if ω_2 was zero (Fig. 3b).
- The forcing mechanism is then activated, and D_2 performs a discrete rotation to compensate for its smaller circumference and match the distance traveled.
- The cycle of switching on/off continues until both disks complete a full revolution.

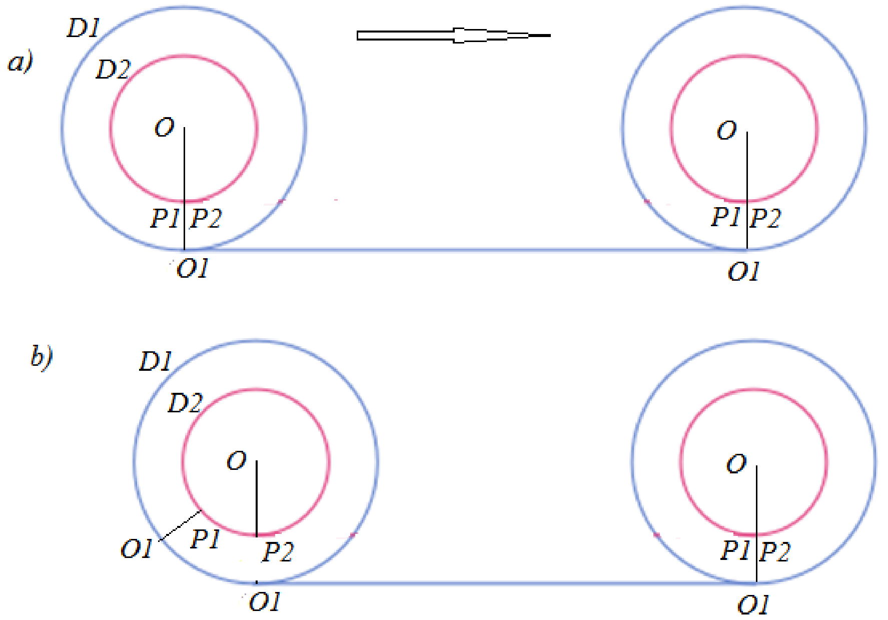

Fig. 3

This discrete model **simulates** how the smaller disk D_2 is forced to perform **additional** rotation (which is realized as slippage in the continuous case) so that its perimeter $2\pi r_2$ is "stretched" over the distance $2\pi r_1$.

P.S. The author has a technical specification for manufacturing a model with discrete slippage.