Analytical Model for Anomalous Transport of The Tokamak Plasma

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Abstract

The anomalous transport of the magnetic confinement plasma has been a long-standing problem for understanding the operation performance of the magnetic confinement fusion reactor, despite many theoretical advances such as the neoclassical theory. In the work, two key physical ideas were revealed for the quasi-neutral plasma under the magnetic fields. One is that the conventional Lorentz forces for the ions and electrons should emerge as the magnetic moment force. The second one is that the free electrons should be referred to as the semi-free electrons, because the electron positions may be represented by the nearest-ion positions in the framework of calculus. Therefore, a simply analytical model was put forward and it can successfully explain the particle diffusion constants, thermal diffusivities, the edge transport barrier and the internal transport barrier of the tokamak plasma. The revealed physical ideas and constructed model may be essential for interpreting the anomalous transport behaviors of the tokamak plasma and improving the operation performance of the fusion reactor.

Keywords：anomalous transport; magnetic confinement; tokamak plasma; diffusion constant; edge transport barrier; internal transport barrier

1. Introduction

The transport behaviors of the plasma under the magnetic fields has been one of the most concerned problem for the magnetic confinement fusion. The radial particle diffusion constant given by the classical theory based on a uniform magnetic field was found to be several orders of magnitude smaller than the experimental results. To tackle the problem, the non-uniformity of the magnetic fields was taken into account and thereby the neoclassical theory was proposed [1], [2]. The neoclassical theory can improve the magnitude of the radial diffusion constant by one or two orders at most [3], but it was still far smaller than the experimental results 1-10 m^2/s [3], [4], [5], [6], [7], [8], [9]. Meanwhile, the radial thermal diffusivities of the ions and electrons obtained by the experiments were found to be two orders of magnitude larger than that given by the neoclassical theory [4]. The related transport behaviors revealed by various experiments cannot be interpreted by the neoclassical theories and were widely regarded as the anomalous transport [3], [4], [10], [11]. To address the anomalous transport problem, many efforts such as the magnetohydrodynamics (MHD) in axisymmetric toroidal geometry [12], [13], turbulent diffusion models due to the micro-instabilities [11], [13], [14] and large-scale collective motions [11], [15], have been made during recent decades. However, due to the very different spatial scales of the turbulent structures and the very distinct timescales of the election and ion motions, it has been a very challenging task for the construction of the turbulence model [16], [17]. Therefore, owing to the complexities and difficulties, the analytical model for the anomalous transport of the magnetic confinement plasma has been lacking and a long-standing problem [3] in the area of the plasma physics.

In the work, a simply analytical model for the anomalous transport behaviors of the magnetic-confinement plasma, in particular the tokamak plasma, was proposed. The paper was designed as follows. Section 2 will give the theory and discussions for the magnetic confinement plasma, wherein Section 2.1 will focus on the transport model for the cylindrical plasma under a longitudinal magnetic field and Section 2.2 will turn to the transport model for the tokamak plasma. Section 3 will present the conclusions.

2. Theory and Discussion

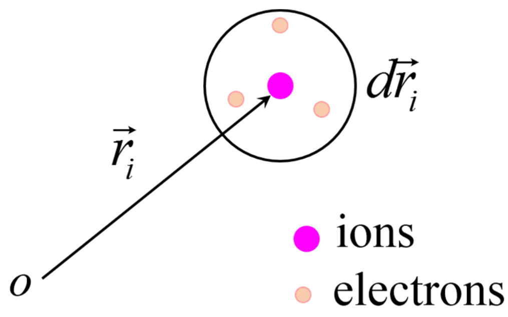

Figure 1 Schematic diagram for the quasi-neutral plasma with electrons (orange circles) and the ions (magenta circles) in the mathematical micro-element.

For the quasi-neutral plasma in steady state, the number of ions and electrons may be nearly equivalent to each other. Besides, owing to the charge neutrality the ion position and the electron position cannot be distinguished in the identical mathematical micro-element in calculus, as shown in Figure 1. And the moment of plasma in the mathematical micro-element is commonly determined by the ions because of their much heavier masses than the electrons. As a result, the free electrons do not exhibit their own position degrees of freedom and the electron positions may be represented by the ion positions in the same mathematical micro-element due to their strong electrostatic force. It looks like the neutral gases where the light electrons are bounded by the heavy nucleus and thereby do not display their position degrees of freedom. Therefore, the free electrons in the plasma should be referred to as semi-free electrons. It may be one of the crucial physical ideas for understanding the transport properties of the quasi-neutral plasma.

So the spatial gradient of the electron distribution function with respect to the electron position is zero, *i.e.*,

$$\nabla_{\vec{r}_e} f_e^0 = 0 \tag{1}$$

where r_e signifies the electron position, f_e^0 denotes the isotropic Maxwell distribution function for the semi-free electrons. It may be very different from the conventional theoretical treatments and may determine the direction of the radial electrostatic field along the minor radius of the tokamak plasma as stated in the following section. The Maxwell distribution functions for electrons and ions can be expressed as $f_i^0 = n_i(\vec{r}_i,\ t)exp(-v_i^2/v_{iT}^2)/\pi^{3/2}v_{iT}^3$ and $f_e^0 = n_e(\vec{r}_i,\ t)exp(-v_e^2/v_{eT}^2)/\pi^{3/2}v_{eT}^3$ where f_i^0 denotes the isotropic Maxwell distribution functions for the ions, $n_i(r_i, t)$, $n_e(r_i, t)$ represent the ion and electron number densities, depending on the ion position r_i and time t, v_{iT} and v_{eT} are the thermal velocities of the ions and electrons, respectively. And they satisfy the following relations $v_{iT} = \sqrt{2k_BT_i/m_i}$ and $v_{eT} = \sqrt{2k_BT_e/m_e}$, where k_B represents the Boltzmann constant, m_i and m_e denote the ion mass and electron mass, respectively, T_i and T_e describe the position-dependent ion temperature and electron temperature.

Based on the Spitzer-Härm (SH) theory [18], the distribution functions for ions and electrons can be expressed as the sum of an isotropic term and an anisotropic term $f_i = f_i^0 + f_i^1$ and $f_e = f_e^0 + f_e^1$, where f_e^1 denotes the anisotropic component of the electron distribution function, f_i^1 represents the anisotropic component of the ion distribution function.

In terms of the non-equilibrium Boltzmann transport equation and the SH theory for electron thermal conductivity, the anisotropic component of the electron distribution function can be given by $e\vec{E}_0 \cdot \nabla_{v_e} f_e^0 / m_e = -f_e^1/\tau_e$, where v_e signifies the electron velocity, e represents the electron charge, E_0 is the electric field, and τ_e is the electron relaxation time owing to the electron-ion Coulomb collisions [18]. Insertion of the electron Maxwell distribution function yields

$$f_e^1 = \tau_e \frac{e\vec{E}_0 \cdot \vec{v}_e}{k_B T_e} f_e^0 \tag{2}$$

The above equation may be very important and will be utilized to treat the transport properties of the plasma in the following sections.

2.1 The cylindrical plasma under a magnetic field

2.1.1 Radial diffusion

Considering a cylindrical plasma in an externally applied magnetic field in the z direction in the cylindrical coordinates (r, θ, z), the plasma density gradient and the temperature gradient are assumed to be in the radial direction. The plasma may radially diffuse from the high-pressure core to the low-pressure edges. The radial diffusion of the plasma may generate a poloidal electric field and an electric current due to the Lorentz force. The poloidal electric current will inversely weaken the magnetic field which may vary slowly across the radius and satisfy the condition

$$\left| r_c \nabla B_z \right| << B_z$$

where $B_z(r)$ denotes the weakened radius-dependent magnetic field owing to the poloidal current, r_c stands for the gyration radius of the charged particle.

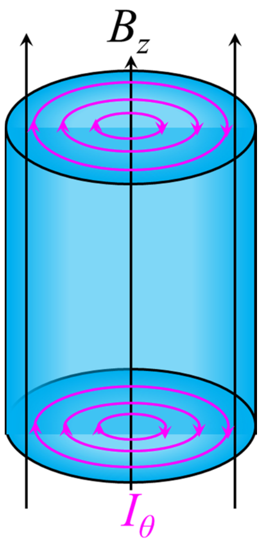

Figure 2. Schematic diagram of the cylindrical plasma under a magnetic field. The magnitude of the magnetic field (black arrows) may be partially canceled by the clockwise currents (magenta arrows) in the plasma.

Under a magnetic field, a single charged particle experiences the Lorentz force and usually moves in a helical orbit. The average movement of the charged particles during the helical period may behave as the movement of the guidance centers and the average drifting-force over the helical period may be the magnetic moment forces [10] $F_{me} = -m_e v_{e\perp}^2 \nabla B_z(r)/2B_z(r)$ and $F_{mi} = -m_i v_{i\perp}^2 \nabla B_z(r)/2B_z(r)$ where F_{me}, F_{mi} denotes the magnetic moment forces for the electrons and ions over the helical period, respectively, $v_{e\perp}$, $v_{i\perp}$ represent the electron velocity and ion velocity perpendicular to the magnetic field. It may be another crucial physical idea for the magnetic-confinement plasma. Therefore, the anisotropic component of the distribution functions for the electrons and ions may be obtained

$$f_e^1 = \frac{\tau_e f_e^0 \vec{v}_e \cdot}{k_B T_e}\left[e\vec{E}_0 - \frac{m_e v_{e\perp}^2}{2B_z(r)} \nabla B_z(r) + \frac{m_e v_{e\theta}^2}{r^2}\vec{r} + eE_\theta(r)\vec{e}_\theta \right]$$

$$f_i^1 = \frac{\tau_i f_i^0 \vec{v}_i \cdot}{k_B T_i}\left[-k_B T_i \nabla_{\vec{r}} \ln p_i(r) - k_B T_i \left(\frac{v_i^2}{v_{iT}^2} - \frac{5}{2} \right) \nabla_{\vec{r}_i} \ln T_i - Ze\vec{E}_0 - \frac{m_i v_{i\perp}^2}{2B_z(r)} \nabla B_z(r) + \frac{m_i v_{i\theta}^2}{r^2}\vec{r} - ZeE_\theta(r)\vec{e}_\theta \right]$$

where E_θ is the poloidal electric field in the plasma, τ_i signifies the ion relaxation time due to the Coulomb collisions [18], Z denotes the net charge number of the ion, the terms $m_i v_{i\theta}^2 \vec{r}/r^2$, $m_i v_{i\theta}^2 \vec{r}/r^2$ represent the inertial centrifugal forces for the electrons and ions, respectively.

The electric current density can be given by $\vec{j}_e = e\int d\vec{v}_e \vec{v}_e f_e^1 - Ze\int d\vec{v}_i \vec{v}_i f_i^1$ where j_e signifies the electric current. Based on the mathematical treatment [18], the radial electric current may be expressed as

$$j_{er} = en_e(\vec{r}_i)\left(\frac{\tau_e}{m_e} + \frac{Z\tau_i}{m_i} \right) e\vec{E}_0 - en_e(\vec{r}_i)\left(\frac{\tau_e T_e}{m_e} - \frac{\tau_i T_i}{m_i} \right) \frac{5k_B}{3} \nabla \ln \frac{B_z(r)}{r} + en_e(\vec{r}_i)\frac{\tau_i}{m_i} k_B T_i \nabla_{\vec{r}} \ln p_i(r)$$

where j_e denotes the radially electric current. The electrical transport processes are typically much faster than thermal transport, leading to the charge neutrality assumption that the radially electric current is zero. This assumption, widely used in related theoretical studies, allows one to obtain the electric field E_0

$$e\vec{E}_0 = -\frac{\tau_i}{m_i}\left(\frac{\tau_e}{m_e} + \frac{Z\tau_i}{m_i} \right)^{-1} k_B T_i \nabla_{\vec{r}} \ln p_i(r) + \left(\frac{\tau_e T_e}{m_e} - \frac{\tau_i T_i}{m_i} \right)\left(\frac{\tau_e}{m_e} + \frac{Z\tau_i}{m_i} \right)^{-1} \frac{5k_B}{3} \nabla \ln \frac{B_z(r)}{r}$$

It means that the electrons and ions display the same mean radial velocity,

$$u_r = -\frac{\tau_e}{m_e}\frac{\tau_i}{m_i}\left(\frac{\tau_e}{m_e} + \frac{Z\tau_i}{m_i} \right)^{-1} k_B T_i(\vec{r}) \nabla \ln p_i(r) - \frac{\tau_e}{m_e}\frac{\tau_i}{m_i}\left(\frac{\tau_e}{m_e} + \frac{Z\tau_i}{m_i} \right)^{-1} \frac{5k_B(ZT_e + T_i)}{3} \nabla \ln \frac{B_z(r)}{r}$$

Once the radius-dependent magnetic field is determined, the radial velocity of the plasma can be obtained.

Based on the Maxwell equations, the magnetic field may depend on the poloidal electric current as shown below

$$\frac{\partial}{\partial r}B_z(r) = -\mu_0\sigma_e E_\theta$$

The poloidal electric field E_θ results from the time-varying enclosed magnetic flux and can be given by

$$E_\theta = -u_r B_z(r) - \frac{1}{r}\int_0^r r'dr'\frac{\partial}{\partial t}B_z(r')$$

When the plasma stay in the steady state, the magnetic field may not vary obviously with time, resulting in the following relation

$$E_\theta \approx -u_r B_z(r)$$

Therefore, the relation between the magnetic field and the radial velocity can be obtained

$$\frac{\partial}{\partial r}\ln B_z(r) = \mu_0\sigma_e u_r \qquad (3)$$

Substituting equation (3) into the expression of the mean radial velocity for the plasma yields

$$u_r = \frac{\frac{\tau_e}{m_e}\frac{\tau_i}{m_i}\left(\frac{\tau_e}{m_e}+\frac{Z\tau_i}{m_i}\right)^{-1}\left[-k_B T_i\frac{\partial}{\partial r}\ln p_i(r) + \frac{5k_B(ZT_e+T_i)}{3r}\right]}{1+\frac{\tau_e}{m_e}\frac{\tau_i}{m_i}\left(\frac{\tau_e}{m_e}+\frac{Z\tau_i}{m_i}\right)^{-1}\frac{5k_B(ZT_e+T_i)}{3}\mu_0\sigma_e}$$

In general the temperature of the Tokamak plasma may be much higher than 10^5 *K*. Thereby the second term in the denominator of the above equation may be far larger than 1, leading to the approximation

$$u_r \approx \frac{1}{\mu_0\sigma_e}\left[-\frac{3}{5}\frac{T_i}{(ZT_e+T_i)}\frac{\partial}{\partial r}\ln p_i(r) + \frac{1}{r}\right] \qquad (4)$$

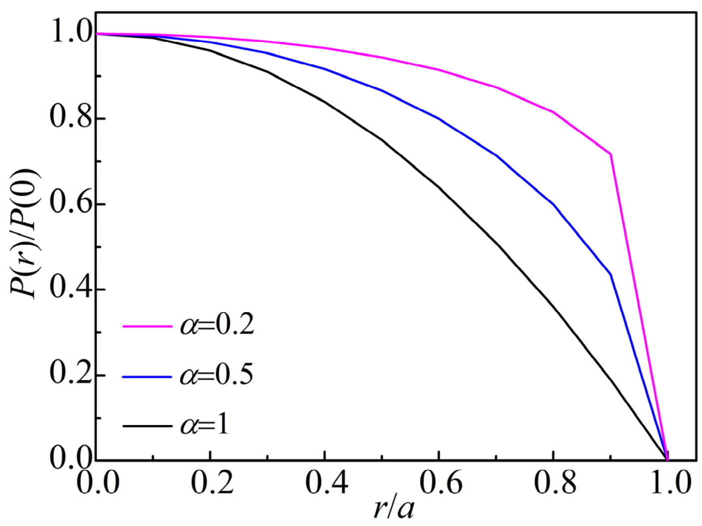

Figure 3. The reduced plasma pressure versus the reduced minor radius. The lines are given by the function $p_i(r)/p_i(0) = (1 - r^2/a^2)^\alpha$, α=1 for the black line, α=0.5 for the blue line and α=0.2 for the magenta line.

The mean radial velocity exhibits several important properties. First, the radial velocity strongly depends on the temperature and varies with the temperature in the manners $u_r \propto T_e^{-1.5}$, because the electron relaxation time obey the relation $\tau_e \propto T_e^{1.5}$ according to the Coulomb scattering mechanism [18]. Second, the mean radial velocity may be proportional to the net charge number Z of the ion, and the ions possessing more net charges may lead to a larger radial velocity. Because the more charges of the ion can reduce the electron relaxation time apparently based on the Coulomb scattering mechanism [18]. Third, the radial velocity at the plasma edge may be monitored by the pressure gradient. In the case that the plasma pressure can be depicted popularly by the function $p_i(r)/p_i(0) = (1 - r^2/a^2)^\alpha$ for many Tokamak plasmas [19], [20], the plasma pressure exhibiting a pedestal at the edge shown in Figure 3, *i.e.* a small value of the index α, will result in a small radial velocity. By up-lifting the temperature at the plasma edge, the radial velocity can be expected to decrease and the confinement performance may be improved, which agreed with the experimental observations on the Tokamak plasma in H-mode [21], [22]. Therefore, both the plasma pressure pedestal and the up-lifted temperature at the plasma edge may descend the mean radial velocity, which may be the physical origin of the edge transport barrier (ETB). Fourth, the radial velocity near the plasma core may be dominated by the second term arising from the inertial centrifugal forces. And the plasma close to the core may display a notable radial velocity which may prohibit the plasma accumulation. But the up-lifted temperature can be implied to decline the radial velocity and the related diffusivity at the plasma core greatly, which agrees with the experimental observations [23], [24]. The steep pressure gradient at the tokamak plasma core means a small value of the index β when the plasma pressure could be described by the function $p_i(r)/p_i(0) = (1 - r^2/r_I^2)^\beta$ (β denotes the dimensionless index, r_I signifies the position of ITB). Both the small index β and the improved temperature in the vicinity of the plasma core can weaken the radial velocity, which may be the working mechanism for the internal transport barrier (ITB). So both the ETB and ITB may be dominated by the same physical mechanism and it could be predicted that the third steep pressure gradient can also meliorate the confinement performance. Fifth, the magnitude of the radial velocity and the related diffusion constant for the plasma at the edge can be estimated in terms of the typical parameters as shown in Figure 4. It shows that the radial velocity in the work is several orders of the magnitude higher than that based on the classical theory. Sixth, it may be the magnetic moment force instead of the Lorentz force that confines the plasma, which may enlighten people that the magnetic field may exist in the mathematic form of the magnetic moment force rather than the Lorentz force in the well-known Fokker-Plank equation. This point may be very different from the situation of a single charged particle under a magnetic field. It is interesting that the radial velocity can be confined by the externally magnetic field conspicuously but its mathematical expression does not exhibit the existence of the magnetic field.

If the plasma reaches the thermal equilibrium state, the electron temperature will be the same as the ion temperature. The concerned diffusion constant can be given by

$$D_r = \frac{1}{\mu_0 \sigma_e}\left[\frac{3}{5(Z+1)}\left(1+\frac{\partial \ln T}{\partial \ln n_i}\right)-\left(\frac{\partial \ln n_i}{\partial \ln r}\right)^{-1}\right] \tag{5}$$

where D_r signifies radial diffusion constant. The most concerned magnitude of the diffusion constant can be estimated. For the plasma at the edge with typical parameters $\partial lnT/\partial lnn_i = 4$, $\partial lnn_i/\partial lnr = -10$, Z=1, the temperature-dependent diffusion constant is shown in Figure 4. It shows that the radial diffusion constant in the work may be several orders of magnitude larger than that based on the classical theory. For the plasma edge with a temperature 10^6 K, the radial diffusion constant may exhibit a value 5 m^2/s whose magnitude of order is consistent with the experimental findings [3]–[8]. Moreover, the plasma temperature continuously increases upon reaching the plasma core, implying a gradually dropping radial diffusion constant towards the plasma core. It may agree with the experimental results that the radial diffusion constant decreases close to the plasma core [3], [4], [7].

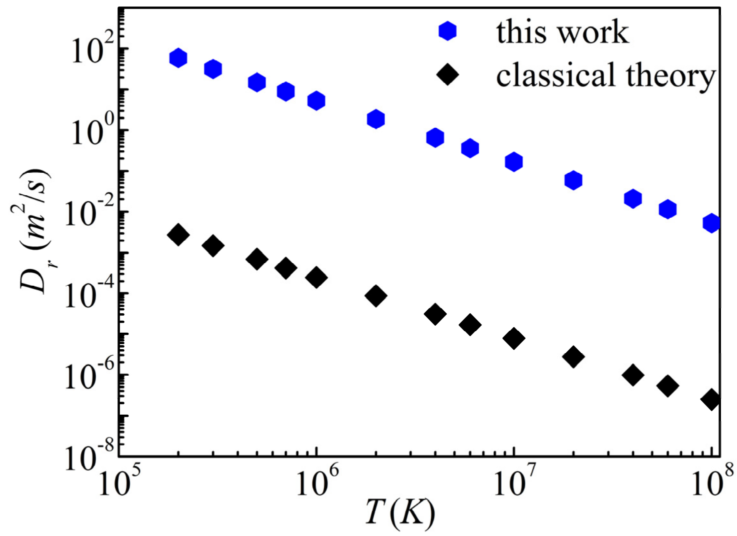

Figure 4. Temperature dependence of the radial diffusion constant for the plasma edge in cylindrical coordinates.

Moreover, the ratio of the above diffusion constant to the diffusion constant in terms of the classical theory can be given by

$$\frac{D_r}{D_{cr}} \approx \frac{3}{5(Z+1)} \frac{B_z^2(r)}{\mu_0 p_i(r)}$$

where D_{cr} denotes the radial diffusion constant based on the classical theory. As is shown, the ratio may reach 10^4 for the ion pressure 10^2 *Pa* at the Tokamak plasma edge under an externally magnetic field 3 *T*.

In terms of the simple calculations, the position-dependent magnetic field can be formulated

$$B_z(r) = B_z(a)\frac{r}{a}\left[\frac{p_i(a)}{p_i(r)}\right]^{\frac{3}{5(Z+1)}} \qquad (6)$$

where $B_z(a)$ denotes the externally magnetic field, *a* signifies the radius of the plasma. It may indicate that the magnetic field continuously decreases upon reaching the plasma core because of the poloidal current. And the large pressure corresponds to the small magnetic field in the steady plasma.

2.1.2 Radial thermal conductivity

The mathematical expression of the mean radial velocity suggest that the radial mobility for the electrons and ions may be

$$\frac{\tau_{em}}{m_e}=\frac{\tau_{im}}{m_i}=\frac{\frac{\tau_e}{m_e}\frac{\tau_i}{m_i}\left(\frac{\tau_e}{m_e}+\frac{Z\tau_i}{m_i}\right)^{-1}}{1+\frac{\tau_e}{m_e}\frac{\tau_i}{m_i}\left(\frac{\tau_e}{m_e}+\frac{Z\tau_i}{m_i}\right)^{-1}\frac{5(Z+1)k_BT_e}{3}\mu_0\sigma_e}$$

where τ_{em}, τ_{im} denotes the modified relaxation times in the radial direction for the electron and ions under the magnetic field, respectively.

The radial electron heat flux and radial ion heat flux may be given by the equations $j_{eqr}=\int d\vec{v}_e f_e^1\vec{v}_e\, m_e v_e^2/2$ and $j_{iqr}=\int d\vec{v}_i f_i^1\vec{v}_i\, m_i v_i^2/2$. In terms of the simple calculation, they can be written as

$$j_{eqr}=\frac{5n_e}{2}\left(k_BT_e\right)^2\frac{\tau_{em}}{m_e}\left[-\frac{T_i}{T_e}\nabla_{\vec{r}}\ln p_i+\frac{7}{3r}\right]$$

$$j_{iqr}=\frac{5}{2}n_i\left(k_BT_i\right)^2\frac{\tau_{im}}{m_i}\left[-\nabla_{\vec{r}}\ln p_i(\vec{r})-\nabla_{\vec{r}}\ln T_i+\frac{7}{3r}\right]$$

where j_{eqr}, j_{iqr} denote the radial electron heat flux and the radial ion heat flux, respectively. Therefore, the electron thermal conductivity and ion thermal conductivity in the radial direction can be obtained

$$\kappa_{er}=\frac{3n_ek_B}{2(Z+1)\mu_0\sigma_e(r)}\left[\left(1+\frac{\partial\ln n_i}{\partial\ln T_i}\right)\frac{\partial T_i}{\partial T_e}-\frac{7}{3}\left(\frac{\partial\ln T_e}{\partial\ln r}\right)^{-1}\right]$$

$$\kappa_{ir}=\frac{3n_ik_B}{2(Z+1)\mu_0\sigma_e(r)}\frac{T_i}{T_e}\left[2+\frac{\partial\ln n_i}{\partial\ln T_i}-\frac{7}{3}\left(\frac{\partial\ln T_i}{\partial\ln r}\right)^{-1}\right]$$

where κ_{er} , κ_{ir} denote the radial thermal conductivities for the electrons and ions, respectively. As is shown, the electron radial thermal conductivity may be comparable with the ion radial thermal conductivity in magnitude, which agrees with the experimental observations [4], [6], [25]. And the thermal conductivity of the plasma can be written as

$$\kappa = \frac{3n_i k_B}{2(Z+1)\mu_0\sigma_e(r)}\left[Z\left(1+\frac{\partial \ln n_i}{\partial \ln T_i}\right)\frac{\partial T_i}{\partial T_e} - \frac{7Z}{3}\left(\frac{\partial \ln T_e}{\partial \ln r}\right)^{-1} + \frac{T_i}{T_e}\left(2+\frac{\partial \ln n_i}{\partial \ln T_i}\right) - \frac{T_i}{T_e}\frac{7}{3}\left(\frac{\partial \ln T_i}{\partial \ln r}\right)^{-1}\right]$$

In the case of the thermal equilibrium for the plasma, *i.e.*, T_e=T_i, the thermal conductivity can be simplified to be

$$\kappa = \frac{3n_i k_B}{2\mu_0\sigma_e(r)}\left[\frac{Z+2}{Z+1} + \frac{\partial \ln n_i}{\partial \ln T_i} - \frac{7}{3}\left(\frac{\partial \ln T}{\partial \ln r}\right)^{-1}\right] \tag{7}$$

The related thermal diffusivities can be formulated

$$\chi_{er} = \frac{1}{(Z+1)\mu_0\sigma_e(r)}\left[\left(1+\frac{\partial \ln n_i}{\partial \ln T_i}\right)\frac{\partial T_i}{\partial T_e} - \frac{7}{3}\left(\frac{\partial \ln T_e}{\partial \ln r}\right)^{-1}\right]$$

$$\chi_{ir} = \frac{1}{(Z+1)\mu_0\sigma_e(r)}\frac{T_i}{T_e}\left[2+\frac{\partial \ln n_i}{\partial \ln T_i} - \frac{7}{3}\left(\frac{\partial \ln T_i}{\partial \ln r}\right)^{-1}\right]$$

where χ_{er}, χ_{ir} signifies the electron thermal diffusivity and the ion thermal diffusivity in the radial direction, respectively. They indicate that the thermal diffusivities may be independent of the magnetic field and the particle density in the mathematical form, which was consistent with the experimental results [26]. Furthermore, the temperature increase of the plasma core may result in a weakened thermal diffusivity, which agreed with the experimental results [23].

Using the classical theory, the radial thermal conductivity in radial direction may be

$$\kappa_c = \frac{5(Z+1)n_i k_B}{4\mu_0\sigma_e}\frac{p_i}{B_z^2/2\mu_0}\left[\frac{Z+2}{Z+1} + \frac{\partial \ln n_i}{\partial \ln T} - \frac{7}{3}\left(\frac{\partial \ln T}{\partial \ln r}\right)^{-1}\right]$$

where κ_c denotes the radial thermal conductivity based on the classical theory. The ratio of the thermal conductivity in the work to the classical value can be given by

$$\frac{\kappa}{\kappa_c}=\frac{3}{5(Z+1)}\frac{B_z^2}{\mu_0 p_i}$$

Substituting equation (6) into the above equation may generate

$$\frac{\kappa}{\kappa_c}=\frac{3}{5(Z+1)}\frac{B_z^2(a)}{\mu_0 p_i(a)}\frac{r^2}{a^2}\left[\frac{p_i(a)}{p_i(r)}\right]^{1+\frac{6}{5(Z+1)}}$$

It indicates that the ratio may continue to decline as the radius decreases. At the Tokamak plasma edge with the typical ion pressure 10^2 *Pa* and the magnetic field 3 *T*, the thermal conductivity ratio may reach 10^4, meaning that the thermal conductivity in the work can be far larger than that based on the classical theory. By comparing the ratio of diffusion constant and the ratio of thermal conductivity, interestingly, the two ratios are the same as each other. The theoretical treatments of the cylindrical plasma may be useful for uncovering the related physics for the tokamak plasma.

2.2 The anomalous transport of tokamak plasma

2.2.1 Diffusion of the tokamak plasma

For the tokamak plasma, the coordinates (r, θ, φ) can be utilized. In the reference, the magnetic field may display the field components in the θ, φ directions, and the Maxwell equations associated with the magnetic field can be written as

$$\mu_0\left(\sigma_e E_\theta \vec{e}_\theta+\sigma_e E_\varphi \vec{e}_\varphi\right)=\frac{1}{r(R+r\cos\theta)}\left\{\vec{e}_r\left[\frac{\partial}{\partial\theta}(R+r\cos\theta)B_\varphi-\frac{\partial}{\partial\varphi}rB_\theta\right]-r\vec{e}_\theta\frac{\partial}{\partial r}(R+r\cos\theta)B_\varphi+(R+r\cos\theta)\vec{e}_\varphi\frac{\partial}{\partial r}rB_\theta\right\}$$

$$0=\nabla\cdot\vec{B}=\frac{1}{r(R+r\cos\theta)}\left[\frac{\partial}{\partial\theta}(R+r\cos\theta)B_\theta+\frac{\partial}{\partial\varphi}rB_\varphi\right]$$

where E_φ denotes the electric field in φ direction, R represents the major radius. Due to the circular symmetry, the magnetic field may be irrelevant with coordinate φ, leading to the following relations

$$\mu_0\sigma_e E_\varphi = \frac{1}{r}\frac{\partial}{\partial r} rB_\theta(r)$$

$$\mu_0\sigma_e E_\theta = -\frac{1}{(R+r\cos\theta)}\frac{\partial}{\partial r}(R+r\cos\theta)B_\varphi(r)$$

As previously shown, the poloidal electric field for the plasma in steady state may be given by $E_\theta = -u_r B_\varphi(r)$. Substituting it into the above equation may generate

$$\mu_0\sigma_e u_r = \frac{\partial}{\partial r}\ln\left[(R+r\cos\theta)B_\varphi(r)\right] \tag{8}$$

In another respect, the toroidal electric field may consist of the magnetically-induced electric field and the self-induced field due to the Lorentz force,

$$\mu_0\sigma_e\left[E_{in} + u_r B_\theta(r)\right] = \frac{1}{r}\frac{\partial}{\partial r} rB_\theta(r) \tag{9}$$

where E_{in} is the magnetically-induced toroidal electric field and the second term comes from the Lorentz force.

The average Lorentz force for the charged particles over the helical periods should be addressed. And the theoretical treatment is the same as that in the reference [10]. The dynamic equation for a charged particle in the uniform plasma may be

$$m\dot{v}_{ir} = q\left(v_\theta B_\varphi - v_\varphi B_\theta\right)$$

where *m*, *q* signifies the mass and the charge of the charged particle, respectively, v_θ denotes the poloidal velocity and v_φ stands for the toroidal velocity. A complete period for the movement of the charged particle may contains *k* poloidal periods and *l* toroidal periods, and the average radial velocity during the complete period should be zero [10], *i.e.*,

$$0 = q_i B_\varphi(r)(\theta_2-\theta_1) + q_i\frac{\partial B_\varphi(r)}{\partial r}k\pi r_\theta^2 - \left[q_i(R+r\cos\theta)B_\theta(r)(\varphi_2-\varphi_1) - q_i\frac{\partial B_\theta(r)}{\partial r}l\pi r_\varphi^2\right]$$

where r_θ, r_φ stands for the round radius of the helical movement. Dividing the complete period may lead to

$$0 = qB_\varphi(r)u_\theta + q\frac{\partial B_\varphi(r)}{\partial r}\frac{k\pi r_\theta^2\omega_\theta}{k2\pi} - \left[q(R + r\cos\theta)B_\theta(r)u_\varphi - q\frac{\partial B_\theta(r)}{\partial r}\frac{l\pi r_\varphi^2\omega_\varphi}{k2\pi}\right]$$

where u_θ, u_φ denotes the average poloidal velocity and average toroidal velocity, respectively, ω_θ, ω_φ represent the related poloidal angular velocity and the toroidal angular velocity. They can be given by $r_\theta = m\sqrt{v_r^2 + v_\theta^2}/qB_\varphi(r)$, $\omega_\theta = qB_\varphi(r)/m$, $r_\varphi = m\sqrt{v_r^2 + v_\varphi^2}/qB_\theta(r)$, $\omega_\varphi = qB_\theta(r)/m$. Substitution of the equations into the above equation will yield

$$qu_\theta B_\varphi(r) - qu_\varphi B_\theta(r) = -\frac{m(v_r^2 + v_\theta^2)}{2B_\varphi(r)}\frac{\partial B_\varphi(r)}{\partial r} - \frac{m(v_r^2 + v_\varphi^2)}{2B_\theta(r)}\frac{\partial B_\theta(r)}{\partial r}$$

Interestingly, the Lorentz forces for the charged particles over the complete period exhibits as the magnetic moment forces in the mathematical form, which is the key in the work. Therefore, the mean radial electron velocity can be derived simply

$$u_{er} = \frac{\tau_e}{m_e}eE_0' - \frac{\tau_e}{m_e}\frac{5k_BT_e}{3}\frac{\partial \ln B_\varphi(r)B_\theta(r)}{\partial r} + \frac{\tau_e}{m_e}\frac{5k_BT_e}{3}\frac{\partial \ln r(R + r\cos\theta)}{\partial r}$$

where E'_0 signifies the electrostatic field in the radial direction. And the mean radial ion velocity can be obtained in the same manners

$$u_{ir} = -\frac{\tau_i}{m_i}k_BT_i\nabla_{\bar{r}}\ln p_i(r) - \frac{\tau_i}{m_i}ZeE_0' - \frac{\tau_i}{m_i}\frac{5k_BT_i}{3}\frac{\partial \ln B_\varphi(r)B_\theta(r)}{\partial r} + \frac{\tau_i}{m_i}\frac{5k_BT_i}{3}\frac{\partial \ln r(R + r\cos\theta)}{\partial r}$$

The neutral plasma assumption requires the mean radial electron velocity be identical to the mean radial ion velocity, generating the relation

$$u_r = \frac{-\frac{\tau_e}{m_e}\frac{\tau_i}{m_i}k_BT_i\nabla_{\bar{r}}\ln p_i(r) - \frac{\tau_e}{m_e}\frac{\tau_i}{m_i}\frac{5k_B(ZT_e + T_i)}{3}\frac{\mu_0\sigma_eE_{in}}{B_\theta(r)} + \frac{\tau_e}{m_e}\frac{\tau_i}{m_i}\frac{10k_B(ZT_e + T_i)}{3}\frac{R + 2r\cos\theta}{r(R + r\cos\theta)}}{\left(\frac{\tau_e}{m_e} + \frac{Z\tau_i}{m_i}\right)\left[1 + \frac{\tau_e}{m_e}\frac{\tau_i}{m_i}\left(\frac{\tau_e}{m_e} + \frac{Z\tau_i}{m_i}\right)^{-1}\frac{10k_B(ZT_e + T_i)}{3}\mu_0\sigma_e\right]}$$

where u_r denotes the same mean radial velocity for the ions and electrons. The following condition can be generally satisfied for the Tokamak plasma,

$$\frac{\tau_e}{m_e}\frac{\tau_i}{m_i}\left(\frac{\tau_e}{m_e}+\frac{Z\tau_i}{m_i}\right)^{-1}\frac{10k_B\left(ZT_e+T_i\right)}{3}\mu_0\sigma_e >> 1$$

Thus, the mean radial velocity can be approximated as

$$u_r \approx \frac{1}{\mu_0\sigma_e}\left[-\frac{3}{10}\frac{T_i}{\left(ZT_e+T_i\right)}\frac{\partial}{\partial r}\ln p_i\left(r\right)-\frac{\mu_0\sigma_e E_{in}}{2B_\theta\left(r\right)}+\frac{1}{r}+\frac{\cos\theta}{\left(R+r\cos\theta\right)}\right] \tag{10}$$

And the radial electric field can be approximated as

$$ZeE_0' \approx -k_B T_i \nabla_{\bar{r}} \ln p_i\left(r\right)-\frac{5k_B T_i}{3}\frac{\partial \ln B_\varphi\left(r\right)B_\theta\left(r\right)}{\partial r}+\frac{5k_B T_i}{3}\frac{\partial \ln r\left(R+r\cos\theta\right)}{\partial r} \tag{11}$$

At the plasma core, the poloidal field may be written as $B_\theta(r) = Cr$, where C is a constant. Substitution of it and equation (10) into equation (9) will yield

$$C \approx \frac{\mu_0\sigma_e E_{in}}{2}$$

At last, the mean radial velocity near the plasma core can be obtained

$$u_r\left(r\right) \approx \frac{1}{\mu_0\sigma_e}\left[-\frac{3}{10}\frac{T_i}{\left(ZT_e+T_i\right)}\frac{\partial}{\partial r}\ln p_i\left(r\right)+\frac{\cos\theta}{\left(R+r\cos\theta\right)}\right] \tag{12}$$

As is shown, the toroidal electric current may greatly weaken the mean radial velocity by virtue of the induced poloidal magnetic field. It is the magnetically-induced electric current density $\sigma_e E_{in}$ that suppresses the radial velocity component due to the inertial centrifugal force. Therefore an appropriate magnitude of the current density $\sigma_e E_{in}$ will be essential for the confinement of the tokamak plasma. And the spontaneous bootstrap current density $\sigma_e u_r B_\theta$ may decline the mean radial velocity component originating from the ion pressure gradient by half. The mean radial velocity can also decrease by improving the ratio of the electron temperature to the ion temperature.

The related diffusion constant can be given by

$$D_r = \frac{1}{\mu_0 \sigma_e}\left[\frac{3}{10}\frac{T_i}{\left(ZT_e + T_i\right)}\left(1 + \frac{\partial \ln T_i}{\partial \ln n_i}\right) - \frac{\cos\theta}{\left(R + r\cos\theta\right)}\left(\frac{\partial \ln n_i}{\partial r}\right)^{-1}\right]$$

If the tokamak plasma reach the thermal equilibrium state, the diffusion constant can be simplified to be

$$D_r = \frac{1}{\mu_0 \sigma_e}\left[\frac{3}{10\left(Z + 1\right)}\left(1 + \frac{\partial \ln T_i}{\partial \ln n_i}\right) - \frac{\cos\theta}{\left(R + r\cos\theta\right)}\left(\frac{\partial \ln n_i}{\partial r}\right)^{-1}\right] \tag{13}$$

It may indicate that the radial diffusion constant may be decreased apparently by the poloidal magnetic field originating from the toroidal current. And the poloidal field component arising from the bootstrap current may reduce the diffusion term owing to the ion pressure gradient by half. Meanwhile another poloidal field component coming from the magnetically-induced electric current may cancel the diffusion term originating from the inertial centrifugal force along the minor radius *r*. It suggests that the diffusion at the plasma core may be mainly confined by the poloidal field component caused by the magnetically-induced electric current. Based on the analytical expression, the concerned magnitude of the diffusion constant can be estimated. For the tokamak plasma edge with the typical parameters $\partial lnT/\partial lnn_i = 4$ $\partial lnn_i/\partial lnr = -10$, Z=1, a/R=1/3, T=10^6 K, the radial diffusion constant may display a magnitude 2.6 m^2/s, which agreed with the experimental results [3], [5]–[8], [27], [28] for the order of the magnitude. The radial diffusion constant may gradually decrease towards the plasma core due to the temperature rising, which agreed with the experimental observations [3], [5], [7], [27], [28]. Owing to the inversely proportionality of the confinement time to the radial diffusion constant [3], the plasma confinement time was anticipated to scale as $T^{3/2}$, which agreed with the experimental findings [29], [30]. Like the aforementioned case of cylindrical plasma, despite the fact that the radial diffusion constant can be reduced noticeably by the toroidal field and the poloidal field, the radial diffusion constant does not exhibit the presence of the magnetic fields in the mathematical form. Therefore, it can be inferred that the confinement time is almost independent of the magnetic fields and is proportional to the electrical conductivity in the mathematical form. The properties may be proved by the experimental scaling law for the confinement time $\tau \propto B_T^{0.15} I_P^{0.93}$ [31], where I_p denotes the toroidal current and B_T signifies the toroidal magnetic field. The weak toroidal field dependence may arise from the field-dependent temperature, which will be addressed in the next section. Moreover, considering the electrical conductivity independent of the plasma density based on the Coulomb collision mechanism, the radial diffusion constant may appear to be insensitive to the plasma density, which was in agreement with the experimental results [8].

Furthermore, the relation between the ion pressure and the magnitude of the magnetic fields can also be revealed for the Tokamak plasma in thermal equilibrium state,

$$B_\varphi(r,\theta)B_\theta(r,\theta) = B_\varphi(a,\theta)B_\theta(a,\theta)\frac{r}{r_0}\frac{(R+r\cos\theta)}{(R+a\cos\theta)}\left[\frac{p_i(a,\theta)}{p_i(r,\theta)}\right]^{\frac{3}{5(Z+1)}} \qquad (14)$$

Substituting the above equation into equation (11) may yield

$$E_0' = -\frac{k_B T_i}{(Z+1)e}\frac{\partial \ln p_i(r,\theta)}{\partial r} \qquad (15)$$

It may indicate that the radial electric field is negative, meaning that the radial electric field points to the plasma core, which was in accord with the experimental measurements [21], [32], [33]. On the contrary, the sign of the radial electric field given by the classical theory is positive. The negative sign of the radial electric field may result from the physics that the semi-free electrons do not possess the position degrees of freedom. By considering the positon-dependent temperature and pressure, the radial electric field may exhibit a U shape and display a value of the order 10^4 V/m at the plasm edge, which was consistent with the observed experimental results [21], [32], [33]. Moreover, the radial electric field can be expressed as the following equation considering the generalized parabolic functions [14], [20] for the ion temperature and ion density profiles $n_i(r) = n_i(0)(1 - r^2/a^2)^{\alpha_n}$, $T_i(r) = T_i(0)(1 - r^2/a^2)^{\alpha_T}$, where $n_i(0)$ and $T_i(0)$ denote the ion density and the ion temperature at the plasma core, α_T, α_n demonstrate the dimensionless indexes,

$$E_0' = -\frac{k_B T_i(0)(\alpha_n + \alpha_T)}{(Z+1)e}\left(1 - \frac{r^2}{a^2}\right)^{\alpha_T - 1}$$

As the tokamak plasma enters the H-mode, the index α_T may decrease dramatically and thereby can result in an enhancement of the radial electrostatic field at the tokamak plasma edge, which agreed with the experimental measurements [33].

Inserting equation (12) into equation (8) will generate

$$\frac{\partial}{\partial r}\ln B_\varphi(r,\theta) + \frac{3}{10(Z+1)}\frac{\partial}{\partial r}\ln p_i(r,\theta) = 0$$

As a result, the position dependence of the toroidal magnetic field can be given by

$$B_\varphi(r,\theta) = B_\varphi(a,\theta)\left[\frac{p_i(a,\theta)}{p_i(r,\theta)}\right]^{\frac{3}{10(Z+1)}} \qquad (16)$$

Substitution of the position-dependent toroidal field into equation (14) yields

$$B_\theta(r,\theta)=B_\theta(a,\theta)\frac{r}{r_0}\frac{(R+r\cos\theta)}{(R+a\cos\theta)}\left[\frac{p_i(a,\theta)}{p_i(r,\theta)}\right]^{\frac{3}{10(Z+1)}} \tag{17}$$

The poloidal velocities of the electrons and ions can be given by

$$u_{e\theta}=\frac{\tau_e}{m_e}\left[eE'_\theta-\frac{5k_BT_e}{3}\frac{\partial\ln B_\varphi(r,\theta)}{r\partial\theta}-\frac{5k_BT_e}{3}\frac{\partial\ln B_\theta(r,\theta)}{r\partial\theta}-\frac{5k_BT_e\sin\theta}{3(R+r\cos\theta)}\right]$$

$$u_{i\theta}=\frac{\tau_i}{m_i}\left[-k_BT_i(\vec{r})\frac{\partial}{r\partial\theta}\ln p_i(r,\theta)-ZeE'_\theta-\frac{5k_BT_i}{3}\frac{\partial\ln B_\varphi(r,\theta)}{r\partial\theta}-\frac{5k_BT_i}{3}\frac{\partial\ln B_\theta(r,\theta)}{r\partial\theta}-\frac{5k_BT_i\sin\theta}{3(R+r\cos\theta)}\right]$$

where E'_θ denotes the poloidal electrostatic field. Upon reaching the steady state, the above poloidal velocities may be the same as each other

$$0=-k_BT_i(\vec{r})\frac{\partial}{r\partial\theta}\ln p_i(r,\theta)-\frac{5k_B(ZT_e+T_i)}{3}\frac{\partial\ln B_\varphi B_\theta(r,\theta)}{r\partial\theta}-\frac{5k_B(ZT_e+T_i)}{3}\frac{\sin\theta}{(R+r\cos\theta)}$$

If the electron temperature is identical to the ion temperature, the following relation can be obtained

$$B_\varphi(r,\theta)B_\theta(r,\theta)=B_\varphi\left(r,\frac{\pi}{2}\right)B_\theta\left(r,\frac{\pi}{2}\right)\frac{(R+r\cos\theta)}{R}\left[\frac{p_i\left(r,\frac{\pi}{2}\right)}{p_i(r,\theta)}\right]^{\frac{3}{5(Z+1)}} \tag{18}$$

The series of equations (16), (17) and (18) can give the position and poloidal angle θ dependence of the magnetic fields.

2.2.2 Thermal conductivity of the tokamak plasma

Due to the confinement of the toroidal and poloidal magnetic fields, the radial relaxation times for the electrons and ions in thermal equilibrium may be modified as

$$\frac{\tau_{im}}{m_i}\approx\frac{3}{10(Z+1)k_BT\mu_0\sigma_e(r)}$$

$$\frac{\tau_{em}}{m_e}\approx\frac{3}{10(Z+1)k_BT\mu_0\sigma_e(r)}$$

where τ_{im}, τ_{em} signifies the modified radial relaxation times for electrons and ions, respectively. Thereby the thermal conductivity of the Tokamak plasma can be calculated. Due to the conservation of the plasma, the radial heat flux contributed by the electrons and ions may be given by

$$j_{eqr} = \frac{5}{2} n_e \left(k_B T\right)^2 \frac{\tau_{em}}{m_e} \left[-\nabla_{\vec{r}} \ln p_i + \frac{7}{3} \frac{\cos\theta}{\left(R + r\cos\theta\right)} \right]$$

$$j_{iqr} = \frac{5}{2} n_i \left(k_B T\right)^2 \frac{\tau_{im}}{m_i} \left[-\nabla_{\vec{r}} \ln p_i\left(\vec{r}\right) - \nabla_{\vec{r}} \ln T + \frac{7}{3} \frac{\cos\theta}{\left(R + r\cos\theta\right)} \right]$$

where j_{eqr}, j_{iqr} signify the radial heat fluxes contributed by the electrons and ions, respectively. So the related radial thermal conductivities may be

$$\kappa_{er} = \frac{3 n_e k_B}{4\left(Z+1\right)\mu_0 \sigma_e\left(r\right)} \left[1 + \frac{\partial \ln n_i}{\partial \ln T} - \frac{7}{3} \left(\frac{\partial \ln T}{\partial r} \right)^{-1} \frac{\cos\theta}{\left(R + r\cos\theta\right)} \right]$$

$$\kappa_{ir} = \frac{3 n_i k_B}{4\left(Z+1\right)\mu_0 \sigma_e\left(r\right)} \left[2 + \frac{\partial \ln n_i}{\partial \ln T} - \frac{7}{3} \left(\frac{\partial \ln T}{\partial r} \right)^{-1} \frac{\cos\theta}{\left(R + r\cos\theta\right)} \right]$$

where κ_{er} , κ_{ir} denote the electron thermal conductivity and the ion thermal conductivity along the minor radius, respectively. Thus, the total radial thermal conductivity can be given by

$$\kappa_r = \frac{3 n_i k_B}{4\mu_0 \sigma_e\left(r\right)} \left[\frac{Z+2}{\left(Z+1\right)} + \frac{\partial \ln n_i}{\partial \ln T} - \frac{7}{3} \left(\frac{\partial \ln T}{\partial \ln r} \right)^{-1} \frac{r\cos\theta}{\left(R + r\cos\theta\right)} \right]$$

where κ_r stands for the total radial thermal conductivity. The related thermal diffusivities can be given by

$$\chi_{er} = \frac{\kappa_{er}}{n_e c_v} = \frac{1}{2\left(Z+1\right)\mu_0 \sigma_e\left(r\right)} \left[1 + \frac{\partial \ln n_i}{\partial \ln T} - \frac{7}{3} \left(\frac{\partial \ln T\left(\vec{r}\right)}{\partial \ln r} \right)^{-1} \frac{r\cos\theta}{\left(R + r\cos\theta\right)} \right]$$

$$\chi_{ir} = \frac{\kappa_{ir}}{n_i c_v} = \frac{1}{2\left(Z+1\right)\mu_0 \sigma_e\left(r\right)} \left[2 + \frac{\partial \ln n_i}{\partial \ln T} - \frac{7}{3} \left(\frac{\partial \ln T\left(\vec{r}\right)}{\partial \ln r} \right)^{-1} \frac{r\cos\theta}{\left(R + r\cos\theta\right)} \right]$$

It indicates that the electron radial thermal diffusivity is comparable to the ion radial thermal diffusivity in magnitude, which agreed with the experimental findings [4], [6],

[25]. The radial thermal diffusivities may be inversely proportional to the electrical conductivity. And they continuously decreases towards the plasma core due to the increasing electrical conductivity, which was consistent with the experimental results [4], [5], [6], [25], [27]. For the Tokamak plasma edge with the previously employed parameters, $\partial lnT/\partial lnn_i = 4$, $\partial lnn_i/\partial lnr = -10$, Z=1, T=10^6 K, a/R=1/3, the estimated magnitude of the thermal diffusivities may be of the order 1 m^2/s, which agreed with the experimental results [4]–[8], [25], [34], [35], [36].

Using the same method, the radial thermal conductivity based on the classical theory may be obtained

$$\kappa_{rc} = \frac{5n_i k_B}{2}\frac{(Z+1)p_i}{\sigma_e B_\varphi^2}\left[\frac{Z+2}{(Z+1)} + \frac{\partial \ln n_i}{\partial \ln T} - \frac{7}{3}\left(\frac{\partial \ln T}{\partial \ln r}\right)^{-1}\frac{r\cos\theta}{(R + r\cos\theta)}\right]$$

where κ_{rc} represents the radial thermal conductivity in terms of the classical theory.

The ratio of the radial thermal conductivity in the work to that in terms of the classical theory may be

$$\frac{\kappa_r}{\kappa_{rc}} = \frac{3}{5(Z+1)}\frac{B_\varphi^2}{2\mu_0 p_i}$$

It indicates that the radial thermal conductivity in the work may be several orders of magnitude higher than that based on the classical theory.

2.2.3 Density limit of the tokamak plasma

As indicated previously, the diffusion of the tokamak plasma may sensitively depend on the plasma temperature. And the time-dependent temperature may be determined by adding the thermal transport equations for the electrons and ions [37] in thermal equilibrium

$$\frac{3}{2}k_B\frac{\partial\left(n_i+n_e\right)T}{\partial t}=\frac{\partial}{r\partial r}\left(\kappa r\frac{\partial}{\partial r}T\right)+\left(1-\gamma\right)p_{in}+p_{OHp}+p_{OHt}-p_{rad}$$

where p_{in} stands for the input power density, γ represents percentage of the input power density driving the electric current, p_{OHp}, p_{OHt} signifies the Ohmic heating power densities arising from the poloidal electric current and the toroidal current, respectively, p_{rad} denotes the radiation power density. Based on the previously obtained mean radial velocity and the toroidal magnetic field, i.e., equation (12) and equation (16), the poloidal Ohmic heating power density p_{OHp} can be formulated

$$p_{OHp}\approx\frac{B_\varphi^2\left(a\right)}{\mu_0^2\sigma_e}\left\{\frac{\partial}{\partial r}\left[\frac{p_i\left(a\right)}{p_i\left(r\right)}\right]^{\frac{3}{10(Z+1)}}\right\}^2$$

The poloidal Ohmic heating may sensitively depend on the magnitude of the toroidal magnetic field and the electrical conductivity. Improvement of the toroidal magnetic field may up-lift the poloidal Ohmic heating power density and the plasma edge temperature, thereby improving the electrical conductivity and reducing the diffusion constant. The poloidal Ohmic heating may gradually increase towards the plasma edge. And its magnitude at the plasma edge may be in the range 10^6 –10^7 W/m^3.

The toroidal Ohmic heating power density may be given by

$$p_{OHt}=\frac{j_p^2}{\sigma_e}$$

where j_p denotes the toroidal electric current density. On contrary to the poloidal Ohmic heating, the toroidal Ohmic heating may increase towards the plasma core. The radiation power density p_{rad} can be given by [38], [39]

$$p_{rad} = n_e n_i L_Z(T)$$

where $L_Z(T)$ denotes the radiation loss rate. It shows that the radiation power density may strongly rely on the electron density and the ion density. The increase of the ion density may significantly enhance the radiation power density, subsequently leading to the dramatic decrease of the plasma temperature and the increase of the dimensionless index α_T. The notable temperature decrease may result in a reduced electrical conductivity, causing an enhanced radial diffusion constant and the breakdown of the tokamak plasma discharging.

For the tokomak plasma core in steady state, the power loss due to the thermal conductivity and the power gain due to the poloidal Ohmic heating could be ignored, resulting in the relation

$$p_{rad} \approx p_{OHt} + (1-\gamma) p_{in}$$

Insertion of the expressions for the radiation power density p_{rad} and the toroidal Ohmic heating power density may yield

$$n_i \approx \sqrt{\frac{j_p^2}{ZL_Z(T)\sigma_e} + \frac{(1-\gamma) p_{in}}{ZL_Z(T)}}$$

For most experimental tokamak plasma [40], [41], the toroidal Ohmic heating power density at the plasma core may be in the range 10^5-10^6 W/m^3 and the average input power density over the whole plasma may also be in the range 10^5-10^6 W/m^3. The simulations showed that the input power density in particular the neutral beam energy may be mainly deposited in the regions r/a >0.2 but much less in the regions r/a <0.2 [42]. Therefore the input power density at the plasma core r/a <0.2 may be much smaller than the average input power density. As a result, the input put power density at the plasma core may be much smaller the toroidal Ohmic heating power density, generating the relation

$$n_i \approx \frac{j_p}{\sqrt{ZL_Z(T)\sigma_e}} \tag{19}$$

It indicates that the ions density limit at the tokamak plasma core sensitively depends on the radiation power density, the electric conductivity and the toroidal electric current density. The electric conductivity may be irrelevant with the electron number density in terms of the Coulomb collision mechanism. Therefore, the ion density limit may be proportional to the toroidal electric current density, agreeing with the empirical Greenwald density limit [43], [44]. The net charge number of the ions may be an interesting parameter. By increasing the net charge number *Z*, the radiation power density can be enhanced [45] but the electrical conductivity conspicuously decreases in the law $\sigma_e \propto Z^{-1}$. As a result, the ion density limit may be weakly dependent of the net charge number *Z*. And its magnitude can be estimated based on the physical parameters of the tokamak plasma core. The radiation loss rate for the hydrogen plasma may be dominated by the Bremsstrahlung radiation and may display the order of the magnitude 10^{-36} $W{\cdot}m^3$ at the temperature 10 *KeV* [39], [41]. The electric conductivity may be of the order 10^8 *S*/*m* at the related temperature. The ion density limit at the plasma core may be estimated to be $n_i \approx 10^{14} j_p$, which was consistent with the empirical Greenwald density limit [43], [44].

The situation may be different for the stellarator where the toroidal Ohmic heating power density at the plasma core may be ignored due to the minority toroidal current, leading to the ion density limit at the plasma core

$$n_i \approx \frac{\sqrt{p_{in}}}{\sqrt{ZL_Z(T)}}$$

which agreed with the experimental scaling law for the stellarators [46].

The steady state plasma edge should be turned to. The estimated thermal loss density may be the order of 10^5 W/m^3 much smaller than the radiation power density. And power gain arising from the input power density and the toroidal Ohmic heating may be much smaller than the poloidal Ohmic heating power density whose estimated magnitude may reach the order of 10^7 W/m^3. Thus, the poloidal Ohmic heating power density p_{OHp} and the radiation power density p_{rad} may dominate the temperature variations of the plasma edge, yielding the relation

$$p_{rad} \approx p_{OHp}$$

Substituting the related expressions may generate the ion density limit for the plasma edge

$$n_i \approx \frac{B_\varphi(a)}{\mu_0\sqrt{ZL_Z(T)\sigma_e}} \frac{3}{10(Z+1)} \left| \frac{\partial}{\partial r} \ln p_i(r) \right| \tag{20}$$

It indicates that the density limit for the plasma edge may be proportional to the magnitude of the magnetic field, which was consistent with the experimental observations of the helicon plasma [47].

Considering the density limits at the plasma core and plasma edge, the density limit may be dominated by the radiation loss rate, agreeing with the theoretical analysis [48]. The radiation loss rate can be enhanced greatly by the introduction of the high-Z impurities in the plasma [49]. Therefore, the impurity content must be controlled carefully to avoid the large radiation loss rate and the resultant radiation collapse.

According to the previous discussion, the underlying points may be important for the tokamak plasma under the steady operation mode. First, a notable electrical conductivity must be sustained by maintaining a high temperature and low density of the high-Z impurities for the plasma. As the nuclear fusion intensifies, the resulting temperature improvement can enhance the confinement performance. Second, the radiation loss rate should be carefully weakened by controlling the impurity density and the temperature. Third, besides the toroidal bootstrap current another electrical current is essential at the plasma core for the long-time operation. Four, the improvement of the toroidal magnetic field can enhance the plasma confinement by virtue of ascending the plasma edge temperature due to the poloidal Ohmic heating.

Last but not least, several scientific comments on the magnetic confinement of the plasma should be made based on the analytical model in the work. First of all, the magnetic fields appear to confine the neutral plasma in terms of the magnetic moment force instead of the classical Lorentz force. It is one of the most important physical idea in the model. Second, the free electrons in the neutral plasma may be held around the ions by the Coulomb force and thereby their positions should be described by the related ion positions within the mathematical framework of calculus, which may be like the bounded electrons in atoms. In a word, the free electrons in a neutral plasma may have the velocity degrees of freedom but lack the position degrees of freedom. Therefore, the free electrons within a neutral plasma should refer to “semi-free electrons”. Third, the classical Fokker-Plank equation should be modified according to the above two points for treating the transport behaviors of the plasma. Fourth, the diffusion mechanism and the Coulomb collision mechanism may be valid for the transport of the tokamak plasma, which was consistent with the theoretical analysis [15] based on experimental results. The micro-turbulences and the instabilities may not be as important as stressed by the conventional work. As a result, the anomalous transport behaviors of the tokamak plasma may not be "anomalous" and can be understood by the above four points.

3 Conclusion

In summary, two main physical ideas were unraveled for the neutral plasma in the work. One is that the Lorentz forces for the charged particles may appear as the magnetic moment forces. The other is that the free electron positions should be represented by the ion positions in the framework of calculus and the free electrons may thereby refer to semi-free electrons. According to the two physical ideas, the conventional Fokker-Plank equation should be modified and thereby a simple analytical model was constructed for the magnetic-confinement plasma in particular the tokamak plasma. And the model may rationally explain the particle diffusion constants, the thermal diffusivities, ETB, ITB and the related properties. The revealed physical ideas and the constructed model may be important for people understanding the related anomalous transport properties and the operational mode of the fusion reactor.

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