

Three fatal errors in quantum physics

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Abstract: The article shows that the basic concepts of quantum physics have three major errors that are fatal to the whole theory. The first error is that in Planck's relation $E_n = nh\nu$ oscillators have only one energy level $E = h\nu$, the article explains how it arises, while the higher energy levels $nh\nu$ must be understood as runs of n wavelengths of an oscillator. The second error is in Einstein's relation $E = pc$, in de Broglie's wavelength $\lambda = h/p$ and in the concept of particle-wave dualism. The error is that p in $E = pc$ is not the momentum of a photon and $p = Ev$ from de Broglie's wavelength $p = h/\lambda$ and Planck's relation $E = h\nu$ is not the momentum of an oscillator. In both cases the correct formula is $p = mv_p$ where v_p is the propagation speed of a matter wave. This error implies that particle-wave dualism is false. The third error is in the substitutions in the Schrödinger equation. The substitution of momentum is in error because it confuses the momentum of an oscillator with the momentum in de Broglie's wavelength formula. The substitution of energy is incorrect because oscillators do not have several energy levels. As a result of the third error it is incorrect to make a Fourier transform from the momentum coordinates to spatial coordinates, which means that a basic method in the quantum field theory fails.

Keywords: blackbody radiation, Planck-Einstein relation, de Broglie wavelength, Schrödinger equation.

1. Introduction

The article shows that quantum physics has serious errors. It is sufficient to point out to three fundamental errors at the very beginning of quantum mechanics.

Quantum physics started in 1900 when Max Planck proposed his blackbody radiation law that made the assumption that energy levels of oscillators in the walls of the blackbody box could only have energy levels $E_n = nh\nu$, $n = 1, 2, 3, \dots$, where h was a new constant (Planck's constant), E_n is energy, ν is frequency and n is a number.

Article [1] by Reidon Renstrøm explains the true history of this discovery, According to [2] blackbody radiation was modelled before 1900 by Wilhelm Wien's empirical law from 1896

$$I(\lambda, T) = c_1 \frac{T}{\lambda^5} e^{-\frac{c_2}{\lambda T}} \quad (1)$$

where T is absolute temperature, λ is wavelength and c_1, c_2 are real numbers. The law agreed well with measurements. Planck tried to derive this law in his papers from 1897-1900. In 1900 Planck got new measurements from Berlin Physikalisch-Technische Reichsanstalt, they showed a small difference between

Wien's law and measurements and Planck presented an improved law that fitted measurements perfectly:

$$I(\lambda, T) = c_1 \frac{T}{\lambda^5} \left(e^{\frac{c_2}{\lambda T}} - 1 \right)^{-1}. \quad (2)$$

Lord Rayleigh did publish a law based on the equipartition theorem in June 1900

$$I(\lambda, T) = c_1 \frac{T}{\lambda^4} \quad (3)$$

but he mentions in the article that the law fails and that it should have the exponential factor as the one in Wien's law. There was no discussion of "infrared catastrophe" before 1911 when Erhenfest coined this term.

According to [1], Einstein in 1905 was the first person to claim that classical physics inexorably leads to Rayleigh's law and that the spectrum of blackbody radiation cannot be explained with classical physics.

Planck's radiation law can be derived by assuming that a wave having the energy E_n in blackbody radiation has the probability given by the Boltzmann distribution

$$Prob(n, \nu) = \frac{e^{-E_n \beta}}{\sum_{j=0}^{\infty} e^{-E_j \beta}} \quad (4)$$

where $\beta = 1/kT$, k is Boltzmann's constant. Then

$$\sum_{j=0}^{\infty} e^{-j h \nu \beta} = \frac{1}{1 - e^{-h \nu \beta}} \quad (5)$$

$$\sum_{j=1}^{\infty} E_j e^{-j h \nu \beta} = \sum_{j=1}^{\infty} j h \nu e^{-j h \nu \beta} = -\frac{d}{d\beta} \sum_{j=0}^{\infty} e^{-j h \nu \beta} = \frac{h \nu e^{-h \nu \beta}}{(1 - e^{-h \nu \beta})^2}. \quad (6)$$

The expectation value of energy in the frequency ν in blackbody radiation depends on ν

$$\langle E_\nu \rangle = \sum_{n=1}^{\infty} E_n Prob(\nu, n) = \frac{\sum_{n=1}^{\infty} E_n e^{-E_n \beta}}{\sum_{j=0}^{\infty} e^{-E_j \beta}} = \frac{h \nu}{e^{h \nu \beta} - 1}. \quad (7)$$

Multiplying this distribution by the number of modes with frequency ν gives Planck's blackbody radiation law.

The blackbody radiation law fits measurements very well, but where does the assumption that $E_n = n h \nu$ come from? The next two sections propose an explanation and point out to two serious errors. In the following sections we will see how this initial error led to other serious errors.

2. The error in Planck's relation $E_n = nh\nu$

As article [1] shows, the the true history may have been very different from what has been told, but it is told, and may be true, that Planck thought about harmonic oscillators emitting radiation to the inside of the box when crafting his blackbody radiation law. What kind of harmonic oscillators he could have considered in 1900?

Planck could not have considered a quantum harmonic oscillator

$$E\psi = -\frac{\hbar^2}{2m} \frac{\partial}{\partial x} \psi + \frac{1}{2} m \omega^2 x^2 \psi \quad (8)$$

where m is mass, ψ is a wavefunction, x is displacement in the x -axis, $\omega = 2\pi\nu$, $\hbar = h/2\pi$ because the Schrödinger equation and the wavefunction were not yet introduced. The solutions to the quantum harmonic oscillator are complicated functions involving Laguerre polynomials and the energy levels are

$$E_n = \left(n + \frac{1}{2} \right) h\nu. \quad (9)$$

They do not agree with Planck's hypothesis of the energy levels $E_n = nh\nu$.

Planck also could not have thought of electromagnetic waves: the power of electromagnetic waves depends on the square of the amplitude only, not on the frequency. The energy of one wavelength depends on frequency as ν^{-1} , not as in Planck's hypothesis. The oscillators that Planck meant were electrons or atoms in the walls of the box generating the waves that were inside the box, that is, material, even mechanical, oscillators.

Planck may have considered one of the following alternatives. The first alternative is the mechanical harmonic oscillator where a mass m oscillates on the x -axis as $x(t) = A \cos(\omega t)$, A is a real constant, and the energy is the sum of kinetic and potential energy

$$E = \frac{1}{2} m \left(\frac{dx}{dt} \right)^2 + \frac{1}{2} m \omega^2 x^2. \quad (10)$$

This oscillator has the constant energy $E = \frac{1}{2} m \omega^2 A^2$. The second alternative is a mechanical wave where a mass is evenly spread along the x -axis and it oscillates in the y -direction $y(x, t) = A \cos(kx - \omega t)$, $kx - \omega t = (2\pi/\lambda)(x - v_p t)$ where v_p is the propagation speed in the x -direction. This oscillator has constant energy for every mass unit $dm = \mu dx$

$$E(x) = \frac{1}{2} dm \left(\frac{\partial y}{\partial t} \right)^2 + \frac{1}{2} dm \omega^2 y^2 \quad (11)$$

$$E(x) = \frac{1}{2} \mu \omega^2 A^2 dx \quad (12)$$

and when integrated over a wavelength, energy is the same as with the oscillating mass m

$$E_\lambda = \frac{1}{2}m\omega^2 A^2. \quad (13)$$

The difference between this and the perious mechanical oscillarot is that energy (13) is available after the oscillation period $T = 1/\nu$ where $\omega = 2\pi\nu$, $\nu = v_p/\lambda$.

As Planck thought of waves inside the box, of the form $A \cos(kx - \omega t)$, he probably thought of the second alternative, but this is not so essential. The essential issue to notice is that a harmonic oscillator has only one energy level, not n energy levels. Thus is the error in Planck's formula $E_n = nh\nu$.

For some reason Planck proposed that the oscillators creating blacbody radiation with the frequency ν have several energy levels $E_n = nh\nu$, $n = 1, 2, 3, \dots$. This is not possible for harmonic oscillators. If we want n energy levels from a harmonic oscillator, then the frequency of the oscillator must have several values $\nu_n = n\nu$, $n = 1, 2, 3, \dots$, but if so, it creates frequencies $n\nu$ to the blackbody radiation inside the box. Yet, in Planck's derivation of the radiation law, all E_n levels give the same frequency ν , not for $\nu_n = n\nu$.

This means that there is only one energy level for the oscillator and the energy levels E_n mean that the oscillator makes n wavelengths for an energy packet of E_n . The oscillator can stop after any period, or it can continue to another period. If the stopping probability is constant, we get a geometric distribution for the probability of runs of length n . The Boltzmann distribution is a geometric distribution.

Let us understand E_n as a run of n periods. There remains the following problem: why

$$E = h\nu = \frac{1}{2}m\omega^2 A^2 \quad (14)$$

should hold? Why the amplitude A cannot have any values? Planck did not give any reason for this assumption, but it is easy to propose a rather understandable reason.

In 1905 Einstein presented a solution to the photoelectric effect and introduced the concept of photon and particle-wave dualism. This paradigm was generalized by de Broglie into the de Broglie wavelength formula:

$$\lambda = \frac{h}{p}. \quad (15)$$

What is p in the de Broglie wavelength? The wavelength λ for a real or complex wave, like

$$y = A \cos(kx - \omega t); \quad \text{or} \quad \phi = e^{i(kx - \omega t)} \quad (16)$$

is a number, not a function. Therefore $p = h/\lambda$ is a number, not a function. From this we can deduce that de Broglie momentum must be the number mass times the propagation speed of the wave

$$p_B = mv_p. \quad (17)$$

The momentum p in the harmonic oscillator (11) is mass times velocity and the mass is composed of small masses dm that have the velocity $v = \partial y / \partial t$ in the y -direction. This momentum is a function of time, not the number. Therefore the momentum from de Broglie's formula is not the momentum of the mass in an oscillator.

From de Broglie wavelength follows

$$\frac{h}{\lambda} = p_B = mv_p \quad (18)$$

$$\frac{h\nu}{\lambda} = mv_p\nu = mv_p \frac{v_p}{\lambda} \quad (19)$$

$$h\nu = mv_p^2 \quad (20)$$

and the equation (14) gets the form

$$E = \frac{1}{2}mv_p^2 \left(\frac{2\pi}{\lambda} \right)^2 A^2 = h\nu \quad (21)$$

$$E = \frac{1}{2}h\nu \left(A \frac{2\pi}{\lambda} \right)^2 = h\nu \quad (22)$$

$$A = \sqrt{2} \frac{\lambda}{2\pi}. \quad (23)$$

The promised rather understandable reason why A should always have a value yielding Planck's energy $E = h\nu$ is that A is proportional to λ .

Planck's error of assuming that the oscillators have n energy levels E_n resulted into Schrödinger creating an energy operator that has n eigenvalues E_n . This is wrong: the n energy levels are runs of an oscillator with only one energy level.

3. The error in the particle-wave duality paradigm

Einstein was the first to propose the idea that light, a wave, also behaves as particles. These light particles were later give the name photons. This was the beginning of the particle-wave dualism paradigm. The concept of photon says that that electromagnetic radiation consists of photons that have the energy $E = h\nu$ and momentum $p = E/c$, If so, then the superposition of any number of photons must depend linearly on ν . This is true in blackbody radiation as Planck's blackbody radiation law shows.

However, the energy of a radio wave with the frequency ν does not depend on the frequency ν in the way as it does in $E = h\nu$, it depends only on the amplitude and the time over which the energy is measured. A sinusoidal wave $f(t) = A_s \sin(2\pi ft)$ can be generated by feeding a current to a sending antenna and it can be received as a weaker signal $f(t) = A_r \sin(2\pi ft)$ by a receiving

antenna. The wave induces a current in the receiving antenna and the received instantaneous power is

$$P(t) = UI = \frac{U^2}{R} = \frac{A_r^2}{R} \sin^2(2\pi ft) \quad (24)$$

where U is the induced voltage, I the induced current and R the resistance of the receiving antenna. The average power over a period T of one oscillation of wavelength λ is

$$P_{ave} = \frac{U^2}{R} = \frac{1}{T} \frac{A_r^2}{R} \int_0^T \sin^2(2\pi ft) dt = \frac{A_r^2}{2R} \quad (25)$$

and the energy in one oscillation period $T = 1/f$ is (denoting the frequency f by ν)

$$E_\lambda = P_{ave}T = \frac{A_r^2}{2R} \frac{1}{\nu}. \quad (26)$$

This energy of one wavelength does depend on the frequency ν , but in the inverse way to $E = h\nu$. So, why do we not see photons in this case?

In both cases, heat emission by blackbox walls and a radio wave emitted by a sending antenna, the emitted wave is caused by oscillating electrons. As electrons have a charge, an oscillating electron creates an oscillating electromagnetic field. Creating this oscillating field requires energy, therefore the sender must be fed with energy. Electrons also have mass and therefore oscillating electrons are like harmonic oscillators where the average power is proportional to both the square of the (very small) amplitude A_e of the movement that an electron makes and the square of the frequency ν .

Blackbody radiation has the same spectrum regardless of what material is used in the walls, therefore the reason why blackbody radiation shows the dependency on ν and a radiowave does not has nothing to do with the material used in the antenna or the walls of the black box. We can assume that the amplitude of electrons depends on the wavelength as in (23) also in the radiowave sending circuit. The reason is that in the blackbody radiation the sending oscillator stops after random number of periods while in the radiowave sender the sender is fed with energy all the time and it will not stop when it is emitting a sinusoidal continuous wave.

Notice that this is one way of saying that an oscillator does not have several energy levels $E_n = n h \nu$, it has one energy level $E = h \nu$ and E_n is energy of a wave consisting of n wavelengths. Otherwise I cannot explain why the energy of a radio wave does not show dependency on the frequency.

Electromagnetic radiation consist of energy packets, that we can call photons, only if it was created by sending oscillators that sent the wave in runs of a number of periods. An electromagnetic wave does not have any inherent particle character, it is a wave. Only when it interacts with matter we may see particle-like effects.

A similar observation is true also for matter. Considering the momentum of a moving mass shows that a moving mass is a particle, not a wave. If the speed v is not close to c , kinetic energy of a mass m is the classical kinetic energy $E_k = (1/2)mv^2 = (1/2)pv$ in quite good accuracy. However, if for a moving mass it is valid to assume that its energy is $E = h\nu$ and that the de Broglie wavelength is $\lambda = h/p$, then the relation between energy and momentum is

$$p = \frac{h}{\lambda} = \frac{h\nu}{\lambda\nu} = \frac{E}{v_p}. \quad (27)$$

In this formula, for a moving mass, v_p , the propagation speed of the wave, must equal the speed v of the mass. The energy E in (27) does not include the rest energy m_0c^2 of the particle. For a moving mass E in (27) must be kinetic energy. We get the result: if a moving mass is considered as a particle it has momentum $p = 2E/v$ while if it is considered as a wave, it has momentum $p = E/v$. The momentum cannot have two different values.

The first observation is that a moving mass cannot be considered as a wave. It is a particle. A free electron behaves as a particle in cathode ray tubes and particle accelerators. The track of an electron can be seen in a bubble chamber trace, it is a track of a charged particle, not a track of a wave. The wave nature of electron, or other particles, only appears when the particle interacts with matter.

This simple observation only shows that particle-wave dualism does not mean that a wave actually is a particle or that a particle actually is a wave. This is not any new discovery, the usual understanding of the particle-wave dualism is that we can see a wave behaving as a particle and particle behaving as a wave only in the interactions of a wave or a particle with mass. Particle-wave dualism is a concept of interaction with mass. This is not the serious error that was promised. The second observation is that there is a serious error.

This error is seen when we look closer at the relation $E = pc$ and the de Broglie variant of it $E = pv$. If $E = pc$ is correct, then the customary analysis of Compton scattering as a collision gives the relativistic kinetic energy formula for the kinetic energy of an electron in Compton scattering. This standard calculation is repeated in [2], but [2] gives two strong theoretical reasons why the relativistic kinetic energy formula is not correct and it gives a plot drawn from Bertozzi's measurements in 1960ies [3] that are supposed to have verified the relativistic kinetic energy formula. It is useful to include figure 1 from [2] also into this article:

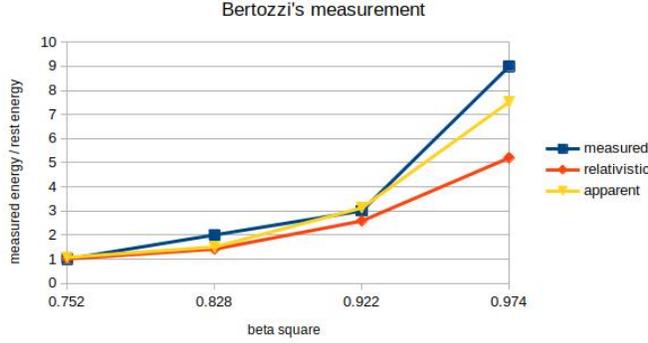


Figure 1 from [2]. The plot that is named *measured* gives the measured energy divided by electron's rest energy in the relativity theory: $E_k/m_e c^2$. The plot named *relativistic* gives the values $\gamma - 1$. If the relativistic kinetic energy formula holds, these plots should be the same. The plot named *apparent* plots the kinetic energy $(1/2)\gamma^{1.5}m_e v^2$, i.e., classical kinetic energy with the apparent mass $\gamma^{1.5}m_e$. We notice that the match with relativistic kinetic energy is not good. Apparent mass gives a better, though not exact, match to Bertozzi's results.

This figure shows that the kinetic relativistic energy formula does not hold. It follows that $E = pc$ does not hold exactly. [4] derives what p is if the relativistic mass of an electron is replaced by the longitudinal apparent mass $m = \gamma^{1.5}m_e$ as in Figure 1. This apparent mass formula also does not give an exact match to Bertozzi's measurements, but it is better. The calculated p is almost $p = E/c$, but slightly larger. We can understand this result as apparent momentum, as [2] suggests, or as the following equation states

$$E = pv_p. \quad (28)$$

As apparent p from the apparent mass $\gamma^{1.5}m_e$ is slightly larger than E/c , the speed v_p is slightly smaller than c . It can be seen as a propagation speed of a mass wave in the electron. A mass wave cannot have the propagation speed c and the formula $E = pc$ is a formula that is valid in the interaction of a wave with matter, it is not a formula for the momentum of light. Light does not have a momentum in the proper sense.

In a similar way, the formula from de Broglie's wavelength $E = mv$ is actually $E = mv_p$ where v_p is the propagation speed of a matter wave. This de Broglie momentum is not the momentum of a moving mass as we already concluded in the explanation to (14).

The serious error that this section points out is that p from de Broglie wavelength is confused with the momentum of a harmonic oscillator of the type in (11). This error causes a serious error in the Schrödinger equation, also repeated in the Dirac equation: the substitution of momentum with the partial derivative of space is not correct.

4. The error in the Schrödinger equation

The Schrödinger equation is not derived, it is a heuristic equation, but it can be rationalized in several ways. One way is that the de Broglie wavelength formula gives a number $p = h/\lambda$ while in an oscillator the momentum p is a function. One way to cope with this problem is to think of the momentum as an operator \hat{p} and p as an eigenvalue of this operator:

$$\hat{p}\psi = p\psi \quad (29)$$

where ψ is a wavefunction, a new concept introduced by Schrödinger. Also energy is both a function in an oscillator and a number $E = nh\nu$. We can make it into an operator \hat{E} and see E as an eigenvalue

$$\hat{E}\psi = E\psi. \quad (30)$$

For a wave $\psi = e^{i(kx - \omega t)}$ the operators take the forms

$$\hat{p} = -i\hbar \frac{\partial}{\partial x} \quad (31)$$

$$\hat{E} = i\hbar \frac{\partial}{\partial t}. \quad (32)$$

We get two separate equations which both equal to zero

$$\left(E - i\hbar \frac{\partial}{\partial t} \right) \psi = 0 \quad (33)$$

$$\left(\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \frac{p^2}{2m} \right) \psi = 0. \quad (34)$$

Combining these equations and rearranging

$$\left(\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \frac{p^2}{2m} \right) \psi = \left(E - i\hbar \frac{\partial}{\partial t} \right) \psi \quad (35)$$

$$\left(i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \right) \psi = \left(E - \frac{p^2}{2m} \right) \psi. \quad (36)$$

Inserting the potential energy

$$V = E - \frac{p^2}{2m} \quad (37)$$

$$\left(i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \right) \psi = V\psi \quad (38)$$

and rewriting it yields the time dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \psi = \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V \right) \psi. \quad (39)$$

There are two errors in this equation. Both are in the substitutions (31) and (32). In the substitution of p in (31) there is the identification of the momentum from the de Broglie wavelength with the momentum of the mass of the oscillator and in the substitution of E in (32) there is an operator that creates n energy levels to the oscillator while the oscillator can only have the energy level $E = h\nu$ and the E_n value means a run of n wavelengths by the oscillator.

In order to see the errors clearer, let us look at the harmonic oscillator (11). Integrating (11) over a wavelength of x changes dm to $m = \lambda dm$ and we can write (11) as

$$E = \frac{1}{2}mv^2 + \frac{1}{2}m\omega^2y^2 = \frac{p^2}{2m} + \frac{1}{2}m\omega^2y^2. \quad (40)$$

Let us choose a special solution for simplicity

$$\psi = Ae^{i(kx - \omega t)}. \quad (41)$$

This ψ satisfies (32). Inserting $\psi = y$ in (40) gives $E = 0$, but as we want (40) to give a nonzero E , y cannot be ψ . The function y corresponding to ψ is $y = Re \psi$.

$$y = A \cos(kx - \omega t). \quad (42)$$

Let us do as Schrödinger did in (31) and confuse p from de Broglie's formula and p as the momentum of the mass m . From de Broglie wavelength we obtain the relation between the momentum p and the wave number k

$$\hbar k = \frac{h}{2\pi} \frac{2\pi}{\lambda} = \frac{h}{\lambda} = p. \quad (43)$$

With this relation we can write (40) as

$$E = \frac{\hbar^2 k^2}{2m} + \frac{1}{2}m\omega^2y^2. \quad (44)$$

Making a heuristic substitution (31) is not mathematical derivation, we look for a mathematically sound way to make this substitution. The multiplier k^2 can come from partial derivation of ψ with respect to x . The form kx is similar to the form ωt in ψ and as we can make a Fourier transform from t to ω , we can also make a Fourier transform from x to k . Therefore, we multiply (44) with the Fourier transform (changing x to k) $\Psi(k)$ of $\psi(x)$. It gives

$$E\Psi = \frac{\hbar^2 k^2}{2m}\Psi(k) + \frac{1}{2}m\omega^2y^2\Psi(k). \quad (45)$$

We make the inverse Fourier transform trying to get the right side of the Schrödinger equation. The Fourier transform of ψ (as $x \rightarrow k$) is $\Psi = Ae^{-i\omega t}\delta(k)$. Therefore $y^2\Psi = A^3 \cos^2(-\omega t)e^{-i\omega t}$. The Fourier inverse $\mathcal{F}^{-1}(y^2\Psi)$ is

$$A^3 \cos^2(kx - \omega t)e^{-i\omega t}\delta(k) = A^3 \cos^2(-\omega t)e^{-i\omega t}. \quad (46)$$

The inverse Fourier transform of (45) is

$$E\psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi + \frac{1}{2} m\omega^2 A^3 \cos^2(-\omega t) e^{-i\omega t}. \quad (47)$$

The wave ψ satisfies the Schrödinger equation (40). It also satisfies separately (31) and (32). Especially, using (32) we can replace the right side

$$i\hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi + \frac{1}{2} m\omega^2 A^3 \cos^2(-\omega t) e^{-i\omega t}. \quad (48)$$

This is not the Schrödinger equation and ψ does not satisfy it.

Seeing what happens to V in (38) we see that y^2 must be seen as a constant. It is not a constant as y is a function of $kx - \omega t$. Interestingly, the quantum harmonic oscillator in (8) does not have this problem. Instead of y^2 in (11), (8) has x^2 and it is not transformed in the inverse Fourier transform, but this does not help: the method must work for all potential energies V .

This is not the end of confusion in the Schrödinger equation. Assume that we do not make the inverse Fourier transform for $y^2\Psi$ but consider y^2 as a constant. Then we get the equation to the form

$$E\psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi + \frac{1}{2} m\omega^2 y^2 \psi. \quad (49)$$

Let us take the real part of this equation:

$$Ey = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} y + \frac{1}{2} m\omega^2 y^3. \quad (50)$$

For $E = 0$ this is an equation of the type

$$y'' + \beta y^3 = 0 \quad (51)$$

which has a solution $y = C(x + b)^{-\alpha}$. The correct solution is $y = Re \psi$. This failure means that we cannot multiply y^2 with $Re \psi$ in this equation.

The only way to get the Schrödinger equation is to do as in (37)-(38) replacing the potential energy term containing y^2 with total energy minus kinetic energy. That is:

$$\frac{\hbar^2 k^2}{2m} \Psi(k) + \frac{1}{2} m\omega^2 y^2 \Psi(k) = 0 \quad (52)$$

$$\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi + \frac{1}{2} m\omega^2 y^2 \psi = 0. \quad (53)$$

Simplified

$$\frac{\partial^2}{\partial x^2} \psi + \frac{m^2 \omega^2}{\hbar^2} y^2 \psi = 0. \quad (54)$$

We have to solve y^2 from (44)

$$y^2 = -\frac{\hbar^2 k^2}{m^2 \omega^2} \quad (55)$$

and insert it to (54)

$$\frac{\partial^2}{\partial x^2}\psi - k^2\psi = 0. \quad (56)$$

The solution is $\psi = C(t)e^{\pm ikx}$ and in this sense the calculation gives the correct result, but this is not any sound mathematical derivation.

Two sound mathematical derivations lead to the same result. The Fourier transform method is mathematically sound. It has its own caveats but here it is not a question of problems in the Fourier transform method. The reason why we do not get the Schrödinger equation after doing the inverse Fourier transform is that the substitution of the momentum of the mass of the oscillator with the momentum from the de Broglie wavelength is not correct. They are not the same entity. This is the fatal error in the Schrödinger equation.

The Fourier transform aspect of these heuristic substitutions is heavily used in quantum field theory. In that theory one often writes

$$e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} = e^{i(\mathbf{p}\cdot\mathbf{x}-E_n t)/\hbar} \quad (57)$$

and transforms between four space x and momentum p dimensions. The result of these 4-dimensional Fourier transforms is that Green functions in the perturbation series of Feynman diagrams diverge and must be renormalized by a method of subtracting infinities from infinities and getting a finite number.

One further indication that the substitution (31) is not justified is that we can equally well find a connection between k and p from the time derivative:

$$\omega = 2\pi\nu = \frac{2\pi v_p}{\lambda} = \frac{p2\pi}{m\lambda} = \frac{pk}{m}. \quad (58)$$

There is no reason to make any use of the partial derivative with respect to x . Notice that in (58) $p = mv_p$, the de Broglie momentum, the momentum that satisfies the following calculations: $E = pv_p$, $h = E/\nu$, $h = p\lambda$, $h = pv_p/\nu = p\lambda$, $E = mv_p^2$. This is not the momentum of the mass m moving along the y -axis.

It seems that p was associated with the partial derivative with respect to x in the Schrödinger equation only because the partial derivative with respect to time was already needed in order to get the energy levels E_n as eigenvalues of the energy operator. This approach was discussed in an earlier section. It is also an incorrect substitution: oscillators creating waves with frequency ν do not have energy levels E_n . They only have one energy level $E = h\nu$ and a run of n wavelengths creates the energy level $E = h\nu$ in blackbody radiation.

Let us briefly look at the energy equation of a harmonic oscillator, integrated over a wavelength of x , in (40)

$$E = \frac{1}{2}mv^2 + \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}m \left(\frac{\partial}{\partial t} \right)^2 + \frac{1}{2}m\omega^2 y^2. \quad (59)$$

Taking the Lagrange-Euler equation from the energy equation gives the force equation

$$m \frac{\partial^2}{\partial t^2} y + m\omega^2 y = 0. \quad (60)$$

The complex valued wave solution of this equation is

$$\psi = Ae^{i(kx-\omega t)} + Be^{-i(kx-\omega t)}. \quad (61)$$

Because the wave solution has the variable $kx - \omega t = (2\pi/\lambda)(x - v_p t)$ we can also solve the equation (60) as

$$mk^2 y + m \frac{\partial^2}{\partial x^2} y = 0 \quad (62)$$

which explains why the Schödinger equation can give correct wave solutions in some situations.

5. Comments on some experiments and paradoxes

The double slot experiment is a confusion created by particle-wave dualism. The way this experiment is described in the literature, light is made of photons. A single photon takes one way or two ways and a photon taking two ways interferes by itself. An observation of one path at a time after the photon has taken one or two ways change this past choice. If it sounds like nonsense, it is nonsense. Light is not composed of photons. Photons do not take one or two ways. No observation can change a past event. All can be explained with light being a wave. A wave takes all possible ways and observing one way disturbs a part of the wave causing the loss of an interference pattern.

There is no need for a wavefunction, at least outside a quantum system. The Schrödinger cat paradox is an example of what confusion arises if one assumes that wavefunctions with mixed states can be applied to the world outside a quantum system. The cat is either dead or alive, the observer only does not know which is the case. Mixed states may be useful in describing the quantum system, but as long as the system has not reached a state that causes a radioactive emission that triggers the release of poison that kills the cat, the cat is alive, after it the cat is dead.

A basic mathematical theorem is not violated in the Bell inequality experiment. A simple solution is that the Bohr measure is incorrect and does not count all probability, but an alternative solution is that the whole model with a wavefunction is invalid. The correct solution is probably the latter one: wavefunctions with mixed states cannot be extended outside a very small quantum system.

Tunneling does not verify existence of a wavefunction: a particle can obtain energy from the environment and pass a potential wall, this does not require any quantum mechanical concepts.

6. Conclusions

Not much of quantum physics can escape the consequences of the three fatal errors pointed out in the article. While this short article only continues up to the Schrödinger equation, noticing that the substitutions in that equation are invalid causes that much of quantum mechanics, like the Heisenberg picture and the relativistic equations by Klein-Gordon and Dirac, and notably quantum field theory are all affected. I can explain this situation only by a direct quote from [1]:

"In the first part of Einstein's famous 1905 paper where he introduced the light quanta hypothesis, he established that physics inexorably led to the law Rayleigh had reached from the equipartition theorem, $c_1 T/\lambda^4$, and Einstein claimed that the spectrum of blackbody radiation could not be explained without a break from classical physics."

Einstein, again.

7. References

- [1] Renstrøm R., The Ultraviolet myth, arXiv:2402.03405v1, 05 Feb 2024.
- [2] Jormakka, J., Apparent mass in Compton scattering, ResearchGate, 2025.
- [3] Bertozzi, W., Speed and Kinetic Energy of Relativistic Electrons. American Journal of Physics, 32(7):551-555 (1964)