**Apparent momentum in Compton scattering**

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Abstract: Compton scattering seemingly verifies the relativistic kinetic energy formula, but the article shows that this is not the case. The relativistic kinetic energy formula can be refuted in several ways, several of these ways are destribed in the text. The article explains how Compton scattering can be understood in the context of apparent mass.

Keywords: Compton scattering, Energy-Momentum Relation, apparent mass.

## 1. Introduction

In Compton scattering light with momentum $p$ hits an electron, changes its direction by the angle $\theta$, and continues with momentum $p'$. The electron is at rest before the event, gets momentum $p_e$ and moves away to the direction given by the angle $\alpha$. The event is modelled as an elastic collision:

$$p = p' \cos(\theta) + p_e \cos(\alpha) \qquad p' \sin(\theta) = p_e \sin(\alpha) \tag{1}$$

$$(p - p' \cos(\theta))^2 = p_e^2 (1 - \sin^2(\alpha)) \qquad p'^2 \sin^2(\theta) = p_e^2 \sin^2(\alpha) \tag{2}$$

$$p^2 + p'^2 - 2pp' \cos(\theta) = p_e^2. \tag{3}$$

Using the less certain relation

$$E = pc \tag{4}$$

we can write (3) as

$$E^2 + E'^2 - 2EE' \cos(\theta) = p_e^2 c^2 \tag{5}$$

$$1 - \cos(\theta) = \frac{1}{2EE'} \left( (E - E')^2 - p_e^2 c^2 \right). \tag{6}$$

In Compton scattering the wavelenght of light changes according to

$$\lambda' - \lambda = \lambda_C (1 - \cos(\theta)) \tag{7}$$

where $\lambda_C$ is the Compton wavelength for an electron. Using the following two relations that have been verified to hold for different light frequencies and different elementary particle Compton wavelengths

$$E = h\nu \qquad h = \lambda_C m_e c \tag{8}$$

and the definition $\lambda = c/\nu$ we get

$$\frac{1}{EE'}(E - E') = \frac{1}{m_e c^2}(1 - \cos(\theta)). \tag{9}$$

$$1 - \cos(\theta) = \frac{1}{2EE'} 2m_e c^2 (E - E'). \tag{10}$$

Eliminating $1 - \cos(\theta)$ from these two equations gives

$$(E - E')^2 + 2m_e c^2 (E - E') - p_e^2 c^2 = 0 \quad (11)$$

which has the solution (only the positive sign is possible here)

$$E - E' = \sqrt{p_e^2 c^2 + m_e^2 c^4} - m_e c^2. \quad (12)$$

Assuming that $p_e = \gamma m_e v$, this is exactly the relativistic mass formula

$$E - E' = \sqrt{\gamma^2 m_e^2 c^2 + m_e^2 c^4} - m_e c^2 = (\gamma - 1) m_e c^2. \quad (13)$$

This looks like a perfect verification of the relativistic kinetic energy formula: Compton scattering is elastic collision with relativistic masses. The article demonstrates that this is not the case.

## 2. The relativistic kinetic energy formula is incorrect

First we will look at Bertozzi's measurements [1]. The following table gives the results of electrons that were accelerated in a linear particle accelerator and the speed and released energy was measured.

Table 1. If relativistic kinetic energy formula holds $E_k/m_e c^2 = \gamma - 1$. In the table $\gamma - 1$ is calculated from the measured $v^2/v^2$. The value of measured $v^2/c^2$ is correct to at least the second digit and the value for the measured $E_k/m_e c^2$ is correct to the first digit according to [1].

| $E_k/m_e c^2$ | measured $v^2/c^2$ | calculated $\gamma - 1$ |
|---|---|---|
| **1** | 0.752 | 1.008 |
| **2** | 0.828 | 1.411 |
| **3** | 0.922 | 2.581 |
| **9** | 0.974 | 5.202 |

The following figure shows how well the relativistic kinetic energy formula matches with measured values.

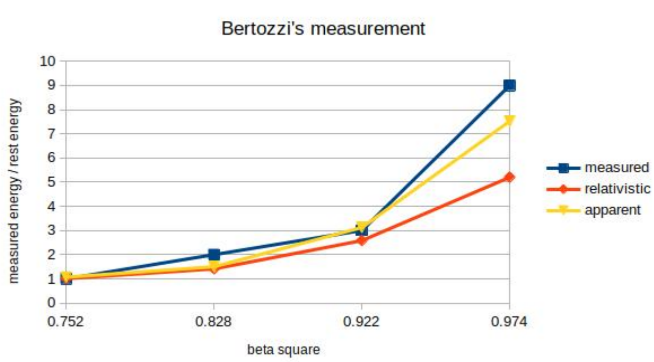

Figure 1. The figure plots $E_k/m_ec^2$ (measured), $0.5\gamma^{1.5}m_ev^2$ (kinetic energy of the apparent mass) and $\gamma - 1$ (relativistic kinetic energy) as an function of $v^2/c^2 = \beta^2$. The y-axis gives the ratio of energy and the rest energy of an electron.

The plot that is named *measured* gives the measured energy divided by electron's rest energy in the relativity theory: $E_k/m_ec^2$. The plot named *relativistic* gives the values $\gamma-1$. If the relativistic kinetic energy formula holds, these plots should be the same. The plot named *apparent* plots the kinetic energy $(1/2)\gamma^{1.5}m_ev^2$, i.e., classical kinetic energy with the apparent mass $\gamma^{1.5}m_e$. We notice that the match with relativistic kinetic energy is not good. Apparent mass gives a better, though not exact, match to Bertozzi's results.

## 3. The relativistic kinetic energy formula is impossible

Mass has two special properties: it causes gravitation and it has inertia. The relativistic mass formula violates both of these properties.

The claim that the gravitational mass grows to infinity when speed approaches $c$ can be refuted simply by noticing that there are stars that have not turned into supernovas, like our sun. If the gravitational mass would approach infinity when the speed approaches $c$, then in an inertial system moving very fast, close to the speed $c$, with respect to the sun, the gravitational mass of the sun would be so large that it would turn into a supernova. This event would be observable in every inertial system provided that the the observer is sufficiently close to the sun because different inertial frames of reference are simply different views to the same physical reality. Anything observable that happens in one inertial frame happens in all. But the sun is there.

The claim that inertial mass follows the relativistic kinetic energy formula is easily refuted by a short calculation. Let the kinetic energy $E_k$ of a mass depend only on $v$ and let this mass slow down by making work $W$ against resisting force $F = dW/ds$. By conservation of energy $E_k = W$. The force that the kinetic energy gives is the second term in the Euler-Lagrange equation for $L(s(t), v(t), t) = E_p(s) + E_k(v)$

$$\frac{\partial}{\partial s}E_p(s) - \frac{d}{dt}\frac{\partial}{\partial v}E_k(v) = 0. \tag{14}$$

As the first term is a force from potential energy $E_p(s)$, the second term must also be force. The force from the kinetic energy must equal the force that makes the work $W$, thus

$$\frac{\partial}{\partial s}W(s) = \frac{d}{dt}\frac{\partial}{\partial v}E_k(v). \tag{15}$$

As $v = v(t), s = s(t)$, $E_k = E_k(v)$, $W = W(s)$ and we can solve $t = t(s)$ there is no need for partial derivatives:

$$\frac{d}{ds}E_k(v) = \frac{d}{dt}\frac{d}{dv}E_k(v) \tag{16}$$

$$\frac{dt}{ds}\frac{dv}{dt}\frac{d}{dv}E_k(v) = \frac{dv}{dt}\frac{d}{dv}\frac{d}{dv}E_k(v). \quad (17)$$

Cancelling $dv/dt$ on both sides and writing $dt/ds = 1/v$ and $y(v) = (d/dv)E_k(v)$ gives

$$\frac{1}{v}y = \frac{dy}{dv} \quad (18)$$

$$\frac{dv}{v} = \frac{dy}{y} \qquad \rightarrow \qquad y = C_1 v \qquad \rightarrow \qquad E_k = \frac{1}{2}mv^2 \quad (19)$$

where we have set $E_k(0) = 0$ and identified $m = C_1$ as the integration constant. Thus, the Newtonian kinetic energy formula is the only possible formula for inertial mass that conserves energy in a situation where it certainly is conserved. The mass $m$ is constant. The proof requires only the definition of work, the Euler-Lagrange equation (which is certainly correct) and the observation that if a mass slows down to zero speed because of a resisting force, the energy must go somewhere, i.e., it will all turn to work whether this work is heat of something else and therefore kinetic energy equals work.

## 4. What is apparent mass, apparent force and apparent momentum?

Kinetic energy $E_k$, momentum $p$ and force $F$ are related by the formulas appearing in the second term in the Euler-Lagrange equation

$$p = \frac{\partial}{\partial v}E_k(v) \qquad F = \frac{d}{dt}p. \quad (20)$$

Force is related to work by the formula

$$W = \int F ds \quad (21)$$

and if energy is conserved when kinetic energy $E_k$ transforms to work $W$, then $W$ equals $E_k$ and force that does work equals force from kinetic energy.

Kaufmann, Buchanan, Lorentz and others (see e.g. [2][3][4]) experimented by speeding electrons in a cathode ray tube to relatively high speeds $v$ and then they deflected the electrons by a static magnetic or electric field transversally to the velocity of the electron. In the transverse direction the electron reached some small speed $w$. Their result was that the electron moves along a classical orbit determined by the Euler-Lagrange equation, but the mass of the electron appeared to be $m = \gamma m_e$ where $\gamma = (1-(v/c)^2)^{-0.5}$. The apparent force in the transverse direction was $F_t = ma$. They called $m$ the transverse mass. In these experiments the speed $v$ was constant when the electron was deflected and the transverse mass depended essentially on $v$ and only very little on $w$. Therefore the transverse mass was essentially constant during the time when the electron was deflected. Consequently the momentum was $\gamma m w$, notice that $\gamma$ depends on $v$, not $w$. This momentum can be called $p_t = mw$ the apparent momentum of the electron in the transverse direction.

If $v$ is constant, the apparent force $F_t = ma$ and the apparent momentum $p_t = mw$ relate as $F_t = (d/dt)p_t$. However, if $w = v$ or $v$ is not constant in time, $F_t = (d/dt)p_t$ does not hold. In this situation the relation is $F_t/\gamma = (d/dt)(p_t/\gamma)$.

In the transverse direction the electron got some kinetic energy in these experiments. As $w$ was small, this kinetic energy was practically equal to $E_{k,t} = (1/2)mw^2$ where $m$ is the transverse mass and $p_t$ is $(\partial/\partial w)E_{k,t}$. However, this is not the main kinetic energy of the electron.

The main kinetic energy of the electron was caused by the longitudinal velocity $v$. In the longitudinal direction the electron was accelerated by the field and appeared to have some mass that was different from $m_e$ and from the transverse mass $m = \gamma m_e$. Lorentz called this mass the longitudinal mass. He did not manage to measure this mass but suggested that it could have been $m_l = \gamma^3 m_0$. Figure 1 suggests that the longitudinal mass in Bertozzi's measurements was about $m_t = \gamma^{1.5} m_e$. There is no reason to assume that the longitudinal mass (or transverse mass) is the same in all situations. In Bertozzi's measurement electrons were accelerared with a changing electric field. It may well be that Lorentz made experiments with a static field and got a longitudinal mass estimate $m_l = \gamma^\alpha m_0$ where $\alpha = 3$.

With the longitudinal mass $m_l$ it is logical to assume that the apparent force $F_l$, apparent momentum $p_l$ and apparent kinetic energy $E_{k,l}$ relate as

$$\frac{F_l}{\gamma^\alpha} = \frac{d}{dt}\frac{p_l}{\gamma^\alpha} \tag{22}$$

$$\frac{p_l}{\gamma^\alpha} = \frac{\partial}{\partial v}\frac{E_{k,l}}{\gamma^\alpha}. \tag{23}$$

This is because the real force, momentum and kinetic energy relate as (20).

Let us compare this to what Einstein did. He took the transverse mass from these early experiments and calculated

$$m = \gamma m_0 \quad \rightarrow \quad m^2\left(1 - \frac{v^2}{c^2}\right) = m_0^2. \tag{24}$$

Differentiating the equation gives

$$m^2 - m^2\frac{v^2}{c^2} = m_0^2 \quad \rightarrow \quad c^2 dm = v^2 dm + mv dv. \tag{25}$$

Then Einstein wrote equation (20) for transverse mass

$$F = \frac{d}{dt}p = \frac{d}{dt}mv = \frac{dm}{dt}v + m\frac{dv}{dt} \tag{26}$$

and used the definition of the work (21)

$$dW = Fds \tag{27}$$

$$dW = \frac{dm}{dt} v ds + m \frac{dv}{dt} ds = \frac{ds}{dt} v dm + m \frac{ds}{dt} dv = v^2 dm + mv dv. \qquad (28)$$

Identifyind the right side as $c^2 dm$ he integrated setting the lower bound to $m_0 c^2$

$$dW = v^2 dm + mv dv = c^2 dm \quad \rightarrow \quad W = mc^2 \qquad (29)$$

and then concluded that $W$ equals the total energy $E$ and the kinetic energy is

$$E_k = E - m_0 c^2 = (\gamma - 1) m_0 c^2. \qquad (30)$$

For some reason Einstein did not check the relation

$$p = \frac{\partial}{\partial v} E_k = \frac{d}{dv} (\gamma - 1) m_0 c^2 = \gamma^3 m_0 v \qquad (31)$$

which would have shown his error in using (20) to an apparent mass in (27) and that this error means that $W$ does not equal $E$. Indeed, $W$ is larger than $E$ and Einstein wrote a formula allowing a perpetual motion machine.

Notice also that (27) gives $F = ma(\gamma + \gamma^3 v^2/c^2)$. This formula was not supported by any experiments. Kaufmann measured $F$ and his measurements showed that $F = \gamma ma$, but here $\gamma = \gamma(v)$ and $a = dw/dt$ where $v$ is the longitudinal speed and $w$ is the transverse speed. There were no experiments where $w$ was the same as $v$, the longitudinal speed. Einstein also did not prove in any way that the rest mass $m_0 c^2$ is energy. The term $m_e c^2$ appears in Compton scattering, equation (12).

Apparent kinetic energy is apparent only in the sense that it is not all kinetic energy. All of it is real energy as is shown by Bertozzi's measurements: the electron does get the energy and it releases the energy when arriving to the sensor. Also in Compton scattering energy $E - E'$ is real energy and the electron gets it. Only, the electron does not get all of it to kinetic energy, there is additional energy that the electron holds in some form. The form might in some situations be heat and for all we know, the electron might emit this energy at some later time. If the electron is in an atom, this additional energy is not heat, it is absorbed in electron orbits as is shown by the fine structure in the spectrum of a hydrogen atom.

## 5. Apparent momentum in Compton scattering

We have seen that Bertozzi's measurements do not support the relativistic kinetic energy formula and that neither gravitational mass nor inertial mass can have a formula where mass grows with speed. However, Compton scattering and early experiments in [2][3][4] clearly show that mass appears to be growing with speed. This is apparent mass. It is caused by weakening of a force that tries to accelerate a mass. If the force comes from a force field that has the local propagation speed $c$ and a mass is moving with a speed close to $c$, the force cannot effect with full strength. It makes work and the energy of this work goes to the mass in some form, but some of this work does not speed up the mass.

In Compton scattering we can define an apparent momentum. For any kinetic energy $E_k$ we can solve the momentum $p_0$ for the mass $m_0$ from the equation (12) coming from Compton scattering, i.e., not from the relativity theory

$$E_k = \sqrt{p_0^2 c^2 + m_0^2 c^4} - m_0 c^2 \tag{32}$$

$$p_0^2 c^2 + m_0^2 c^4 = (E_k + m_0 c^2)^2 \tag{33}$$

$$p_0 = \sqrt{c^{-2}(E_k + m_0 c^2)^2 - m_0^2 c^2}. \tag{34}$$

Figure 1 suggests that $E_k = 0.5\gamma^{1-5} m_0 v^2$ is a better approximation for $E_k$ than $m_0 c^2(\gamma - 1)$, setting $E_k$ to $0.5\gamma^{1-5} m_0 v^2$ gives

$$p_0 = m_0 v \gamma^{\frac{3}{4}} \left(1 + \frac{1}{4}\gamma^{\frac{3}{2}} \frac{v^2}{c^2}\right) \tag{35}$$

$$p_0 = m_0 v \left(1 + \frac{1}{2}\frac{v^2}{c^2} + 0.34375\frac{v^4}{c^4} + \cdots\right). \tag{36}$$

Noticing that

$$p_{relativistic} = \gamma m_0 v = m_0 v \left(1 + \frac{1}{2}\frac{v^2}{c^2} + 0.375\frac{v^4}{c^4} + \cdots\right) \tag{37}$$

we see that for $v$ on the thermal range $v \sim 10^6$ m/s $p_0$ is practically the relativistic momentum. In reality the collision need not be exactly elastic, so equation (3) may be approximate and the momentum may be exactly the relativistic momentum.

The relation $E = pc$ between energy and momentum is the weak point in the derivation in Section 1. It is more reasonable to interprete the momentum as apparent momentum and to write

$$p = \frac{E}{\gamma c} \qquad p' = \frac{E'}{\gamma c} \tag{38}$$

instead of

$$p = \frac{E}{c} \qquad p' = \frac{E'}{c}. \tag{39}$$

Then the momentum equation (3)

$$p^2 + p'^2 - 2pp' \cos(\theta) = p_e^2. \tag{40}$$

is an equation where $p_e$ is the apparent momentum $p_e = \gamma m_e v$ that may or may not be exact but works as a good approximation.

## 6. The relativistic correction to the fine structure constant remains

Notice that

$$\frac{1}{2}\gamma^{1.5}m_0v^2 = \frac{1}{2}m_0v^2 + \frac{3}{8}m_0\frac{v^4}{c^2} + \frac{21}{64}m_0\frac{v^6}{c^4} + \cdots \tag{41}$$

while the relativistic formula is

$$(\gamma - 1)m_0c^2 = \frac{1}{2}m_0v^2 + \frac{3}{8}m_0\frac{v^4}{c^2} + \frac{20}{64}m_0\frac{v^6}{c^4} + \cdots \tag{42}$$

We see that for small $v/c$ the apparent mass (41) gives almost the same energy as the relativistic kinetic energy, but figure 1 shows that this is not the case for $v/c$ close to one.

Notice also that the relativistic momentum (25) and apparent momentum in (23)(24) give practically identical values for $v$ in the thermal range.

Consequently the correction term to the fine structure constant is the same whether it is calculated from the apparent kinetic energy or from the relativistic kinetic energy. In both cases the equation (20) is expressed as a series

$$E_k = m_0c^2\left(\sqrt{1 + \frac{p_0^2}{m_0^2c^2}} - 1\right) = \frac{p_0^2}{2m_0} - \frac{1}{8}\frac{p_0^4}{m_0^3c^2} + \cdots \tag{43}$$

and the second term is taken as a perturbation to the Hamiltonian of the non-relativistic calculation. There is no difference in the calculation. However, there is a question what happens to the additional energy in Compton scattering.

Using relativity theory in the derivation of the Dirac equation is not any error since we can get the form of kinetic energy from Compton scattering, equation (12), and not from the relativity theory. Additionally, the Dirac equation is not even Lorentz invariant, unlike it is often claimed (see [5]). The Dirac equation is not seriously dependent on the relativity theory even though Diract constructed the equation to fully comply with the relativity theory.

Of course, the Schrödinger and Dirac equations are not properly derived, they are heuristic and at least the dependence of atomic oscillation time on the gravitational field (gravitational time dilation) should be explicit in these equations so that time is not a part of space-time geometry.

## 7. Is Compton scattering the source of cosmic microwave radiation?

In Compton scattering an electron gets more energy than its kinetic energy receives. There are no orbits where this energy could be stored unlike in atoms. What happens to this energy? The logical answer is that it is almost immediately emitted as radiation.

Let us estimate the wavelength of such hypothetical radiation. Thermic electrons have speed on the range $10^6$ m/s. We will assume that $v$ of the electron in

Compton scattering is on this range. Apparent kinetic energy is

$$E_k = \frac{1}{2}\gamma^{1.5} m_e v^2 = \frac{1}{2} m_e v^2 + \frac{3}{8} m_e \frac{v^4}{c^2} + \cdots \tag{44}$$

The second term would be the additional energy, the later terms are too small to matter. This additional energy would turn into radiation with the wavelength $\lambda$

$$E_k = E = h\nu = h\frac{c}{\lambda}. \tag{45}$$

Then

$$\lambda = \frac{8hc}{3}\frac{c^2}{m_e v^4} = \frac{8 * 6.6256 * 10^{-34} * 2.9979^3 * 10^{24}}{3 * 9.1091 * 10^{-31}} * v^{-4}\left(\frac{m^5}{s^4}\right) \tag{46}$$

$$\lambda = 5.2 * 10^{22} v^{-4}\left(\frac{m^5}{s^4}\right). \tag{47}$$

Consmic microwave radiation has the maximim at the wavelength about 1.9 mm. Then $v$ would be

$$v = \left(\frac{5.2 * 10^{22}}{1.9 * 10^{-3}}\right)^{\frac{1}{4}} m/s = \left(27.5 * 10^{24}\right)^{\frac{1}{4}} m/s = 2.3 * 10^6 m/s. \tag{48}$$

That is a perfectly fine speed range for thermal electrons, but let us leave this only as a question: how strong actually are the arguments that identify cosmic background radiation as a relict of the Big Bang? We may in a similar way ask about cosmic redshift. In Compton scattering the frequency of light does decrease and it could decrease linearly with distance.

### 8. de Broglie wavelength

The Schrödinger equation is derived with the heuristic insertations

$$E = h\nu = h\frac{v}{\lambda} \qquad \lambda = \frac{h}{p} = \frac{h}{\gamma m_0 v}. \tag{49}$$

The same insertations are made in the Dirac equations. The first equation should hold for any wave and second equation is de Broglie wavelength for a matter wave. The concept of a matter wave and particle-wave duality suggests that an electron is a wave. Can this be so?

An electron can well behave as a wave in an atom, but does a free equation propagate as a wave? In Compton scattering an electron behaves as a classical particle with apparent/relativistic mass in the sense that the trajectory can be solved from the Euler-Lagrange equation. In the experiments [1][2][3][4] where electrons are accelerated they also behave as classical particles with apparent mass in this sense. We can even see traces of trajectories of electrons in bubble chamber images.

One confusing issue is that equations (49) give the relation

$$E = hv\frac{p}{h} = pv \tag{50}$$

but in all of these experiments [1][2][3][4] the relation between energy and momentum is not $E = pv$, it is $E \sim (1/2)pv$ in the first order.

This makes one question all applications, including quantum field theory, where a free spinor field is a part of the Lagrangean: the Dirac equation is valid only in a quantum system, not necessarily at all for free electrons or other spin 1/2 particles.

The evidence of de Broglie wavelength is that electrons and neutrons diffract and de Broglie wavelength gives the correct diffraction. This does not imply that these elementary particles are waves when they are free particles. It describes the interaction with a slit of double slit, i.e., interaction with matter. We only have to explain how an elementary particle can get at the same time to two slits in order to create the interference pattern. This seems to imply that there is some field around the elementary particle. Let us try some simple reasoning.

It seems that the force, momentum and kinetic energy of a mass are given by (20), but forces are weakened by their finite propagation speed and forces see the mass as an apparent mass. This is why $p$ in de Broglie wavelength is apparent momentum, larger than the real momentum of the mass. Force makes work, but all of the work does not transfer to kinetic energy of the mass, yet it has to go somewhere. It goes to the mass object but not as gravitational or inertial mass, as then it would change the kinetic energy of the mass.

When a mass, like an electron, gets accelerated, the field accelerating it makes more work than the amount that is transferred to the kinetic energy of the mass. This means that the mass becomes excited. It may have an energy field around it, a different concept than an electro-magnetic or gravitational field and also a different concept than a probability distribution such as a wavefunction. This energy appears in charged electrons and in uncharged neutrons, therefore it probably has nothing to do with a charge. Gravitation is a very weak force and it is also unlikely that this energy has anything to do with a gravitational field. A probability distribution does not have energy, therefore this energy is not connected with a probability distribution. But such an energy field may explain why electrons and neutrons diffract with de Broglie wavelength.

It could be that a free mass can emit this additional energy and a fast electron does not always need to have more energy than its real kinetic energy and such a mechanims might give an alternative explanation to cosmic background radiation.

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