# Neutrino Masses, Gravitational Coupling Constant And Cosmological Constant

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## Abstract

Masses of the three neutrino mass eigenstates are predicted to be $m_0$, $4m_0$ and $22m_0$ where $m_0 = 2.281\,\text{meV/c}^2$. These predictions are arrived at by applying two ad-hoc postulates to neutrino oscillation data. The first postulate is that the mass $m_0$ of the lightest neutrino mass eigenstate is the smallest quantum of mass, and that the masses of all the massive elementary particles are positive integer multiples of $m_0$. The dimensionless gravitational coupling constant $\alpha_g$ is then defined as, $\alpha_g = \dfrac{m_0}{M_p}$, where $M_p$ is the Planck Mass. The second postulate is that the dark energy driving the accelerated expansion of the universe is represented by a cosmological constant $\Lambda$, or, equivalently, a vacuum energy with constant density $\rho_\Lambda = {\alpha_g}^4 \rho_P$, where $\rho_P$ is the Planck Density. These postulates also lead to the prediction of (dimensionless) vacuum energy density to be $\Omega_\Lambda h^2 = 0.3344$, in agreement with the $\nu\Lambda$CDM model. Furthermore, the effective electron neutrino mass is predicted to be $m_\beta = 9.185\,\text{meV/c}^2$. We also predict the effective Majorana mass of the electron neutrino to be $m_{\beta\beta} \leq 5.803\,\text{meV/c}^2$. This upper bound on $m_{\beta\beta}$ is then used to calculate lower bounds on half-lifes of various isotopes expected to undergo a neutrinoless double beta decay ($0\nu\beta\beta$). A new natural system of units is proposed in which both $m_0$ and $\rho_\Lambda$ are unity.


## I. INTRODUCTION

Experimenters have discovered three flavours of neutrinos: electron neutrino $\nu_e$, muon neutrino $\nu_\mu$ and tau neutrino $\nu_\tau$. They also observed that these neutrinos keep changing flavours, i.e. they keep transforming into each other, for example, $\nu_e \rightleftharpoons \nu_\mu$. This phenomenon of neutrino oscillations conclusively proves that the neutrinos have mass. In fact, various experiments indicate that the neutrinos have sub-electronvolt masses, making them the lightest known massive elementary particles. However, their masses are not known yet. The aim of this article is to predict the masses of neutrinos by introducing two postulates — one concerning the neutrinos themselves and the other concerning the dark energy responsible for the accelerated expansion of the universe.

In the first postulate, we propose a new analogy between mass and charge. We postulate that the masses of massive elementary particles are 'quantized' — just like their charges —

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and that they are positive integer multiples of the 'smallest quantum of mass'. We then use this postulate to define a dimensionless fundamental constant, the gravitational coupling constant $\alpha_g$.

The second postulate adds a new dimension to the cosmological constant problem.[1] Quantum Field Theory (QFT) provides a natural explanation for the existence of dark energy in the form of a constant vacuum energy. But the observed value of the vacuum energy differs from the value expected in QFT by a factor of about $10^{-123}$ This is the well-known cosmological constant problem. In the second postulate, we elevate this factor of $10^{-123}$ to the status of a fundamental constant by assuming that it is equal to the gravitational coupling constant raised to the power 4, i.e. ${\alpha_g}^4 \sim 10^{-123}$.

The rest of this article is arranged as follows. We will begin in section [II] with a brief overview of the $3\nu$ mixing model of neutrino oscillations. In section [III], we will present the Quantum of Mass (QoM) postulate and define the gravitational coupling constant $\alpha_g$. In section [IV], we will present the Cosmological Constant (CC) postulate. We will also discuss how the ongoing Hubble Tension constrains our ability to make predictions using the cosmological data. In section [V], we will predict the masses of neutrino mass eigenstates and the vacuum energy density using the neutrino oscillation data. We will also predict the effective electron neutrino mass $m_\beta$ in $\beta$-decay. In addition, we will make predictions about the effective Majorana mass $m_{\beta\beta}$ of electron neutrino and half-lifes of various isotopes expected to undergo a neutrinoless double beta decay ($0\nu\beta\beta$). In section [VI], we will construct a new natural system of units. In section [VII], we will discuss the implications of the two postulates and their predictions.

## II. THE $3\nu$ MODEL

The phenomenological three-neutrino mixing model ($3\nu$ model) provides a good fit to the outcome of the neutrino oscillation experiments. In this model, each of the three neutrino flavour eigenstates $\nu_e, \nu_\mu$ and $\nu_\tau$ is considered to be an admixture of the three mass eigenstates $\nu_1, \nu_2$ and $\nu_3$ having respective mass eigenvalues $m_1, m_2$ and $m_3$. The eigenstates are related by

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} {U^*}_{\alpha i} \, |\nu_i\rangle \tag{1}$$

where $U$ is the neutrino mixing matrix.[2] If neutrinos are Majorana particles, then $U$ depends on six independent parameters which are conveniently chosen to be 3 mixing angles $\theta_{21}, \theta_{31}, \theta_{32} \in \left[0, \frac{\pi}{2}\right]$ and 3 phases $\delta, \eta_1, \eta_2 \in [0, 2\pi]$ so that,

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{32} & s_{32} \\ 0 & -s_{32} & c_{32} \end{bmatrix} \begin{bmatrix} c_{31} & 0 & s_{31}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{31}\, e^{i\delta} & 0 & c_{31} \end{bmatrix} \begin{bmatrix} c_{21} & s_{21} & 0 \\ -s_{21} & c_{21} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

where, $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$.

If neutrinos are Dirac particles, then the Majorana phases $\eta_i$ can be absorbed into the neutrino state and then $U$ is a function of the remaining 4 parameters,

$$U = \begin{bmatrix} c_{21}\, c_{31} & s_{21}\, c_{31} & s_{31}\, e^{-i\delta} \\ -s_{21}\, c_{32} - c_{21}\, s_{32}\, s_{31}\, e^{i\delta} & c_{21}\, c_{32} - s_{21}\, s_{32}\, s_{31}\, e^{i\delta} & s_{32}\, c_{31} \\ s_{21}\, s_{32} - c_{21}\, c_{32}\, s_{31}\, e^{i\delta} & -c_{21}\, s_{32} - s_{21}\, c_{32}\, s_{31}\, e^{i\delta} & c_{32}\, c_{31} \end{bmatrix} \tag{3}$$

Application of the $3\nu$ model to the data obtained from various neutrino oscillation experiments provides best fit values of these 4 parameters — the 3 mixing angles $\theta_{ij}$, the CP phase $\delta$ — as well as the squared mass differences,

$$\begin{aligned} \Delta_{21} &= {m_2}^2 - {m_1}^2 \\ \Delta_{31} &= {m_3}^2 - {m_1}^2 \\ \Delta_{32} &= {m_3}^2 - {m_2}^2 \end{aligned} \tag{4}$$

It is not possible to measure the Majorana phases $\eta_i$ and the absolute neutrino masses $m_i$ from the oscillation data. Even the ordering of these masses is not known yet. There are two possible mass orderings:

i Normal Ordering (NO): $m_1 < m_2 < m_3$.

ii Inverted Ordering (IO): $m_3 < m_1 < m_2$.

In Table I, we show the best fit values and $3\sigma$ ranges of some of the oscillation parameters needed in this article for both the normal and inverted ordering as provided by NuFIT 6.0 (IC24 with SK atmospheric data).[3] We choose the NuFIT 6.0 data for no particular reason. Choosing other recent dataset by Capozzi et al (2020)[4] will not change the outcome of this article.

| NuFIT 6.0 (w/ atmospheric data) | Best Fit | $3\sigma$ Range | Ordering |
|---|---|---|---|
| $\Delta_{21}$ $(\text{meV/c}^2)^2$ | 74.9 | $69.2 \to 80.5$ | Both |
| $\Delta_{31}$ $(\text{meV/c}^2)^2$ | 2513 | $2451 \to 2578$ | Normal |
| $\Delta_{32}$ $(\text{meV/c}^2)^2$ | -2484 | $-2547 \to -2421$ | Inverted |
| $\sin^2\theta_{21}$ | 0.308 | $0.275 \to 0.345$ | Normal |
| $sin^2\theta_{31}$ | 0.02215 | $0.02030 \to 0.02388$ | Normal |
| $\delta_{CP}$ | 212° | $124° \to 364°$ | Normal |

**TABLE I:** The best fit values and $3\sigma$ ranges of the neutrino oscillation parameters (needed in this article) from NuFIT 6.0 (with SK atmospheric data).

Since $\Delta_{21}, \Delta_{31}$ and $\Delta_{32}$ are non-zero, the heavier two masses ($m_2$ and $m_3$ for NO, $m_1$ and $m_2$ for IO) are non-zero. It is not yet known if the lightest mass is zero or not. For later convenience, let's denote this lightest mass by $m_0$. Then for NO, $m_1 = m_0$ whereas, for IO, $m_3 = m_0$.

## III. QUANTUM OF MASS AND GRAVITATIONAL COUPLING CONSTANT

All charged particles have constant charges. In addition, these charges are 'quantized', i.e. they are integer multiples of $\frac{e}{3}$, the charge of the down antiquark, where $e$ is the charge of a proton. We may say that $e_0 = \frac{e}{3}$ is the 'smallest quantum of charge'. The charge $q$ of a particle is then given by $q = N_q\, e_0$ for some fixed integer $N_q$. Just like charges, all particles have constant masses. But it is not known whether these masses are quantized in the same sense. In this article, we assume that this is indeed the case. We assume that there exists a 'smallest quantum of mass' and that the masses of massive elementary particles are positive integer multiples of it. Since neutrinos are the lightest known massive elementary particles, they provide a natural candidate for this smallest quantum of mass.

### A. Quantum of Mass Postulate

We postulate that

I $m_0$, the mass of the lightest neutrino mass eigenstate, is non-zero.

II $m_0$ is the smallest quantum of mass.

III The mass eigenvalues of all the massive elementary particles — leptons, quarks, W and Z bosons and the Higgs boson — are positive integer multiples of $m_0$.

We will call this postulate the Quantum of Mass (QoM) postulate.

Note that the QoM postulate does not predict the masses of the elementary particles. These masses need to be measured experimentally. For example, the QoM postulate asserts that the mass of an electron is $m_e = N_e\, m_0$ for some fixed natural number $N_e$, but it does not provide us with the value of $N_e$. This value can only be determined experimentally. In section [V], we will determine neutrino masses using the experimentally obtained values of neutrino oscillation parameters.

### B. Gravitational Coupling Constant

The dimensionless gravitational coupling constant is defined as[5],

$$\alpha_g = \frac{m_p}{M_P} \tag{5}$$

where $m_p$ is the mass of an elementary particle like electron and $M_P = \sqrt{\dfrac{\hbar\, c}{G}}$ is Planck mass.[1]. Different choices of $m_p$ lead to different definitions of $\alpha_g$. In [5], the mass of a proton was used for $m_p$ but an electron was also acceptable. And in [6], the mass of an

[1] This definition is 'square root' of the definition in [5].

electron was used, although any other elementary particle would have done equally well. Since the QoM postulate accords a special status to the mass $m_0$, we choose $m_p = m_0$, therefore,

$$\alpha_g = \frac{m_0}{M_P} \tag{6}$$

This relation between $\alpha_g$ and $m_0$ is analogous to the relationship between the fine structure constant $\alpha$ and proton charge $e$,

$$\sqrt{\alpha} = \frac{e}{Q_P} \tag{7}$$

where $Q_P = \sqrt{\frac{4\,\pi\,\hbar}{\mu_0\,c}} = \sqrt{4\,\pi\,\epsilon_0\,\hbar\,c}$ is the Planck charge. We can make this analogy more obvious by defining the electromagnetic coupling constant $\alpha_e$ as,

$$\alpha_e = \frac{e_0}{Q_P} = \frac{\sqrt{\alpha}}{3} \tag{8}$$

Like $\alpha_e$ (or, $\alpha$), we expect $\alpha_g$ to play the role of a fundamental constant of nature.

## IV. COSMOLOGICAL CONSTANT

The origin and composition of dark energy is yet unknown. The ΛCDM model is the simplest phenomenological model of the dark energy in good agreement with cosmological observations. According to this model, the dark energy is represented by a cosmological constant $\Lambda$ or, equivalently, by a vacuum energy with constant density $\rho_\Lambda$, where,

$$\rho_\Lambda\,c^2 = \frac{c^4}{8\,\pi\,G}\,\Lambda \tag{9}$$

If the present-day observed value of the Hubble constant is $H_0$ and that of the vacuum energy density parameter is $\Omega_\Lambda$, then,

$$\rho_\Lambda = \frac{3}{8\,\pi\,G}\,\Omega_\Lambda\,{H_0}^2 \approx \Omega_\Lambda\,h^2 \times 1.8688 \times 10^{-26}\,\mathrm{kg/m^3} \tag{10}$$

where $h = \frac{H_0}{100\,\mathrm{km/s/Mpc}}$ is the reduced Hubble constant.

### A. Cosmological Constant Postulate

In this article, we will assume a simple relationship between the vacuum energy density $\rho_\Lambda$ and the gravitational coupling constant $\alpha_g$. As a fundamental constant of nature, $\alpha_g$ sets the value of the smallest quantum of mass $m_0$ via definition (6). In addition, we claim that $\alpha_g$ also sets the value of the vacuum energy. We postulate that the vacuum energy density is related to the gravitational coupling constant by,

$$\rho_\Lambda = {\alpha_g}^4 \, \rho_P \tag{11}$$

where $\rho_P = \dfrac{c^5}{\hbar \, G^2}$ is the Planck density. We will call this postulate the Cosmological Constant (CC) postulate. And we will refer to both the QoM and the CC postulates together as the postulates regarding Gravitational Coupling Constant or GCC postulates in short.

An immediate consequence of the CC postulate is that the vacuum energy density is proportional to the fourth power of the lightest neutrino mass. Substituting from (6) in (11), we get,

$$\rho_\Lambda = \frac{c^3}{\hbar^3} \, {m_0}^4 \tag{12}$$

This kind of a relationship between the vacuum energy and the neutrino mass has been speculated in the past and has sparked new ideas.[7–9] Further, using (9), (11) and (12)), we obtain the relation between the cosmological constant and $\alpha_g$ (and $m_0$) as,

$$\Lambda = \frac{8\,\pi}{{L_P}^2} \, {\alpha_g}^4 = \frac{8\,\pi\,G\,c}{\hbar^3} \, {m_0}^4 \tag{13}$$

where $L_P = \sqrt{\dfrac{\hbar \, G}{c^3}}$ is the Planck length.

Both the GCC postulates are ad hoc. Their merit lies in their ability to make accurate predictions. Using these postulates we can calculate the neutrino masses. In fact, the easiest way of calculating $m_0$ is by first combining (10) and (12) to get,

$$m_0 = \sqrt[4]{\frac{3\,\hbar^3}{8\,\pi\,G\,c^3}} \, \sqrt[4]{\Omega_\Lambda \, {H_0}^2} \approx 3 \, \sqrt[4]{\Omega_\Lambda \, h^2} \, \mathrm{meV/c^2} \tag{14}$$

and then substituting the value of $\Omega_\Lambda \, h^2$ obtained from cosmological observations. But the ongoing Hubble tension gets in way.

### B. Hubble Tension

Planck 2018 data[10] provides the most precise value of $\Omega_\Lambda \, h^2$. For the $\nu\Lambda$CDM model[2], it reports $h = 0.6710^{+0.0120}_{-0.0067}$, $\Omega_b \, h^2 = 0.02236(15)$ and $\Omega_c \, h^2 = 0.1201(13)$ so that $\Omega_\Lambda \, h^2 = 0.3078^{+0.0177}_{-0.0104}$. A combined analysis of data from Planck, ACT and DESI gives $h = 0.6836(29)$

and $\Omega_m = 0.3009(37)$ so that $\Omega_\Lambda h^2 = 0.3267(45)$ for the $\nu\Lambda$CDM model.[11] However, these values of $h$ are in significant disagreement with the model independent, direct measurements of $h$ obtained by late time probes using distance ladders. For example, Breuval et al[12] measured $h = 0.7317(86)$. This disagreement in the measured values of $h$ is known as the Hubble Tension.

A resolution of this tension in favour of Breuval can lead to a value of $\Omega_\Lambda h^2$ that is significantly different from the corresponding Planck value. It may even lead to a modification of the $\Lambda$CDM model itself. Therefore, we will not use the values of $\Omega_\Lambda h^2$ obtained from cosmological observations to measure the neutrino masses. Instead, we will use the experimentally obtained values of neutrino oscillation parameters to predict the value of $\Omega_\Lambda h^2$. The only restriction we will place on $\Omega_\Lambda h^2$ is that it lies in the interval $[0.2, 0.5]$. This interval includes the Planck and the DESI values of $\Omega_\Lambda h^2$ mentioned above and is large enough to encompass any possible future modifications of it.

## V. THE PREDICTIONS

The QoM postulate asserts that $m_1, m_2$ and $m_3$ are positive integer multiples of $m_0$. Let

$$\begin{aligned} m_1 &= N_1\, m_0 \\ m_2 &= N_2\, m_0 \\ m_3 &= N_3\, m_0 \end{aligned} \tag{15}$$

where $N_1, N_2$ and $N_3$ are natural numbers. Substituting (14) and (15) in (4) we get,

$$\Delta_{21} = \left({N_2}^2 - {N_1}^2\right)\, m_0^2 \approx 9\, \left({N_2}^2 - {N_1}^2\right)\, \sqrt{\Omega_\Lambda\, h^2}\, \left(\text{meV/c}^2\right)^2 \tag{16}$$

$$\Delta_{31} = \left({N_3}^2 - {N_1}^2\right)\, m_0^2 \approx 9\, \left({N_3}^2 - {N_1}^2\right)\, \sqrt{\Omega_\Lambda\, h^2}\, \left(\text{meV/c}^2\right)^2 \tag{17}$$

$$\Delta_{32} = \left({N_3}^2 - {N_2}^2\right)\, m_0^2 \approx 9\, \left({N_3}^2 - {N_2}^2\right)\, \sqrt{\Omega_\Lambda\, h^2}\, \left(\text{meV/c}^2\right)^2 \tag{18}$$

Exactly one of $N_1, N_2$ and $N_3$ is equal to 1, depending upon the mass ordering. Their values can be calculated by substituting the experimentally obtained values of $\Delta_{21}, \Delta_{31}, \Delta_{32}$ and $\Omega_\Lambda h^2$ in (16), (17) and (18). But, as discussed in the previous section, we will assume

[2] We consider the $\nu\Lambda$CDM ($\Lambda\text{CDM} + \sum m_i$) model instead of the $\Lambda$CDM model because in this model, the value of the sum of the neutrino masses $\sum m_i$ is allowed to vary.

that $\Omega_\Lambda h^2$ lies in the range $[0.2, 0.5]$. In addition, we will use the $3\sigma$ ranges of the neutrino oscillation parameters in Table I to calculate the allowed values of the natural numbers $N_1, N_2$ and $N_3$. Since the neutrino mass ordering is not yet known, we will do this for both NO and IO separately, starting with IO.

### A. Inverted Ordering (IO)

As per IO, $\nu_3$ is considered to be the lightest mass eigenstate so that $m_3 = m_0$, $1 = N_3 < N_1 < N_2$ and equation (18) becomes,

$$\Delta_{32} \approx 9 \left(1 - {N_2}^2\right) \sqrt{(\Omega_\Lambda \, h^2)} \left(\mathrm{meV/c^2}\right)^2 \tag{19}$$

If we apply the constraint $\Omega_\Lambda \, h^2 \in [0.2, 0.5]$ to equations (16) and (19), then, for the $3\sigma$ ranges $\Delta_{21} \in [69.2, 80.5]$ $\left(\mathrm{meV/c^2}\right)^2$ and $\Delta_{32} \in [-2547, -2421]$ $\left(\mathrm{meV/c^2}\right)^2$ they must satisfy the requirements,

$$\begin{aligned} {N_2}^2 - {N_1}^2 &\in [10.9, 20.0] \\ \& \; N_2 &\in [19.53, 25.18] \end{aligned} \tag{20}$$

There are no natural numbers $N_1$ and $N_2 \, (> N_1)$ that satisfy these requirements. Thus, if $\Omega_\Lambda \, h^2 \in [0.2, 0.5]$ and $\Delta_{21} \in [69.2, 80.5]$ $\left(\mathrm{meV/c^2}\right)^2$ and $\Delta_{32} \in [-2547, -2421]$ $\left(\mathrm{meV/c^2}\right)^2$, then the GCC postulates lead to the conclusion that IO is not a viable mass ordering. This conclusion is well supported by experimental data, which shows a preference for NO over IO.[3]

### B. Normal Ordering (NO)

As per NO, $\nu_1$ is considered to be the lightest mass eigenstate so that $m_1 = m_0$, $1 = N_1 < N_2 < N_3$ and equations (16) and (17) become,

$$\Delta_{21} = \left({N_2}^2 - 1\right) \, {m_0}^2 \approx 9 \, \left({N_2}^2 - 1\right) \sqrt{(\Omega_\Lambda \, h^2)} \left(\mathrm{meV/c^2}\right)^2 \tag{21}$$

$$\Delta_{31} = \left({N_3}^2 - 1\right) \, {m_0}^2 \approx 9 \, \left({N_3}^2 - 1\right) \sqrt{(\Omega_\Lambda \, h^2)} \left(\mathrm{meV/c^2}\right)^2 \tag{22}$$

If we apply the constraint $\Omega_\Lambda \, h^2 \in [0.2, 0.5]$ to (21), then, for the $3\sigma$ range $\Delta_{21} \in [69.2, 80.5] \, (\mathrm{meV/c^2})^2$, we get,

$$N_2 \in [3.45, 4.58]$$
$$\therefore \; N_2 = 4$$

And equation (21) simplifies to,

$$\Delta_{21} = 15 {m_0}^2 \approx 135 \sqrt{(\Omega_\Lambda \, h^2)} \, \left(\mathrm{meV/c^2}\right)^2 \tag{23}$$

Next, dividing (22) by (23) we get

$$\frac{\Delta_{31}}{\Delta_{21}} = \frac{\left({N_3}^2 - 1\right)}{15} \tag{24}$$

Applying the $3\sigma$ ranges $\Delta_{21} \in [69.2, 80.5] \, (\mathrm{meV/c^2})^2$ and $\Delta_{31} \in [2451, 2578] \, (\mathrm{meV/c^2})^2$, we get,

$$N_3 \in [21.4, 23.7]$$
$$\therefore \; N_3 = 22 \, \mathrm{or} \, 23$$

Thus, if $\Omega_\Lambda \, h^2 \in [0.2, 0.5]$ and $\Delta_{21} \in [69.2, 80.5] \; (\mathrm{meV/c^2})^2$ and $\Delta_{31} \in [2451, 2578] \; (\mathrm{meV/c^2})^2$ then the GCC postulates assert that NO is valid mass ordering with,

$$\begin{aligned} N_2 &= 4 \\ N_3 &= 22 \, \mathrm{or} \, 23 \end{aligned} \tag{25}$$

We cannot choose between the two possible values of $N_3$ with the current level of precision of the parameters $\Delta_{21}$ and $\Delta_{31}$. We need more precise measurements. In NuFIT 6.0, the $3\sigma$ relative precision of $\Delta_{21}$ is 15 % and that of $\Delta_{31}$ is 6 %. The ongoing JUNO experiment will measure these parameters with better precision ($< 1\,\%$, corresponding to a $3\sigma$ relative precision of $< 6\,\%$) possibly leading to the settlement of this matter.[13]

Further, substituting the values of $N_3$ in (24) we get,

$$\frac{\Delta_{31}}{\Delta_{21}} = 32.2 \, \mathrm{or} \, 35.2 \tag{26}$$

This is an exact prediction that is directly testable through the neutrino oscillation experiments.

Also, from (15) we get the three neutrino masses in terms of $m_0$ as,

$$\begin{aligned} m_1 &= m_0 \\ m_2 &= 4m_0 \\ m_3 &= 22m_0 \,\text{or}\, 23m_0 \end{aligned} \tag{27}$$

We can calculate $m_0$ either by substituting for $\Delta_{31}$ in (22) or for $\Delta_{21}$ in (23). We choose (22) because the $3\sigma$ relative precision of $\Delta_{31}$ is better than that of $\Delta_{21}$. With the value of $m_0$ thus obtained, we can calculate the values of neutrino masses as well as $\alpha_g, \Omega_\Lambda\, h^2, \rho_\Lambda$ and $\Lambda$ using (27), (6), (14), (12) and (13). The results of these calculations for both the possible values of $N_3$ are presented in Table II.

Even though we have refrained from using $\Delta_{21}$ in these calculations, the value of $\Delta_{31}$ itself depends on the value of $\Delta_{21}$. In the global analysis of the neutrino oscillation data from various accelerator and reactor experiments, the best fit value of $\Delta_{31}$ is obtained after fixing the value of $\Delta_{21}$ to the one obtained from various solar neutrino experiments and KamLAND.

In Table II, we also present the sum of the three neutrino mass eigenvalues. This sum is consistent with the cosmological constraint of $\sum m_i < 64.2\,\mathrm{meV/c^2}$ for the $\nu\Lambda$CDM model.[11]

*1. $\beta$-Decay*

Beta decay allows for direct measurement of the effective electron neutrino mass $m_\beta$ using the energy momentum conservation principle. It is given by[2],

$$m_\beta = \sqrt{\sum_{i=1}^{3} \left|U_{ei}\right|^2 {m_i}^2} \tag{28}$$

$m_\beta$ is the same for both Dirac and Majorana neutrinos as only the magnitudes of the components of $U$ enter the equation. Substituting (3) and (27) in (28) and simplifying we get,

$$m_\beta = m_0 \sqrt{\left({N_3}^2 - 1\right)\, {s_{31}}^2 + 15\, {s_{21}}^2\, \left(1 - {s_{31}}^2\right) + 1} \tag{29}$$

Last row of table II shows the best fit values and the corresponding $3\sigma$ ranges of $m_\beta$ for $N_3 = 22$ and 23 calculated by substituting the best fit values and $3\sigma$ ranges of $m_0, s_{21}$ and

|  | Formula | $N_3 = 22$ |  | $N_3 = 23$ |  |
|---|---|---|---|---|---|
|  |  | Best Fit | $3\sigma$ Range | Best Fit | $3\sigma$ Range |
| $\frac{\Delta_{31}}{\Delta_{21}}$ | $\frac{{N_3}^2 - 1}{4^2 - 1}$ | 32.2 | - | 35.2 | - |
| $m_1 = m_0$ $(\mathrm{meV/c^2})$ | $\sqrt{\frac{\Delta_{31}}{{N_3}^2 - 1}}$ | 2.281 | $2.253 \to 2.310$ | 2.182 | $2.155 \to 2.210$ |
| $m_2$ (meV/c2) | $4m_0$ | 9.124 | $9.011 \to 9.241$ | 8.726 | $8.618 \to 8.839$ |
| $m_3$ $(\mathrm{meV/c^2})$ | $N_3\, m_0$ | 50.18 | $49.56 \to 50.83$ | 50.18 | $49.55 \to 50.82$ |
| $\sum m_i$ $(\mathrm{meV/c^2})$ | $(N_3 + 4 + 1)\ m_0$ | 61.59 | $60.82 \to 62.38$ | 61.09 | $60.33 \to 61.87$ |
| $\alpha_g$ $(\times 10^{-31})$ | $\frac{m_0}{M_p}$ | 1.868 | $1.845 \to 1.892$ | 1.787 | $1.765 \to 1.810$ |
| $\Omega_\Lambda\, h^2$ | $\left(\frac{m_0}{3 \times 10^{-3}}\right)^4$ | 0.3344 | $0.3181 \to 0.3519$ | 0.2798 | $0.2662 \to 0.2945$ |
| $\rho_\Lambda$ $(\times 10^{-27}\,\mathrm{kg/m^3})$ | ${\alpha_g}^4\, \rho_P$ | 6.281 | $5.975 \to 6.610$ | 5.256 | $5.000 \to 5.531$ |
| $\Lambda$ $(\times 10^{-52}\,\mathrm{m^{-2}})$ | ${\alpha_g}^4\, \frac{8\pi}{{L_p}^2}$ | 1.172 | $1.115 \to 1.234$ | 0.9809 | $0.9331 \to 1.032$ |
| $m_\beta$ $(\mathrm{meV/c^2})$ | $\sqrt{\sum \lvert U_{ei}\rvert^2 {m_i}^2}$ | 9.185 | $8.680 \to 9.688$ | 9.051 | $8.553 \to 9.545$ |

**TABLE II:** Predicted values and $3\sigma$ ranges (for both $N_3 = 22$ and 23) of $\frac{\Delta_{31}}{\Delta_{21}}$ (26), neutrino mass eigenvalues $m_1 = m_0$ (22), $m_2$ and $m_3$ (27), sum of neutrino masses, gravitational coupling constant $\alpha_g$ (6), vacuum energy density $\Omega_\Lambda\, h^2$ (14), $\rho_\Lambda$ (12), cosmological Constant $\Lambda$ (13) and effective electron neutrino mass $m_\beta$ (29). Predictions for $\frac{\Delta_{31}}{\Delta_{21}}$ are exact. For all the other predictions we use the best fit value and $3\sigma$ range of $\Delta_{31}$ given in table I. For $m_\beta$, we also use the parameters of the $U$ matrix in table I. $M_P$ is Planck mass, $\rho_P$ is Planck density and $L_P$ is Planck length.

$s_{31}$ in (29). These predictions are well below the current upper limit of $m_\beta < 450\,\mathrm{meV/c^2}$ set by KATRIN[14]. It is also below the estimated sensitivity limit of about $300\,\mathrm{meV/c^2}$ for KATRIN.

### 2. *Neutrinoless Double Beta Decay ($0\nu\beta\beta$)*

Multiple experiments are searching for the first evidence of a neutrinoless double beta ($0\nu\beta\beta$) decay, $(A, Z) \longrightarrow (A, Z+2) + e^- + e^-$.[2] Observation of a $0\nu\beta\beta$ decay will conclusively prove that the neutrinos are Majorana particles. The corresponding decay half-life $T_{1/2}$ depends upon the effective Majorana mass $m_{\beta\beta}$ of $\nu_e$ as,

$$T_{1/2} {m_{\beta\beta}}^2 = K \tag{30}$$

where the constant $K$ is different for different isotopes undergoing the decay and,

$$m_{\beta\beta} = |\sum_{i=1}^{3} {U_{ei}}^2\, m_i| \tag{31}$$

Substituting from (2) and (27) and simplifying we get,

$$m_{\beta\beta} = m_0\, |\left(1 - {s_{21}}^2\right)\left(1 - {s_{31}}^2\right) + 4\, {s_{21}}^2\, \left(1 - {s_{31}}^2\right)\, e^{2i(\eta_2 - \eta_1)} + N_3\, {s_{31}}^2\, e^{-2i(\delta + \eta_1)}| \tag{32}$$

Since the Majorana phases are not yet known, we cannot calculate the best fit value of $m_{\beta\beta}$. Instead we calculate the allowed range of values of $m_{\beta\beta}$ for $\eta_i \in [0, 2\pi]$ and for the $3\sigma$ ranges of $m_0, s_{21}, s_{31}$ and $\delta$. For $N_3 = 22$, we get $m_{\beta\beta} \leq 5.803\,\mathrm{meV/c^2}$ and for $N_3 = 23$, we get $m_{\beta\beta} \leq 5.603\,\mathrm{meV/c^2}$. The lower bound of $m_{\beta\beta}$ is practically zero. Table III shows the predicted lower bounds on the half-lifes of various isotopes corresponding to the upper bounds of $m_{\beta\beta}$. The spread in the values of these lower bounds is due to uncertainties involved in value of the constant $K$.

## VI. UNITS OF NATURE

A system of units defined using fundamental constants of nature is termed as a natural system of units. For example, Planck units are defined by setting $c = \hbar = G = \dfrac{e}{\sqrt{\alpha}} = 1$. The special status accorded to $m_0$ and $\rho_\Lambda$ by the GCC postulates allows us to define a new

| Isotope | Experiment | $0\nu\beta\beta$ Half Life $\left(\times 10^{27}\text{ years}\right)$ | | |
|---|---|---|---|---|
| | | Current Limit | Prediction | |
| | | | $N_3 = 22$ $m_{\beta\beta} \leq 5.803\,\mathrm{meV/c^2}$ | $N_3 = 23$ $m_{\beta\beta} \leq 5.603\,\mathrm{meV/c^2}$ |
| $^{48}$Ca | ELEGANT-IV[15] | $> 5.8 \times 10^{-5}$ | $\geq 21 \; - 830$ | $\geq 23 \; - 890$ |
| $^{76}$Ge | GERDA[16] | $> 0.18$ | $\geq 33 \; - 170$ | $\geq 36 \; - 190$ |
| $^{82}$Se | CUPID-0[17] | $> 4.6 \times 10^{-3}$ | $\geq 9.4 \; - 41$ | $\geq 10 \; - 44$ |
| $^{96}$Zr | NEMO-3[18] | $> 9.2 \times 10^{-6}$ | $\geq 14 \; - 100$ | $\geq 15 \; - 110$ |
| $^{100}$Mo | CUPID-Mo[19] | $> 1.8 \times 10^{-3}$ | $\geq 4.2 \; - 13$ | $\geq 4.5 \; - 14$ |
| $^{116}$Cd | NEMO-3[20] | $> 2.2 \times 10^{-4}$ | $\geq 6.5 \; - 19$ | $\geq 7.0 \; - 20$ |
| $^{130}$Te | CUORE[21] | $> 3.5 \times 10^{-2}$ | $\geq 5.1 \; - 65$ | $\geq 5.5 \; - 70$ |
| $^{136}$Xe | KamLAND-Zen[22] | $> 0.38$ | $\geq 8.8 \; - 170$ | $\geq 9.5 \; - 180$ |
| $^{150}$Nd | NEMO-3[23] | $> 2.0 \times 10^{-5}$ | $\geq 1.5 \; - 17$ | $\geq 1.6 \; - 18$ |

**TABLE III:** Predicted lower bounds on half-lifes of various isotopes expected to undergo a neutrinoless double beta decay ($0\nu\beta\beta$) for both $N_3 = 22$ and 23. Latest lower bounds on their half-lifes from different experiments are also shown.

natural system of units in which these quantities have unit magnitude. We construct the new system of units by setting,

$$m_0 = \rho_\Lambda = \hbar = e_0 \left(= \frac{e}{3}\right) = 1 \tag{33}$$

In these units,

$$\begin{aligned} c &= 1, \qquad & G = \frac{\Lambda}{8\,\pi} &= {\alpha_g}^2, \\ e &= 3, \qquad & \frac{1}{4\,\pi\,\epsilon_0} = \frac{\mu_0}{4\,\pi} &= {\alpha_e}^2 \end{aligned} \tag{34}$$

The unit of length is defined to be,

$$l_0 = \sqrt[3]{\frac{m_0}{\rho_\Lambda}} \tag{35}$$

Using (12), we find that it is equal to the reduced Compton wavelength of $m_0$, i.e.

$$l_0 = \frac{\hbar}{m_0\,c} \tag{36}$$

The unit of time is defined to be,

$$t_0 = \frac{l_0}{c} = \frac{\hbar}{m_0\,c^2} \tag{37}$$

In the new system of units,

$$l_0 = t_0 = 1 \tag{38}$$

The Coulomb force between two charges $q_1$ and $q_2$ separated by a distance $r$ takes a simple form in the new units,

$$F_e = {\alpha_e}^2 \, \frac{q_1 \, q_2}{r^2} \tag{39}$$

Similarly, the Newtonian gravitational force between two masses $M_1$ and $M_2$ separated by a distance $r$ takes the form,

$$F_g = {\alpha_g}^2 \, \frac{M_1 \, M_2}{r^2} \tag{40}$$

The Einstein-Dirac-Maxwell action is,

$$\begin{aligned} S = & \frac{c^3}{16 \, \pi \, G} \int \sqrt{|g|} \, (R \, (g) - 2 \, \Lambda) \; d^4x \\ & + \frac{1}{c} \int \sqrt{|g|} \, \bar{\psi} \left( i \, \hbar \, c \, \gamma^\mu \, \nabla_\mu - c \, q \, \gamma^\mu \, A_\mu - m \, c^2 \right) \psi \, d^4x \\ & - \frac{1}{4 \, \mu_0 \, c} \int \sqrt{|g|} \, g^{\alpha\beta} \, g^{\gamma\mu} \, F_{\alpha\gamma} \, F_{\beta\mu} \, d^4x \end{aligned} \tag{41}$$

In the new units, the action takes the form,

$$\begin{aligned} S = & \frac{1}{16 \, \pi \, {\alpha_g}^2} \int \sqrt{|g|} \, \left( R \, (g) - 16 \, \pi \, {\alpha_g}^2 \right) \; d^4x \\ & + \int \sqrt{|g|} \, \bar{\psi} (i \, \gamma^\mu \, \nabla_\mu - q \, \gamma^\mu \, A_\mu - m) \, \psi \, d^4x \\ & - \frac{1}{16 \, \pi \, {\alpha_e}^2} \int \sqrt{|g|} \, g^{\alpha\beta} \, g^{\gamma\mu} \, F_{\alpha\gamma} \, F_{\beta\mu} \, d^4x \end{aligned} \tag{42}$$

Here, the charge $q$ of the elementary fermion is an integer and it's mass $m$ is a natural number.

## VII. REMARKS

The CC postulate requires that the dark energy is represented by a cosmological constant. This implies that the 6 parameter $\Lambda$CDM model and its 6+ parameter extensions like o$\Lambda$CDM ($\Lambda$CDM + $\Omega_K$), $\nu\Lambda$CDM etc. are viable models of the universe whereas others like wCDM are not. In addition, this rules out quintessence, MOND etc. as viable models of dark energy.

### A. Vacuum Energy and Oscillation Parameters

Figure 1 shows the vacuum energy densities $\Omega_\Lambda h^2$ obtained using neutrino oscillation data as well as cosmological data. The bottom three plots in the figure depict the best fit values and $3\sigma$ ranges of $\Omega_\Lambda h^2$ calculated using (22) and (23). We can see that the $3\sigma$ range $\Omega_\Lambda h^2 \in [0.263, 0.356]$ calculated from (23) using $\Delta_{21} \in [69.2, 80.5]\,(\text{meV}/\text{c}^2)^2$ is large enough to encompass the $3\sigma$ ranges $\Omega_\Lambda h^2 \in [0.3181, 0.3519]$ and $\Omega_\Lambda h^2 \in [0.2662, 0.2945]$ calculated from (22) using $\Delta_{31} \in [2451, 2578]\,(\text{meV}/\text{c}^2)^2$ for both $N_3 = 22$ and 23. This illustrates why we can not choose between the two possible values of $N_3$ from the neutrino oscillation data alone.

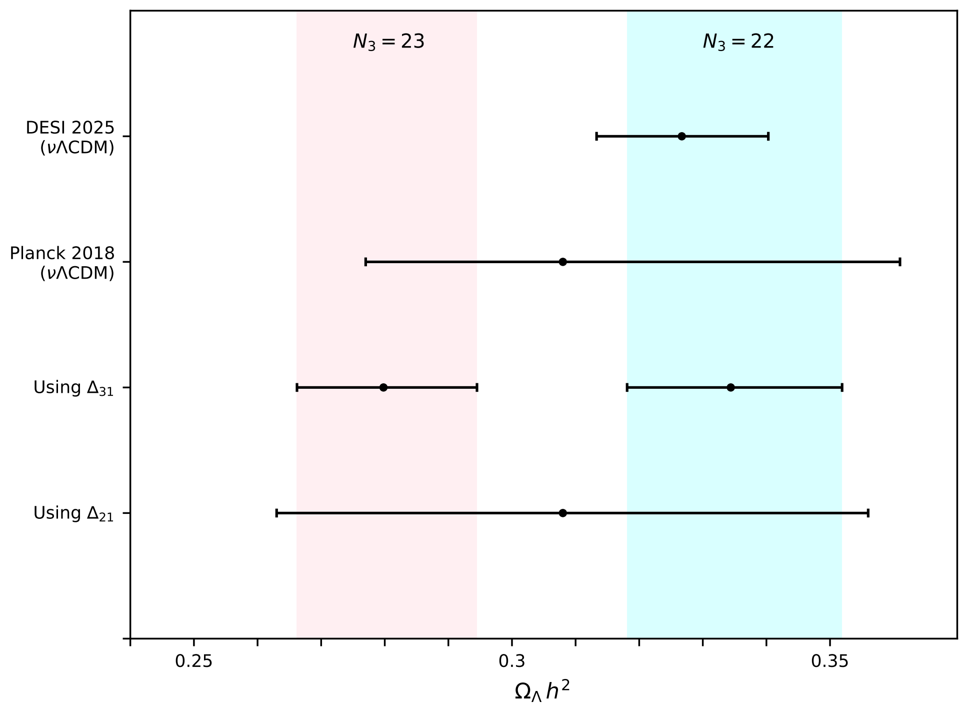


**FIG. 1:** Whisker plots of vacuum energy density $\Omega_\Lambda h^2$. The bottom three plots are created using best fit values and $3\sigma$ ranges of neutrino oscillation parameter $\Delta_{21}$ in (23) and $\Delta_{31}$ in (22) for both $N_3 = 22$ (cyan vertical band) and $N_3 = 23$ (light pink vertical band). The top two plots are created using the values and $3 \times 1\sigma$ ranges of $\Omega_\Lambda h^2$ provided by DESI [11] and Planck [10] for $\Lambda$CDM model.

But the cosmological data do show preference for $N_3 = 22$ as can be seen from the top two plots in the figure. In particular, the value $\Omega_\Lambda h^2 = 0.3344$ calculated using (22) for $N_3 = 22$ is closer to the value $\Omega_\Lambda h^2 = 0.3267$ given by DESI at $1.704\sigma$ than the value $\Omega_\Lambda h^2 = 0.2798$ for $N_3 = 23$ at $10.47\sigma$.

Notably, the GCC postulates enforce a strong correlation between the $3\nu$ model and the $\nu\Lambda$CDM model when we choose $N_2 = 4$ and $N_3 = 22$. Especially for $\Delta_{21} = 74.9\,(\mathrm{meV/c^2})^2$ in (23) with $N_2 = 4$, we get $\Omega_\Lambda h^2 = 0.308$ consistent with the Planck 2018 and the DESI values for the $\nu\Lambda$CDM model. For any other natural number $N_2$, $\Omega_\Lambda h^2$ is either too small (0.120 for $N_2 = 5$) or too large (1.08 for $N_2 = 3$) as compared to the observed values.

### A. Remarks On QoM Postulate

The masses of composite particles like protons are not equal to the sums of masses of their component elementary particles, the quarks. Therefore the masses of the composite particles are not integer multiples of $m_0$.

An immediate consequence of the QoM postulate is that there are no massive particles with mass smaller than $m_0$. Any hypothetical massive particle like Axion, Prion, Inflaton etc. must have mass equal to positive integer multiple of $m_0$.

In the QoM postulate, the mass of the lightest neutrino mass eigenstate is taken as the smallest quantum of mass (i.e., $m_1 = m_0$). If we drop this requirement and instead assume that the mass of the lightest neutrino mass eigenstate is a positive integer multiple of the smallest quantum of mass (i e., $m_1 = N_1\, m_0$ where, $N_1$ is a natural number greater than 1), then we get other possibilities for the neutrino masses — for example, $7m_0, 8m_0$ and $23m_0$ (or $24m_0$ ). In this case, is $m_0$ the mass of an axion?

Although the QoM postulate applies to all the elementary particles, we have used it only for neutrinos. We need to test its viability for other elementary particles.

### B. Running Gravitational Coupling Constant

In Quantum Electrodynamics (QED), the fine structure constant is a function of the energy scale, $\alpha(E)$. It is possible that in a theory of quantum gravity, even $\alpha_g$ is a function of the energy scale, $\alpha_g(E)$. Then owing to (6), (11) and (13), $m_0, \rho_\Lambda$ and $\Lambda$ are also functions of the energy scale.

This possible running of $\alpha_g(E)$ may not require any significant correction to the $\Lambda$CDM model. The $\Lambda$CDM model is based on observations pertaining to the period since the time of photon decoupling. In this period, the temperature of the universe has decreased from about 3000 K ($z = 1100$) to 2.728 K ($z = 0$) and the energy scale of the universe has decreased by a factor of only 1000. Hence it is reasonable to assume that $\alpha_g(E)$ may have remained approximately constant in this time.

But the running of $\alpha_g(E)$ may have implications for the period right after the big bang when the universe was too hot. It is possible that just after the big bang, $\alpha_g(E)$ and hence $\Lambda$ was much larger than its current value. This large value of $\Lambda$ may provide a natural explanation of the initial inflationary phase.

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