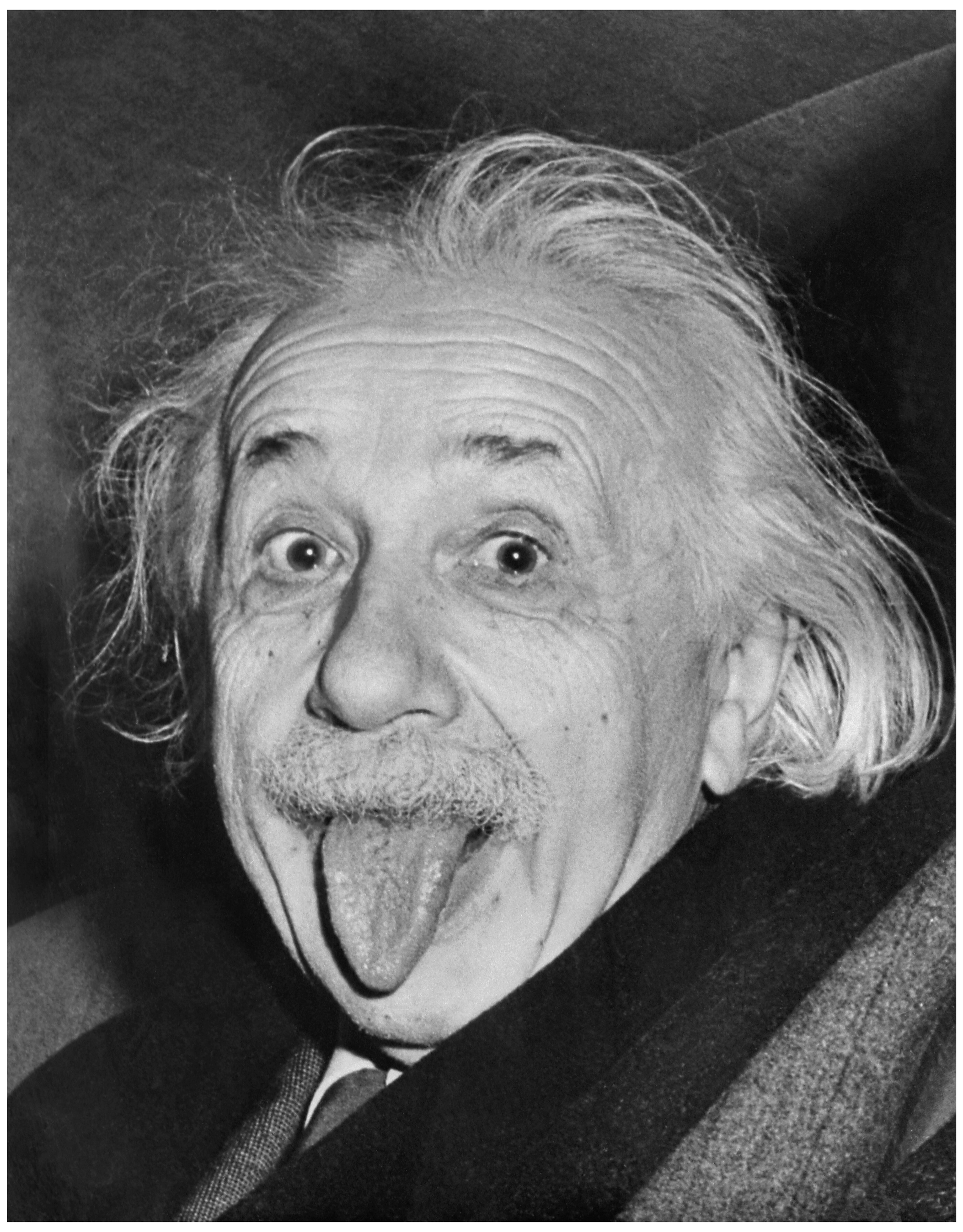
General Covariance as a Spontaneous Subsidiary Symmetry of Lorentz-Covariant Gravity



Albert Einstein

Did Einstein lose the baby and keep the bath water to formulate GR?

The profound insights of Albert Einstein in 1905 allowed him to formulate the theory of Special Relativity where the Lorentz transformation combines two postulates – the principle of relativity and the invariance of the speed of light. There is abundant evidence to validate these premises, which have passed all experimental tests. This is a cornerstone in our work, which makes Lorentz covariance fundamental and general covariance an emergent or subsidiary property. Einstein found the correct answer in 1915, in a fortuitous departure from his 1913 ideas for General Relativity, but then failed to realize the importance of the conventions he adopted to arrive at that result, and resumed his original course. Is it possible that in so doing – and assuming that general covariance is fundamental while formulating General Relativity – Einstein lost the baby and kept the bath water? We explore how Einstein's temporary foray can be combined with his insights for SR to create a Lorentz-covariant gravity theory by proceeding in analogy with the derivation for Lorentz-covariant electrodynamics.

Einstein's application of the coordinate condition $\det(g_{\mu\nu}(x)) = -1$ for all x in his landmark November 18, 1915 paper produced the correct values for both Mercury's remnant perihelion shift and the deflection of starlight by the sun's gravity, but before adopting that coordinate condition, Einstein had spent approximately two years struggling with results for Mercury's remnant perihelion shift which were substantially too small. Unfortunately, his failure to recognize the crucial importance of the highly successful November 18, 1915 coordinate condition $\det(g_{\mu\nu}(x)) = -1$ for all x paved the way for Alexandre Friedmann's 1922 promotion of his Galilean-relativity-consistent coordinate condition $g_{00}(x) = 1$ for all x, which cannot accommodate the empirically well-established phenomenon of gravitational time dilation and as part of the FLRW formula fails to predict the accelerating expansion of the universe. This accelerated expansion occurs naturally in our Lorentz-covariant gravity theory, as explained in our other paper and poster "Should Einstein's 1915 Coordinates Replace Friedmann's for Cosmology?"

Our Motivation and Derivation

There was a distinct change in approach of Einstein's earlier work on Relativity leading to the theory of SR and the work from 1913 onward that led to formulating the theory of GR as we know it today – with the exception of his landmark paper published November 18, 1915! This reflects the contrasting and somewhat incompatible sensibilities of Felix Klein and Bernhard Riemann about the proper relationship between Mathematics and Physics – when they must be used together to solve a problem. Should we favor mathematical consistency or physical realism when the two appear to be in conflict? Klein favored an approach where mathematical assumptions are added in as needed to make the model represented in the Math physically realistic while Riemann assumed as little as possible and felt that the Math would guide us toward physical realism – if we chose the right initial assumptions. We favor an approach where the Physics guides the Math, whenever the two are at odds, rather than the reverse.

Given recent observational data from JWST and DESI, we think nature is telling us Lorentz covariance is a fundamental property of physical reality, which makes Einstein's original conception of Relativity correct and with his 1915 coordinate condition lets us formulate a theory where general covariance arises spontaneously in the low-energy limit. We proceed by analogy to the Lorentz-covariant theory of electromagnetism as follows. Electromagnetism is taken to be the result of a conserved scalar entity Q called charge. The four-vector density-flux of charge Q, i.e., the current density $j^\lambda(x)$, is the source of the four-vector electromagnetic potential $A_\mu(x)$. Local charge conservation implies that,

$$\partial j^\lambda(x)/\partial x^\lambda = 0. \qquad (1.1)$$

In the static limit $j^\lambda(x) \to (c\rho(\mathbf{x}), 0)$, where $\rho(\mathbf{x})$ is the static charge density, and in that limit $A_0(\mathbf{x})$ is assumed to be the electrostatic potential that follows from $\rho(\mathbf{x})$ by Coulomb's Law,

$$-\nabla_x^2 A_0(\mathbf{x}) = 4\pi\rho(\mathbf{x}) = 4\pi(j^0(\mathbf{x})/c). \qquad (1.2)$$

The nonrelativistic equation of motion of a test body of mass m and charge e in the electrostatic potential $A_0(\mathbf{x})$ is of course,

$$md^2\mathbf{x}/dt^2 = -e\nabla_x A_0(\mathbf{x}), \qquad (1.3a)$$

and the Eq. (1.3a) nonrelativistic equation of motion corresponds to the nonrelativistic Lagrangian,

$$L(dx/dt, A_0(\mathbf{x})) = (m/2)|dx/dt|^2 - eA_0(\mathbf{x}). \qquad (1.3b)$$

Making use of the Lorentz invariance of differential proper time $d\tau := dt\sqrt{1 - |(dx/dt)/c|^2}$, and assuming the Lorentz covariance of the dynamic electromagnetic four-vector potential $A_\mu(x)$, we readily extend the nonrelativistic Eq. (1.3b) Lagrangian to a Lorentz-invariant Lagrangian, i.e.,

$$L(dx^\mu/d\tau, A_\nu(x)) = -(m/2)\eta_{\mu\nu}(dx^\mu/d\tau)(dx^\nu/d\tau) - (e/c)A_\nu(x)(dx^\nu/d\tau), \quad (1.4a)$$

where $\eta_{\mu\nu}$ is the Minkowski metric,

$$\eta_{00} = 1, \eta_{11} = \eta_{22} = \eta_{33} = -1 \text{ and } \eta_{\mu\nu} = 0 \text{ if } \mu \neq \nu. \quad (1.4b)$$

In the nonrelativistic regime where $|dx/dt| << c$ the Eq. (1.4a) Lorentz-invariant Lagrangian plus the dynamically-inert constant term $(m/2)c^2$ goes over into the Eq. (1.3b) nonrelativistic Lagrangian. When the electromagnetic four-vector potential $A_\mu(x)$ vanishes, the Eq. (1.4a) Lorentz-invariant Lagrangian of course becomes the Lorentz-invariant Lagrangian for the free mass-m test body, namely,

$$L(dx^\mu/d\tau) = -(m/2)\eta_{\mu\nu}(dx^\mu/d\tau)(dx^\nu/d\tau). \quad (1.4c)$$

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Richard Feynman

What would Feynman do? How would he re-imagine gravity given what we know today?

The approach of Richard Feynman to problems in Physics was at best unconventional and at worst something completely different from what others had tried before. He was bright, brave, and innovative – a giant in his time. He taught that we should not care overmuch about what others think, but instead we should learn how to figure things out for ourselves. And he was not a particular fan of General Relativity, as a robust explanation of gravitation. After attending what is now known as GR1 in Chapel Hill, NC; Feynman reportedly told his wife to remind him not to attend any more GR conferences, because Relativists tend to fantasize too much. This reflects his well-known tendency to think about things his own way, and not to adopt ideas that don't fit into *his* notion of what makes sense physically. And the same is true of his student, Steven K. Kauffmann, whose derivations are the backbone of our current work.

James Gleick's book "Genius" details how Feynman attempted to convey a method of learning and exploration that spawns inspiration, creativity, and discovery. Recent observations challenge accepted theory and demand that we do more of this. We can't make progress or advances in our understanding, if we only learn what is already known and try to understand things the same way everyone else does. These insights are echoed by another Nobel laureate, Doug Osheroff, in his lectures on how advances are made. Osheroff suggests that we need to look outside the well-explored parameter space to find something new, and that we should not rely overmuch on current theory because it is only people's best guess for now. So we should not rule out other possibilities that remain viable. This is why we are working to re-imagine gravity theory so that it respects Einstein's original conception of Relativity, along with his 1915 coordinate condition, which makes Lorentz covariance fundamental and allows General Covariance to arise naturally in all Lorentz-covariant frames.

If he was alive and knew what we do now, we imagine Richard Feynman would have a very different conception of gravity from what the mainstream has adopted. He likely would use a field-theoretic approach or derive gravity using force analysis rather than geometry – which is the norm for general relativists. Our analysis uses a methodology similar to Feynman's , which offers a unique derivation for various expressions familiar to gravity theorists. Feynman also thought we should let the Physics inform us and help to shape our mathematical formulations, rather than letting the Math guide us overmuch. People tend to get caught up in the formalism or mathematical description of physical phenomena and forget that it is only a model, and even if it is a good one it only tells part of the story and the physical analysis must supply the rest. In the introduction to the Calculus of Variations in her book "Mathematical Methods for the Physical Sciences," Mary L. Boas explains a fundamental tenet – where the 1st derivative goes to zero we find a maximum, minimum, or optimum/inflection point, but we don't know which – only the Physics tells us the rest of the story.

Though he is no longer with us, the spirit of Feynman's work lives on in the many people he inspired and through their ongoing efforts. His unique approach to Physics is even more valuable than his many contributions to the field – which are still widely used today. So we are happy that one of his students is here among us to keep that inspiration alive. While we freely acknowledge that Feynman was a controversial character both in his career and in his personal life, we believe that his quirks were understandable considering his prodigious ability and his intensity as a personality. Even if his character flaws have been a subject of recent critiques, his ways of pursuing knowledge remain as valuable and valid as ever, for anyone who wants to learn the subject of Physics. He will still be an example to *us*, for learning about how to learn more and better. Feynman taught that we should not believe something is true just because it came from a reputable textbook, but instead we should learn to derive and prove things for ourselves. We all believe strongly in that ideal.

Steven Weinberg

Why do we use Weinberg's textbook as a reference for this work?

Steven Weinberg's classic textbook "Gravitation and Cosmology" is one of the two standard references, along with Misner, Thorne, and Wheeler's book "Gravitation," which form the basis for a large number of later introductory texts on the subject of Relativity, as well as several more advanced textbooks. While both texts have a lot to offer readers who want a detailed and rigorous understanding of gravity, their approach is quite different. Many Relativists like MT&W better, but Weinberg's book is greatly favored by particle physicists and others because it makes gravity understandable in familiar language. It uses the analysis of forces, rather than geometry, as its main explanatory tool. This is closer to the spirit of Einstein's earlier work and is much more like how Mr. Feynman would like to see gravity explained. But he attempts to give the reader a larger toolkit than is offered elsewhere, so they are not overly dependent on Riemannian geometry. Acknowledging the limitations of tensor calculus, Weinberg also includes a section on exterior derivatives. These are some of the reasons why Weinberg's book is our reference of choice for this work.

Weinberg presents the subject of gravity in a way that does not leave the reader with a singular view, but instead offers options and explains when it is appropriate to use various models, methods, or analytic tools. We find that this is more helpful for those bold souls who want to create alternatives to GR rather than supplements for its limitations. General Relativity is a very powerful and extremely well-accepted theory that is effective over a wide range of scale, but it is known – at least – to be incompatible with Quantum Mechanics – which is also a very successful theory – and to have other flaws which make theorists continue to search for a more complete theory which would explain a wider range of phenomena. But while the search goes on for a theory of Quantum Gravity, and for ways to keep GR intact by adding or changing other explanatory factors, we imagine it might be possible to fix the problems with Relativity theory at the root. This kind of radical re-think of GR could make marrying gravity theory and quantum mechanics easier, because it removes some of the incompatibilities before they arise. And likewise, it could solve problems in Cosmology, possibly even making add-ons like Dark Energy unnecessary.

Our Motivation and Derivation continued

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Exploiting an unexpected feature of Eq. (1.4a) we obtain the Lorentz covariant dynamical equation,

$$m\eta^{\lambda\nu}(d^2x^\nu/d\tau^2) = (e/c)[(\partial A_\nu(x)/\partial x^\lambda) - (\partial A_\lambda(x)/\partial x^\nu)](dx^\nu/d\tau), \quad (1.5a)$$

a gauge transformation invariant result. In the normal form for Lorentzian dynamics Eq. (1.5a) reads,

$$m(d^2x^\mu/d\tau^2) = (e/c)\eta^{\mu\lambda}[(\partial A_\nu(x)/\partial x^\lambda) - (\partial A_\lambda(x)/\partial x^\nu)](dx^\nu/d\tau), \quad (1.5b)$$

where $\eta^{\mu\lambda}$ is the matrix inverse of the Minkowski metric $\eta_{\lambda\nu}$ defined by Eq. (1.4b); in fact, of course, $\eta^{\mu\lambda} = \eta_{\mu\lambda}$. Eq. (1.5b) is the well-known gauge-invariant Lorentz Force Law of dynamic electromagnetism.

In the gravitational case, we assume gravity to be the result of conserved energy-momentum, a four-vector P^μ. This symmetric-second-rank-tensor, which is the energy-momentum tensor $T^{\mu\nu}(x)$, is the source of the symmetric-second-rank-tensor gravitational potential $\varphi_{\mu\nu}(x)$. Gravity itself, however, *also contributes to the total conserved energy-momentum*, a nonlinear effect that can be neglected in the weak-gravity static limit in accord with the static differential Newtonian Law of Gravity,

$$\nabla_x^2\varphi_{00}(x) = 4\pi G\,(T^{00}(x)/c^2). \qquad (2.1)$$

We obtain a Lorentz-invariant Lagrangian that reduces to a beautifully simple dynamic gravitational metric form,

$$L(dx^\mu/d\tau, g_{\mu\nu}(x)) = -(m/2)g_{\mu\nu}(x)(dx^\mu/d\tau)(dx^\nu/d\tau), \quad (2.3b)$$

where,

$$g_{\mu\nu}(x) := \eta_{\mu\nu} + (2/c^2)\,\varphi_{\mu\nu}(x). \qquad (2.3c)$$

This expression, in addition to being Lorentz-invariant has the form of an invariant under general coordinate transformations. Thus, just as gauge invariance is a spontaneous subsidiary effect of Lorentz-covariant four-vector electromagnetic theory, general coordinate transformation covariance is a spontaneous subsidiary effect of Lorentz-covariant symmetric-second-rank-tensor gravity theory.

Abstract

Physical phenomena other than gravity are customarily assumed to be described by Lorentz-covariant theories, and the validity of the Lorentz transformation has been empirically verified to very high accuracy. But if all nongravitational phenomena really are Lorentz covariant, it would challenge physical consistency for gravity not to be Lorentz covariant as well. Here we work out the Lorentz-covariant dynamics of test bodies that interact with, respectively, any electromagnetic four-vector potential and any gravitational symmetric-second-rank-tensor potential. A subsidiary symmetry spontaneously emerges in each case: gauge invariance in electromagnetism, and general coordinate covariance in gravitation. We then work out field equations for the electromagnetic and gravitational potentials which incorporate their respective subsidiary symmetries; such field equations unavoidably have an infinite number of candidate solutions for their potentials, but candidate potentials which aren't Lorentz covariant are of course excluded, as are candidate potentials which violate causality or introduce singularities.

This poster is based on a paper of the same title by

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See also "Should Einstein's 1915 Coordinates Replace Friedmann's in Cosmology"

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