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Elementary energy excitation of correlated electrons and mechanism of tunnel effect

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Abstract We present an alternative explanation of the mechanism of tunneling from both quantum and statistical perspective that tunneling is related to the existence of ensemble of entangled and correlated electrons. The purpose of our work is to elucidate the true mechanism of tunneling consistent with quantum theory and energy conservation. We explain the quantum nature of tunneling on the basis of the exchange-correlation interaction of entangled electrons around the barrier and the quantum current resonance in the barrier. Our approach enables a satisfactory explanation of the quantum aspects of tunneling relative to the barrier height and width, keeping the energy conservation. It is explained that tunneling has the statistical aspect, since there exist electrons able to overcome the barrier thanks to the elementary energy excitation of the Fermi liquid that is formed as a result of the exchange-correlation interaction between correlated electrons. Our approach satisfies both the principle of energy conservation and fundamentals of quantum mechanics. Eventually, we provide a consistent and general explanation of the mechanism and characteristics of tunneling that is essentially a quantum and statistical hybrid phenomenon.

Keywords Quantum tunneling dynamics · Entangled state · Exchange-correlation interaction · Elementary energy excitation · Quantum statistics

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1 Introduction

Tunneling is famous for the understanding that electrons with energy insufficient to surmount a potential barrier can cross it. One of two typical tunneling experiments is the tunneling between two superconductors separated by an insulator. As the other type of tunneling, the Giaever tunneling (single-particle tunneling) is the tunneling of a single quasielectron from an ordinary metal to a superconducting metal [1]. Tunneling that is regarded as a pure quantum-mechanical phenomenon, above all, is significant for elucidating the physical processes in microscopic scales. Physical phenomena in semiconductor device including new quantum Hall devices and tunneling diode are explained based on tunneling [2,3]. It is recognized that tunneling lies behind the α–decay and the cold-field emission as well.

Moreover, tunneling is significant for studying a wide range of physical world beyond microscopic world. Several kinds of tunneling and their subtle properties which extend the coverage of tunneling have been discovered and reported. Tunneling rises as an interesting subject even in cosmology, as tunneling is regarded as an indispensable element for solving the problem of black hole [4,5]. As a phenomenon of significance, the evaporation of black holes as a result of the Hawking radiations has also been considered to be due to tunneling of particles from the horizons of black holes. Brabec's theory of tunnel ionization in complex systems yielded reasonable results in agreement with experiments [6]. Thus, tunneling leads to an understanding of the essence of ionization. Vindel-Zandbergen's studies of tunneling-induced electron transfer between separated protons showed the relevance of tunneling in nuclear reactions [7].

The recent researches have confirmed that tunneling depends on several factors purely beyond quantum realm.
Jürgensen reported the observation of density-induced tun-

neling which breaks the symmetric behavior for attractive and repulsive interactions predicted by the Hubbard model [8]. Perrin proposed that theoretical predictions and models based on other mechanisms such as asymmetric tunneling barriers or asymmetric charging are possible [9]. Tokieda's study on quantum tunneling in a one-dimensional potential in the presence of energy dissipation showed that it is possible to calculate the tunneling probability using a time-dependent wave packet method [10]. An analytical study of relativistic tunneling through opaque barriers demonstrated that it is necessary to consider the relativistic aspect of tunneling [11]. Chuprikov showed a deep relevance of tunneling time and superposition principle, whose gist is that scattering of a quantum particle in a one-dimensional potential barrier violates the superposition principle and thus the potential barrier and the layered structure play role of nonlinear elements [12].

In order to explain the mechanism of tunneling in intuitive ways, there have been proposed and developed several methods. The non-tunnel model for a physical particle consisting of a bare particle and its virtual decay cloud was assumed. In this theory, it is supposed that the barrier is more transparent for virtual particles than for the bare particle and the barrier width is less than the size of virtual cloud. Then tunneling is explained in such a view that virtual particles regenerate the primary physical particle behind the barrier [13]. Jonson explained a mechanism of tunneling by using the exchange-correlation potential felt by an electron tunneling from a metal through a classically forbidden region into vacuum [14].

The quasi-classical approaches of quantum mechanics in phase space is an important paradigm, which uses the Wigner or Husimi function useful for physical picture [15–23]. The quasi-classical approaches based on trajectory have been used as an effective mean to treat the quantum dynamical processes relevant to tunneling [24,25]. As a version of quasi-classical approaches, the entangled-trajectory molecular dynamics (ETMD) has been newly developed based on the quantum phase-space theory [24,26]. This approach has become a powerful tool for elucidating the dynamical processes of quantum characteristics such as the tunnel effect by using entangled trajectories in phase space. Results obtained by applying ETMD approaches in various fields of quantum dynamics have been reported [24,27]. The Husimi function-based approach is useful for obtaining the correct information related to tunneling, since the Husimi function is positive at every point and guarantees accuracy. Several works using this approach have been reported [25,28].

Despite the advances, several difficulties in interpreting the tunnel effect by using clear physical intuition still remain unsolved. Concerning the conventional interpretation, it should be noted that the solution of the Schrödinger equation in the barrier region violates the law of energy conservation and the continuity of probability current density. This big problem makes the conventional explanation of tunneling unreliable.

It still remains an important problem to elucidate the mechanism of tunneling in a consistent way. The problem is whether tunneling is a quantum effect by a single electron or quantum statistical effect by an ensemble of correlated electrons or entangled electronic states. If tunneling is due to the ensemble of electrons or ensemble state of an electron, then it should depend on a kind of temperature defined in a statistical way. Noticeably, the conventional interpretation of tunneling is not associated with temperature. However, many studies of tunneling in chemical and biochemical systems at room temperature as well as tunneling at cryogenic temperature have been reported [29]. Some researchers claim that the most general and exact approach is to apply quantum dynamics, i.e., to solve the time-dependent Schrödinger equation starting from the reactant state ensemble [29].

It is necessary to turn our attention to the fact that actually, experiments on tunneling are related not to an independent single electron but to an ensemble of electrons or of electron states. Moreover, it should be noted that the conventional theory cannot explain as yet in a general way the behavior of tunnel current in the whole domain of variability of factors such as bias voltage and temperature.

The purpose of our work is to elucidate that tunneling is not a pure quantum mechanical effect but a quantum statistical hybrid effect and the interpretation of tunneling should keep the energy conservation law and the fundamental principle of quantum mechanics. We aim to explain the main characteristics of tunneling, taking into consideration all the quantum statistical factors. Specifically, we aim to elucidate the nature and the quantum statistical characteristics of tunnel current, based on the exchange-correlation interaction of electrons.

The remaining paper is organized as follows. In Sect. 2, we explain the quantum nature of the tunneling. In Sect. 3, we explain how to describe tunneling in a statistical way by introducing the electron-excitation temperature. In Sect. 4, we describe why the conventional interpretation of the tunnel effect is inconsistent with the law of energy conservation and fundamentals of quantum mechanics. The paper is concluded in Sect. 5.

2 Quantum nature of tunneling

2.1 Exchange-correlation interaction and collective behavior of electrons around barrier

We consider that the exchange-correlation interaction causes collective behaviours of entangled electrons. Electrons present in entangled states before and after the barrier are correlated. The interaction between them helps electrons of low energy to acquire the additional energy necessary for overcoming the barrier from the correlated electrons of high energy behind the barrier. According to the superposition principle, electrons are in a superposed state entangled by eigenstates and the interaction of entangled electrons results in the exchange-correlation coupling of electrons. This coupling depends on the distance between the incident electrons and the transmitted electrons, i.e., the barrier width: the thinner the barrier width is, the stronger the exchange-correlation interaction. Thus, electrons as an integrated entity around the barrier cooperate with one another to overcome the barrier.

Based on Ref.[30], it is possible to evaluate the exchange-correlation energy combining two electrons which are separated by the barrier as

$$E_{xc} = -\frac{1}{2}\sum_{j\neq i}\left[\delta\left(m_{s_i}, m_{s_j}\right)\cdot \int\int \psi_i^*(\mathbf{r}_1)\,\psi_j^*(\mathbf{r}_2)\,\hat{H}_{12}\psi_i(\mathbf{r}_2)\,\psi_j(\mathbf{r}_1)\,\theta_{ij}(\mathbf{r}_1,\mathbf{r}_2)\mathrm{d}\tau_1\mathrm{d}\tau_2\right], \tag{1}$$

where ψ_i and ψ_j are the one-electron wave functions for states before and after the barrier, m_{s_i}, m_{s_j} the spin quantum numbers for those states, respectively, $\hat{H}_{12}$ the operator of interaction between two electrons, and $\theta_{ij}(\mathbf{r}_1, \mathbf{r}_2)$ the correlation hole function for electrons in two states i and j, respectively. Setting $\psi_i(\mathbf{r}) = e^{\mathrm{i}k_i x}, \psi_j(\mathbf{r}) = e^{\mathrm{i}K_j x}, \hat{H}_{12} = \frac{e^2}{x_2-x_1}$ where k_i and K_j are the wave numbers of electron before and after the barrier, respectively, we write the energy of the exchange-correlation interaction as

$$E_{xc} = -\frac{1}{2}\sum_{j\neq i}\left[\delta\left(m_{s_i}, m_{s_j}\right)\cdot \int\int e^{-\mathrm{i}k_i x_1} e^{-\mathrm{i}K_j x_2}\frac{e^2}{x_2-x_1} e^{\mathrm{i}k_i x_2} e^{\mathrm{i}K_j x_1}\theta_{ij}(x_1,x_2)\mathrm{d}x_1\mathrm{d}x_2\right]. \tag{2}$$

Assuming the mean value of the correlation hole function $\theta_{ij}(x_1, x_2)$, we write Eq. (2) as

$$E_{xc} = -\frac{e^2}{2}\sum_{i,j}\delta\left(m_{s_i}, m_{s_j}\right)\bar{\theta}_{ij}\int\int \frac{e^{\mathrm{i}(k_i-K_j)(x_2-x_1)}}{x_2-x_1}\mathrm{d}x_1\mathrm{d}x_2.$$

Taking into consideration that the exchange-correlation interaction is the interaction between electrons which are spaced about the barrier width apart, we set $x_2 - x_1$ approximately as the barrier width, l, to rewrite

$$E_{xc} = -\frac{e^2}{2l}\sum_{i,j}\delta\left(m_{s_i}, m_{s_j}\right)\bar{\theta}_{ij}\int_0^l\int_0^l e^{\mathrm{i}(k_i-K_j)(x_2-x_1)}\mathrm{d}x_1\mathrm{d}x_2.$$

Again, taking the mean value to be placed out of the sum denotation for $\delta\left(m_{s_i}, m_{s_j}\right)\bar{\theta}$, we have

$$E_{xc} = -\frac{Be^2}{2l}\sum_{i,j}\int_0^l\int_0^l e^{\mathrm{i}(k_i-K_j)(x_2-x_1)}\mathrm{d}x_1\mathrm{d}x_2.$$

By computing the integral, we obtain the final result,

$$E_{xc} = -\frac{Be^2}{l}\sum_{i,j}\frac{\left\{1-\sin[(k_i-K_j)l]\right\}}{l(k_i-K_j)^2} \approx -\frac{Be^2}{l}\sum_{i,j}\frac{1}{(k_i-K_j)^2}, \tag{3}$$

where $\sin[(k_i - K_j)l] \ll 1$ has been taken into consideration.

It is possible to explain the relation between the barrier height and $\sum_{i,j}\frac{1}{(k_i-K_j)^2}$. Since electrons passing through the barrier lose as much energy as the barrier height, the relation

$$\frac{\hbar^2 k_i^2}{2m} - \frac{\hbar^2 K_j^2}{2m} = \Phi$$

should hold. From

$$\frac{\hbar^2 k_i^2}{2m} - \frac{\hbar^2 K_j^2}{2m} = \frac{\hbar^2}{2m}(k_i^2 - K_j^2) = \frac{\hbar^2}{2m}(k_i - K_j)(k_i + K_j),$$

we find

$$\frac{\hbar^2}{2m}(k_i - K_j)(k_i + K_j) = \Phi.$$

Hence, we have

$$(k_i - K_j) = \frac{2m}{\hbar^2(k_i + K_j)}\Phi. \tag{4}$$

Inserting Eq. (4) into Eq. (3), we obtain

$$E_{xc} \approx -\frac{Be^2\hbar^4}{4m^2}\frac{1}{l\Phi^2}\sum_{i,j}(k_i + K_j)^2.$$

Every electron is correlated with many electrons that are clustered by the exchange-correlation energy. Therefore, introducing B_{xc} in consideration of all factors including many-electron correlation together with $\frac{Be^2\hbar^4}{4m^2}\sum_{i,j}(k_i + K_j)^2$, it is possible to give the concise expression:

$$E_{xc} = -\frac{B_{xc}}{l\Phi^2}. \tag{5}$$

In this case, E_{xc} depends on the number of electrons involved in the cluster of correlated electrons. Hence, it turns out that the exchange-correlation energy is inversely proportional to the barrier width and the square of the barrier height. The exchange-correlation energy couples electrons as a single entity. This implies that tunneling is related to the exchange-correlation energy and should be considered as a problem relative to ensemble of electrons or of electronic states.

2.2 Elementary energy excitation of correlated electrons, and electron-excitation temperature

One of the most interesting concerns of many-electron system is the problem of the internal energy excitation modes of electrons. The energy excitation modes produce excited particles from the vibrating electronic system. Electrons, for instance, in metals can be thought of to be an interacting system or open quantum system, where electrons are in entangled states and at the same time are so mutually correlated. Thus they lose their individual identities and thus outcome to a single entity. Ensemble of such single entities forms statistical distribution. Such an interaction between electrons enables the introduction of a new temperature that is defined by statistical distribution. As a result of the statistical distribution, there exist electrons which have energies able to surmount the barrier with a certain probability even if the additional energy which an electron acquires under applied voltage is lower than the barrier height. The current by such electrons apparently would look like the tunneling. We believe that the nature of tunneling lies in many-body effect by interaction of electrons.

It is viewed that the strongly interacting electrons in a metal form a normal Fermi liquid. We view that the exchange-correlation interaction causes single entities of clustered electrons. With the help of exchange-correlation interactions of attraction, electrons before and after the barrier are combined with one another and behave collectively, sharing their energies like molecules in liquid. Such correlated electrons form the Fermi liquid that can be compared with bodies that are connected by a chain. With the help of this chain of interaction, electrons before the barrier that have insufficient energy to surmount the barrier can receive necessary energy from electrons behind the barrier and electrons on the same side. This mechanism gives rise to the elementary energy excitation.

The elementary energy excitations in the Fermi liquid can be dealt with based on the concept of the so-called Landau quasiparticles or quasielectrons. For every electron, there is a quasielectron. Not all quasielectrons are important. Only electrons that are near the Fermi level in energy are detected in most experiments, since these quasielectrons have fairly long lifetimes. Quasielectrons do not interact with one another and form a statistical ensemble. Therefore, quasielectrons can be dealt as ideal gas. The statistical distribution of quasielectrons can be characterized by a kind of temperature that is defined by an ensemble of the exchange-correlation energy. We define the electron-excitation temperature which characterizes the elementary energy excitation by means of the mean value of the exchange-correlation energy $\bar{E}_{xc}$ as

$$\bar{E}_{xc} = \frac{3}{2}kT_e, \tag{6}$$

where $\bar{E}_{xc}$ is the mean value of E_{xc}. Combining Eqs. (5) and (6), we find

$$T_e = \frac{2\bar{B}_{xc}}{3kl\Phi^2}, \tag{7}$$

where $\bar{B}_{xc}$ is the mean value of B_{xc}. Eq. (7) manifestly shows that the smaller the barrier height and width are, the higher the electron-excitation temperature.

Meanwhile, the lattice vibrations scatter electrons. This is because the lattice periodicity is distorted by the lattice vibrations and the distortion scatters electrons. In fact, a system of electrons is not of a static structure. At any finite temperature, there will be thermal vibrations. Even at absolute zero, according to quantum mechanics, there will be zero-point vibrations. These lattice vibrations can be described using the totality of normal modes constituting the vibration of atoms. The quanta of these normal modes are phonons. The phonons interact with quasielectrons. This electron-phonon interaction abates the exchange-correlation interaction of electrons.

Whatever the form of the interaction, it vanishes when there are no atomic displacements. For small displacements, the interaction should be linear in the displacements. Thus the phenomenological interaction part of the Hamiltonian of electron-phonon interaction is written as

$$\mathcal{H}_{\mathrm{ep}} = \sum_{\boldsymbol{l},\boldsymbol{b}} \left(\boldsymbol{x}_{\boldsymbol{l},\boldsymbol{b}} \cdot \nabla_{\boldsymbol{x}_{\boldsymbol{l},\boldsymbol{b}}} U \right),$$

where $\boldsymbol{x}_{\boldsymbol{l},\boldsymbol{b}}$ are the displacements of atoms. If assuming that the electrons can be treated by a one-electron approximation, and that only harmonic terms are important for the lattice potential, we represent a typical matrix element of the exchange-correlation potential as

$$T_{\boldsymbol{k},\boldsymbol{k}'} = -\langle n_{q,p}| \int \int \psi^*_{\boldsymbol{k}}(\boldsymbol{r_1})\psi^*_{\boldsymbol{k}'}(\boldsymbol{r_2})\mathcal{H}_{\mathrm{ep}}\psi_{\boldsymbol{k}}(\boldsymbol{r}_2)\psi_{\boldsymbol{k}'}(\boldsymbol{r}_1) \, \mathrm{d}\tau_1 \mathrm{d}\tau_2 |n_{q,p} - 1\rangle,$$

where $|n_{q,p}\rangle$ are phonon eigenkets and $\psi_{\boldsymbol{k}}$ are electron eigenfunctions with the wave vector $\boldsymbol{k}$. The displacement $\boldsymbol{x}_{\boldsymbol{l},\boldsymbol{b}}$ increases with temperature. This means the increase in the distance between electrons which are exposed to the exchange-correlation interaction. Meanwhile, the increase in the distance between electrons leads to the decrease of the overlap of wave functions. Therefore, the exchange-correlation interaction decreases with temperature. In other words, the increase in real temperature results in the decrease in the electron-excitation temperature.

2.3 Quantum resonance of tunnel current by barrier

The quantum constraint in the barrier affects the tunnel current because electrons have wavelike property. This mechanism of the resonant current through the barrier attributed purely to quantum property satisfies energy conservation unlike the conventional interpretation of tunneling.

Based on the formalism of quantum mechanics in phase space, we can explain reasonably the resonance characteristics of tunnel current related to the barrier. This temperature-independent, purely quantum effect should be considered to be associated with the quantum resonance.

Let us start from quantum mechanics in phase space. The de Broglie relation helps us presume the form of the wave function. The explanation runs as follows. It is obvious that the de Broglie's relation defines the frequency and the wave vector of the de Broglie wave. According to the general relation for a wave, the phase of the de Broglie wave is represented as the function with variable upper limits, $\mathbf{q}, t$,

$$\Phi = \int_0^{\boldsymbol{q}} \boldsymbol{k}(\boldsymbol{q}')\, \mathrm{d}\boldsymbol{q}' - \int_0^t \omega(t')\mathrm{d}t'. \tag{8}$$

The phase of a free particle is expressed simply as

$$\Phi = \boldsymbol{k}\, \boldsymbol{q} - \omega t.$$

For a period, the phase relation becomes

$$\int_0^{\boldsymbol{q}} \boldsymbol{k}(\boldsymbol{q}')\, \mathrm{d}\boldsymbol{q}' - \int_0^t \omega(t')\mathrm{d}t' = 2\pi. \tag{9}$$

The above relation specifies the minimal condition necessary for a wave. In order for a wave to exist, it should satisfy at least this physical condition. According to the de Broglie relation:

$$\boldsymbol{p} = \hbar \boldsymbol{k}, \qquad E = \hbar\omega,$$

we get from Eq. (9)

$$\int_0^{\boldsymbol{q}} \boldsymbol{p}(\boldsymbol{q}')\, \mathrm{d}\boldsymbol{q}' - \int_0^t E(t')\mathrm{d}t' = 2\pi\hbar = h, \tag{10}$$

where we have supposed that even though the momentum of a particle is time-dependent, the de Broglie relation holds as in the case of a free particle. Hence, we can adopt the general condition of periodicity as

$$\int_0^{\boldsymbol{q}} \boldsymbol{p}(\boldsymbol{q}')\, \mathrm{d}\boldsymbol{q}' - \int_0^t E(t')\mathrm{d}t' = nh,$$

where n is an integer. The condition of periodicity for a free particle is represented as

$$\boldsymbol{p}\boldsymbol{q} - Et = nh.$$

This is nothing but the Bohr-Sommerfeld quantization condition. Consequently, we in general can write the phase of the de Broglie wave as

$$\Phi = 2\pi \frac{\int_0^{\boldsymbol{q}} \boldsymbol{p}(\boldsymbol{q}')\mathrm{d}\boldsymbol{q}' - \int_0^t E(t')\mathrm{d}t'}{h}. \tag{11}$$

Here $\int_0^{\boldsymbol{q}} \boldsymbol{p}(\boldsymbol{q}')\mathrm{d}\boldsymbol{q}' - \int_0^t E(t')\mathrm{d}t'$ obviously is the action function with variable upper limit, which is represented in terms of the Hamilton function as

$$S\,(\boldsymbol{q}, \boldsymbol{p}, t) = \int_0^{\boldsymbol{q}} \boldsymbol{p}\,(\boldsymbol{q}')\, \mathrm{d}\boldsymbol{q}' - \int_0^t H(t')\mathrm{d}t'. \tag{12}$$

The fact that momentum is described as an independent variable is related to the introduction of ensemble of paths. A variety of virtual paths that represents the probabilistic behavior of microscopic particles produces a spectrum of momenta. Therefore, the action function must be defined in phase space. From Eq. (12), the action function for a free particle is

$$S\,(\boldsymbol{q}, \boldsymbol{p}, t) = \boldsymbol{p}\, \boldsymbol{q} - Ht.$$

Consequently, the phase of the probability wave is written in terms of the action function as

$$\Phi = \frac{S\,(\boldsymbol{q}, \boldsymbol{p}, t)}{\hbar}. \tag{13}$$

In the end, the de Broglie wave can be represented as

$$\psi\,(\boldsymbol{q}, \boldsymbol{p}, t) = \varphi(\boldsymbol{q}, \boldsymbol{p}, t) \exp\left(\mathrm{i}\frac{S\,(\boldsymbol{q}, \boldsymbol{p}, t)}{\hbar}\right), \tag{14}$$

where $\varphi\,(\boldsymbol{q}, \boldsymbol{p}, t)$ as the probability amplitude is a real-valued function.

We therefore can conclude that the de Broglie relation determines the quantum phase and subsequently, the form of the wave function. Meanwhile, Eq. (10) shows essential contents of the uncertainty relation reflecting ensemble in phase space and the broad context of classical mechanics and quantum mechanics. Expressions (10) and (11) shows that the quantum of the action is h. At the same time, it shows the necessity and validity of the simultaneous determination of position and momentum, and of time and energy. This is because if it were not to be possible, we could not imagine the phase of a wave.

From Eq. (10), the quantum in phase space should be represented as

$$h = \int_0^{\boldsymbol{q}} \boldsymbol{p}(\boldsymbol{q}')\, \mathrm{d}\boldsymbol{q}' \tag{15}$$

and the quantum in energy-time space should be represented as

$$h = \int_0^t H(t')\,\mathrm{d}t'. \tag{16}$$

These relations really explain the nature of space-time quantization. It is natural to interpret these relations as characterizing an ensemble that consists of pairs of position and momentum, and of time and energy. The relations (15) and (16) show that the greater the momentum is, the more the space is localized, and the greater the energy is, the shorter the time of quantum process.

According to quantum mechanics in phase space [31], the solution of the wave equation for stationary states, in general, takes the following form:

$$\psi(x,p) = \psi_0(x,p)\exp\left[\frac{\mathrm{i}}{\hbar}\int_0^x p(x')\mathrm{d}x'\right], \tag{17}$$

where the probability amplitude $\psi_0(x,p)$ is a real function. It is to be noted that the phase part of this expression is identified with that of the wave function in quasi-classical approximation. From Eq. (17), it is obvious that the barrier region should satisfy the quantum constraint:

$$\int_0^l p(x)\mathrm{d}x = \int_0^l \sqrt{2m\,[E-U(x)]}\,\mathrm{d}x = nh, \tag{18}$$

where E is the total energy of an electron determined by bias voltage and $U(x)$, the barrier potential. This constraint stands for the periodicity condition of the wave function within the barrier, which is distinguished from the Wentzel-Kramer-Brillouin approximation (WKB approximation) according to which the transmission factor of a barrier is proportional to $\exp\left(-\frac{2}{\hbar}\int\sqrt{U(x)-E}\mathrm{d}x\right)$.

With the help of this constraint, the quantum characteristic of tunneling can be explained. From Eq. (18), it follows that the kinetic energy of an electron passing through the barrier should be higher than the barrier height. For the purpose of considering qualitatively, we define the mean value of momentum of an electron in a barrier region as

$$\bar{p} = \frac{\int_0^l p(x)\mathrm{d}x}{l} = \frac{\int_0^l \sqrt{2m[E-U(x)]}\mathrm{d}x}{l}.$$

According to Eq. (18), the following condition should be satisfied.

$$\bar{p}l = nh.$$

From this, the mean momentum of an electron surmounting the barrier should be quantized as

$$\bar{p}_n = \frac{h}{l}n. \tag{19}$$

This condition relative to momentum characterizes the current resonance in the barrier. The minimum momentum of an electron able to pass through the barrier is represented as

$$\bar{p} = \frac{h}{l}. \tag{20}$$

Hence, it follows that the thinner a barrier is, the higher the velocity of an electron passing through it should be. A barrier can be regarded as a quantum filter which connects the two free-motion regions and defines quantized momenta necessary to pass through the barrier. With sufficient reasons, it can be considered that the electron density in the barrier region is almost in a saturation state and thus, is constant. This is because the interaction between correlated electrons and the bias voltage play the role of a source complementing the electrons satisfying the quantum constraint in the barrier. From the relation of current density:

$$j = \rho v, \tag{21}$$

the quantity contributing to the current density is assessed principally as the velocity of electron, v. According to the quantum constraint, Eq. (19), the thinner the barrier width is, the larger the momentum which the barrier filters, and as a consequence the current is increased.

3 Statistical description of tunnel current

Quantum statistical approach is indispensable for explaining tunneling through the barrier because we have defined the electron-excitation temperature as a statistical temperature distinguished from the real temperature corresponding to phonon.

First, the Fermi energy should be taken into consideration, since only electrons in the vicinity of the Fermi level can be excited. The electron density $\mathrm{d}n$ in a small energy interval $\mathrm{d}E$ is

$$\mathrm{d}n = f(E)g(E)\mathrm{d}E, \tag{22}$$

where $g(E)$ is the density of states. The electrons near the Fermi level can take part in the elementary energy excitation and thus the work function decreases as much as the Fermi energy. Therefore, we necessarily should allow for the Fermi energy when investigating the tunneling through the barrier. By the Fermi-Dirac distribution, the electron density can be expressed as

$$n = \int_0^\infty f(E)g(E)\mathrm{d}E = \frac{4\pi(2m^*)^{3/2}}{h^3}\int_0^\infty \frac{E^{1/2}}{\exp\left(\frac{E-E_f}{kT}\right)+1}\mathrm{d}E, \tag{23}$$

where $f(E)$ is the distribution function and m^* is the effective mass of an electron that undergoes the exchange-correlation interaction. The further calculation gives

$$n = \frac{4\pi(2m^*)^{3/2}}{h^3}\int_0^\infty \frac{E^{1/2}}{\exp\left(\frac{E-E_f}{kT}\right)+1}\mathrm{d}E = \frac{2\pi(2m^*kT)^{3/2}}{h^3}F_{1/2}\left(\frac{E_f}{kT}\right), \tag{24}$$

where the Fermi integral $F_{1/2}(x)$ is represented as

$$F_{1/2}(x) = \frac{2}{\sqrt{\pi}}\int_0^\infty \frac{\varepsilon^{1/2}}{e^{\varepsilon-x}+1}\mathrm{d}\varepsilon. \tag{25}$$

Thus, the density of electrons is the function of the Fermi energy and temperature. Meanwhile, the Fermi level is the function of the density of electrons and temperature. In the absolute zero, every state is occupied by one electron and the highest energy level is the Fermi level. Therefore, the density of electrons at the absolute zero is represented as

$$n_0 = \int_0^{E_f} g(E)\mathrm{d}E = \frac{4\pi(2m^*)^{3/2}}{h^3}\int_0^{E_f} E^{1/2}\mathrm{d}E = \frac{1}{3\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}E_f^{3/2}. \tag{26}$$

Accordingly, the Fermi energy is given as

$$E_f = \frac{\hbar^2}{2m^*}\left(3\pi^2 n_0\right)^{2/3}. \tag{27}$$

This shows that the Fermi energy depends on the density of electrons. For the thermionic emission, the current density per unit area j_{the} of hot electrons through the barrier is determined by

$$j_{the} = \int_\Phi^\infty -e\mathrm{v}_x \mathrm{d}n. \tag{28}$$

The integral starts at the lowest possible energy, the top of the barrier.

As a result of calculation, we obtain the Richardson-Dushmann equation for current density for the thermionic emission:

$$j_{the} = AT^2\exp\left(-\frac{E_w}{kT}\right) = AT^2\exp\left(-\frac{\Phi - E_f}{kT}\right), \tag{29}$$

where E_w is the work function and in case there is a vacuum after the barrier, A is $120\mathrm{Acm}^{-2}\mathrm{K}^{-2}$. From Eq. (29), it follows that E_f decreases the work function, in other words lowers the barrier. Therefore, according to Eq. (27), the thermionic emission depends on the density of electrons.

In the case of dealing with the elementary energy excitation in a Fermi liquid, temperature T related to phonon should be replaced by the electron-excitation temperature T_e. In the absence of bias voltage, the cross flow of electrons in the barrier balances. In the presence of bias voltage, the balance breaks down, so the drift current in the barrier occurs. Fig. 1 shows the changes in the barrier heights on the both sides.

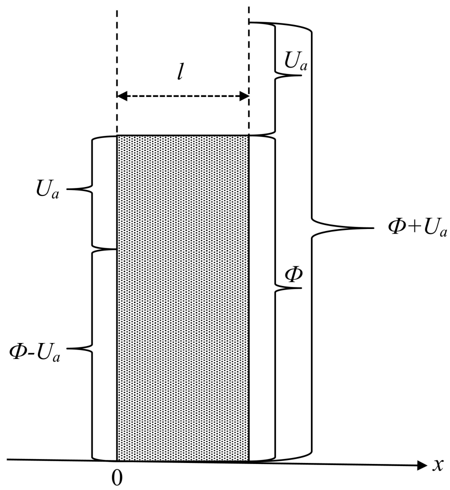

Fig. 1 Variation in height of barrier on both sides due to bias voltage. U_a denotes the variation in barrier height by bias voltage.

By a bias voltage V_a, electrons acquire the energy $U_a = eV_a$. Hereafter, we shall call $U_a = \mathrm{e}V_a$ the bias energy. Suppose that the bias energy is lower than the barrier height Φ. Then for the electrons on the left side of the barrier, the barrier height becomes $\Phi - E_f - U_a$ and for the electrons on the right side of barrier, $\Phi - E_f + U_a$. If a bias voltage is applied, then the currents through the barrier from both sides are shifted to a new equilibrium, so the total current is not zero. In case the forward bias is applied, the current density in the forward direction is

$$j_\Rightarrow = AT_e^2\exp\left(-\frac{\Phi - E_f - U_a}{kT_e}\right), \tag{30}$$

while the current density in the backward direction,

$$j_\Leftarrow = AT_e^2\exp\left(-\frac{\Phi - E_f + U_a}{kT_e}\right). \tag{31}$$

Then the resultant tunnel current density in the forward direction is determined by

$$j_{tun} = j_\Rightarrow - j_\Leftarrow = 2AT_e^2\exp\left(-\frac{\Phi - E_f}{kT_e}\right)\sinh\left(\frac{U_a}{kT_e}\right). \tag{32}$$

In case there is a vacuum after the barrier, the tunnel current density is represented as

$$j_{tun} = j_\Rightarrow = AT_e^2\exp\left(-\frac{\Phi - E_f}{kT_e}\right)\exp\left(\frac{U_a}{kT_e}\right). \tag{33}$$

Meanwhile, it is possible to explain the current-voltage characteristic of the tunnel effect. Originally, according as the momentum of electrons due to the bias voltage approaches $\bar{p}$, the current through the barrier increases in compliance with the quantum constraint Eq. (19) but at the next stage according as the increasing momentum of electrons is deviated from $\bar{p}$ the current through the barrier decreases gradually as a result of the breach of the quantum constraint. This gives a good explanation to the question on why a peak occurs on the current-voltage characteristics for tunneling.

It is possible to show this fact in a semi-quantitative way. With Eq. (32), the current-voltage characteristic of the tunnel current density is visualized in Fig. 2. We set $T_e = 1000\text{K}$, $E_f = 2\text{eV}$, $\Phi = 6\text{eV}$, $C_1 = 5$. The tunnel current

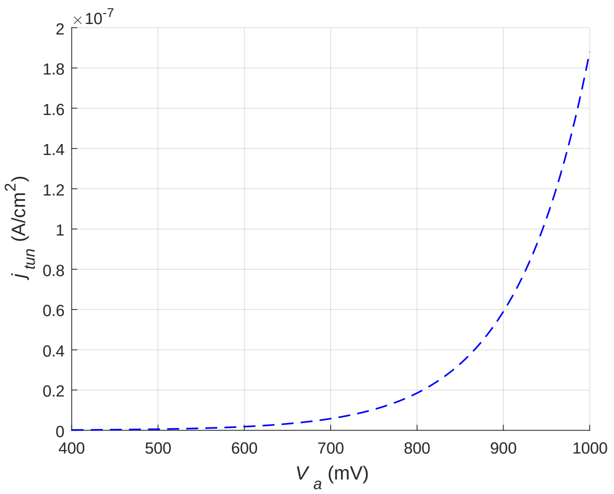

Fig. 2 Tunnel current density in the barrier versus bias voltage

of statistical origin undergoes a kind of filtering due to the quantum constraint in the barrier as determined by Eq. (18). This filtering effect modulates the statistical current density. In line with physical meaning, we can suppose the modulation coefficient by which the statistical current is to be multiplied. The modulation coefficient characterizing the filtering of statistical current can be represented as the following mathematically modeled relation:

$$\eta = 1 + \sum_n C_n \exp\left(-\alpha_n \left(p^2 - \bar{p}_n^2\right)^2\right),$$

where the modulation coefficient η is a non-dimensional quantity and p_n denotes the extreme values of momentum of electron corresponding to the quantum constraint. With $U_a = \frac{p^2}{2m}$, the above relation can be recast as the modulation-voltage relation:

$$\eta = 1 + \sum_n C_n \exp\left(-\beta_n \left(U_a - \frac{1}{2m}\left(\frac{h}{l}n\right)^2\right)^2\right). \tag{34}$$

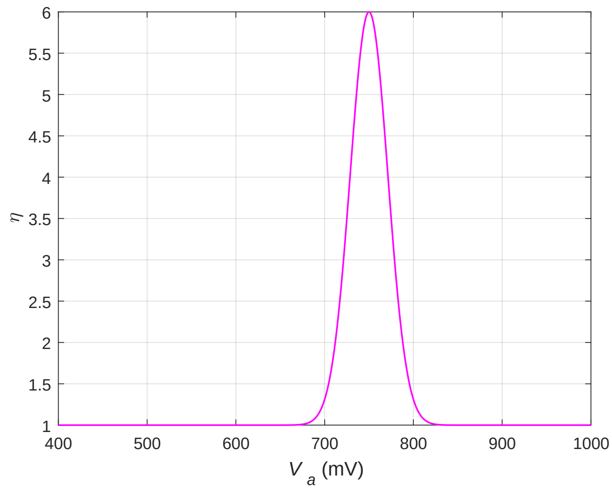

Fig. 3 Modulation-voltage curve due to condition of quantum resonance in the barrier

Eq. (34) is sketched in Fig. 3. Considering that the exchange-correlation interaction is effective within the range comparable to the de Broglie wavelength, we took one peak for the modulation corresponding to current resonances. According to $\frac{1}{2m^*}\left(\frac{h}{l}\right)^2$, the first maximum point was taken as 750mV and $\frac{1}{\beta_1} = 30\text{mV}$ was given in consideration of the dispersion due to interactions of electrons.

The resultant tunnel current density, filtered by the condition of current resonance, is evaluated from

$$j_{res} = \eta j_{tun}. \tag{35}$$

This must be just the tunnel current density in the sense of the conventional theory. The resultant tunnel current density according to bias voltage is visualized in Fig. 4. For com-

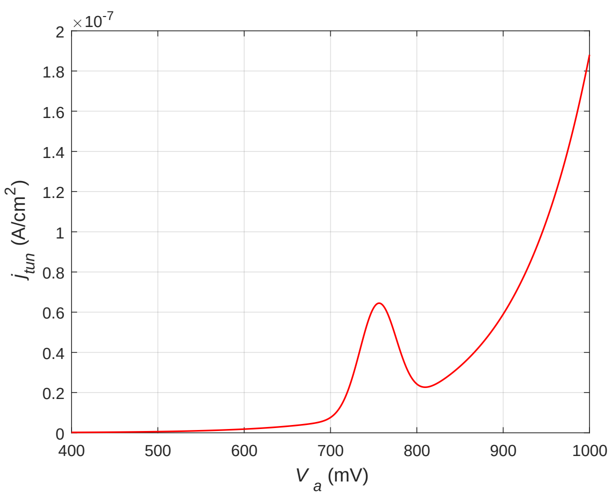

Fig. 4 Resultant tunnel current density versus bias voltage

parison, the overlap of Figs. 2 and 4 is given in Fig. 5. Fig. 5

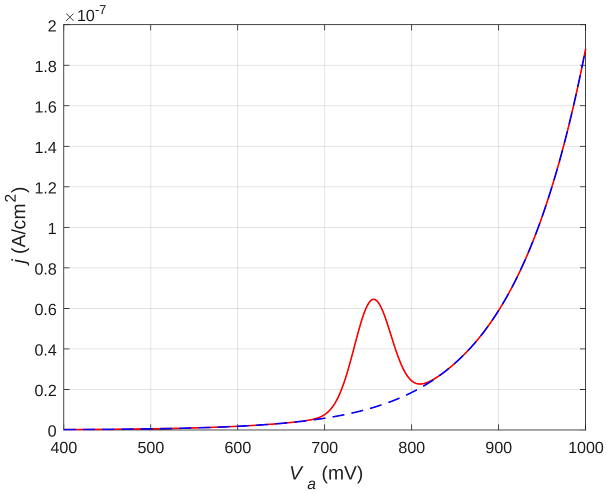

Fig. 5 Overlap of Fig. 2 and Fig. 4

is in good agreement with the current-voltage characteristic of the tunneling diode that exhibits a maximum followed by a minimum and subsequently an exponential increase [3]. This demonstrates that our quantum statistical analysis is reasonable. Thus, it is revealed that we have referred to the current phenomena due to such a quantum cause as tunneling. Unlike the conventional theory, our approach reasonably explains the characteristic of current change in case the energy by applied voltage is higher than the barrier height as well.

This approach gives a reasonable explanation for the resonant tunnel effect as well. From the quantum constraint, Eq. (18), it is obvious that in case barriers and wells alternate periodically, for every region the quantum constraints should be satisfied, i.e.,

$$\int_{l_i^{(k)}}^{l_f^{(k)}} p(x)\mathrm{d}x = \int_{l_i^{(k)}}^{l_f^{(k)}} \sqrt{2m[E-U(x)]}\mathrm{d}x = n_k h, \tag{36}$$

where $l_i^{(k)}$ is the first boundary of the kth barrier or well and $l_f^{(k)}$, its final boundary. Only electrons satisfying these constraints all together can pass through the whole region of barriers and wells. Therefore, barriers and wells should be regarded as playing the role of a kind of resonance which filters electrons in agreement with the quantum constraint. For this reason, it is possible that in a multiple barrier-well region, the quantum-selective effect, i.e., the sharp resonance effect occurs.

Ultimately, all the properties of tunneling have been explained consistently without violating energy conservation. Based on our work, the tunnel effect should be considered to be the phenomenon related to the current through the barrier occurring in case the bias energy is lower than the barrier height. In this case, the current resonance due to the quantum constraint and electronic correlation plays a key role in the increase in current through the barrier. However, also in the future we shall call this phenomenon simply tunneling according to the convention.

4 Discussion

4.1 Why to have to improve understanding of tunneling

Obviously, there is a diversity of views on the mechanism of tunneling. Since these views disagree with one another, it is necessary to achieve consensus on the mechanism of tunneling. The conventional explanation of the tunnel effect is based on a solution of the Schrödinger equation subject to boundary conditions related to a barrier region and two free-motion regions. Fig. 6 shows these regions in the case of the square barrier. The transmission factor for tunneling through the square barrier:

$$D(E) = \left\{1 + \frac{1}{4}\left[\frac{\Phi^2}{E(\Phi - E)}\right]\sinh^2 \alpha l\right\}^{-1} \tag{37}$$

gives a plausible explanation of the main characteristics of the tunnel effect. Here, $\alpha = \frac{\sqrt{2m(\Phi-E)}}{\hbar}$. For high, wide barriers, we simplify the formula as

$$D = D_0 e^{-\frac{2}{\hbar}\sqrt{2m(\Phi-E)}\, l}. \tag{38}$$

Thus, for barriers of an arbitrary form, we have

$$D = D_0 e^{-\frac{2}{\hbar}\int_0^l \sqrt{2m[U(x)-E]}\mathrm{d}x}.$$

However, the conventional interpretation of the tunnel effect causes serious problems related to physical foundations such as the violation of the fundamentals of quantum mechanics and of the law of energy conservation as a universal law of physics. Moreover, it does not reflect the statistical states of physical system, and cannot explain the characteristics of the tunneling current in the whole range of applied voltage. To review this problem, it is necessary to reconsider the solution of the Schrödinger equation in the region of potential barrier.

The Schrödinger equation in a square barrier region shown in Fig. 6 is represented as

$$-\frac{\hbar^2}{2m}\Delta\psi + \Phi\psi = E\psi, \tag{39}$$

where Φ is the barrier height. This equation reflects the correspondence principle for the energy relation:

$$E = \frac{p^2}{2m} + \Phi. \tag{40}$$

In other words, the total energy of an incident electron before a barrier should equal the sum of the kinetic energy of the electron in the barrier and the energy which the electron loses in the barrier. Therefore, the kinetic energy must be positive also in the barrier. If the barrier height is higher than the total energy of a particle, its kinetic energy in the barrier should be negative.

For the negative kinetic energy, two cases are possible: one is the case where the momentum of a particle is purely imaginary and the other is the case where the mass of the particle is negative. But neither of the cases is allowed because they are physically meaningless. Above all, the momentum of a particle is never imaginary because of the Hermiticity of the momentum operator. Thus, the kinetic energy of negative value is inconsistent with quantum theory. This fact is enough to understand that the conventional method for describing tunneling has an incorrect starting point.

Let us consider the problem of negative kinetic energy in more detail. According to the conventional theory, the wave

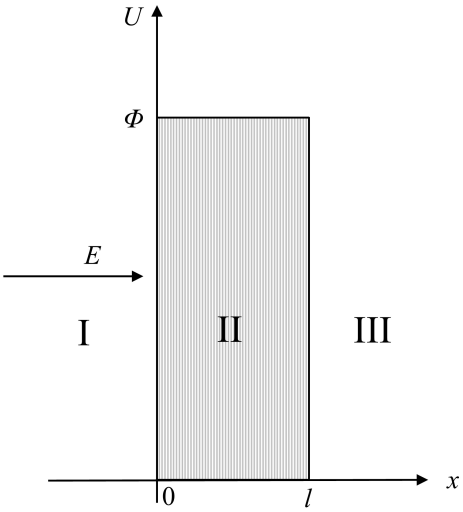

Fig. 6 A barrier (II) and two free-motion regions (I and III): A free electron with total energy E in region I is incident upon the barrier of potential Φ, i.e., region II . The transmitted electron moves freely in region III.

functions for the regions in Fig. 6 are represented as

$$\left.\begin{array}{l}\psi_{\mathrm{I}} = e^{\mathrm{i}kx} + B_1 e^{-\mathrm{i}kx} \\ \psi_{\mathrm{II}} = A_2 e^{\mathrm{i}Kx} + B_2 e^{-\mathrm{i}Kx} \\ \psi_{\mathrm{III}} = A_3 e^{\mathrm{i}kx}\end{array}\right\}, \tag{41}$$

respectively. Here, $k = \frac{\sqrt{2mE}}{\hbar}$ and $K = \frac{\sqrt{2m(E-\Phi)}}{\hbar}$. In fact, Eq. (41) states that particles do not lose their energy in the barrier and the barrier only performs the role of reflecting particles.

At its face value, Eq. (41) says that at point 0, an electron loses energy as much as Φ, and then at point l, it gains energy as much as Φ. Therefore, the energy of the electron remains unchanged after having passed through the barrier. In the end, the barrier has no effect on the dynamical state of the electron besides changing the probability of transmission. ψ_{III} in Eq. (41) illustrates this fact. In fact, k is the same before and after the barrier.

But in the true sense, an electron in region III should possess kinetic energy $E - \Phi$, since it has already passed through the barrier. To describe this process correctly, Fig. 6 should be replaced by Fig. 7. Correspondingly, Eq. (41)

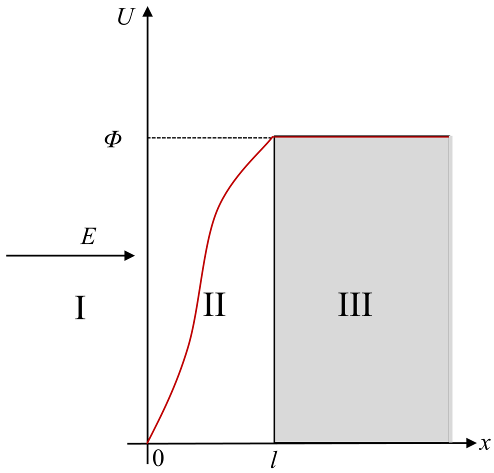

Fig. 7 A barrier (II) and two free-motion regions (I and III): A free electron with total energy E in region I is incident upon the barrier of potential Φ, i.e., region II . The transmitted electron moves freely with kinetic energy $E - \Phi$ in region III. The red line denotes the potential energy.

should be replaced by

$$\left.\begin{array}{l}\psi_{\mathrm{I}} = e^{\mathrm{i}kx} + B_1 e^{-\mathrm{i}kx} \\ \psi_{\mathrm{II}} = A_2 e^{\mathrm{i}Kx} + B_2 e^{-\mathrm{i}Kx} \\ \psi_{\mathrm{III}} = A_3 e^{\mathrm{i}Kx}\end{array}\right\}. \tag{42}$$

On the other hand, it is necessary to reconsider the continuity condition. We conventionally take the continuity condition as

$$\left.\begin{array}{l}\psi_{\mathrm{I}}|_{x=0} = \psi_{\mathrm{II}}|_{x=0} \\ \psi_{\mathrm{II}}|_{x=l} = \psi_{\mathrm{III}}|_{x=l} \\ \psi'_{\mathrm{I}}|_{x=0} = \psi'_{\mathrm{II}}|_{x=0} \\ \psi'_{\mathrm{II}}|_{x=l} = \psi'_{\mathrm{III}}|_{x=l}\end{array}\right\}. \tag{43}$$

However, it is more reasonable to impose the continuity condition on the boundaries in terms of the continuity of the wave function and probability current density. Then we can write the continuity condition as

$$\left.\begin{array}{l}\psi_{\mathrm{I}}|_{x=0} = \psi_{\mathrm{II}}|_{x=0} \\ \psi_{\mathrm{II}}|_{x=l} = \psi_{\mathrm{III}}|_{x=l} \\ j_{\mathrm{I}}|_{x=0} = j_{\mathrm{II}}|_{x=0} \\ j_{\mathrm{II}}|_{x=l} = j_{\mathrm{III}}|_{x=l}\end{array}\right\}. \tag{44}$$

Evidently, Eqs. (43) and (44) are not identical. Really, Eq. (44) is more reasonable than Eq. (43). This shows that the starting point of the conventional interpretation is not correct.

The negative kinetic energy implies that in the Schrödinger equation in the barrier:

$$-\frac{\hbar^2}{2m}\Delta\psi = (E - \Phi)\psi,$$

$E - \Phi$ is negative. According to the law of energy conservation, the total energy of the electron behind the barrier, i.e., in region III should be $E - \Phi$. This indicates that the kinetic energy of the electron behind the barrier should be negative. However, the conventional theory supposes that the kinetic energy of the electron in region III is equal to that before the barrier. Obviously, it is the violation of the law of energy conservation, thus physically inconsistent.

It is instructive to recall the following fact. If negative $E - \Phi$ were permitted, for the wave equation for a free particle:

$$-\frac{\hbar^2}{2m}\Delta\psi = E\psi,$$

a negative E should be admitted. Obviously, such an event is physically impossible because it means that the de Broglie wave damps out or increases spontaneously in a free space. Therefore, the Schrödinger equation has physical meaning only for positive $E - \Phi$. Of course, purely from the mathematical point of view, we may obtain a solution of the Schrödinger equation violating the energy conservation law. However, the case is no more than a purely mathematical instance, so we must necessarily examine whether obtained solutions satisfy physical requirements.

On the other hand, the fact that the Hamilton operator is commuted with itself is sufficient to understand that the energy conservation law should hold exactly at every instant in microscopic systems. In fact, for stationary states, we have

$$\frac{\mathrm{d}\hat{H}}{\mathrm{d}t} = \frac{1}{\mathrm{i}\hbar}[\hat{H}, \hat{H}] = 0.$$

This indicates energy conservation.

It is necessary to examine the physical meaning of the square potential barrier. The square potential barrier carries the meaning of the loss of energy at the beginning of the barrier and then the gain of energy at the end of the barrier. This indicates the lack of the change in dynamical state of transmitted electrons. In the end, the square barrier is meaningless. On the other hand, the square potential barrier purports a constant potential inside the barrier. Since the constant potential does not generate force, it cannot affect the energy of a particle inside the barrier regardless of the barrier width. However, according to the conventional interpretation, the probability of tunneling depends on the barrier width. A possible effect of the barrier width in the quantum sense is the constraint relevant to the de Broglie wavelength of an electron. In fact, the barrier width only plays the role of selecting the de Broglie wavelengths of electrons commensurate to the barrier. Consequently, an electron in the potential barrier should be considered to be in a state of free motion except for the quantum constraint due to the wave nature. But the conventional theory does not consider this point. The potential in the barrier, in general, should depend on coordinate.

According to the conventional theory, the wave function inside the barrier has the form of $\psi_{\mathrm{II}} = A_2 e^{-\chi x} + B_2 e^{\chi x}$ where $K = \frac{\sqrt{2m(E-\Phi)}}{\hbar} = \mathrm{i}\chi$. Hence, it follows that since χ is real and thus ψ_{II} is real, the probability current density inside the barrier vanishes according to

$$j_{\mathrm{II}} = \frac{\mathrm{i}\hbar}{2m}\left(\psi_{\mathrm{II}}\frac{d\psi^*_{\mathrm{II}}}{dx} - \psi^*_{\mathrm{II}}\frac{d\psi_{\mathrm{II}}}{dx}\right).$$

The current in the barrier becomes absent, so the case violates the continuity condition.

The conventional interpretation of tunneling cannot give the reasonable explanation for electron of higher energy than the top of barrier. Actually, with the transmission factor for tunneling through the barrier, i.e., Eqs. (37) and (38), it is not possible to give reasonable interpretation of tunneling. This is because in case E is higher than Φ, the transmission factor becomes complex number. In fact, Eqs. (37) and (38) are derived from the ratio of transmitted current density j_t to incident current density j_i:

$$D = \frac{j_t}{j_i},$$

so they should be real. Eventually, D of a complex value is inconsistent with its definition.

As an important problem, it is necessary to reveal whether the tunnel effect is associated with a single electron or statistical ensemble of electrons. In this connection, it should be noted that we always make an experiment with electron ensemble but not one electron. Even for one electron confined in a certain region, such an electron cannot be regarded as a free particle, since the electron interacts ceaselessly with surrounding system, i.e., many-particle system that confines it, so we should consider that it exists in a set of statistical states undergoing quantum fluctuation as a result of the interaction with the surrounding system. Such a state should be represented with the help of the density matrix, $\Psi = \sum_n W_n |\psi_n\rangle\langle\psi_n|$. Even in the case of cryogenic phenomena such as the tunnel effect in superconductors, it means not exactly the absolute zero-point state, since the electron-excitation temperature is not zero.

It is important to consider that every electron does not possess the total energy as determined only by a bias voltage. Electrons produce a statistical ensemble which is characterized by a statistical distribution by interacting with one another and therefore, it is natural to think that in the sense of probability there exist the electrons that possess higher energy than that given by the applied voltage. In contrast to this, there exist the electrons of lower energy than that by applied voltage.

Obviously, tunneling is attributed to the quantum factor. But thanks to the wave property of electrons there can never occur such a miracle that electrons with lower energy than the barrier height might surmount a barrier. If tunneling is possible, it means that work is performed but there is no energy consumption. In the conventional interpretation, one uses the solution to the Schrödinger equation with negative kinetic energy and purely imaginary momentum in the barrier region. This is inconsistent with the fundamentals of quantum mechanics, for example, the real value property of observables. In any case, the Schrödinger equation must satisfy the energy conservation law and fundamentals of quantum mechanics.

4.2 Consistent and general results

To explain the mechanism of tunneling in a consistent way, we adopted the quantum statistical approach where the electron-excitation temperature T_{exc} is essential. For the purpose of explaining tunneling, the conventional theory on tunneling states that although the energy of an electron is lower than the barrier height, the electron can tunnel the barrier with a definite probability thanks to the wavelike property of electron. As it is, this implies that we should take the physically meaningless solution of the Schrödinger equation admitting of even the negative kinetic energy. For this reason, this interpretation is not consistent with the quantum theory and violates energy conservation.

In this work, we have satisfactorily explained the two important characteristics of tunneling related to the height and barrier width in accordance with the law of energy conservation and quantum theory. We treated the tunneling current based on collective behavior of correlated electrons around the barrier and the current resonance in the barrier. Our view is that tunneling is attributed to not only quantum but also statistical characteristics of electrons due to the elementary energy excitation. In this connection, our method should be considered to belong to such a category of picture similar to that presented in [15–23].

According to our interpretation, the tunnel current depends on four factors: the electron-excitation temperature, the Fermi energy, the barrier height and width, and bias voltage. First, the elementary energy excitation arises due to the exchange-correlation interaction of electrons around the barrier. Interactions form a statistical ensemble of correlated electrons, by which the electron-excitation temperature is determined. Second, the Fermi energy plays the role of lowering the work function of electrons necessary to pass through the barrier. Therefore, the work function decreases with increasing density of electrons. Third, the barrier height and width affect the exchange-correlation energy of electrons according to Eq. (5). This features the tunnel effect related to the barrier height and width.

Once the bias voltage is applied, the tunnel current immediately flows. The reason is as follows. The statistical ensemble of correlated electrons is in an equilibrium state before applying voltage. The electrons passing through the barrier exist with a definite probability because the electron ensemble has a statistical distribution. In an equilibrium state, the cross flows from both sides of a barrier balance. If a voltage is applied, then the barrier heights on the both sides which electrons feel become different. Consequently, the momentum distributions of electrons able to surmount the barrier on both sides change, and as a result the current in the forward direction occurs. As shown in Eq. (32), the current increases with the behavior of the exponential function according as the difference between the barrier height and the energy of electron due to applied voltage decreases. This is the first origin responsible for the current phenomenon in the barrier, which assumes statistic.

Meanwhile, the barrier imposes the quantum constraint on the current through the barrier. The quantum constraint which is determined by the relation between the momentum of a electron passing through the barrier and the barrier width plays the role of making the momentum distribution of the passing electrons be raised up abruptly. In other words, the thinner the barrier width is, the higher the mean velocity of electrons passing through the barrier. Thus, the barrier performs the role of a quantum filter to make the current increase. Based on this interpretation, we can describe the effect of barrier width on the current through the barrier. The exchange-correlation interaction around the barrier leads to an ensemble of electrons obeying quantum statistics, and thus electrons form a Fermi liquid. These two effects are the main origin of the tunnel current phenomenon. The quantum condition given by Eq. (18) and the exchange-correlation interaction cause the effect increasing the current in the barrier. Essentially, the quantum constraint reflects the periodicity of quantum phase given by the barrier. This condition is in common with studies for interpreting tunneling by applying the quasi-classical approach in terms of Wigner or Husimi function [15–17, 24, 25]. The current of statistical origin is filtered in the barrier and amplified by the ensemble of electrons correlated by the exchange-correlation interaction.

Our work gives reasonable explanations to a wide range of previous researches. The problem of tunneling in black holes [4, 5] may be considered to be related to internal processes of interactions that occur inside the black hole. As a result of the internal processes, the black hole can be considered as a Fermi liquid characterized by the elementary energy excitation. Therefore, it is possible to apply quantum statistics to problems of the black hole, based on our paradigm of mechanism of tunneling. Also, the idea for the elementary energy excitation explains sufficiently Brabec's theory of tunnel ionization in complex systems [6] and Vindel-Zandbergen's studies of tunneling-induced electron transfer between separated protons and the relevance of tunneling in nuclear reactions [7]. This is because tunneling, in essence, is due to the elementary energy excitation that occurs in a statistical way and this elementary energy excitation gives the possibility of the instantaneous appearance of electrons acquiring large energy from a system consisting of electrons with small energy. Jürgensen's observation of density-induced tunneling [8] is explained by the fact that the work function is decreased by the Fermi energy. In fact, since the Fermi energy increases with the density of electrons, the dependence of the tunnel effect on the density of electrons is natural. Chuprikov's study that showed that based on a relevance of tunneling time and the superposition principle, the superposition principle is violated in the barrier seems to be unreasonable. This is because the superposition principle is fundamental for quantum mechanics and moreover according to this principle it is possible to imagine the exchange-correlation energy that couples electrons as a single entity. The assumption about virtual particles that was proposed in order to explain the non-tunnel model of the mechanism of tunneling in an intuitive and reasonable way [13] is explained if we consider the virtual particles to be particles which have passed through the barrier by the elementary energy excitation. Meanwhile, our work is in common with Jonson's explanation of a mechanism of tunneling given by using the exchange-correlation potential felt by an electron tunneling from a metal into vacuum [14], as our work is based on the exchange-correlation interaction. Moreover, from the viewpoint on entangled states, the quasi-classical approaches of the entangled-trajectory molecular dynamics [24,26] are in common with our approach, since our theory is based on entangled states of electron.

It is possible to explain the dependence of tunnel current on temperature. If the electron-excitation temperature is very high, the tunnel current related to the difference $\Phi - E_f - U_a$ is insignificant, since Eq. (32) is reduced to

$$j_{tun} = AT_e^2 \exp\left(-\frac{\Phi - E_f - U_a}{kT_e}\right)\left[1 - \exp\left(-\frac{2U_a}{kT_e}\right)\right] \approx AT_e^2\left[1 - \exp\left(-\frac{2U_a}{kT_e}\right)\right] \approx AT_e^2\frac{2U_a}{kT_e} = \frac{2AT_eU_a}{k}. \quad (45)$$

Therefore, the tunnel current increases with the electron-excitation temperature T_e and the energy by bias voltage U_a, and the role of barrier is insignificant. This means that in this case, the characteristic current phenomenon due to the relation between the energy by applied voltage and the barrier height vanishes completely as a consequence of being quenched by the increasing drift current due to barrier-surmountable electrons.

In the case of very low electron-excitation temperature too, the characteristic current related to the difference $\Phi - E_f - U_a$ vanishes, since as $\frac{\Phi - E_f - U_a}{kT_e} \to \infty$, the tunnel current density becomes

$$j_{tun} = AT_e^2 \exp\left(-\frac{\Phi - E_f - U_a}{kT_e}\right)\left[1 - \exp\left(-\frac{2U_a}{kT_e}\right)\right] \to 0. \quad (46)$$

From Eq. (46), it is evident that the tunnel current of statistical origin decreases with lowering the electron-excitation temperature, which corresponds to the case of very low density of electrons.

Eq. (32) shows that according as the applied voltage increases, the influence of barrier gradually vanishes and thus in the case of $U_a \gg \Phi$, the tunnel current is dominated purely by the bias voltage, i.e.,

$$j_{tun} = AT_e^2\left[\exp\left(-\frac{\Phi - E_f - U_a}{kT_e}\right) - \exp\left(-\frac{\Phi - E_f + U_a}{kT_e}\right)\right] \approx AT_e^2 \exp\left(\frac{U_a}{kT_e}\right)\left[1 - \exp\left(-\frac{2U_a}{kT_e}\right)\right] \approx AT_e^2 \exp\left(\frac{U_a}{kT_e}\right). \quad (47)$$

The real temperature relative to phonon affects the tunnel effect that is due to the electron-excitation temperature. In case there is a vacuum behind the barrier, with the real temperature, the thermionic current density is represented as

$$j_{the} = AT^2 \exp\left(-\frac{\Phi - E_f - U_a}{kT}\right). \quad (48)$$

Meanwhile, the tunnel current density is

$$j_{tun} = AT_e^2 \exp\left(-\frac{\Phi - E_f - U_a}{kT_e}\right). \quad (49)$$

The total current density through the barrier is the sum of the thermionic current density and the tunnel current density. Namely

$$j_{tot} = j_{the} + j_{tun}. \quad (50)$$

We define the relation for evaluating the contribution of the tunnel current density to the total current density:

$$\beta = \frac{j_{tun}}{j_{tun} + j_{the}}. \quad (51)$$

By Eqs. (48) and (49), we have

$$\beta = \frac{j_{tun}}{j_{tun} + j_{the}} = \left(1 + \frac{j_{the}}{j_{tun}}\right)^{-1} = \left[1 + \left(\frac{T}{T_e}\right)^2 \exp\left(-\frac{\Phi - E_f - U_a}{kT} + \frac{\Phi - E_f - U_a}{kT_e}\right)\right]^{-1}. \quad (52)$$

If $j_{the} \ll j_{tun}$, i.e., $T \ll T_e$, since $\beta \to 1$, the tunnel effect is predominant over thermionic emission. Conversely, if $j_{the} \gg j_{tun}$, i.e., $T \gg T_e$, since $\beta \to 0$, the tunnel effect is suppressed by thermionic emission. Meanwhile, it should be taken into consideration that due to the electron-phonon interaction, the increase in real temperature results in the decrease in the electron-excitation temperature.

The main purpose of this work has been to reveal that the phenomenon of tunnel current is actually a quantum statistical hybrid phenomenon beyond pure quantum realm. Our work has unraveled some imperfect aspects of the conventional interpretation and has offered a consistent and general formulation for explaining the true mechanism of tunneling.

5 Conclusions

Our aim has been to elucidate the true mechanism of tunneling and to explain general characteristics of tunneling, based on the quantum and statistical theory. For this purpose, we adopted a new approach distinct from the conventional one. Based on an alternative quantum statistical interpretation, we have satisfactorily explained the statistical and quantum aspects of the tunnel current phenomenon. The adopted approach has taken all together into consideration the relevant factors such as the correlation of entangled electrons, the exchange-correlation interaction, the electron-excitation temperature of the Fermi liquid, and the quantum resonant current.

As a main motivation of our work, it has been analyzed that the conventional explanation of the tunnel effect violates the universal law of energy conservation and is incompatible with quantum theory. Contrary to this, our theory satisfies the energy conservation and quantum theory all together. This is because we took into consideration the statistical nature of the tunnel current phenomenon, based on the correlation of entangled electrons by the elementary energy excitation. Our research concludes that tunneling is the phenomenon related to the current through the barrier occurring in case the bias energy is lower than the barrier height. Essentially, there is no tunneling. This is because it is inconsistent with fundamentals of physics including the law of energy conservation. Behind seeming tunneling, there is an important mechanism of interaction. These are the elementary energy excitation and quantum resonance.

The main conclusion of our work is that the tunnel current should be explained based on quantum statistics together with quantum mechanics. Moreover, our work has successfully explained the characteristics of tunnel current in the barrier in the whole range of bias voltage and the dependence of tunnel current on the electron-excitation temperature. The adopted perspective enabled us to elucidate the statistical and quantum aspects of the tunnel current, thus leading to unraveling the nature of tunneling. Importantly, our work makes it possible to solve some inconsistent problems such as the violation of the law of energy conservation arising from the conventional approach to tunneling and the direct contradiction to quantum theory, and offers the possibility of leading to the innovation in the conception of tunneling. We expect our work to substantially contribute to the elucidation of the physical nature of tunneling and also to the researches on tunneling of complex systems including the resonant tunneling effect.

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