"Binary Calendar" and Global Warming

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Abstract. The "binary calendar" is "Years divisible by 4 but not by 128 are leap years." I point out this suggestion should be more accurate than the "Gregorian calendar." We also look at the idea of using precise measurements of sidereal day length (Earth's rotation period≈86164.1 SI seconds) as a tool to help assess "climate change." We find it could be helpful but is not trivial to assess.

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1. Introduction

The Gregorian calendar (so-called since introduced by Pope Gregory XIII in October 1582) is an international standard solar calendar designed to maintain synchrony with the mean tropical year. Its rule:

Every year divisible by four is a leap year, except for years divisible by 100, except in turn for years also divisible by 400.

For example, 1600, 2000, and 2004 were leap years, but 1800, 1900, and 2001 were not. Leap years add February 29 to the calendar. This cycles every 400 Gregorian years, i.e. every 146097 solar days. Each cycle repeats the months, dates, and weekdays. The average length of a Gregorian year is exactly

$$146097/400 = 365+1/4-1/100+1/400 = 365+97/400 = \mathbf{365.2425}$$

solar days.

However, the Gregorians were incorrect in the sense that the mean tropical year as of 1 January 2000, instead was **365.2421897** solar days. The mean solar day length was pretty exactly 86400 SI seconds (to within less than 10 microseconds) during the year 2019 AD, as well exactly valid during at least some times between 1862 and 1967 assuming the validity of the transition between the original international definition of the second as 1/86400 of a mean solar day, to its 1967 (and current) atomic-clock-based SI definition as "9192631770 cycles of the unperturbed ground-state hyperfine transition frequency of the caesium 133 atom."

Both the mean tropical year and the mean length of the solar day, as measured in SI seconds, vary. The year-length would remain constant according to the Kepler-Newton solution of the sun-earth 2-body problem, but changes because the solar saystem involves *more* than two bodies. The day length would remain constant if the earth were a rigid rotating body not subject to any externally applied torques. But in fact, it is not rigid, and tidal friction from the moon and sun both cause torques, while temperature changes, volcanos, earthquakes, and weather cause expansion and contraction and mass-redistribution of the earth's outer layers (e.g. atmosphere, ocean, ice caps), plus winds and ocean currents and internal-earth convection currents all alter the earth's angular momentum, all changing the length of the solar day. These effects are complicated and nonuniform, and the more precisely you try to model them, the more complicated they become.

According to solar system modeling by Jacques Laskar, an expression suitable between 8000 BC and 12000 AD for calculating the length of a tropical year in "SI days" is

$$\text{Year} = 365.2421896698 - 6.15359\times10^{-6}T - 7.29\times10^{-10}T^2 + 2.64\times10^{-10}T^3$$

where T is measured in "Julian centuries" with T=0 on 1 January 2000. (Definitions: an "SI day" is 86400=24×60×60 SI seconds, and a "Julian century" is 36525 SI days. Laskar's expression is valid for $|T|\le100$.) According to this formula in 8000 BC the year length was 365.2425337, in 2000 AD was 365.2421896698, and in 12000 AD will be 365.2418310 SI days, in between averaging to 365.24218724. The furthest deviations from this average are +0.00035843 at T=-87.230 and -0.00036490 at T=+89.071; the mean |deviation| is 0.0002417; and the RMS deviation is 0.0002668. That is, the mean value of Laskar's expression when integrated for $-100\le T\le+100$ is **365.24218724**.

During the 66 years [1960, 2025] AD, the solar day *shortened* by about 1.3 SI milliseconds from 86400.00125 to 86399.99995 SI seconds. That on average is a shrinking of solar day length by about 20 SI microseconds per year. However over the long term, the solar day should be gradually *lengthening* due to tidal friction from the moon slowing the Earth's rotation and pushing the moon's orbit outward. E.g. daily and annual growth-feature evidence from ancient coral fossils (first measured by John W. Wells in 1963) clearly indicate that about 350 Myr ago the solar day was a bit less than 23 SI hours long, and 430 Myr ago was a bit less than 21 SI hours. Analysis by M.L.Lantink et al in 2022 of "Milankovitch-cycle" features in banded iron formations in sedimentary rocks in Western Australia suggest, less clearly, that when those geological formations formed 2.46 Gyr ago, solar days were only 17 hours long. Mitchell & Kirscher 2023 attempted to plot of solar day length versus year over the last 2700 Myr, and concluded that solar day length increased roughly linearly with time *except* that between about 1100 and 2200 Myr ago (roughly coinciding with the biological-evolution era known as the "boring billion"), it stalled, remaining approximately constant at about 19 SI hours, due to a "tidal resonance" in which the opposed torques from the lunar semidiurnal oceanic tide and solar semidiurnal atmospheric tide canceled out. (Deitrick & Goldblatt 2024 have some interesting followup speculations.)

Those indicate the long term trend that the solar day *lengthened* about +26 SI microseconds per year on average during the last 430 Myr. An atomic clock counting SI seconds, even if it got the solar day length exactly correct during year 2019, would after 10000 years (if the solar day length increased 20μsec/year during that time) mispredict midnight by 101.5 hours = 4.2 days.

That makes it clear that it is not really possible, given today's (lack of) predictive understanding of solar-day-length variation, to design a calendar system that will remain valid to within ±2 solar days for 20000 years. If the solar day length in SI seconds were linearly increasing or decreasing with time (which it isn't – it actually looks more like plots of the stock market) at 20 μsec/year, that would throw off the year/(solar day) ratio by ±0.0008455 at the two ends of Laskar's 20000-year interval, with the mean |deviation| due to this effect being 0.0004227 and the RMS deviation 0.0004881. So it is reasonable to design a calendar for use during 8000 BC to 12000 AD which regards the year/(solar day) ratio during Laskar's 20000-year interval, as 365.24218724 to within RMS error ±0.00056. The Gregorian ratio 365.2425 unfortunately lies toward the high end of that error interval.

2. Binary calendar

I want to point out the astonishing fact that

$$365 + 1/4 - 1/128 = 365 + 31/128 = \mathbf{365.2421875}$$

happens to equal (up to error 2.6 in its final decimal place!) Laskar's estimate 365.24218724 of the length of the mean tropical year in SI days between 8000 BC and 12000 AD. And as we said, during 2019 AD, the SI and mean solar days were equal to within a few microseconds. This suggests this "**binary calendar**" as a better-than-Gregorian calendar system:

Years divisible by 4 but not by 128 are leap years.

This should be substantially more accurate than the Gregorian calendar, plus arguably is simpler. The reason I call this "binary" is that $4=2^2$ and $128=2^7$. So our rule is equivalent to

Years whose binary representation ends in 2-6 zeros are leap years.

3. Global warming assessment from day length?

In this section I want to explore the effects of climate change and various astrophysical effects on the Earth's rotation period. Earth's rotation period can be measured very precisely by atomic clocks and astronomical observations of, e.g, distant quasars. The "event horizon telescope" claimed an angular resolution of 19 micro-arcseconds, from which we infer that it ought to be feasible to measure the earth's rotation period (each day) accurate to ±1.26 microseconds. Converted to a distance along Earth's equator, this accuracy is equivalent to ±0.47 millimeters.

If we could deduce the **mean temperature** of earth's atmosphere from earth's rotation period (possibly combined with tidal energy-loss models) then we would have a very easy way to measure that temperature and thus track climate change – without needing even a single thermometer!

Earth's moment of inertia is $I_{earth}=8.0358\times10^{37}$ meter^{2}kg. The moment of inertia of the atmosphere alone is about $I_{atm}=(2/3)M_{atm}R_{atm}^2\approx1.3955\times10^{26}$ meter^{2}kg. where $M_{atm}\approx5.1441\times10^{18}$ kg is the mass of the atmosphere, while $R_{atm}=6379$ km is the solid+liquid Earth's radius plus an extra 8000 meters to get us up to about the mean height above sea level of a molecule in the atmosphere.

So if the atmosphere's **mean temperature** increases by 1°C, expanding it by a factor of 1/288, that should increase I_{atm} by a factor of about 1.00000871, i.e. 8.71 parts per million, thus increasing I_{earth} by a factor of $1+1.5126\times10^{-17}$, thus increasing the day length by 1.31 picoseconds.

Continental drift speeds range from 1-10 cm per year. If the entire liquid part of the Earth's interior convected unidirectionally parallel to Earth's equator speed equivalent (if measured at the surface) to 10 cm/year, that would alter day length by about 150 picoseconds. (Obviously such unidirectionality does not occur, but this should be useful as a rough upper estimate on the magnitude of this effect.)

Due to **ice cap melting**, Antarctica is losing ice at an average rate of about 135 billion tons per year (about 1 cm in average ice cap height per year), while Greenland is losing about 266 billion tons per year. That mass is not "lost," but rather "mixed into Earth's oceans," thus moving it radially outward from the earth's rotation axis from about 2000 to about 5200 km (RMS). This increases I_{earth} about 9×10^{27} meter^{2}kg annually, which is a nearly-2-orders-of-magnitude greater |effect| on I_{earth} each year than completely eliminating the entire atmosphere would have had! Fractionally this is about 1.15×10^{-10} annual increase in I_{earth}, which corresponds to an annual increase in day length by about 9.9 microseconds, which should be quite feasible to measure.

However, the day length instead *shrank* between 1960 and 2025 AD by about 20 microseconds per year. So even assessing ice cap melting is not so easy via day-length measurements, because other effects (such as winds) evidently are larger.

If the entire atmosphere had unidirectional **prevailing winds** parallel to the equator at, say, 10 km/hour mean windspeed, then that would decrease the rotational angular momentum 5.8598×10^{33} Joule-sec of the earth by a factor$\approx1+1.27\times10^{-8}$ versus if there were no winds, thereby decreasing the day length by 1 millisecond, which would be a 2-orders-of-magnitude greater effect than ice cap melting!

If all seawater were flowing at 1 km/hour on average unidirectionally parallel to the equator (due to **prevailing ocean currents**) then that would decrease the rotational angular momentum of the earth enough to decrease the day length by 29 milliseconds versus an earth with no ocean currents. That would be a nearly-2-orders-of-magnitude greater effect than winds!

We conclude that the day-length method of measuring earth's atmosphere's temperature will not work, i.e. the noise will greatly exceed the signal; but it nevertheless could be a valid way to assess climate change by measuring combined effects from ice cap melting, prevailing wind and ocean current changes, etc provided we were capable of modeling all the main effects, which, unlike all our analysis in this paper, certainly is not trivial.

4. Numerology Update

I received a 10 Dec. 2025 email from Don Page (physics professor, Univ. of Alberta) pointing out that on 6 Mar. 2024 Page had also noticed the same remarkable "numerical coincidence" I did – Page's version was that the Particle Data Group's 2020 Astrophysical Constants and Parameters data compilation (revised by D.E.Groom & Douglas Scott, but their number just copied from *Astronomical Almanac*), says that the "tropical year (equinox to equinox, 2020 AD)" is "31 556 925.1 seconds = **365.242189** [solar] days," versus Laskar's claimed 20000-year average **365.24218724** and my binary calendar's 365+1/4-1/128=**365.2421875**. These numbers if rewritten in binary are:

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Binary Calendar                  101101101.001111100 00000000 0000
Groom/Scott/PDG2000              101101101.001111100 00000000 0011
Laskar Average                   101101101.001111011 11111111 1111
Gregorian Calendar               101101101.001111100 00101000 1111
   5^(-1/2)128^(-2)             000000000.000000000 000001110 0101
Laskar RMS dev. from constancy   000000000.000000000 00100011 0000
```

where the binary calendar's number continues with 0000... forever, whereas the astronomers' measured numbers do not. The top three numbers agree in their most significant 28 bits but disagree in bits 29-31. Page then points out that if their most significant 27 bits are partitioned into three nonads, each nonad is **palindromic**!

Is this really merely a lucky numerical "coincidence," or is there some evil plot (e.g. celestial "resonance") behind it? Certainly rotational and orbital periods can get synchronized to rational ratios, the earth's moon (1:1 ratio) being the best known example. Such 1:1 "tidal locking" is common, exhibited by Mars' moons Phobos and Deimos, Jupiter's Galilean moons Ganymede, Europa, Io, and Callisto, and all 20 known solar system moons large enough to be round. Pluto and Charon (and Eris & Dysnomia) even enjoy "mutual tidal locking" with 1:1:1 ratio of their two rotation rates with their orbit. Mercury orbits the sun every 2 of its solar days (3 spins per 2 orbits). It is suspected that 2:1 and 5:2 spin-orbit resonances also should occur somewhere in Nature, but no celestial examples are known.

The **relationship** between a planet's "solar day" duration S, its (sidereal) "rotation period" R, and its orbital period, aka "year," Y is 1/S+1/Y=1/R, where note R must have negative sign if the planet rotates retrograde, as does Venus.

Here is a table of the inner 4 planets giving their Y,R,S in SI hours, and then stating Y/S, a rational approximation P/Q to its fractional part, and the additive error E in that approximation. My rotation periods are from wikipedia rotation period except that Campbell et al 2019 say based on 29 years of radar data that Venus's rotation period is 5832.5088±0.0144 SI hours, while wikipedia follows the Margot et al's 2021 claim that "The average sidereal day on Venus in the 2006-2020 interval is 243.0226±0.0013 Earth days," i.e. 5832.5424±0.0312 hours. Mercury's orbital period is from wikipedia Mercury, but Venus' and Mars' from NASA with earth-year normalization. Note that Earth's "orbital period" has been claimed slightly longer than its "tropical year," I presume mainly due to slow precession of Earth's rotation axis and/or its perihelion. (If planet rotation axes and their orbital ellipses stay at fixed orientation the two concepts should coincide, and I think measurement precision never got good enough to distinguish the two for any planet besides Earth.) For Earth I use the tropical year; for the other planets I use the orbital period.

Planet	Y	R	S	Y/S	P/Q	E
Mercury	2111.262	1407.509	-4222.533	2.000004	2/1	4×10^{-6}
Venus	5392.6100	-5832.5088 or 5832.5424	2801.9699 or 2801.9777	1.9245781 or 1.9245728	380/411	$(3.9 \text{ or } 1.4)\times10^{-6}$
Earth	8765.8125520752	23.934469592	24	365.24218724	31/128	2.6×10^{-7}
Mars	16486.871	24.6229622	24.659791	668.57301	51/89	0.00002

All three of the ratios 1, 2, and 365+31/128, have "nice" binary expansions, e.g. are rationals with power-of-2 denominators. A well known theorem in Diophantine approximation (A.Hurwitz 1891, E.Borel 1903) states that any irrational real number X has infinitely many distinct rational approximations P/Q obeying $|X-P/Q|<5^{-1/2}Q^{-2}$; and the constants "2" and "√5" both are maximum possible. For Mercury and all those Moons the accuracy is tremendously better than Hurwitz-Borel. In the Earth's case, that best rational approximation has Q=128, so it is not surprising to achieve additive error<$5^{-1/2}128^{-2}$≈0.0000273 solar days, which is nearly 10× smaller than Lasker's RMS deviation ±0.0002668 from constancy over his 20000-year span.

How impressive are the facts that the Groom/Scott and Laskar-average errors versus the binary calendar's exact rational 365+31/128 respectively are 18 and 105 times smaller than $5^{-1/2}128^{-2}$? My computer tells me that a random-uniform real number in (0,1) is approximable by at least one rational P/Q with 1≤Q≤256, such that |X-P/Q|<1/max(500000, $18Q^2\sqrt{5}$), with probability=**0.04**02487±0.0000054. That isn't very impressive, but if, say, Mars and Venus also had p≈0.04, then I'd probably be impressed. (That conceivably could happen in future if clearer and better data becomes available for Mars and Venus.) If we further-demand that Q must be a power of 2, then this probability drops to $(5.474\pm0.0070)\times10^{-4}$.

If this is not mere luck, i.e. the earth's rotation is trapped in resonance with its orbit, then the "binary calendar" should work *better* than naive expectations – but 0.04 is not lucky enough to convince me of that. If we also examine the other 3 inner planets, I'm convinced Mercury's is a genuine 2:1 resonance; but Venus and Mars' days seem less-convincingly resonant than Earth, and Earth's solar day clearly seems to be changing. Nevertheless it looks as though Earth's rotation period may genuinely have been resonantly trapped during a 1100 Myr Mid-Proterozoic timespan (Mitchell & Kirsher 2023).

Ravsky 2003 proves that every n-bit binary word can be partitioned into K(n) or fewer palindromes, where K(n)=⌊n/6⌋+⌊(n+4)/6⌋+1 for every n≥1, except that K(11)=5. Page's palindrome observation considerably outperforms Ravsky's K(27)=10 and K(30)=11. While this is enough to make me suspicious, again it is not enough evidence to convince me there is any celestial conspiracy favoring palindromes. My computer tells me that a random 30-bit word is partitionable into 3 or fewer palindromes with probability=21600066/1073741824≈**0.02**0116629, while for random 27-bit words this probability is 5967048/134217728=**0.04**4457972.

Here is a table compiling all(?) known orbital-period resonances ordered from most to least commonly observed:

P	Q	#examples	Comment
1	1	>7000	Trojan asteroids & Jupiter.
3	2	>6000	Pluto & Orcus orbit twice while Neptune orbits 3 times. Hilda asteroids orbit twice while Jupiter orbits 3 times.
2	1	>100	Neptune & "twotinos"; Enceladus & Dione; Europa & Ganymede
5	2	57	Neptune & various objects such as (26375)_1999_DE9
7	4	55	Neptune & various objects such as "385446 Manwë"
4	3	50	Neptune & various objects; Titan & Hyperion
5	3	47	Neptune & various objects such as "(530839) 2011 UK_{411}."
5	4	17	Neptune & various objects such as "(432949) 2012 HH_2"
7	2	17	Neptune & various objects such as 471143 Dziewanna
7	3	17	Neptune & various objects such as "(495297) 2013 TJ_{159}"
3	1	14	Neptune & various objects such as "(136120) 2003 LG7"

4	1	9	Neptune & objects; Io & Ganymede
9	5	6	Neptune & object
12	5	6	Neptune & object
9	4	5	Neptune & object
8	3	5	Neptune & object
11	3	5	Neptune & object
5	1	4	Neptune & object
6	1	4	Neptune & object
8	5	4	Neptune & object
11	6	4	Neptune & object
9	2	3	Neptune & object
11	2	3	Neptune & object
13	4	3	Neptune & object
9	1	2	Neptune & object
10	3	2	Neptune & Gonggong
9	7	1	Kepler-29 exoplanets
7	5	1	Neptune & object
8	1	1	Neptune & object
10	1	1	Neptune & object
10	7	1	Neptune & object
11	5	1	Neptune & object
11	7	1	Neptune & object
12	7	1	Neptune & object; also perhaps Neptune & Haumea
21	5	1	Neptune & object

Neptune's innermost moon, Naiad, is in 73:69 resonance with the next outward moon, Thalassa. Sometimes to recognize resonances in planetary systems involving *more* than than 2 planets, it helps to switch to an appropriately-rotating sun-centered coordinate frame. (Or for moons, planet-centered.) Examples:

- Moons of Jupiter: 4:2:1 for Io, Europa, and Ganymede. Triple conjunctions do not occur.
- Moons of Pluto: There are 5 conjunctions of Styx and Hydra and 3 conjunctions of Nix and Hydra for every 2 conjunctions of Styx and Nix; triple conjunctions do not occur.
- Kepler-223 has four planets in resonance with 8:6:4:3 orbit ratio.
- Kepler-80: in a frame of reference that rotates with the conjunctions, planet orbit ratio of 9:6:4:3:2.
- TOI-178: in a rotating frame of reference 18:9:6:4:3 orbit ratios.
- HD 110067 has six known planets, in a 54:36:24:16:12:9 resonance ratio.

Which numbers occur in resonances? For whatever it is worth, I point out that *every* integer ratio P:Q in the above list, including putative Venus, Earth, and Mars Year:SolarDay ratios, have either P or Q (or both) in the following ultimately-sparse (zero density) subsequence of the integers:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 21, 27, 31, 32, 33, 34, 37, 41, 51, 62, 64, 65, 69, 73, 74, 81, 82, 85, 119, 127, 128, 129, 133, ... 411, ...
Numbers which are either prime powers or semiprimes (i.e. products of exactly 2 primes), and have binary representation in which all runs of 1s have equal length.

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