Calculation of the hyperfine splitting in the ground-state of atomic hydrogen

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Abstract

In terms of the 1S–2S transition, defined as the 15-digit frequency 2 466 061 413.187 01 MHz, the hyperfine splitting in the ground-state of the neutral atom hydrogen (H) is predicted to be 1420.405 751 768 78 MHz, which should undergo experimental testing with sub-mHz accuracy for consistency. The conclusion is founded on the premise that the proton's unbound magnetic moment in Bohr magneton and the binding correction are simple algebraic expressions that include integers or fractions of integers, linking them through a basic inner building block to the structure of the proton. The use of an alternative fine structure constant in the modeling necessitates a mass correction factor to the mass of the electron recommended by CODATA, which is established through both experimental data and a theoretical derivation.

Keywords: Relativistic quantum mechanics, hyperfine splitting, proton magnetic moment, atomic hydrogen, proton structure.

Introduction

In [1], the author outlined a technique for modeling transition energies among nS states in atomic hydrogen. The study, using the method of least squares, utilized optically measured data from two extraordinarily well known 1S–2S transitions and three not so accurate 1S–3S transitions for the analysis. Below is a summary of the result.

The electron binding energy in hydrogen associated with a metastable nS state is [2: eq.1]

$$E(nS1/2) = E(n,0,1/2) = E_D(n,1/2) \cdot \gamma_{zero} \cdot \{1 + B/n\} \tag{1}$$

E_D is the relativistic Dirac binding energy and B illustrates the dimensionless Lamb shift of the 1S state in hydrogen adjusted by [1: eq.1, 2, 3]

$$B \equiv \frac{\mu}{\mu_B} \cdot \left(-\frac{1}{2}\right) \cdot \gamma_{diamag\_shielding} \cdot (2\alpha)^2$$

with

$$\frac{\mu}{\mu_B} = \frac{\mathbf{5}}{\left(\frac{\lambda_{e\_bar}}{L_2(L)} + \mathbf{3}\right)} \approx 0.023\,025 \quad ; \quad \frac{\lambda_{e\_bar}}{L_2(L)} = 2^{16}\pi^{-5}$$

and

$$\gamma_{diamag\_shielding} \equiv \frac{1}{3}\left(1 + 2\sqrt{1 - (\mathbf{3}\alpha)^2}\right) \approx 1 - \frac{(3\alpha)^2}{3}$$

The speculative magnetic monopole $\frac{\mu}{\mu_B}$ and the screening factor $\gamma_{diamag\_shielding}$ are dimensionless fundamental constants associated with the free proton, while α represents the fine structure constant given within the geometrized framework by the axiomatic number constant [6: p.42]

$\alpha \equiv 2^{-6}\pi^{-\frac{2}{3}} \approx 0.00\,72\,843 \; (\alpha^{-1} \approx 137.28)$

The magnitude of the scaling factor γ_{zero} changes depending on the atom due to the recoil effect and the unit of energy used for the measurement of the transitions. It can be linked with the precisely known 1S–2S transition $\Delta E_{1S-2S}^{meas} \equiv 2\,466\,061\,413.187\,01$ MHz, applying

$$\gamma_{zero} = \frac{\Delta E_{1S-2S}^{meas}}{X + B \cdot Y} \quad with \quad X \equiv E_D(2,1/2) - E_D(1,1/2) \quad and \quad Y \equiv \frac{E_D(2,1/2)}{2} - E_D(1,1/2) \qquad (2)$$

The experimental mass correction to the electron mass derived from the measured 1S1/2 hyperfine interval

In analogy to [2: eq.6], the hyperfine splitting of the 1S1/2 level $(\Delta E)_{1S1/2}^{hfs}$ in atomic hydrogen (H) can be calculated using

$$(\Delta E)_{1S1/2}^{hfs} = -E(1,0,1/2) \cdot \frac{16}{3} \cdot \tilde{B} \qquad (3)$$

where the energy $E(1,0,1/2)$ is defined by formula (1) and the parameter $\tilde{B}$ by [2: eq.7]

$$\tilde{B} \equiv \alpha^2 \cdot \frac{\mu_p}{\mu_e} \cdot \frac{\mu_e}{\mu_B} \cdot \gamma_{corr} \qquad (4)$$

The precise measurements of

$$(\Delta E)_{1S1/2}^{hfs\_meas} = 1420.405\,751\,768(2)\; MHz \qquad [3]$$

$$\frac{\mu_e}{\mu_p} = 658.210\,706(6) \qquad [4]$$

$$\frac{\mu_e}{\mu_B} = 1 + a_e = 1.001\,159\,652\,1884(43) \qquad [5]$$

enable to estimate ${\gamma_{corr}}^{hfs}$ using formula (1), (2), (3) and (4). In terms of the 1S–2S reference transition, defined as the 15-digit frequency 2 466 061 413.187 01 MHz , the calculation straightforwardly yields ${\gamma_{corr}}^{hfs} \approx 1.003\,594\,936$.

Winkler and colleagues determined the proton electron magnetic ratio $\frac{\mu_e}{\mu_p}$ by analyzing the spectrum of an atomic hydrogen maser in a magnetic field and observing both the proton and the electron spin flip frequencies. The ratio is the most fragile aspect of formula (4), as only one literature precision value [4] is available, and it heavily relies on the theoretical framework used to interpret the energy levels of atomic hydrogen in an external magnetic field. The modeling process is difficult, and the technique is complex, requiring careful management of the experimental setting.

The parameter a_e represents the unbound electron's anomalous magnetic moment, which the Dyck-Schwinberg-Dehmelt group at the University of Washington [5] measured with great precision.[1]

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[1] The calculation of a_e within the geometrized framework is available in [6: p.130]. Three simple algebraic interaction terms are adequate to match the precise experimental value of a_e.

The theoretical mass correction to the electron mass obtained within the geometrized framework

The values of the natural constants h and c are precisely established by committee decision with absolute certainty due to the redefinition of the kilogram, in contrast to the electron mass m_e, which is derived from theoretical interpretations utilizing experimental data and whose true origin is still unclear. CODATA presently connects it to the Rydberg constant and the fine-structure constant [7: eq. 22]. In Penning trap experiments, the mass of the electron is determined by calculating the g-factor of the electron by bound-state QED. In every instance, it is impossible to determine the mass of the electron without a theoretical foundation. At present, the theoretical framework is free or bound-state QED and CODATA regularly supplies an adapted value with ever-higher precision for m_e that is consistent with the dominating QED, despite the theory's complexity (renormalization techniques to eliminate infinities), lack of physical clarity, and insensible mathematics. The upcoming content makes use of the CODATA values for h, c, and m_e from [8] without restating it again.

In [6], the author examined simple axioms to shed light on the origin of fundamental constants and suggested that the mass of the electron within the geometrized framework is inter-related to h and c according to [6: p.56]

$$m_{e\_geom} \equiv 2^{\frac{15}{4}} \cdot \pi^{\frac{26}{3}} \cdot h^{\frac{3}{4}} \cdot c^{-\frac{5}{4}}$$

Comparing m_e with m_{e_geom}, along with the value ${\gamma_{corr}}^{hfs}$ experimentally obtained earlier, prompts the hypothesis

$${\gamma_{corr}}^{theo} \equiv \left(\frac{m_e}{m_{e\_geom}}\right)^{\frac{3}{4}} \approx 1.003\,595\,267 \qquad (5)$$

on the theoretical mass correction factor to the electron mass, which raises the question whether the alignment of ${\gamma_{corr}}^{theo}$ with ${\gamma_{corr}}^{hfs}$ is coincidental, or if formula (5) is applicable for any unit set $\{m_e, h, c\}$. The relative difference between ${\gamma_{corr}}^{theo}$ and ${\gamma_{corr}}^{hfs}$ amounts to 3.3 parts per 10^7.

Theoretical hfs prediction: The magnetic moment of the unbound proton in Bohr magneton and the binding correction

The Standard Model has failed to provide a reliable explanation for the magnetic moment of the proton. By relying on the theoretically predicted factor ${\gamma_{corr}}^{theo}$ and ensuring that $(\Delta E)_{1S1/2}^{hfs}$ corresponds with experimental data, the magnetic moment of the unbound proton in Bohr magneton $\frac{\mu_p}{\mu_B}$ and the binding correction $\gamma_{binding}$ can be established through numerical testing using

$$\tilde{B} = \alpha^2 \cdot \frac{\mu_p(H)}{\mu_B} \cdot {\gamma_{corr}}^{theo} = \alpha^2 \cdot \frac{\mu_p(free)}{\mu_B} \cdot \gamma_{binding} \cdot {\gamma_{corr}}^{theo} \qquad (6)$$

To better understand the complex mechanisms involved, the magnetic moment in atomic hydrogen $\frac{\mu_p(H)}{\mu_B}$ has been decomposed into the physically meaningful product $\frac{\mu_p(free)}{\mu_B}\gamma_{binding}$. The binding correction $\gamma_{binding}$ serves as a convenient adjustment that conceals our lack of understanding regarding the true generation of the magnetic moment within the bound proton.

Numerical modeling yields

$$\frac{\mu_p(free)}{\mu_B} = \frac{\mathbf{\frac{1}{3}}}{\left(\frac{\lambda_{e\_bar}}{L_2(L)} + \mathbf{5}\right)} \approx 0.001\,520\,985 \quad ; \quad \frac{\lambda_{e\_bar}}{L_2(L)} = 2^{16}\pi^{-5}$$

$$\gamma_{binding} \equiv 1 + \mathbf{\frac{4}{7}}\alpha^2 + (\mathbf{3} \cdot \mathbf{5} + \mathbf{\frac{5}{9}})\alpha^4 \tag{7}$$

Using the relations (1), (2), (3), (5), (6), and (7), the theoretical value of $(\Delta E)_{1S1/2}^{hfs\_theo}$ is determined to be **1420.405 751 768 78 MHz**. This falls within the acceptable error margins (2 mHz) and should undergo experimental testing with sub-mHz accuracy for consistency to fulfill the criteria of the philosopher Popper. Just as in the case of the Lamb shift [1], the energy building block $\frac{1}{L_2(L)}$ in the energy unit $\frac{1}{\lambda_{e\_bar}}$, where λ_{e_bar} corresponds to the reduced Compton wavelength of the electron, is connected to the saturation density of matter established through elastic electron scattering experiments. Likewise, the conjectural factors such as 3, 5, 1/3, 4/7, 5/9 in formula (7) are consistent with the concept that symmetries in quantum theory result in simple mathematical equations that include integers or fractions of integers. In [6: p.77, p.125], supplementary examples are provided that emphasize the databased significance of the numbers 4/7 or 5/9.

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