

# A Kinetic Route to the Lorentz Transform and Beyond

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December 8, 2025

## Abstract

We model an elementary particle as a closed, lightlike intrinsic motion with rest-cycle period  $\tau$  that can undergo bodily translation without ever exceeding speed  $c$ . A local triangle construction and cycle averaging yield the Pythagorean time-share relation  $T^2 = \tau^2 + \bar{T}^2$  and standard time dilation. Interpreting the reallocation between intrinsic cycling ( $T$ ) and translation ( $\bar{T}$ ) as a symmetric two-channel kinetics with rate  $k(t)$  integrates to a hyperbolic rotation (Lorentz boost) with rapidity  $\phi = \int k dt$  and  $v/c = \tanh \phi$ . In the small-signal limit this identifies  $k = F/(mc)$ , linking the kinetic picture to Newton's second law while the  $\tanh$  nonlinearity enforces the  $c$  bound. We also give a physical reading of *relative rapidity* as the net logarithmic bias in time-share needed to map between motion states.

## 1 A Closed Wave Undergoing Bodily Motion

Established models of matter posit that elementary-particle substrates undergo an intrinsic oscillatory motion at speed  $c$  (for example: Dirac's zitterbewegung; the Higgs mechanism's chiral to-and-fro; the Williamson-van der Mark toroidal-photon model of the electron; and, by allusion, Einstein's light clock). Here we analyze ramifications of this feature and show how some properties of motion of such systems follow from simple geometry, and how the time of bodily translation must kinetically couple with the intrinsic cycle period for it to preserve its shape in its frame. We do not commit to a specific topology: we only require that the circulating wave always has instantaneous speed magnitude  $c$  and an intrinsic closed-cycle period  $\tau$  at true rest. A circulating wave is at true rest when the circulation is truly closed in a hypothesized special frame of reference. If not at true rest, the circulation orbit opens up just like the closed to-and-fro of a light clock opens up in a zig-zag when seen in motion, or how the earth's closed orbit looks like an open helix in light of the solar system's galactic motion. We show below that this simple model of a circulating wave gives rise to the Lorentz transform and (approximately) Newton's second law without additional postulates.

Consider a short intrinsic motion clip of duration  $\delta\tau$  at true rest (as defined above) and impose a bodily translation in some direction  $D$ . The translation cannot be naively added to the intrinsic wavefront velocity without violating the fixed speed magnitude  $c$ , which is a key departure from the Galilean picture. In the Galilean setting one can add a bodily velocity to internal motions, but here the wavefront speed is pinned to  $c$  by physics outside the present scope. To superimpose the translation we instead introduce a time-share: during the clip the wavefront divides its activity between intrinsic cycling and bodily advance. For the intrinsic segment  $\delta\tau$ , denote the time portion devoted to translation by  $\delta\bar{T}$  (a "motion penalty") and the total elapsed time by  $\delta T$ . The wave orbit remains closed in its own frame, just the way the earth's orbit is closed in the solar-system's frame, but it takes longer to traverse because some time is spent on bodily shift.

The three intervals will relate like the sides of a triangle; crucially it is **not** true that  $\delta T = \delta\tau + \delta\bar{T}$  despite the temptation to view them as a simple partition of time. The reason arises from the wave nature of the substrate. By Huygens' construction every point of the front acts as a secondary source, and advance of the front occurs along the direction that arrives first (Fermat's principle). When intrinsic cycling and bodily translation are superposed, the effective advance from a point  $A$  to a point  $C$  during the same Newtonian budget is not the concatenation of local legs  $A \rightarrow B$  and  $B \rightarrow C$  with summed times; the front reaches  $C$  along the shortest-time resultant. With local speed constrained to be  $c$  in both channels, this entails vector addition of displacements and hence a cosine-law relationship among the three durations  $c\delta T$ ,  $c\delta\tau$ , and  $c\delta\bar{T}$ . This is algebraically similar to Einstein's light-clock construction but slightly more general (in that the direction  $D$  need not be perpendicular to the "rest" motion of the wavefront, as is the case with the light clock), but is physically distinct in that a special rest frame is implied.

The small intervals  $\delta\tau$  and  $\delta T$  are not arbitrary; they are proportional discretizations of the respective cycle periods, e.g.  $\delta\tau = \tau/N$  and  $\delta T = T/N$  for sufficiently large  $N$  so that the intrinsic path curvature is negligible within a clip. Take a true-rest clip in which the wavefront goes from  $A$  to  $B$  with length  $|AB| = c\delta\tau$ . In the bodily moving case the equal phase progress takes time  $\delta T$ , so the corresponding phase point must lie on the sphere of radius  $c\delta T$  centered at  $A$ . The bodily direction in space is fixed; extend a ray from  $B$  along that bodily direction until it intersects the sphere at  $C$ . Then  $|AC| = c\delta T$  and  $|BC| = c\delta\bar{T}$ , which fully specifies the infinitesimal triangle whose sides encode the time-share. The angle  $\theta$  referred to below is precisely the angle between the intrinsic leg  $AB$  and the bodily leg  $BC$ . If the bodily speed  $v$  is introduced here, we might as well define  $\delta\bar{T}$  in terms of  $v$  as  $\delta\bar{T} = \frac{v}{c}\delta T$ , since the wave's overall bodily headway for the said phase progress may be accounted for in two ways, as  $v\delta T$  and as  $c\delta\bar{T}$ .

So the wave displacements over the three time intervals form a triangle with side lengths  $c\delta T$ ,  $c\delta\tau$ , and  $c\delta\bar{T}$ . If  $\theta$  is the instantaneous angle between the intrinsic segment direction and the bodily translation direction, the law of cosines of the triangle gives

$$c^2 \delta T^2 = c^2 \delta\tau^2 + c^2 \delta\bar{T}^2 - 2c^2 \delta\tau \delta\bar{T} \cos \theta. \quad (1)$$

Over a full intrinsic cycle, averaging over  $\theta$  eliminates the cross term, irrespective of the detailed shape of the closed path. Summing the segments over a cycle yields

$$c^2 T^2 = c^2 \tau^2 + c^2 \bar{T}^2, \quad \text{i.e.} \quad T^2 = \tau^2 + \bar{T}^2. \quad (2)$$

Here  $\tau$  is the closed-cycle period at true rest,  $T$  is the prolonged cycle time in the presence of bodily motion, and  $\bar{T}$  is the total time-share devoted to translation during that cycle.

**Time dilation from the triangle.** Define the bodily headway per prolonged orbital time as

$$v = \frac{c\bar{T}}{T}, \quad \beta = \frac{\bar{T}}{T} = \frac{v}{c}, \quad (3)$$

so (2) implies

$$c^2 T^2 = c^2 \tau^2 + v^2 T^2 \quad \Rightarrow \quad T = \frac{\tau}{\sqrt{1 - v^2/c^2}}, \quad (4)$$

which is the standard time-dilation relation obtained here from the closed-path generalization of the light-clock argument.

In this notation,  $\beta = \bar{T}/T$  is the true bodily speed (as a fraction of  $c$ ) of the particle with respect to the implied special frame singled out by the time-sharing variables. Equivalently,  $v = c\beta$  is that bodily speed in conventional units.

## 2 Differential Kinetics of Motion Modes

Treat  $\tau$  as an intrinsic constant of the substrate (closed-cycle period in its own frame) and let both  $T$  and  $\bar{T}$  vary in response to external influence over Newtonian time  $t$ . Differentiating (2) with respect to  $t$  gives

$$2T\dot{T} = 2\bar{T}\dot{\bar{T}} \quad \Rightarrow \quad T\dot{T} = \bar{T}\dot{\bar{T}}, \quad (5)$$

where dots denote  $\frac{d}{dt}$ . Rearranging,

$$\frac{\dot{T}}{\bar{T}} = \frac{\dot{\bar{T}}}{T} =: k(t), \quad (6)$$

defines a scalar rate  $k(t)$  governing a symmetric two-channel kinetics:

$$\dot{T} = k(t)\bar{T}, \quad \dot{\bar{T}} = k(t)T. \quad (7)$$

## 3 Integrating over a finite Newtonian time interval

Let  $\mathbf{u}(t) := \begin{bmatrix} \bar{T}(t) \\ T(t) \end{bmatrix}$ . The linear system (7) integrates over any interval  $[t_0, t_1]$  to a hyperbolic rotation (Lorentz boost in 1+1D):

$$\mathbf{u}(t_1) = \begin{bmatrix} \cosh \phi & \sinh \phi \\ \sinh \phi & \cosh \phi \end{bmatrix} \mathbf{u}(t_0), \quad \phi = \int_{t_0}^{t_1} k(t) dt. \quad (8)$$

Using (3) and the standard identification,

$$\frac{v}{c} = \tanh \phi, \quad \gamma = \cosh \phi = \frac{1}{\sqrt{1 - v^2/c^2}}, \quad (9)$$

so the finite-time map (8) is exactly the Lorentz boost matrix acting on the two-component time-share vector  $[\bar{T}, T]^\top$ . Identifying spatial displacement and elapsed orbital time via  $X = c\bar{T}N$  and  $T_{\text{orb}} = TN$  for  $N$  intrinsic cycles reproduces the standard coordinate transform  $(X/c, T_{\text{orb}})$  under a boost of rapidity  $\phi$ .

## 4 Three-Dimensional Time-Share Kinetics

Allow the bodily translation to point in an arbitrary spatial direction. Over a full intrinsic cycle, let the accumulated bodily displacement be the vector  $(x, y, z)$ , so the triangle relation (2) generalizes to

$$c^2T^2 = c^2\tau^2 + x^2 + y^2 + z^2 = c^2\tau^2 + c^2(\bar{T}_x^2 + \bar{T}_y^2 + \bar{T}_z^2), \quad (10)$$

where  $\bar{T}_x := x/c$ ,  $\bar{T}_y := y/c$ , and  $\bar{T}_z := z/c$  are the time-shares associated with bodily headway along each axis. Differentiating (10) with respect to Newtonian time  $t$  gives the constraint

$$T\dot{T} = \bar{T}_x\dot{\bar{T}}_x + \bar{T}_y\dot{\bar{T}}_y + \bar{T}_z\dot{\bar{T}}_z. \quad (11)$$

Introducing axis-specific exchange rates

$$k_x := \frac{\dot{\bar{T}}_x}{\bar{T}}, \quad k_y := \frac{\dot{\bar{T}}_y}{\bar{T}}, \quad k_z := \frac{\dot{\bar{T}}_z}{\bar{T}}, \quad (12)$$

the differential system becomes

$$\dot{T} = k_x \bar{T}_x + k_y \bar{T}_y + k_z \bar{T}_z, \quad \dot{\bar{T}}_x = k_x T, \quad \dot{\bar{T}}_y = k_y T, \quad \dot{\bar{T}}_z = k_z T. \quad (13)$$

Let  $\mathbf{u}(t) := [T \ \bar{T}_x \ \bar{T}_y \ \bar{T}_z]^\top$  and write (13) compactly as  $\dot{\mathbf{u}} = \mathbf{K} \mathbf{u}$  with

$$\mathbf{K} := \begin{bmatrix} 0 & k_x & k_y & k_z \\ k_x & 0 & 0 & 0 \\ k_y & 0 & 0 & 0 \\ k_z & 0 & 0 & 0 \end{bmatrix}. \quad (14)$$

The powers of  $\mathbf{K}$  close after cubic order:

$$\mathbf{K}^3 = \chi^2 \mathbf{K}, \quad \chi := \sqrt{k_x^2 + k_y^2 + k_z^2}. \quad (15)$$

Over any interval where the rates  $k_x, k_y, k_z$  are constant, the finite-time evolution is

$$\mathbf{u}(t_1) = \exp(\Delta t \mathbf{K}) \mathbf{u}(t_0), \quad \Delta t := t_1 - t_0. \quad (16)$$

Defining the normalized generator  $\hat{\mathbf{K}} := \mathbf{K}/\chi$  with  $\hat{\mathbf{K}}^3 = \hat{\mathbf{K}}$ , the exponential evaluates to a closed form analogous to the 1+1 Lorentz boost:

$$\boxed{\exp(\Delta t \mathbf{K}) = \mathbb{I}_4 + \sinh(\chi \Delta t) \hat{\mathbf{K}} + (\cosh(\chi \Delta t) - 1) \hat{\mathbf{K}}^2} \quad (17)$$

with  $\hat{\mathbf{K}}^2$  projecting onto the three translation channels. Setting two of the  $k$ 's to zero recovers the 1+1 system (7), while general  $(k_x, k_y, k_z)$  produce the 3-D analogue of the Lorentz transform acting on the time-share vector  $[T, \bar{T}_x, \bar{T}_y, \bar{T}_z]^\top$ .

**Time-varying rates.** When  $k_{x,y,z}$  vary with  $t$ , the exact propagator is the time-ordered exponential  $\mathcal{T} \exp(\int_{t_0}^{t_1} \mathbf{K}(t) dt)$ . If the rates commute over the interval (e.g. fixed direction  $\hat{k}$  with changing magnitude), the integral reduces to (17) with  $\chi \Delta t$  replaced by  $\int \chi(t) dt$ . Otherwise one may (i) integrate numerically, or (ii) approximate by slicing into short intervals where  $k_{x,y,z}$  are quasi-constant and multiply the corresponding closed-form factors.

## 5 Einstein's Derivation and Extra Branches

Consider two inertial frames  $S$  and  $S'$  with relative speed  $v$  along  $x$ . Let light emitted from the tail of the train at  $t = 0$  reach a detector at the head at event  $(x, t)$  in  $S$  and  $(x', t')$  in  $S'$ . Postulate 1 (invariance of light speed) gives  $x = ct$  and  $x' = ct'$ . Assume a linear transform along the motion axis,

$$x' = a(v)x + b(v)t, \quad t' = A(v)x + B(v)t, \quad (18)$$

with reciprocity ensuring the inverse is obtained by  $v \rightarrow -v$ :

$$x = a(-v)x' + b(-v)t', \quad t = A(-v)x' + B(-v)t'. \quad (19)$$

At time  $t$  in  $S$ , the origin of  $S'$  sits at  $x = vt$ , so setting  $x' = 0$  yields  $0 = a(v)vt + b(v)t$  and hence  $b(v) = -a(v)v$ . Substituting,

$$x' = a(v)(x - vt), \quad x = a(-v)(x' + vt'). \quad (20)$$

Enforcing  $x = ct$  and  $x' = ct'$  gives

$$ct' = a(v)t(c - v), \quad ct = a(-v)t'(c + v), \quad (21)$$

so eliminating  $t$  and  $t'$  produces

$$a(v)a(-v) = \frac{c^2}{(c + v)(c - v)}. \quad (22)$$

If one additionally assumes  $a$  is even, (22) yields the standard  $a(v) = 1/\sqrt{1 - v^2/c^2}$ . Without that assumption, two asymmetric branches also satisfy (22):

$$a(v) = \frac{1}{1 - v/c}, \quad a(v) = \frac{1}{1 + v/c}. \quad (23)$$

Einstein (1905) and many later expositions (Pauli, Bohm, Susskind) invoke behavior in transverse or opposite directions to discard (23). From a kinetic standpoint this reliance on perpendicular information is ad hoc for a phenomenon confined to one axis. Rigorous treatments necessarily

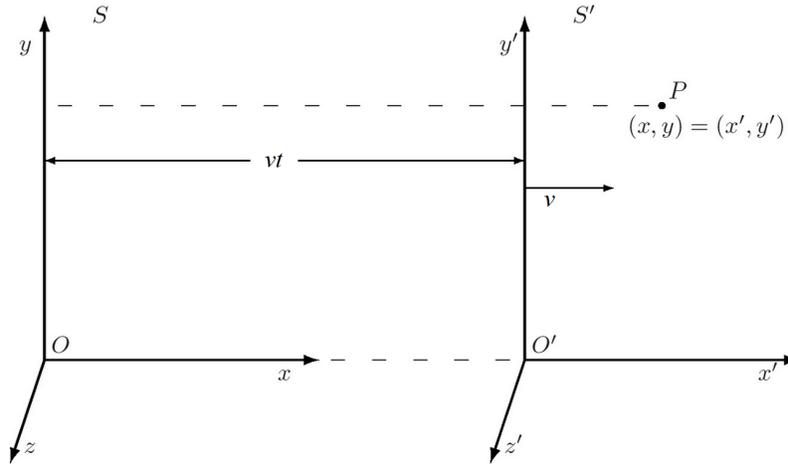


Figure 1: Two inertial frames with relative speed  $v$  along  $x$ .

introduce information from directions perpendicular or opposite to the motion to fix coefficients. If one restricts attention to linearity, reciprocity, and light-speed invariance along the motion axis, the functional equations allow additional asymmetric branches such as  $a(v) = 1/(1 \pm v/c)$  alongside the even solution  $a(v) = 1/\sqrt{1 - v^2/c^2}$ . The transverse-direction appeal then serves as an extra axiom that removes the unwanted branches. From a kinetic viewpoint this is unsatisfying: a phenomenon confined to one direction should not need deus-ex-machina input from unused axes. The time-share kinetics closes the ambiguity internally—by tying the coefficients to a symmetric exchange between intrinsic cycling and bodily advance—so the standard Lorentz form emerges without auxiliary perpendicular arguments. Most modern textbooks conceal the ambiguity by writing  $a$  as if it were a constant, not a function  $a(v)$ , so  $a(v)$  and  $a(-v)$  become indistinguishable and the extra branches in (23) never surface.

## 6 Special Frame and Lorentz Invariance

For  $N$  intrinsic cycles, the total elapsed times are  $NT$  in the singled-out special frame,  $N\tau$  as perceived by the particle, and  $N\bar{T}$  allocated to bodily motion. Because the evolution is linear

and integrates to matrix exponentials, successive boosts associate: if frame  $\mathcal{F}_1$  is boosted from the special frame by  $L_1$  and the particle has relative boost  $L_{12}$  within  $\mathcal{F}_1$ , then its net boost in the special frame is  $L_1 L_{12}$ . This mirrors exponential growth/decay where each increment acts on the current state regardless of when the process began, so the Lorentz structure holds from any frame even though a special frame exists.

The special frame is defined by coincidence of the internal tick count and Newtonian time ( $N\tau = NT$ ). In any other frame the proper time  $N\tau$  lags the Newtonian time. Detecting the special frame is nontrivial; one handle is directional anisotropy of observed time dilation. If a special frame exists, bodily motion in different directions relative to it can yield the same local clock rate while still harboring a net relative speed, implying measurable directional differences in accumulated lag. Careful directional comparisons of clock rates (or other intrinsic oscillations) could thus reveal the anisotropy and point to the special frame.

## 7 Why Maxwell's Equations Look Invariant

The time-share kinetics also gives a microscopic reason for why a moving observer still finds Maxwell's equations valid. Take a frame moving along  $z$  relative to the special frame. During the lightlike slice devoted to bodily advance, two features hold: (i) that slice does not register on the internal clock (so proper time omits it), and (ii) the particle moves at speed  $c$  along  $z$ . Consider a Maxwell equation involving  $\partial/\partial t$  and  $\partial/\partial z$ ; in the moving frame the measured time acquires an unclocked offset  $\delta t = \delta z/c$  and the measured  $z$  coordinate gains  $\delta z$  over that slice, while transverse components remain unchanged. Applying Maxwell's equation to the experienced fields over the lightlike slice shows that the time offset on  $\partial/\partial t$  is exactly balanced by the spatial offset on  $\partial/\partial z$ , so the relation still holds. Thus the apparent invariance of Maxwell's equations in moving frames follows from the time-share structure and does not require forbidding a special frame; the invariance persists because the offsets cancel at the microscopic level.

## 8 Connection with Newton's Second Law

Using (9) with  $\phi = \int k(t) dt$ , the state speed is

$$v = c \tanh\left(\int k(t) dt\right). \quad (24)$$

For small excursions ( $\tanh x \approx x$ ),

$$\Delta v \approx c \int k(t) dt. \quad (25)$$

Newton's second law over the same interval gives  $\Delta v = \int F(t)/m dt$ . Equating the small-signal forms identifies

$$k(t) = \frac{F(t)}{mc} \quad (\text{longitudinal}). \quad (26)$$

Equivalently, the accumulated rapidity is approximately impulse per  $mc$ :

$$\phi = \int k dt \approx \frac{1}{mc} \int F dt. \quad (27)$$

Thus, at low speeds the kinetic two-channel response reduces to proportionate acceleration; at higher speeds the hyperbolic nonlinearity enforces the  $c$  bound while preserving rapidity additivity.

## 9 Parting Thoughts

Starting from a closed-path intrinsic motion at speed  $c$  with rest-cycle  $\tau$  and superposed bodily translation, the local triangle construction and cycle averaging yield  $T^2 = \tau^2 + \bar{T}^2$  and the standard time dilation. Modeling the time-share reallocation as a symmetric two-channel kinetics with rate  $k(t)$  integrates to a hyperbolic rotation of rapidity  $\phi = \int k dt$ , i.e., the Lorentz boost with  $v/c = \tanh \phi$ . In this view the boost is a finite-time state map of a kinetic exchange between intrinsic cycling and translation, independent of the detailed topology of the intrinsic closed path.

Crucially, the Lorentz transform that emerges here is a transfer function between two speed states of the same substrate, not a symmetry operation relating two relatively moving reference frames as in textbook special relativity. Framed this way, the boost records how the internal time-share rebalances between intrinsic cycling and bodily advance when moving from one state to another, rather than asserting an equivalence of distinct frames. This perspective sidesteps standard reciprocity-driven paradoxes and offers a direct kinetic lens on experimental discrepancies that hinge on frame symmetry assumptions.

The preservation of a wave's internal closedness in its own frame under this symmetric exchange is the natural complement to rotational rigidity: antisymmetric (rotational) flows preserve shape under changes of direction, while the proposed symmetric exchange preserves internal structure under changes of speed (magnitude). This provides a coherent, geometry-led, kinetic origin for Lorentz boosts and suggests why persistent substrates would be selected to maintain their internal shape while developing bodily headway.