Making a Cayley Group Table
With a TI84 Calculator

Timothy W. Jones

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Abstract

We quickly define permutation group S_3 and then make a Cayley Table for it. We use a surprisingly simple program written on a TI84-CE calculator.

Introduction

Birkoff and MacLane [1] introduces groups, Chapter 6, with *Symmetries of the Square*. In the problem set for this Section 6.1, they ask readers to emulate the ideas using the symmetries of a triangle.

One doesn't need to create a triangle to get the idea. The idea of permuting (123) to (231) is given by a rotation: 1 to 2; 2 to 3; and 3 to 1. All the 3! permutations correspond to the *symmetries* of the triangle.

One can immediately understand that the possible permutations of three books on a shelf are such that any permutation can be combined with any other permutation to yield a third permutation. From (123) I use (231) to arrive at (231), a rotation. I can also then subject this result to a flip, say (132) and arrive at (213); another flip this time (213) returns the original, identity order: (123). Such operations are then closed, have an identity, and each book arrangement has an inverse that returns the book order to the original, the identity. Such things are called groups. There are $3! = 6$ such permutations of three objects taken three at a time for $6 \times 6 = 36$ such calculations in this S_3 group.

We can use the matrix and programming features of the TI84 calculator to make a *multiplication* table of the 6×6 products. Such a table in the context of abstract algebra are called Cayley Tables.

A Cayley Table for S_3

As the permutations are three digit numbers, we can use the trick of multiplying a number by 100, then 10, and then 1 and then adding them up. So if the permutation is (123), we can generate this with $100{*}1{+}10{*}2{+}3 = 123$. First store all six permutations in matrix $[A]$; see Line 003 in Figure 1, Left. Then, its a bit of trick, use the I and J variables in for loops to index into this matrix and form the product table, Lines 004 - 009, Figure 1, Right.

```
001 {6,6}→dim([B])
002 {6,3}→dim([A])
003 [[1,2,3][1,3,2][2,1,3][2,3,1][3,1,2][3,2,1]]→[A]
004 For(I,1,6)
005 For(J,1,6)
006 100[A](J,[A](I,1))→A
007 A+10[A](J,[A](I,2))→A
008 A+[A](J,[A](I,3))→A
009 A→[B](I,J)
010 End
011 End
012 Disp [B]
```

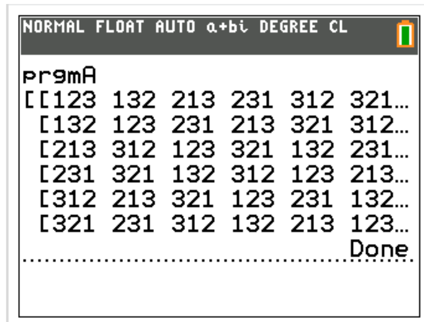

Figure 1: Left: Code for generating S_3 Cayley table; Right, Output of the code.

Let's try a couple of checks. How about $(312) \times (231)$. Our mental algorithm goes like this: 1 goes to 3 and then 3 goes to 1; write (1; next 2 goes to 1 and 1 goes 2; continue writing for (12; finally, 3 goes to 2 and 2 goes to 3; write (123). We find this in the table. How about $(231) \times (132)$? Our mental algorithm gives (321) and the table confirms this.

Conclusion

Try modifying the code and making a Cayley table for S_4. This will be a 24×24 table. Its getting big!

Can you find a subgroup that gives the symmetries of the square? Draw a square with center at the origin and label vertices in the first through fourth quadrant with 1, 2, 3, and 4. There are eight permutations: (1234) is the identity; then three rotations of 90 degree increments for (2341), (3412), and (4123). Next are flips. Flips that preserve the square shape are along horizontal and vertical axes (4321) and (2143) and diagonals from first to third quadrants and second to fourth: (1432) and (3214). Do some modifications of the program in Figure 1 and you should be able to do it.

Congratulations you have been introduced to the world of abstract groups.

References

[1] Birkoff, G., MacLane, S. (1977). *A Survey of Modern Algebra*, 4th ed. New York: Macmillan.