Adventures in **Visual Mathematics.**

A Universal Notation for Plotting Lines and the World of Diranuls

A book by **Faiz Qamar**

Introduction

Curiosity is, perhaps, what makes a human being truly human. It leads us to intellectual pursuits and new ideas. One such group of curious people, responsible for much of what we have today, were the natural philosophers.

Over time, the ancient field of natural philosophy evolved into distinct fields, one of which we now know as Mathematics. But the mathematics we know today isn't the one they developed at the time; mathematics was, in its earliest form, no more than geometry. Over time, geometry evolved, and the physical world began to be described with growing mathematical precision. Thus was born Physics, perhaps a mathematical study of the physical world surrounding us.

In this book, we return to geometry, not merely to explore its earliest forms, but to provide it with a new perspective. This book proposes a system that aims to introduce a universal notation for geometrical and multidimensional objects, providing a way to express shapes and physical quantities in a coherent and symbolic *l*anguage. Along the way, we shall encounter novel mathematical entities, not previously formalised, that arise naturally within this framework.

This is an exploration of what may lie at the very core of ancient geometrical mathematics. It is meant for the curious mind, just as geometry was born from many such...

(*June*, MMXXV)

— Faiz Qamar

Acknowledgements

Writing this book has been a remarkable experience, and many have helped and supported me throughout this journey, to whom I am grateful.

I am grateful to the readers who have taken their valuable time to read this book. I am truly honoured by your presence.

I am deeply grateful to my parents, Farzana Anjum, my mother, and Shaz Qamar, my father, who have been a constant source of support, helping me wherever possible. Their contribution to me, and my gratitude, cannot be expressed in words.

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I am grateful to all the critics who have pointed out errors in this book and have helped me improve it, and welcome any further suggestions and/or feedback.

Finally, I acknowledge all the natural philosophers, physicists and mathematicians who have advanced the mathematical-scientific fields thus far, for it is them upon whose work my ideas and books are based.

(*October 27*, MMXXV)

— Faiz Qamar

About the Author

The author, Faiz Qamar, is an aspiring physicist and an enthusiast in the field of Mathematics, especially concerning Geometry. He is a researcher, delving into some of his major interests, including astrophysics, astronomy, aeronautics and aerospace, palaeontology, myrmecology, genetics, philosophy, Ottoman history, linguistics, literature, poetry, psychology, relativistic physics, particle physics and quantum mechanics.

In his free time, Faiz enjoys expanding his knowledge, thinking, and formulating hypotheses to describe the world, and creating new systems, much like how this book was born.

The Author began writing his first book at the age of twelve: 'A Brief History of the Sublime Ottoman State', and published his first book in September 2025, titled 'Entangled Particles: The Basics of Quantum Entanglement', which he began writing in August 2024 at the age of fourteen.

Faiz has a deep passion for the physical and mathematical fields. With a keen interest in the workings and functioning of the universe, the author has decided to dedicate his entire life to research and shall continue doing so for the rest of his life.

(*October 26*, MMXXV)

— Faiz Qamar

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Reference to Symbols Used

Throughout the book, various symbols and font styles have been used to denote specific meanings, and those that have remained consistent throughout the text are provided here for the reader's reference.

x_n*-axis* —

The n^{th} axis of the coordinate system

$(x_1, x_2, x_3, \ldots, x_n)$ —

Coordinates in n dimensions with values (x_i) corresponding to respective axes (x_i-axis), $i \in [1, n]$.

l —

Length of a finite straight line or a section of a line.

Θ *or* θ —

Angle measured in radians.

$\mathscr{P}$ —

A point in an appropriate coordinate system.

$\int(\delta)$ —

To donate the expression of δ, that is, an expression of the form $\theta\delta_{a_1}^{a_2}(A_1, A_2)[E]\omega e$

r, $\boldsymbol{r}$ —

Used as a varying parameter of length

c, *C*, *k*, κ —

To denote constants.

a, b —

Constants used in the equations of an ellipse and a hyperbola, such that $b^2 = a^2(1 - e^2)$, where e is the eccentricity.

$\mathfrak{R}$ —

A ratio of two or more quantities.

Chapter 1
Introduction to the Notation

Over the centuries, from early geometry, mathematics has evolved into something beautifully profound and elaborate. However, with this evolution, we have ended up with an intricate patchwork of systems for describing shapes and functions, lacking overall uniformity. In this chapter and those that follow, we will embark on a quest to create a unified way to describe geometry and coordinate systems that might just be easier to interpret.

Why a Universal Notation?

The simplest way to move towards a general notation for describing all of geometry is to start at the most fundamental, a point (·). However, a true point, being dimensionless, is much smaller than that and thus cannot be observed. But perhaps a point doesn't need any defining more than that, so we move on to the next most fundamental thing — lines. The simplest lines are the ones that are straight and infinite in length, existing in only a single dimension, unlike the point, which has none.

A line consists of an infinite number of points that together form it, but one cannot arrange points 'next to' each other to form a line. Because of this, even a line with a finite length, or one with the shortest of the lengths possible, would always have an infinite number of points, as long as it has a dimension. It is this line that all of geometry is based on.

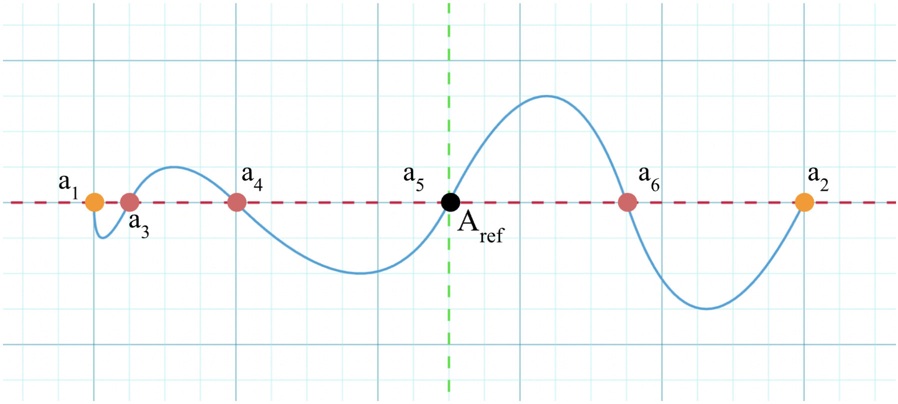
A simple example of an elementary line in a flat two-dimensional Euclidean coordinate system, such as the Cartesian coordinate system with two axes being the x_1-axis and the x_2-axis, would be $x_1 = x_2$, or perhaps $x_1 = -x_2$, or of the form $x_1 = k$ or $x_2 = C$; k and C being some constant. These simple lines can be tilted or moved by multiplying x_1 and/or x_2 by different constants and adding values to the equation, respectively. From this, one obtains the general equation for a straight line: $ax_1 + bx_2 + c = 0$.

But what of lines that are not straight, rather curved, like the graph of the sine of an angle, or a circle? Those, too, are formed in the same manner as a straight line, as a collection of infinite points, but cannot be described by the same expressions. The only significant difference between a straight line and a curved one is that the latter needs at least two dimensions to exist.

A simple circle may be described by the equation $x_m^2 + x_n^2 = r^2$, an ellipse by $\frac{x_m^2}{a^2} + \frac{x_n^2}{b^2} = 1$, or a parabola by $ax_m^2 + bx_n + c = 0$. But these are only a few simple curves that one rarely ever encounters; most curves are too complex to be defined through elementary means. Some withhold just enough complexity to make it an absurd idea to try describing them.

Towards a Universal Notation

When faced with curves that defy standard expressions, those that one may regard as, in simpler terms, chaotic or random, this may prompt a question: Is there a way to describe them uniformly, without patching new rules each time? In this chapter, we will explore one such attempt, but first, one must understand what makes any line truly unique and different from others. Let us take an example of a random line that may appear to be somewhat difficult to describe using the methods one may generally use.



In this graph, one may see that the line has several curves, and the curvature varies from point to point. If one could describe all its *curvatures*, *dimensions*, *lengths*, and the *position* and *slope* of its points relative to an assumed origin, then one would have described this line in a unique manner different from any other line.

Now that we know the essential characteristics that distinguish one line from another, we can begin our exploration of this new system, or rather a notation, which for now shall be called '*The Universal Notation for Line Plotting*'. In the figure, one can see that the line is finite in length, and thus has a starting and ending point, which are called a_1 and a_2. The straight, infinite line joining them is called the '*Principal Line*' *(the red dotted line in the figure)*. And, subsequently, all the points where the line intersects the principal line are named as a_1, a_3, a_4, a_5, … a_2. The midpoint of the principal line *(or the segment* $\overline{a_1a_2}$*)* is called A_{ref} *(the reference point)*, which is what one may assume as the origin in *most* cases. Just having this information about the line tells us a lot about it, the basis on which the Universal Notation is built. All that we now require is to find at what angles the line and the Principal line intersect and the curvatures, which would be simple enough and would be described in later parts of the book, as we go. For now, let us express the portion of this notation uncovered thus far, symbolically —

$$\theta\delta_{a_1}^{a_2}[A]e$$

or

$$\theta\delta_{(x_{1,1}, x_{2,1})}^{(x_{1,2}, x_{2,2})}[A]e,$$

expanded as— $\{\theta_1, \theta_3, \theta_4, \dots \theta_{ref}, \dots \theta_{n-1}, \theta_n, \theta_2\}\delta_{a_1}^{a_2}\{a_1, a_3, a_4, \dots A_{ref}, \dots a_{n-1}, a_n, a_2\}e$

Here, $\boldsymbol{\delta_i^f}$ is the Line Plotting Function, the true soul of the notation, that plots a line from an initial point ($a_1(x_{1,1}, x_{2,1})$) to the final point ($a_2(x_{1,2}, x_{2,2})$) according to the conditions described by the notation, A is the set of the points where the line intersects the principal line, including A_{ref}, irrespective of whether the two lines intersect at A_{ref}, $\boldsymbol{\theta}$ is the set of angles corresponding to each value in the set A ($\boldsymbol{\theta_A}$) and B ($\boldsymbol{\theta_B}$) and **e** is the set that defines the curvature for every length of the line in the set *B*, something which we will cover later on. The curvature, e, can be defined for a line in two ways —

- As eccentricity of a conic section over some length 'b' of a line, written as ***e*** or,
- As a function of predefined lines, represented by the Greek symbol **ε**.

Thus, e can either be written as *e* or as ε, depending on the kind of line. We shall discuss e in the next chapter and ε in the third chapter in greater detail.

Summary and what's next

In the first chapter, we have discussed some properties of a line and its characteristics, hinting at the need for a universal notation and a possible approach to the problem.

We came to know some fundamental parts of the notation, which included θ, δ_i^f, [A], A_{ref}, θ_{ref} and e. In the chapters that follow, we shall build upon this foundation. We will introduce [E] and discuss more about e, delve into practical uses of the notation, and consider both its strengths and limitations. We will explore how to represent lines of varying lengths, finite, infinite, and semi-infinite (rays), and gradually extend the notation to accommodate multidimensional lines. This journey will bring us closer to understanding geometry as a unified system.

Chapter 11
Describing 2-D Structures

In the previous chapter, we explored the foundational ideas behind the Universal Notation for Line Plotting, establishing the basic components necessary to describe a wide variety of lines. In this chapter and those that follow, we shall begin testing the notation further by applying it to standard geometric figures and conic sections. We will aim to form general expressions for many fundamental lines and shapes, including those that exist in up to three dimensions.

To refine and enhance the descriptive power of our system, we shall also introduce three new components into the notation — B, (A_1, A_2) and ω — to increase its accuracy in capturing the full behaviour of curves and figures.

[B]

In the previous chapter, important characteristics of a line were listed as —

- *curvature*
- *dimension*
- *lengths of different sections, and*
- *position and slope*

Despite this, a method for representing lengths had not yet been introduced in the notation. That is precisely what [B] does. B is the set of the lengths of different sections of the line, taken in series, in the most *convenient* manner. However, identifying and measuring these segments can sometimes be rather tedious. This is why [B] is associated with the changes in curvature, which differentiate the divisions along the line. The individual lengths of the set B are named as — b_1, b_2, b_3 …

This may not make complete sense right now, as you read this, but you will understand as more shapes are described in this chapter.

Including this new component, our notation thus becomes —

$$\theta\delta_{a_1}^{a_2}[A][B]e,$$

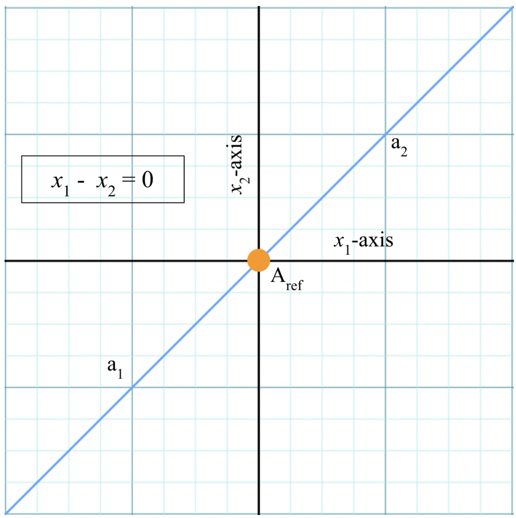
expanded as— $\{\theta_1, \theta_3, \ldots \theta_{ref}, \ldots \theta_n, \theta_2\}\delta_{a_1}^{a_2}\{a_1, a_3, \ldots A_{ref}, \ldots a_n, a_2\}\{b_1, b_2, \ldots b_n\}\{e_1, e_2, \ldots e_n\}$

Where each element of the set B has a corresponding element of e, and each element of the set A has a corresponding element of the set θ, such that, in both cases, every element of the first set corresponds to only one element of the second and vice versa.

Describing Elementary Lines

Infinite straight line —

From my experiments on this notation, one can easily describe a lot of weird and indescribable shapes, but describing a straight line is a perplexing task. Within this framework there was no easy way to differentiate between two straight lines at different angles, such as the lines described by the equations $x_1 - x_2 = 0$ and $x_1 + x_2 = 0$, unless one lets go of the principal line as the base axis, and defines their observations and all the sets in the notation with the x_1-axis (or x_2).



Because the line is infinite, it has no starting or ending points, which is why a_1 and a_2 are defined as *assumed* starting and ending points.

So, the best way to describe the line, without violating the conditions set down in the notation, would be —

$$\{\theta_1, \theta_{ref}, \theta_2\}\delta_{a_1}^{a_2}\{a_1, A_{ref}, a_2\}\{b\}\{e\}$$

$$\{\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{4}\}\delta_{a_1}^{a_2}\{a_1, A_{ref}, a_2\}\{\infty\}\{\infty\},$$

$$\text{or simply—}\{\frac{\pi}{4}\}\delta_{a_1}^{a_2}\{a_1, A_{ref}, a_2\}(e = \frac{1}{0})$$

This expression would have imaginary starting and ending points, had one not assumed them. In this case of an infinite, straight line, one can ignore [B] since it has no defined length. However, it is much easier to describe a straight line simply by its angle and starting and ending points, since it has no other features, and even more so if the angles are disregarded as well.

$$\theta\delta_{a_1}^{a_2}(e = \frac{1}{0}) \text{ or just } \delta_{a_1}^{a_2}$$

And from this, one could also describe $x_1 - x_2 = 0$ as —

$$(\theta = \frac{\pi}{4})\ \delta_{a_1}^{a_2}$$

Having done so, we have arrived at the general expression for an infinitely long, straight line —

$$\theta\delta_{a_1}^{a_2}(e = \frac{1}{0})$$

Finite straight line —

Having derived the general expression of an infinitely long straight line, for a finite straight line, all one has to do is define its length.

$$\theta\delta_{a_1}^{a_2}\{l\}(e = \frac{1}{0})$$

Where b = l, and l is the length of the line.

(A_1, A_2)

Before moving on to describing two-dimensional shapes, I must first introduce the next component of the notation, as mentioned in the chapter's introduction — **(A_1, A_2)**. In certain cases, it is hard to describe a line that intersects the line perpendicular to the principal line, thus, the sets A_1 (the set of all points lying on the line perpendicular to the principal line at A_{ref}, on the defined primary side of the principal line) and A_2 (the set of all points lying on the line perpendicular to the principal line at A_{ref}, on the defined secondary side of the principal line) have been introduced.

With this new component, our standard notation changes to —

$$\theta\delta_{a_1}^{a_2}(A_1, A_2)[A][B]e,$$

expanded as —

$$\{\theta_1, \theta_3, \dots \theta_{ref}, \dots \theta_n, \theta_2\}\ \delta_{a_1}^{a_2}\ (\{A_{1,1}, A_{1,2}, \dots A_{1,n}\}, \{A_{2,1}, A_{2,2}, \dots A_{2,n}\})$$

$$\{a_1, a_3, \dots A_{ref}, \dots a_n, a_2\}\{b_1, b_2, \dots b_n\}\{e_1, e_2, \dots e_n\}$$

Describing Elementary 2D Shapes

The *'universal notation of line plotting'* extends beyond simply representing standard lines; it can also be used to create any shape that can be formed by a continuous line (a continuous locus of points). This includes shapes such as circles, quadrilaterals, triangles, pentagons, hexagons … every shape that has one or fewer breaks, *literally any shape one could think of!* But in the case that is not found to be true, then that would rid the notation of its *'universality'*.

That being said, like almost all assumptions or theorems to be proven in mathematics, the universal notation for line plotting is no different in the fact that it requires a lot of cases where it holds true, that is, it is hard to prove, but it needs just one case to show that it may not work. However, I have not yet encountered any instances where the notation does not hold.

And now shall begin, in the next section, our description of the 2-dimensional shapes, starting with the square.

A Square —

When working with two-dimensional shapes such as a square, there are a lot of possibilities for describing a shape, for example —

- *One could describe a square as four individual lines*
- *One could describe it as two sets of adjacent lines*
- *One could define a square as a single line looping back on itself*
- *One could even take different configurations for a_1 and a_2 on the square's perimeter.*

I shall first begin by describing a square as four individual lines that join to form one.

Let there be, in a regular, two-dimensional Cartesian plane of coordinates, a square at any arbitrary angle θ' between any two opposite sides and the x_1–axis, such that it has a centre at the coordinates (0, 0), A_{ref}, and the respective vertices be a_1, a_2, a_3 and a_4. One may see that, in the order shown in the figure, the vertices would have the coordinates —

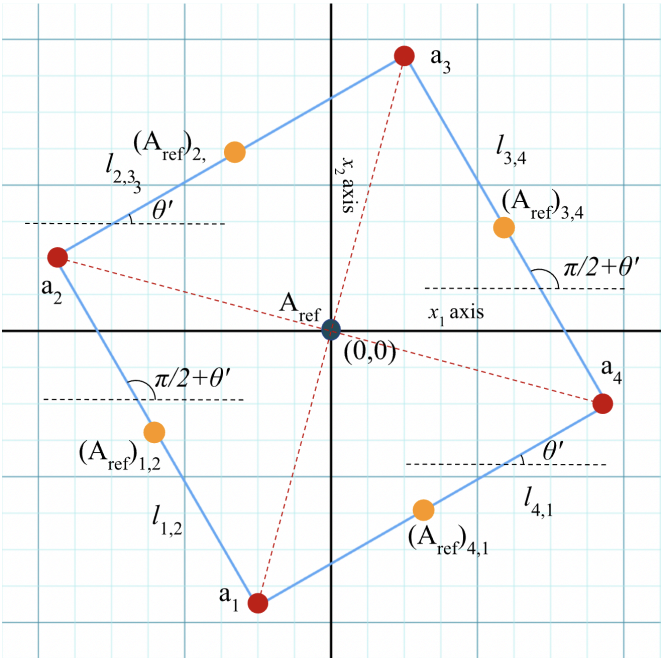
$a_1(-\frac{l}{\sqrt{2}}\cos\{\frac{\pi}{4}+\theta'\}, -\frac{l}{2}\{\cos\theta'+\sin\theta'\})$

$a_2(-\frac{l}{\sqrt{2}}\cos\{\frac{\pi}{4}-\theta'\}, \frac{l}{\sqrt{2}}\sin\{\frac{\pi}{4}-\theta'\})$

$a_3(\frac{l}{\sqrt{2}}\cos\{\frac{\pi}{4}+\theta'\}, \frac{l}{2}\{\cos\theta'+\sin\theta'\})$

$a_4(\frac{l}{\sqrt{2}}\cos\{\frac{\pi}{4}-\theta'\}, -\frac{l}{\sqrt{2}}\sin\{\frac{\pi}{4}-\theta'\})$

Where l is the length of each line or side of the square.



In this arrangement of four lines, I shall call the lines joining a_1 and a_2, a_2 and a_3, a_3 and a_4, and a_4 and a_1, $l_{1,2}$, $l_{2,3}$, $l_{3,4}$ and $l_{4,1}$ in that order, with their own reference points, such as that of a line described as A_{ref}, as $(A_{ref})_{1,2}$, $(A_{ref})_{2,3}$, $(A_{ref})_{3,4}$ and $(A_{ref})_{4,1}$.

The square with four separate lines being its sides would then be described as—

$$\theta\delta_{a_i}^{a_f}[A][B]e\ \{\theta_{a_i,a_f}\}\delta_{a_i}^{a_f}\{a_i, (A_{ref})_{i,f}, a_f\}\{l_{i,f}\}e$$

$$\{\frac{\pi}{2}+\theta'\}\delta_{a_1}^{a_2}\{a_1, (A_{ref})_{1,2}\ a_2\}\{l\}(e=\frac{1}{0}) \qquad \{\theta'\}\delta_{a_2}^{a_3}\{a_2, (A_{ref})_{2,3}\ a_3\}\{l\}(e=\frac{1}{0})$$

$$\{\frac{\pi}{2}+\theta'\}\delta_{a_3}^{a_4}\{a_3, (A_{ref})_{3,4}\ a_4\}\{l\}(e=\frac{1}{0}) \qquad \{\theta'\}\delta_{a_4}^{a_1}\{a_4, (A_{ref})_{4,1}\ a_1\}\{l\}(e=\frac{1}{0})$$

Having given a possible description of a square as four different lines of a ‘desired configuration’, one may use similar methods to describe any such shape in any number of lines, as one pleases, with any configurations one may desire. Thus, I will not show here how to do that.

I shall now describe a square as *1.* a single line looping back on itself, *the starting and the ending points being the same*, and *2.* as a single line ‘moving’ in two directions along the square’s perimeter only partially, combining to form a single square, or in other words, *the starting and ending points being different*.

Describing a square as a single line with the same starting and ending points, to give a rather general expression, we, like earlier, suppose that a side of the square is tilted at an angle θ'

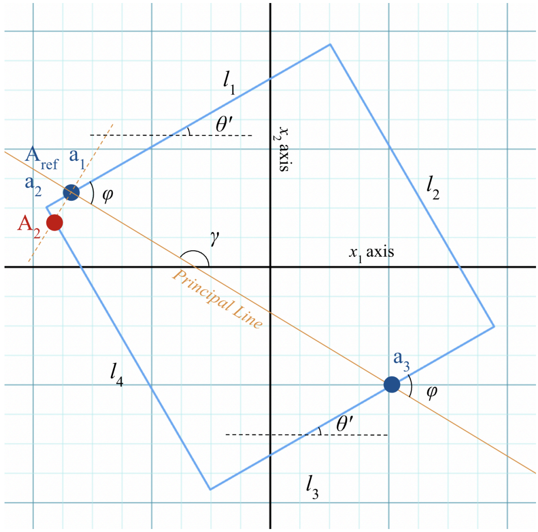
when viewed from the x_1–axis, and that the vertices of the square are —

$$(\frac{l}{\sqrt{2}}\cos\{\frac{\pi}{4}+\theta'\}, \frac{l}{2}\{\cos\theta'+\sin\theta'\}),$$
$$(\frac{l}{\sqrt{2}}\cos\{\frac{\pi}{4}-\theta'\}, -\frac{l}{\sqrt{2}}\sin\{\frac{\pi}{4}-\theta'\}),$$
$$(-\frac{l}{\sqrt{2}}\cos\{\frac{\pi}{4}+\theta'\}, -\frac{l}{2}\{\cos\theta'+\sin\theta'\}) \text{ and}$$
$$(-\frac{l}{\sqrt{2}}\cos\{\frac{\pi}{4}-\theta'\}, \frac{l}{\sqrt{2}}\sin\{\frac{\pi}{4}-\theta'\})$$

We also suppose, because a_1 and a_2 lie on the same line, as a result of which, no one principal line can be defined, that the principal line is at an angle γ when viewed from the x_1–axis, and that the angle between the side of the square tilted at θ′ and the principal line is φ.

An illustration of the above description has been provided in the post-given figure. The orange line depicted is the principal line. a_1, a_2 and A_{ref} are at the same point at a random point on the square's perimeter. I shall call this point ℵ. The orange dotted line perpendicular to the principal line at ℵ is to show the points A_1 and A_2, but here only one such point exists, which, by convention, has been taken as A_2.

Looking at this figure, it can seem intimidating to be able to represent it in the notation, but one must also remember that we have no use for the axes other than to know the coordinates, which we already know from this figure. Thus, to make things tidier, one could rid the shape of any axes and tilt it as we please.



[ω]

However, to be able to describe a square like this, we would require the third component of the notation that was to be introduced in this chapter, **ω**. ω is the set of all coordinates where the different sections, as described in [B], meet. ω_1 is defined as the point of intersection of b_1 and b_2, ω_2 of b_2 and b_3 … and so on.

With the addition of ω, the general expression of our notation has become —

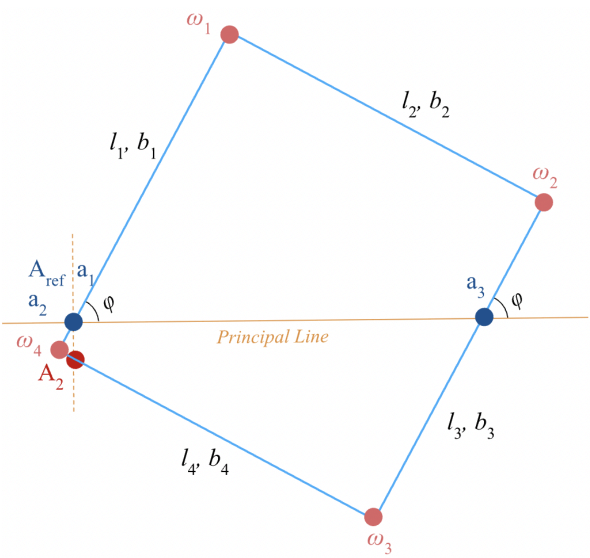
$$\theta\delta_{a_1}^{a_2}(A_1, A_2)[A][B]\omega e,$$

expanded as —

$$\{\theta_{a1}, \theta_{a3}, \ldots \theta_{Aref}, \ldots \theta_{an}, \theta_{a2}\}\{\theta_{b1}, \theta_{b2}, \ldots \theta_{bn}\}\delta_{a_1}^{a_2}(\{A_{1,1}, A_{1,2}, \ldots A_{1,n}\}, \{A_{2,1}, A_{2,2}, \ldots A_{2,n}\})\{a_1, a_3, \ldots A_{ref}, \ldots a_n, a_2\}\{b_1, b_2, \ldots b_n\}\{\omega_1, \omega_2, \ldots \omega_n\}\{e_1, e_2, \ldots e_n\}$$

Thus, for the square that is to be described, ω includes the coordinates of the square's vertices.

In the previous chapter, it was mentioned that the θ has two types, θ_A and θ_B. Until now, we have only been using the former and have not needed θ_B because there was no need to use [B].



θ_A is the set of angles corresponding to the values in the set A, whereas θ_B is the set of angles corresponding to the values in the set B.

A square, as a continuous line with the same starting and ending points, can then be described as —

$$\theta_A\theta_B\delta_{a_1}^{a_2}(A_1, A_2)[A][B]\omega e,$$

$$\{\theta_{a1}, \theta_{a3}, \theta_{Aref}, \theta_{a2}\}\{\theta_{b1}, \theta_{b2}, \theta_{b3}, \theta_{b4}\}\ \delta_{a_1}^{a_2}\{A_{2,1}\}\ \{a_1, a_3, A_{ref}, a_2\}\{b_1, b_2, b_3, b_4\}$$

$$\{\omega_1, \omega_2, \omega_3, \omega_4\}\{e_1, e_2, e_3, e_4\}$$

expanded as —

$$\{\theta', \theta', \theta', \theta'\}\{\theta', \frac{\pi}{2}+\theta', \theta', \frac{\pi}{2}+\theta'\}\ \delta_{\aleph}^{\aleph}\{A_2\}\{\aleph, a_3, \aleph, \aleph\}\{l, l, l, l\}$$

$$\{(\frac{l}{\sqrt{2}}\cos\{\frac{\pi}{4}+\theta'\}, \frac{l}{2}\{\cos\theta'+\sin\theta'\}), (\frac{l}{\sqrt{2}}\cos\{\frac{\pi}{4}-\theta'\}, -\frac{l}{\sqrt{2}}\sin\{\frac{\pi}{4}-\theta'\}),$$

$$(-\frac{l}{\sqrt{2}}\cos\{\frac{\pi}{4}+\theta'\}, -\frac{l}{2}\{\cos\theta'+\sin\theta'\}), (-\frac{l}{\sqrt{2}}\cos\{\frac{\pi}{4}-\theta'\}, \frac{l}{\sqrt{2}}\sin\{\frac{\pi}{4}-\theta'\})\}$$

$$\{\frac{1}{0}, \frac{1}{0}, \frac{1}{0}, \frac{1}{0}\}$$

Now that we have described a square as a single line looping, and having the same starting and ending points, we can easily describe it as one having different starting and ending points by letting a_1 and a_2 be random points on the square's perimeter and A_{ref} as their midpoint. And thus, with this, we conclude the chapter here.

Summary

Till now, I have defined the use of θ, δ_i^f, (A_1, A_2), [A], [B], ω, and e, and that θ is of two kinds, θ_A and θ_B, and so is e, the curvature, as e and ε. In this chapter, through the examples of different squares, we formed some generalised expressions and were able to find methods to define almost all standard 2D shapes using the notation. In the next chapter, I shall define methods to define conic sections and some infinite patterns, and talk about ε in detail.

$$\boldsymbol{\int(\delta) = \theta\delta_{a_1}^{a_2}(A_1, A_2)[E]\omega e}, \qquad [E]=[A][B][C][D][F][G]$$

$$\int(\delta) = \theta_A\theta_B\delta_{a_1}^{a_2}(A_1, A_2)[A][B]\omega e$$

Chapter III

Describing Conic Sections

Throughout the last chapter, we explored two things: a straight line and a square, because they are just complex enough, yet still fundamental, to allow similar methods to be extended to a rather large group of standard 2D polygonal structures.

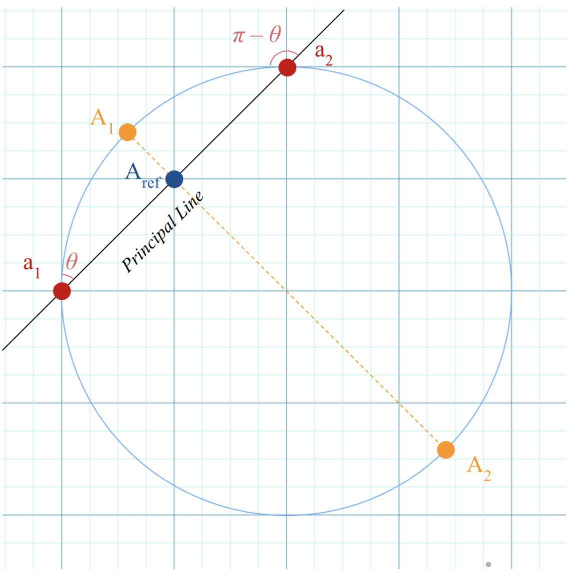
In this chapter, we will primarily focus on conic sections and, from them, define the ε and use it to describe some common trigonometric functions and other common patterns.

Describing Conic Sections

Conic sections are perhaps the most important part of this notation, essential for describing many things and for defining many curvatures (ε). Their beauty and efficacy stem from their natural emergence from the intersection of a plane and a cone. Let us now describe all the conic sections using the part of our notation thus uncovered, moving further on in our journey, where these conic sections will be used to describe some familiar patterns.

Circle

A circle, the most fundamental conic section, and something that is, too complicated, requiring just the right conditions — an infinite number of points, all exactly the same distance from a point (the centre), all of them spread uniformly in all directions, yet confined to only two dimensions, yet the easiest to describe.



Just as it has been shown with a square, however not tidy, a circle too can be represented in many ways. Still, I shall describe it only as a single looping back on itself, and A_{ref} being the midpoint of a_1 and a_2. A circle would thus then be represented as —

$$\theta_A\theta_B\delta_{a_1}^{a_2}(A_1, A_2)[A][B]\omega e,$$

$\theta_A = \{\theta_{a1}, \theta_{Aref}, \theta_{a2}\}= \{\theta, \{0, 0\}, \pi - \theta\}$
$A = \{a_1, A_{ref}, a_2\}$
$B = \{b_1, b_2\} = \{2r\theta, 2r(\pi - \theta)\}$
$\omega = \{a_1, a_2\}$
$e = e = \{0, 0\}$

Where r is the radius of the circle.

Here, one can, in no easy manner, define θ_B, but one does not need to either, just as one did not need to define A because it only has a_1, a_2, and A_{ref}, and B, because of which one did not need to define the two values of *e* or the set *ω*

Thus, expression is now simplified as —

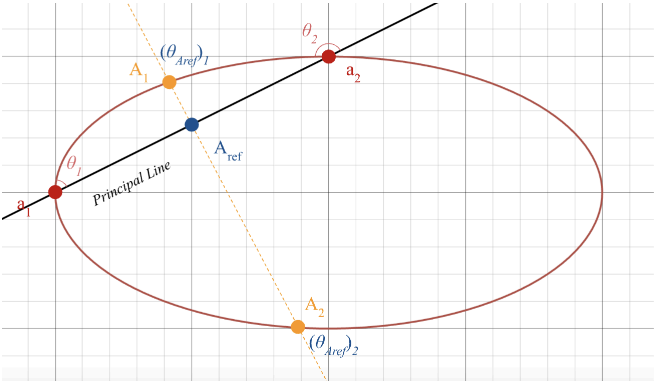
$$\theta_A\delta_{a_1}^{a_2}(A_1, A_2)e,$$

expanded as —

$$\{\theta, \{0, 0\}, \pi - \theta\}\delta_{a_1}^{a_2}(A_1, A_2)(e = 0)$$

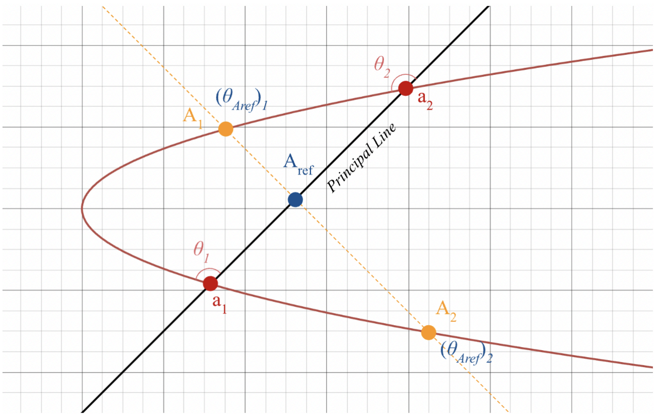
Just as the description of a circle, all conic sections, except for the hyperbola, can be described in a similar manner using the same base expression as the circle.

Ellipse



$\{\theta_1, \{(\theta_{Aref})_1, (\theta_{Aref})_2\}, \theta_2\}\ \delta_{a_1}^{a_2}(A_1, A_2)\ (0 < e < 1)$

Parabola



$\{\theta_1, \{(\theta_{Aref})_1, (\theta_{Aref})_2\}, \theta_2\}\ \delta_{a_1}^{a_2}(A_1, A_2)\ (e = 1)$

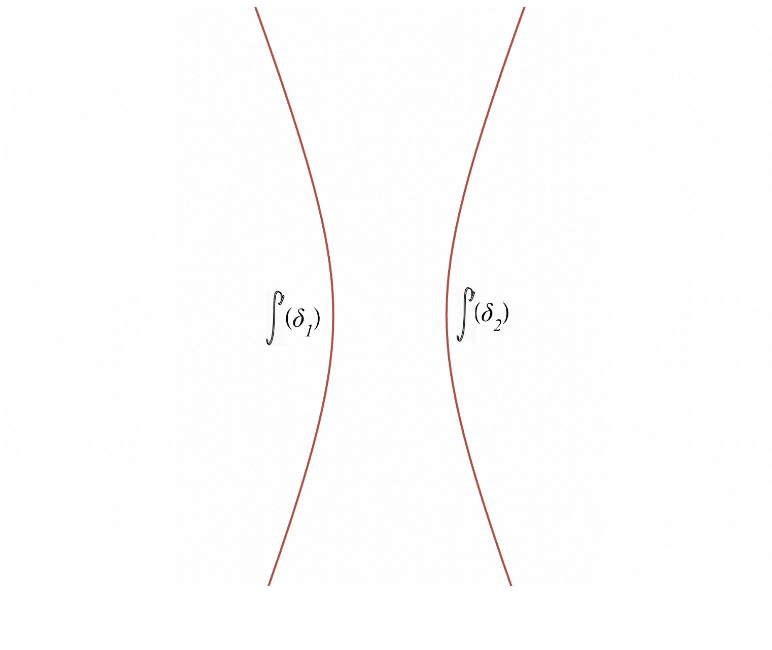
Hyperbola

However, one does not need to describe a hyperbola, since it is not a continuous locus of points, but rather two lines. Though if one does have to describe one, one can describe it as two separate lines — $\int(\delta_1)$ and $\int(\delta_2)$ —

$$\int(\delta_1) = \int(\delta_1), \int(\delta_2)$$

$$\int(\delta_1) = \{\theta_1, \{(\theta_{Aref})_1, (\theta_{Aref})_2\}, \theta_2\}_1\ \delta_{a_{1,1}}^{a_{2,1}}(A_1, A_2)_1\ (e > 1)_1,$$

$$\{\theta_1, \{(\theta_{Aref})_1, (\theta_{Aref})_2\}, \theta_2\}_2\ \delta_{a_{1,2}}^{a_{2,2}}(A_1, A_2)_2\ (e > 1)_2$$



Curvature (e)

I have hinted towards this a few times before, but now I formally introduce e. In this notation, 'The Universal Notation for Plotting Lines', as we have been calling it for the time being, we use the curvature, e, to describe how the locus of points, which was called a line, curves between a_1 and a_2. The curvature, e, is of two kinds — *e* and ε.

- *e is the curvature one may describe using eccentricities, that is, the conic sections, or their parts.*
- *ε is defined using ratios of one or more distances in conic sections, which we shall discuss in detail in this chapter.*

$$e = \{e_1, e_2, e_3, \ldots\}$$

e_n can be *e* or ε, depending on the nature of the curvature of b_n.

Variable Curvature (ε)

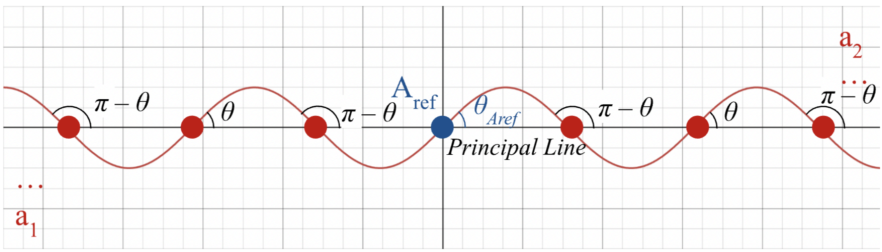
The curvature ε is used wherever one cannot define a section of a line as a standard curvature (eccentricity). The ε is defined as the change in a ratio of two or more lengths about the changing coordinate of a point in a line as a point traverses the perimeter of a defined standard shape, usually a conic section.
We shall describe ε in the form —

$$e_n = \varepsilon = \{e = \epsilon, \frac{\Delta\Re}{\Delta\theta}\}, \theta = \kappa\chi + c,$$

where e_n is the curvature of b_n, which is for the form ε of the two possible forms, e is the eccentricity of the conic section with which ε is defined, and $\frac{\Delta\Re}{\Delta\theta}$ is the change in the ratio $\Re$ as the angle θ changes, which corresponds to an axis χ, the values of which are dependent on θ, and κ is their relation, and c is a constant.

We shall see how ε works with a few examples as we describe some continuous patterns with different eccentricity at different points, such as $sine(x_1) = x_2$

$sine(x_1) = x_2$



Because the line is infinite in length, I have chosen A_{ref} at a point about which the graph is symmetrical, and such that one does not need (A_1, A_2), and also such that b can be chosen as the length of a single wave between alternating points in the set A, and because of this I have chosen a_1 and a_2 to be at infinities.

A sine wave would thus be represented as —

$$\theta_A\, \delta_{a_1}^{a_2} [A][B]\omega e,$$

expanded as —

$$\{ \ldots \theta, \pi - \theta, \theta, \pi - \theta, \theta, \ldots \}\, \delta_{a_1}^{a_2} \{a_1, \ldots (-3\pi, 0), (-2\pi, 0), (-\pi, 0), (0, 0), (\pi, 0), (2\pi, 0), (3\pi, 0) \ldots a_2\}\{ \ldots b, b, b, b, \ldots\}\{ \ldots (-2\pi, 0), (0, 0), (2\pi, 0) \ldots \}\{ \ldots \varepsilon, \varepsilon, \varepsilon, \varepsilon, \ldots \}$$

One may see that the sine wave requires all the bs to have a curvature ε. The ε is defined as —

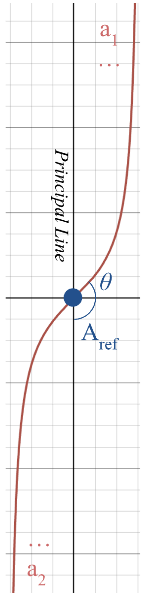
$$\varepsilon = \{e = 0, \frac{\Delta \frac{p}{R}}{\Delta\theta}\}, \theta = x_1$$

We know that the curvature of a sine wave corresponds to the change in the ratio of the perpendicular and the hypotenuse of a right-angled triangle in a circle, the hypotenuse being the circle's radius. That is how one may define ε in a circle. I only use p for perpendicular and R for radius here; a more formalised notation for ratios shall be provided later on.

One can similarly define the cosine wave, and thus, the description is not being provided here.

$tangent(x_1) = x_2$

The graph of a tangent is not a single line, so one can describe only one line on the graph, which has been depicted in the image that follows. Thus, I have, like before, chosen a_1 and a_2 at infinities and A_{ref} accordingly, obtaining the thus shown principal line and the angle.



$$\theta_A\, \delta_{a_1}^{a_2}\, [A][B]e,$$

$$\{\theta\}\delta_{a_1}^{a_2}\, \{a_1, A_{ref}, a_2\}\{\infty\}\{\varepsilon\}, \quad \varepsilon = \{e = 0, \frac{\Delta\frac{p}{b}}{\Delta\theta}\}, \theta = x_1$$

Defining Ratios of ε

Whilst ε, we used ratios of lines in conic sections that changed as the angle θ changed, which corresponded to some value on a graph; however, a way to describe these ratios has not yet been defined. To define some common lines to be used in the ratios, we use the symbols —

$p_{1,\mathscr{P}}$ $p_{2,\mathscr{P}}$ $p_{3,\mathscr{P}}$ $p_{4,\mathscr{P}}$ $p_{5,\mathscr{P}}$

$l_{\mathscr{P}}$ p_P p_R p_D

While defining these lines, the expressions used for the conic section are —

$x_1^2 + x_2^2 = 1$ for a circle,

$x_1^2 + \frac{x_2^2}{1-e^2} = 1$ for an ellipse, $0 < e < 1$,

$x_2^2 = x_1$ for a parabola, and

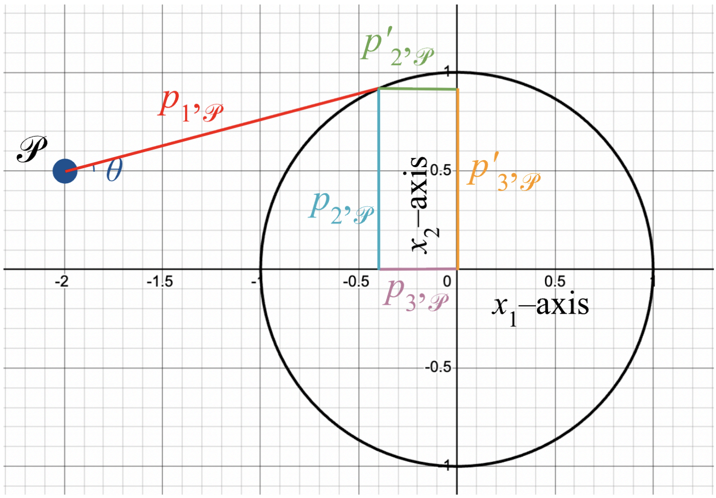
$x_1^2 + \frac{x_2^2}{1-e^2} = 1$ for a hyperbola, $e > 1$,

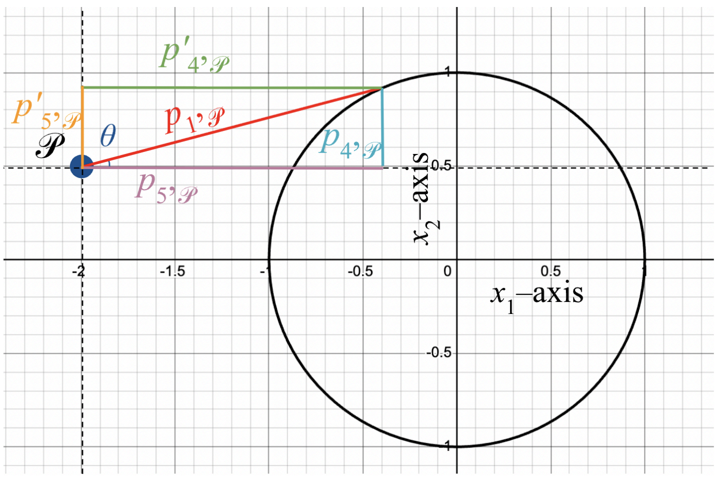
$p_{1,\mathscr{P}}$ is the line from a point $\mathscr{P}$ to the conic at an angle θ to the x_1–axis. $p_{2,\mathscr{P}}$ is the line from the x_1–axis, perpendicular to it, to the point where a line (say $p_{1,\mathscr{P}}$) meets the conic, and $p_{3,\mathscr{P}}$ is the line from the origin (or the centre of the conic) to $p_{2,\mathscr{P}}$, where it meets the x_1–axis. Similarly, $p'_{2,\mathscr{P}}$, and $p'_{3,\mathscr{P}}$ are defined for the x_2–axis respectively, in place of $p_{2,\mathscr{P}}$, and $p_{3,\mathscr{P}}$. $p_{4,\mathscr{P}}$, $p_{5,\mathscr{P}}$, $p'_{4,\mathscr{P}}$ and $p'_{5,\mathscr{P}}$ are, instead of the x_1– and x_2–axes, defined for the lines parallel to the axes and passing through $\mathscr{P}$, and $\mathscr{P}$ instead of the origin, respectively, in place of $p_{2,\mathscr{P}}$, $p_{3,\mathscr{P}}$, $p'_{2,\mathscr{P}}$ and $p'_{3,\mathscr{P}}$.

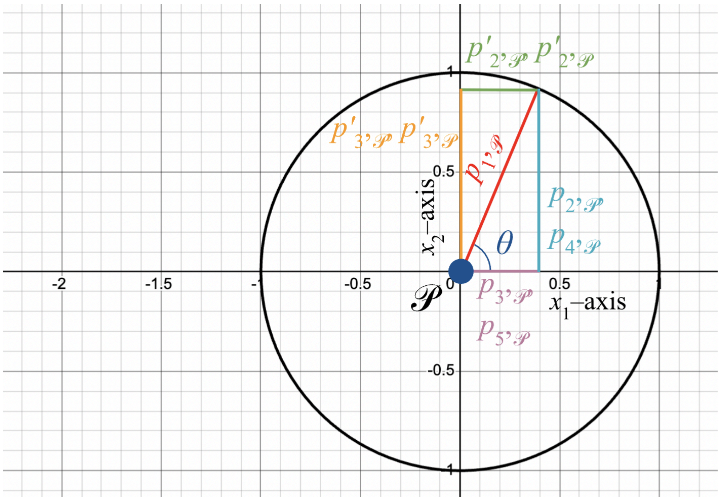
$l_{\mathscr{P}}$ is the chord of the conic that passes through a point $\mathscr{P}$. $p_{2,\mathscr{P}}$, $p_{3,\mathscr{P}}$, $p'_{2,\mathscr{P}}$ and $p'_{3,\mathscr{P}}$ are defined for $l_{\mathscr{P}}$ in the same way as they were for $p_{1,\mathscr{P}}$. p_P is the line perpendicular to the line that meets the conic (any line that could also be $p_{1,\mathscr{P}}$, or $l_{\mathscr{P}}$) at a point where it meets the conic. Similarly, p_R is defined like p_P, but instead of being perpendicular to the line, it is its 'reflection' on the perimeter of the conic. p_D too is like p_P and p_R, but is parallel to the x_2–axis. p'_D is like p_D, but instead defined for the x_2–axis instead of the x_1–axis.

It is to be noted that throughout this description, $\mathscr{P}$ is assumed to be a random point in the x_1–x_2 plane and that all the lines defined are not defined (to be) dependent on each other.

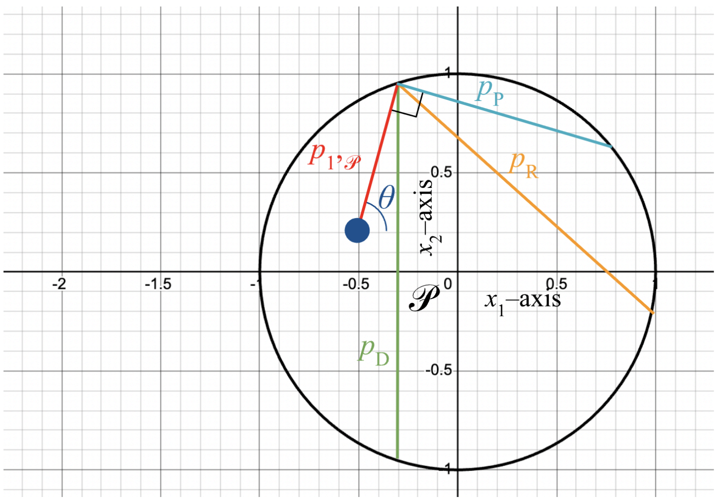
A visual depiction of the above has been provided here with a circle —



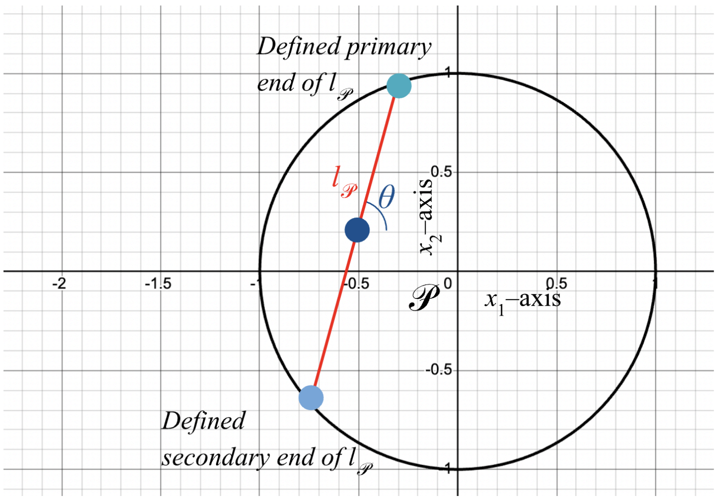




If 𝒫 is at the origin, $p_{2,\mathscr{P}}$ *and* $p_{4,\mathscr{P}}$ *are the same line, and so are* $p_{3,\mathscr{P}}$ *and* $p_{5,\mathscr{P}}$



When one uses $l_{\mathscr{P}}$ in the ratio, because $l_{\mathscr{P}}$ has two ends, 'touching' the conic, the end on the positive side of the x_1–axis, when θ is equal to zero, is defined to be the primary end of the chord $l_{\mathscr{P}}$, from where all the other lines shall be defined.



Summary

In this chapter, it has been defined and shown, albeit not in much detail, the usage of variable curvature ε, with which we are capable of describing almost all continuous 2-dimensional structures. It was also shown how conic sections are represented and how they play a valuable role in this notation, especially in describing something as simple as a sine wave.

In the chapters that follow, we shall introduce some more parts of the notation to reduce the length of the expressions for more complicated geometries, and later talk in detail about what $\langle A \rangle$ and $\langle A, [\int(\vec{\delta})] \rangle$ mean.

Chapter IV
3-Dimensional Objects

In the previous chapters, the notation had been defined and used to show how most standard 2-dimensional and lower 'lines' or shapes could be described. Thus, it may seem only natural to describe most standard 3-dimensional shapes, which is what we shall do in this chapter.

One could do that by simply defining all the points, lengths and angles, like we have done thus far, but that may appear to be a rather daunting task, especially when we move on to even higher dimensions and have to deal with different curves, considering that the expressions for the square were not that short in themselves, which is why, in this chapter, we shall introduce some functions to reduce our work — [C] and [D].

[C]

The function [C] allows one to translate or sweep a line, $\int \delta$, along a defined line or an axis, $\int \delta'$, producing a new line (or a shape) of a dimension, the same as, or higher than, that of the original.

For example, a point can be swept to produce a line, a line, along a straight path of a length equal to the line's length, can be used to form a 'filled' square, and similarly, a solid cube from that square.

The general way to define [C] is —

$$[\int \delta', \mathscr{P}_1, \mathscr{P}_2]$$

This represents that δ is to be moved along δ′, forming a new line, from the point $\mathscr{P}_1$, until it reaches $\mathscr{P}_2$.

With C, the general expression of our notation thus becomes —

$$\int \delta = \theta\delta_{a_1}^{a_2}(A_1, A_2)[E]\omega e$$

$$\int \delta = \theta_A\theta_B\delta_{a_1}^{a_2}(A_1, A_2)[A][B][C]\omega e,$$

expanded as —

$$\{\theta_{a1}, \theta_{a3}, \ldots \theta_{Aref}, \ldots \theta_{an}, \theta_{a2}\}\{\theta_{b1}, \theta_{b2}, \ldots \theta_{bn}\}\ \delta_{a_1}^{a_2}\ (\{A_{1,1}, A_{1,2}, \ldots A_{1,n}\}, \{A_{2,1}, A_{2,2}, \ldots A_{2,n}\})$$
$$\{a_1, a_3, \ldots A_{ref}, \ldots a_n, a_2\}\ \{b_1, b_2, \ldots b_n\}\ [\int \delta', \mathscr{P}_1, \mathscr{P}_2]\ \{\omega_1, \omega_2, \ldots \omega_n\}\ \{e_1, e_2, \ldots e_n\}$$

Solid Cube

Having defined [C], I describe a square, using it as a line of length l, translating about another line of the same length perpendicular to it.

Let $\int \delta$ be

$$\int \delta = \theta\delta_{a_1}^{a_2}(A_1, A_2)[E]\omega e$$

$$= \{\theta_{a1}, \theta_{Aref}, \theta_{a2}\}\ \delta_{a_1}^{a_2}\ \{a_1, A_{ref}, a_2\}\{b\}[\int \delta', \mathscr{P}_1, \mathscr{P}_2](e = \infty)$$

$$= \{\theta\}\ \delta_{a_1}^{a_2}\ \{l\}\ [\{\theta'\}\ \delta_{a'_1}^{a'_2}\ (e = \infty)\ , \mathscr{P}_1, \mathscr{P}_2]\ (e = \infty)$$

θ and θ' are such that the lines are perpendicular, and $\mathscr{P}_1$ and $\mathscr{P}_2$ are such that the given conditions are satisfied, and $\overline{\mathcal{P}_1\mathcal{P}_2} = l$

Note that this square is different from those that were defined in the second chapter, because it has a filled area, while those had only a perimeter.

With this, one could also extend this square to form a solid cube, which we call $\int \delta^*$

$$\int \delta^* = \theta\delta_{a_1}^{a_2}(A_1, A_2)[E]\omega e$$

$$= \{\theta_{a1}, \theta_{Aref}, \theta_{a2}\}\ \delta_{a_1}^{a_2}\ \{a_1, A_{ref}, a_2\}\{b\}[\{\int \delta', \mathscr{P}_1, \mathscr{P}_2\}, \{\int \delta'', \mathscr{P}_3, \mathscr{P}_4\}](e = \infty)$$

$$= \{\theta\}\delta_{a_1}^{a_2}\{a_1, A_{ref}, a_2\}\{l\}\ [\{\{\theta'\}\ \delta_{a'_1}^{a'_2}\ (e = \infty)\ , \mathscr{P}_1, \mathscr{P}_2\}, \{\{\theta''\}\ \delta_{a''_1}^{a''_2}\ (e = \infty)\ , \mathscr{P}_3, \mathscr{P}_4\}]\ (e = \infty)$$

θ, θ′ and θ″ are such that the lines are perpendicular, and $\mathscr{P}_1$ and $\mathscr{P}_2$, and $\mathscr{P}_3$ and $\mathscr{P}_4$ are such that the given conditions are satisfied, and $\overline{\mathcal{P}_1\mathcal{P}_2} = l$, and $\overline{\mathcal{P}_3\mathcal{P}_4} = l$.

In this expression, we have first translated a line of length l, say $\int \delta_l$, along a line $\int \delta'$, perpendicular to it, through a length l, resulting in the square $\int \delta$, and then translated $\int \delta$ along the line $\int \delta''$, perpendicular to both $\int \delta_l$ and $\int \delta'$, through a length l yet another time, having obtained a solid cube $\int \delta^*$.

A solid cuboid can be produced similarly by changing the lengths by which the lines translate; thus, it is not described here how that may be done and is left to the reader.

And because it had been shown how 2-dimensional shapes can be described, and that they can be translated to form different prisms, that too has been left to the reader.

[D]

The function [D] is similar to [C], but unlike [C], which sweeps a line along another line, [D] rotates a line about an axis defined by another line.

For example, if you were to have the expression for a circle, you could rotate it about a diameter by an angle of π radians and obtain a hollow sphere.

The general form of [D] is —

$$[\int \delta', \Theta, \theta = \kappa l + c]$$

Here, δ' is the expression of the axis, Θ is the angle up to which the line is to be rotated, and $\theta = \kappa l + c$ is the relation between the angle by which the line rotates about an axis (θ) in [D] and the length by which it is swept in [C].

With the addition of [D], the general form of our notation becomes —

$$\int \delta = \theta\delta_{a_1}^{a_2}(A_1, A_2)[E]\omega e$$

$$\int \delta = \theta_A\theta_B\delta_{a_1}^{a_2}(A_1, A_2)[A][B][C][D]\omega e,$$

expanded as —

$$\{\theta_{a1}, \theta_{a3}, \ldots \theta_{Aref}, \ldots \theta_{an}, \theta_{a2}\}\{\theta_{b1}, \theta_{b2}, \ldots \theta_{bn}\}\ \delta_{a_1}^{a_2}(\{A_{1,1}, A_{1,2}, \ldots A_{1,n}\}, \{A_{2,1}, A_{2,2}, \ldots A_{2,n}\})$$

$$\{a_1, a_3, \ldots A_{ref}, \ldots a_n, a_2\}\ \{b_1, b_2, \ldots b_n\}\ [\int \delta', \mathscr{P}_1, \mathscr{P}_2]\ [\int \delta', \Theta, \theta = \kappa l + c]$$

$$\{\omega_1, \omega_2, \ldots \omega_n\}\ \{e_1, e_2, \ldots e_n\}$$

Hollow Sphere

Let there be a circle,

$$\int \delta = \{\theta, \{0, 0\}, \pi - \theta\}\ \delta_{a_1}^{a_2} (A_1, A_2)(e = 0)$$

And a line along its diameter (the axis),

$$\int \delta' = \{\theta'\}\ \delta_{a'_1}^{a'_2} (e = \infty)$$

Then the hollow sphere, say $\int \delta^*$, obtained upon rotating the circle about the axis, will be —

$$\int \delta^* = \theta\ \delta_{a_1}^{a_2}(A_1, A_2)\ [D]\ e$$

$$= \{\theta, \{0, 0\}, \pi - \theta\}\ \delta_{a_1}^{a_2} (A_1, A_2)\ [\{\theta'\}\ \delta_{a'_1}^{a'_2} (e = \infty), \pi]\ (e = 0)$$

Here, because the rotation is independent of [C], one may not need to define its relation with [D].

Hollow Cone

Having described a hollow sphere, I now describe a cone, first a hollow one without a base.

To describe one, one may use a straight line and rotate it about an axis, a line that touches it but is not parallel to it, by an angle of 2π radians.

Let there be a line,

$$\theta\delta_{a_1}^{a_2}(A_1, A_2)[E]\omega e$$

$$\theta\delta_{a_1}^{a_2}(A_1, A_2)[A][B][C][D]\omega e$$

$$\theta\delta_{a_1}^{a_2}[B]e$$

$$\{\theta\}\delta_{a_1}^{a_2}\{a_1\bar{a}_2\}(e=\infty),$$

that is to be rotated along the axis,

$$\{\theta'\}\delta_{a'_1}^{a_2}(e=\infty),$$

touching it at a_2, thus forming the hollow, baseless cone —

$$\theta\delta_{a_1}^{a_2}[B][D]e$$

$$\{\theta\}\delta_{a_1}^{a_2}\{a_1\bar{a}_2\}[\{\theta'\}\delta_{a'_1}^{a_2}(e=\infty), 2\pi](e=\infty)$$

With this, one could also describe a hollow cone with a base by adding, to the expression, a 'filled' circle, whose plane is perpendicular to the axis, the axis passing through the centre, and whose radius is equal to the maximum distance between the line and the axis, touching the line, that is it touches the line at a_1. However, we have not yet described a filled circle; we shall do so now.

Let there be a point a_3 such that a line $a_1\bar{a}_3$ may be rotated to form the base of the cone. Then the expression for the base of the circle would be —

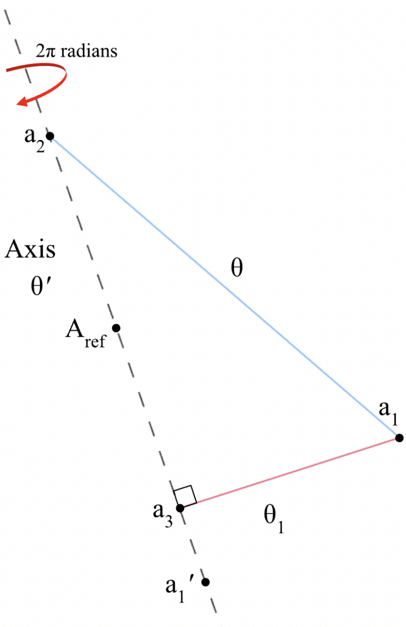
$$\theta\delta_{a_1}^{a_2}[B][D]e$$

$$\{\theta\}\delta_{a_1}^{a_3}\{a_1\bar{a}_3\}[\{\theta'\}\delta_{a'_1}^{a_2}(e=\infty), 2\pi](e=\infty)$$

Thus, the expression for a hollow cone would be —

$$\{\theta\}\delta_{a_1}^{a_2}\{a_1\bar{a}_2\}[\{\theta'\}\delta_{a'_1}^{a_2}(e=\infty), 2\pi](e=\infty),\ \{\theta_1\}\delta_{a_1}^{a_3}\{a_1\bar{a}_3\}[\{\theta'\}\delta_{a'_1}^{a_2}(e=\infty), 2\pi](e=\infty)$$

But instead of describing a hollow cone as two separate lines, one could also form one single expression by describing $a_1a_2a_3$ as a single line and rotating it about the axis.



The expression for the combined line would be —

$$\theta\delta_{a_1}^{a_2}(A_1, A_2)[E]\omega e$$

$$\theta\delta_{a_1}^{a_2}(A_1)[A][B]\omega e$$

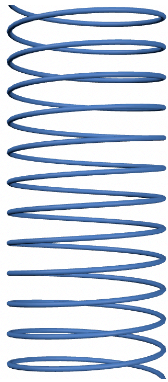
$$\{\theta, \theta, \theta_1\}\delta_{a_2}^{a_3}(a_1)\{a_2, A_{ref}, a_3\}$$

$$\{a_1\bar{a}_2, a_1\bar{a}_3\}\{a_1\}\{\infty, \infty\}$$

And the hollow cone would thus be —

$$\{\theta, \theta, \theta_1\}\delta_{a_2}^{a_3}(\mathrm{a}_1)\{\mathrm{a}_2, \mathrm{A}_{\mathrm{ref}}, \mathrm{a}_3\}\{a_1\bar{}a_2, a_1\bar{}a_3\}[\{\theta'\}\delta_{a'_1}^{a_2}(e=\infty), 2\pi]\{\mathrm{a}_1\}\{\infty, \infty\}$$

Helix



Having described the methods to describe certain elementary three-dimensional shapes, I now move on to a helix, which I shall do by translating and rotating a point about a line.

This would thus be the first line described here that includes both [C] and [D], and they are mutually dependent.

Let there be a point —

$$\mathrm{a}_o(x_1, x_2, x_3),$$

that is to be translated, and rotated about, a line —

$$\{\theta\}\delta_{a'_1}^{a_2}(e=\infty)$$

Then the expression for the helix would be —

$$\mathrm{a}_o[\mathrm{C}][\mathrm{D}]$$
$$(x_1, x_2, x_3)\ [\{\theta\}\delta_{a'_1}^{a_2}(e=\infty), \mathscr{P}_1, \mathscr{P}_2][\{\theta\}\delta_{a'_1}^{a_2}(e=\infty), \infty, \theta=\kappa l]$$

Here, in the description of the helical line, one may observe that the point rotates about and translates along the same line, but does not lie on it.

In the case that a point or a line is swept along a line on which it does not lie, the line moves such that it is equidistant from the line, maintaining its configuration unless changed by [D].

Higher Dimensional Lines

Just as I have described two-dimensional lines with one-dimensional lines, and those with points, of no dimensions, and used two-dimensional lines to form three-dimensional lines, by the sole means of rotation and *'sweeping'*, one can use similar methods to extend them to the fourth dimension, then the fifth, sixth, … and so on, until one reaches a dimension one pleases.

And depending on the methods one may use, decide whether or not the thus obtained higher-dimensional line is hollow or filled, and in what dimensions, something which shall be discussed in the next chapter in greater detail.

We do this merely because the general human mind cannot imagine spaces greater in dimensions than three, or at most five, for the highly educated being, simply because one has never seen those dimensions, even if they exist, that one has to extend lower-dimensional lines to higher dimensions, instead of taking the arduous efforts of describing everything by hand in those dimensions. This would also mean that it may not be that easy to know the possibilities for lines in higher dimensions, those that can not be produced from lower dimensions.

Summary

In this chapter, we have explored and seen examples of the uses of [C] and [D], through the descriptions of a solid cube, a hollow sphere, a hollow cube and a hollow cone, thus extending the notation to include further complexities.

In the next chapter, we shall explore some other functions that make working with higher dimensions even easier, and we will have completely described the notation required to describe all lines, the universal notation, until I find results suggest otherwise.

Chapter v
Some Other Functions

In the previous chapter, we introduced to the notation two functions — [C] to sweep lines, and [D] to rotate lines, and also remarked that they may not be enough to describe all three-dimensional structures through simple methods.

Thus, in this chapter, we introduce three more functions [F], [G], and [H] to complete this notation.

[F]

In the last chapter, a hollow sphere and a hollow cone were described, but one could not describe filled ones, because one did not have an expression of lower-dimensional filled shapes that one could use with [C] and [D] to produce them.

This is why the function [F], the filler, was introduced. The [F] is used to fill any line in a specified dimension.

(F = 0) fills a line in the zeroth dimension, that is, the line disappears. (F = 1) fills a line in the first dimension, such that only the edges are visible. (F = 2) fills a line similarly fills all the faces, or two-dimensional parts of the line, and (F = 3) does the same with volumes, and so on until one reaches the desired dimension one pleases.

For example, in the description of a hollow sphere, in the last chapter, one could obtain a solid sphere using (F = 3).

The [F], however, can be of two kinds — f and $\not{f}$ — depending on how one defines filling a line in different dimensions.

The f fills a line in the number of dimensions specified, and the resulting line is independent of the number of dimensions the original line was filled in. The $\not{f}$, on the other hand, is dependent on the number of dimensions the original line was filled in.

As an example, take a hollow cube (a line filled in two dimensions). Applying both (f = 3) and ($\not{f}$ = 3) would result in the same line, a solid cube, filled in three dimensions, but for (F = 1), f would result in a cube with empty faces, just the edges, whereas $\not{f}$ would bring no change to the original line.

This is because f can change the number of dimensions with no limits, but $\not{f}$ can only fill, that is, add to the given line. However, both f and $\not{f}$ are limited to the maximum number of dimensions the original line exists in, for one cannot define a line being filled in dimensions it does not exist.

In general, the [F] is defined as —

$$(F = \eta)$$

Where η is a whole number between 0 and n, including 0 and n, and n is the maximum number of dimensions the original line exists in.

$$\int \delta = \theta \delta_{a_1}^{a_2} (A_1, A_2)[E]\omega e$$

$$\int \delta = \theta_A \theta_B \delta_{a_1}^{a_2} (A_1, A_2)[A][B][C][D][F]\omega e,$$

expanded as —

$$\{\theta_{a1}, \theta_{a3}, \ldots \theta_{Aref}, \ldots \theta_{an}, \theta_{a2}\} \{\theta_{b1}, \theta_{b2}, \ldots \theta_{bn}\}\ \delta_{a_1}^{a_2} (\{A_{1,1}, A_{1,2}, \ldots A_{1,n}\}, \{A_{2,1}, A_{2,2}, \ldots A_{2,n}\})$$
$$\{a_1, a_3, \ldots A_{ref}, \ldots a_n, a_2\}\ \{b_1, b_2, \ldots b_n\}\ [\int \delta', \mathscr{P}_1, \mathscr{P}_2]\ [\int \delta', \Theta, \theta = \kappa l + c]$$
$$(F = \eta)\ \{\omega_1, \omega_2, \ldots \omega_n\}\ \{e_1, e_2, \ldots e_n\}$$

Solid Sphere

Let there be a hollow cube, as defined in the previous chapter —

$$\theta\ \delta_{a_1}^{a_2}(A_1, A_2)\ [D]\ e$$

$$\{\theta,\ \{0, 0\}\ ,\ \pi - \theta\}\ \delta_{a_1}^{a_2}\ (A_1, A_2)\ [\{\theta'\}\ \delta_{a'_1}^{a'_2}\ (e = \infty),\ \pi]\ (e = 0)$$

A solid sphere can then be described with [F] by filling the hollow sphere in the third dimension. Here, because the hollow sphere (original line) is three-dimensional, *n* (the maximum value of η) has a value of 3, and because the original line was filled in two dimensions, which is to be filled in three dimensions, both f and $\not{f}$ would result in the same line when given a value of 3.

A solid sphere would then be —

$$\theta\ \delta_{a_1}^{a_2}(A_1, A_2)\ [D][F]\ e$$

$$\{\theta,\ \{0, 0\}\ ,\ \pi - \theta\}\ \delta_{a_1}^{a_2}\ (A_1, A_2)\ [\{\theta'\}\ \delta_{a'_1}^{a'_2}\ (e = \infty),\ \pi]\ (F = 3)\ (e = 0)$$

One may observe that if one were to use [F] to reduce the hollow sphere to being filled in up to only one dimension, both f and $\not{f}$ would still have resulted in the same line, that is, the original line, because a hollow sphere does not have any one-dimensional edges, but still can be thought of as having infinitely many with infinite two-dimensional faces between them. But if [F] had a value of 0, f would have resulted in there being no line, while $\not{f}$ would not have changed anything, due to reasons that may be regarded with how one defined them, as they were earlier in this chapter.

Cube

In the previous chapter, a solid cube had been defined as —

$$\theta\delta_{a_1}^{a_2}[A][B][C]e$$

$$\{\theta\}\delta_{a_1}^{a_2}\{a_1, A_{ref}, a_2\}\{l\}\,[\{\{\theta'\}\,\delta_{a'_1}^{a'_2}(e=\infty), \mathscr{P}_1, \mathscr{P}_2\}, \{\{\theta''\}\,\delta_{a''_1}^{a''_2}(e=\infty), \mathscr{P}_3, \mathscr{P}_4\}]\,(e=\infty)$$

Here, the cube is solid and cannot be filled in any further dimensions with [F], thus can only be reduced to be filled only in lower ones, because of which one shall only use f and not $\not{f}$.

Hollow cube —

$$\theta\delta_{a_1}^{a_2}[A][B][C][F]e$$

$$\{\theta\}\delta_{a_1}^{a_2}\{a_1, A_{ref}, a_2\}\{l\}\,[\{\{\theta'\}\,\delta_{a'_1}^{a'_2}(e=\infty), \mathscr{P}_1, \mathscr{P}_2\}, \{\{\theta''\}\,\delta_{a''_1}^{a''_2}(e=\infty), \mathscr{P}_3, \mathscr{P}_4\}]$$
$$(f=2)\,(e=\infty)$$

Cube filled in one dimension —

$$\theta\delta_{a_1}^{a_2}[A][B][C][F]e$$

$$\{\theta\}\delta_{a_1}^{a_2}\{a_1, A_{ref}, a_2\}\{l\}\,[\{\{\theta'\}\,\delta_{a'_1}^{a'_2}(e=\infty), \mathscr{P}_1, \mathscr{P}_2\}, \{\{\theta''\}\,\delta_{a''_1}^{a''_2}(e=\infty), \mathscr{P}_3, \mathscr{P}_4\}]$$
$$(f=1)\,(e=\infty)$$

[G] and [H]

[G] and [H] are similar to how [C] and [D] were defined in the previous chapter, to the extent that one may even call them modified [C] and [D], but unlike them, [G] and [H] are only defined for axes that are straight lines.

[G] is the function that translates a line along a straight line axis δ', reducing the line in the dimension the original line exists in by a parameter '$\imath$' that is a function of the length it has translated.

In General, [G] is defined as —

$$\int \delta', \mathscr{P}_1, \mathscr{P}_2, \imath = \kappa l + c]$$

And similarly, [H] rotates a line along a straight line axis, reducing it by the parameter '$\imath$' that is a function of the angle through which the line has rotated.

In General, [H] is defined as —

$$[\int \delta', \Theta, \theta = \kappa l + c, \imath = \gamma l + C]$$

I explain these functions further with a few examples.

Solid Cone

In the last chapter, a hollow cone was described. To describe a solid cone, one could fill the hollow cone in the third dimension using [F], or one could translate a filled circle along an axis, reducing it with [G].

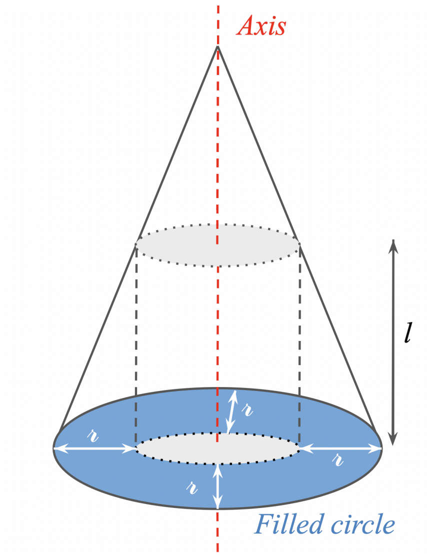
To describe one through the latter, one may define a filled circle as —

$$\begin{matrix} \theta\delta_{a_1}^{a_2}(A_1, A_2)[F]e \\ \{\theta,\ \{0, 0\}\ ,\ \pi - \theta\}\delta_{a_1}^{a_2}(A_1, A_2)(F = 2)(e = 0) \end{matrix},$$

which is translated along an axis —

$$\begin{matrix} \theta\delta_{a_1}^{a_2}e \\ \{\theta'\}\delta_{a'_1}^{a'_2}(e = \infty) \end{matrix},$$

passing through the circle’s centre and perpendicular to its plane.



Then the solid cone obtained would be described as —

$$\begin{matrix} \theta\delta_{a_1}^{a_2}(A_1, A_2)[F][G]e \\ \{\theta,\ \{0, 0\}\ ,\ \pi - \theta\}\delta_{a_1}^{a_2}(A_1, A_2)(F = 2)\ [\{\theta'\}\delta_{a'_1}^{a'_2}(e = \infty),\ \mathscr{P}_1,\ \mathscr{P}_2,\ \mathscr{r} = \kappa l]\ (e = 0) \end{matrix}$$

Summary

In this chapter, the final components of the notation were defined, which is claimed to have allowed one to describe all lines in any number of dimensions.

Hitherto, in this book, we have defined the notation, and all its components — θ, θ_A and θ_B, δ_i^f, (A_1, A_2), [A], [B], [C], [D], [F], *f* and *𝒻*, [G], [H], ω, e, and *e* and ε

$$\int \delta = \theta \delta_{a_1}^{a_2}(A_1, A_2)[E]\omega e$$

$$\int \delta = \theta_A \theta_B \delta_{a_1}^{a_2}(A_1, A_2)[A][B][C][D][F][G][H]\omega e,$$

expanded as —

$$\{\theta_{a1}, \theta_{a3}, \ldots \theta_{Aref}, \ldots \theta_{an}, \theta_{a2}\} \{\theta_{b1}, \theta_{b2}, \ldots \theta_{bn}\} \, \delta_{a_1}^{a_2} (\{A_{1,1}, A_{1,2}, \ldots A_{1,n}\}, \{A_{2,1}, A_{2,2}, \ldots A_{2,n}\})$$

$$\{a_1, a_3, \ldots A_{ref}, \ldots a_n, a_2\} \; \{b_1, b_2, \ldots b_n\} \, [\int \delta', \mathscr{P}_1, \mathscr{P}_2] \, [\int \delta', \Theta, \theta = \kappa l + c]$$

$$(F = \eta)[\int \delta', \mathscr{P}_1, \mathscr{P}_2, \mathscr{r} = \kappa l + c][\int \delta', \Theta, \theta = \kappa l + c, \mathscr{r} = \gamma l + C]$$

$$\{\omega_1, \omega_2, \ldots \omega_n\} \; \{e_1, e_2, \ldots e_n\}$$