Generalized equations of thermodynamics with nonspecific hidden variables

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Abstract

For the collective gravithermodynamic Gibbs microstates the connection between all thermodynamic potentials and parameters of matter have been found. This connection is realized with the help of four hidden wave functions that can take arbitrary values with certain probability. The possibility of obtaining the known equations of thermodynamic state of real gases is shown based on the use of both their the limit velocities of individual (separate) motion and the mathematical expectations precisely of these four nonspecific hidden parameters (wave functions) and functions of them. It is substantiated that in a quasi-equilibrium state, a real gas has spatial homogeneity not only of its entropy but also of the resulting extensive parameter (an indicator the compressibility coefficient). But the radial values of resulting intensive parameter (an indicator of hierarchical complexity and of quasi-equilibrium of cooling down) of a real gas are invariant in time.

Keywords: thermodynamics, gravity, gravitational field, inert free energy, Gibbs microstate, hidden variables, wave functions.

1. Introduction

Equations of state of matter are a necessary complement to the laws of thermodynamics. They allow the application of the laws of thermodynamics to specific substances and systems, since the laws of thermodynamics by themselves do not provide complete information about the state of the system. Equations of state cannot be derived from the laws of thermodynamics alone. They are obtained experimentally or theoretically, using ideas about the structure of matter, for example, methods of statistical physics.

The most famous equations of state for real gases are the generalized Clapeyron–Mendeleev equation, the van der Waals virial equation (1873) [1], the Dieterici equation (1898) [2], the Berthelot equation (1900–7) [3], the Kamerlingh-Onnes virial equation (1901), the Beattie–Bridgeman equation (1927) [4, 5], the Benedict–Webb–Rubin equation (1940–42) [6 – 9], the Redlich–Kwong equation (1949) [10], the Soave–Redlich–Kwong equation (1972) [11, 12], the Peng-Robinson EOS (1976) [13], etc. [14 – 35].

Studies of the spatially uniform compressibility coefficient of gases and liquids are important [21, 23, 27, 36 – 39]. It may also be important for the analysis of the cooling process of the hot Universe (when the Universe was uniformly filled mainly with hydrogen) to find out the value of the time-invariant intensive thermodynamic parameter $A_\rho = T^2 S / pV$. The most popular, practical, and perfect are the van der Waals virial equation [1, 15 – 17, 24] and the Benedict–Webb–Rubin

equation of state [6 – 9, 18, 20, 25 – 35]. But they are also purely empirical and artificial. After all, they are based on the use of only coefficients and corrections, and not thermodynamic nonspecific hidden parameters, which are wave functions capable of taking any values with a certain probability. Therefore, these equations of the thermodynamic state of matter do not allow us to obtain a set of multitude Gibbs microstates for matter. Moreover, they do not allow us to obtain equations of spatially inhomogeneous quasi-equilibrium thermodynamic states of astronomical gas clusters that gradually cool down. It is to the solution of these important and urgent problems that the proposed in the article results of careful theoretical research of the author are devoted.

Internal energy U of real gases, liquids and solid matters depends on many pairs of their intensive A_i and extensive a_i thermodynamic specific hidden parameters. And there are a very large number of those parameters in solid substances, and therefore the internal energy in these substances is very significant. Those facts prompt us (in the general relativity (GR)) to falsely identify the inert free energy $\hat{E}_0 = \hat{m}_{00} c v_{cv}$ of matter with the multiplicative component $U_0 = U_{00} c / v_{cv}$ of its thermodynamic internal energy due to the use (in the GR) of the eigenvalue of the hybrid enthalpy $\hat{H}_{T00} = \hat{m}_{00} c^2 + p_{00} V_{00} = \hat{H}_{T0} v_{cv} / c = \mathbf{const}(r)$ that is invariant along the radial coordinate r (in the gravitational quantum time of the matter [41]). Similarly, in relativistic gravithermodynamics (RGTD), the ordinary rest energy $W_0 = m_{00} c^3 / v_l \equiv G_0$ of matter is identified with the multiplicative component G_0 of the Gibbs thermodynamic free energy G [41]. Here: $\hat{m}_{00}$ and m_{00} are the eigenvalues of the mass of matter that is not under pressure and the true mass of the matter, respectively; $p_{00} V_{00} = p V v_{cv} / c = \mathbf{const}(r)$; $U_{00} = \mathbf{const}(r)$, $p_{00} \neq \mathbf{const}(r)$ and $V_{00} \neq \mathbf{const}(r)$ are eigenvalues of the multiplicative component U_0 of the internal energy U, of the pressure p and of the molar volume V of the matter, respectively; v_{cv} and v_l are the coordinate pseudo-vacuum velocity of light of the GR and the equivalent (but not identical) limit velocity of individual (separate) motion of matter (which at the same point in space may be different in the RGTD for different matters) respectively.

However, what is considered here are not at all specific hidden parameters characterizing thermodynamic macrostates of matter, but rather non-specific hidden variables that are mutually related to thermodynamic natural parameters (pressure, molar volume, temperature, and entropy). In addition, non-specific hidden variables that form Gibbs microstates, unlike specific internal macroscopic parameters, can instantly take any values with a certain probability.

Internal energy can also be shown as a sum of internal energy of hypothetic ideal gas (liquid) U_{id} and output of multiplication of resulting intensive $A_\rho(r) = TS / R_T(t) = T^2 S / pV$ and extensive $a_\rho(t) \equiv R_T = pV / T$ thermodynamic parameters:

$$U = U_{id} + \sum_{i=2}^{n} A_i a_i = U_{id} + A_\rho a_\rho, \qquad dU = T_{id} dS_{id} + A_\rho da_\rho - pdV = TdS - pdV,$$

where: $T_{id} = TR_T / R_{UT}$, $S_{id} = SR_{UT} / R_T$, $A_\rho a_\rho = T_{id} S_{id} = TS$. For gases: $a_i = R_{UT} B_i V^{1-i}$, B_i is virial coefficients that depend on both temperature and individual gas properties [40], while R_{UT} is universal gas constant and $R_T(r) = pV / T = \mathbf{const}(t)$ is thermodynamic parameter of gas, that determines the compressibility coefficient $Z = R_T / R_{UT}$ of the gas and does not vary in space at conditions of quasi-equilibrium cooling down of gas (is the same on any radial distance r from the gravitational attraction center in the comoving to it frame of reference of spatial coordinates and time t (FR)). And exactly this invariability in space of $R_T(t)$ is responsible for the fact that properties of real gases that gradually cools are close to properties of hypothetical ideal gas.

"Ideal" component U_{id} of internal energy is de facto identical to Helmholtz free energy F_T, while "ideal" component H_{Tid} of enthalpy is identical to the Gibbs free energy G:

$$U_{id} = U - a_\rho A_\rho = U - ST = F_T, \qquad H_{Tid} = H_T - a_\rho A_\rho = H_T - ST = G,$$

$$dU_{id} = T_{id} dS_{id} - a_\rho dA_\rho - p dV = (TdS - \frac{TS}{R_T} dR_T) - (SdT + TdS - \frac{TS}{R_T} dR_T) - pdV = -SdT - pdV = dF_T,$$

$$dH_{Tid} = T_{id} dS_{id} - a_\rho dA_\rho + Vdp = -SdT + Vdp = dG.$$

This, of course, is caused by the absence of binding energy ($\sum_{i=2}^{n} A_i a_i = A_\rho a_\rho = 0$) in ideal gas and ideal liquid due to the absence of electromagnetic interaction of their molecules and atoms. Self-organization of hierarchically more complicated interactions and interconnections in matter is in the tendency of Helmholtz and Gibbs free energies to their minimum.

Lower layers of matter, loaded by its upper layers form the extended system. The energy of such extended system [40] that consists of the whole gravithermodynamically bonded matter is indeed equivalent to enthalpy of a supercooled matter. Therefore, to obtain integral values of the ordinary rest energy and the equivalent to it gravitational mass of any astronomical body, integration must be performed using the spatial distribution of the density of the true mass (equivalent in GR to the enthalpy and in RGTD to the Gibbs free energy of matter), and not at all using the spatial distribution of the density of the "thermal" mass, which is equivalent only to the thermodynamic internal energy of hot matter.

Moreover, as it is shown further, parameter a_ρ (in contrast to A_ρ parameter) takes the same value in the whole space filled by matter that gradually cools ($(\partial a_\rho / \partial r)_t = 0$). And, therefore, Gibbs free energy "behaves" as it is expected: it only changes in space along the radial coordinate r together with the gravitational potential. And when Gibbs energy changes in time together with the gravitational potential, it "behaves" like multiplicative component of enthalpy (like the energy of extended system). This is quite logical and reflected in static equations of GR gravitational field.

The study of specific hidden intensive and extensive thermodynamic parameters (internal variables) corresponding to specific properties of matter is really important. However, what is considered here is not at all specific hidden parameters characterizing thermodynamic macrostates of matter, but rather non-specific hidden parameters characterizing thermodynamic microstates of matter, linking thermodynamic natural variables (pressure, molar volume, temperature, and entropy) and which, unlike specific internal macroscopic variables, can instantly take any values with a certain probability.

2. Maximum possible velocity of definite matter individual (separate) motion

The limitation of the velocity of physical bodies is indeed exists in rarefied gas-dust matter. However, this limitation is not related to the velocity of light in the matter or in hypothetic absolute vacuum. In airspace, as well as in dense matter, the charged micro objects (protons) can propagate faster than the velocity of light. That is confirmed by the origination of the radiation, found by Cherenkov, in this case.

On the other hand, hypothetical frequency of intranuclear interaction (alternative to the pseudo-vacuum velocity of light of GR) decreases while approaching the gravitational attraction center unlike the real frequency of electromagnetic interaction in matter, which increases. This is in a good correspondence with the fact that thermodynamic and gravity-evolutionary processes have opposite directions and is related to the fact that frequency of electromagnetic interaction in matter is greater when the temperature of the matter is greater. Due to the same reason the physical processes flow faster not on the surface but in hotter bowels of the astronomical objects despite the gravitational slowing down is predicted by GR for those processes.

The reason for limitation of velocity of physical bodies is indeed the nature of matter movement in the space. Physical vacuum is not carried away by physical body. Matter is only the non-mechanic excitement of physical vacuum (space-time modulations of its physical characteristics). Therefore, the perception of high-frequency discrete movement of the body in the space as the continuous motion is similar to cinematographic perception of discrete change of image frame. The limitation of body velocity can be related to the fact that it is impossible to reach infinitely high frequency of discrete

change of Gibbs collective thermodynamic microstate (quantum «hologram») of the whole its RGTD-bonded matter and to the fact that it is impossible to reach the zero value of the length of spatial step shift (quantum micromovement) of the body. This frequency and this micromovement are de facto the de Brogile frequency $\mathbf{v}_B$ and wave length λ_B of the moving body. That is why instead of denying the possibility of moving body to overcome the velocity of light we should state the principal impossibility to reach the extremely big velocity $v_l = \sqrt{v v_B} = v_{B\min}$ of individual (separate) motion of its homogeneous matter. And this corresponds to the tending of $\mathbf{v}_B$ to infinity, and tending of $\lambda_{B\min} = v_l / \mathbf{v}_B = h / m v_l \Gamma = 0$ ($v = v_l$, $\mathbf{v}_B = \infty$, $\Gamma = \infty$) to zero, when phase velocity of de Brogile wave propagation v_B reaches its minimal value, equal to maximal possible velocity $v_{\max}$ of individual motion of the homogeneous matter of the body ($v_{B\min} = v_{\max} \equiv v_l$).

Thus, the greater the gravitational mass $m_{gr0} = m_{00} c / v_l$ of an astronomical body, the greater the frequency with which de Broglie waves run on him, and therefore, the greater the frequency with which step-by-step movement of this body can be carried out. And this means that the inertial mass $m_{in0} = m_{00} v_l / c = m_{gr} v_l^2 c^{-2}$ of an astronomical body, on the contrary, becomes as many times smaller. And therefore, the inertial mass of matter can be identical to its gravitational mass only according to the own clock of matter, when $v_l = c$. According to this, the GR currently actually uses an inertial mass that is equivalent only to the inert free energy $E_0 = m_{in} c^2 = m_{00} c v_l$ of matter, instead of a gravitational mass that is equivalent to the ordinary rest energy $W_0 = m_{gr} c^2 = m_{00} c^3 / v_l$, identical to the multiplicative component of the thermodynamic Gibbs free energy of matter.

In addition, gravitational fields set only the gradients of the intrinsic values of the limit velocity matter individual (separate) motion, and not the intrinsic values of the limit velocity of matter individual motion themselves, which are determined purely by the thermodynamic parameters of matter. And this also applies to distant galaxies due to the logarithmic potential $\varphi_H = c^2 \ln(v_{lH} / c)$ of the dynamic gravitational field. And, therefore, the thermodynamic parameters and potentials, which are determined purely by the intrinsic values of the limit velocity $v_{lH} = v_l \Gamma_H = c T_{00} / T$ of definite matter individual motion, are not only relativistically, but also gravitationally invariant. Here $\Gamma_H = (1 - v_H^2 v_l^{-2})^{-1/2}$, v_H is Hubble velocity of the radial motion of the galaxy, and T_{00} is a constant that is intrinsic in a definite substance and is independent on external gravitational fields. Since the constant T_{00ind} may be different for different matter, then at the same point in space at the same absolute temperature T they will also have different intrinsic values of the limit velocity v_{lHind} of individual motion of galactic definite substance.

Dense substances, in which the limit velocities of individual (separate) motion are less than the actual velocity, cannot move separately. They must be pushed by substances, in which the limit velocities of individual motion are greater than the actual velocity of their collective motion. Therefore, very dense substances must be contained mainly in the bowels of planets, stars and galaxies, so that they can move at extremely high velocities. And therefore, it is not at all accidental that very dense neutron stars are located only in the centers of galaxies. Because of this, the frequency of de Broglie waves that fall on them corresponds not only to their mass, but to the mass of the entire galaxy.

This is what allows the use of a single limit velocity of motion (in the RGTD for the group motion of many substances), which is inherent, for example, to the most common substance – the hydrogen that is the surface layer of stars (similar to the use of a single coordinate pseudo-vacuum velocity of light, which is supposedly inherent to all substances, in the GR). If such a limit velocity of group motion is known, for example, for a star moving with a known maximum orbital velocity, then the equations of the gravitational field of the galaxy allow us to determine the limit velocities of group motion of a similar substance for stars moving with other orbital velocities.

In distant galaxies which are not conditionally cooled, as well as in any bodies moving in a gravitational field by inertia, the value $v_{lHind} = cT_{00ind}/T = \textbf{const}(t)$, corresponding to a particular substance will be the same as in the same substance on Earth only at the same absolute temperature. In the hot matter of distant galaxies, as in the matter in the bowels of the Earth, it is much smaller and gradually increases with a decrease in its temperature. And the much smaller value v_{lH} in the matter of distant galaxies is well consistent with the gradual cooling of the very hot matter of the Universe in the distant past.

Unlike gases, the internal energy of solid matter includes not only thermal energy, but also consists of many products of intensive and extensive thermodynamic parameters. Therefore, it would seem logical to identify gravitational energy, which is equivalent to the gravitational mass of matter, with it. But in fact, it is not this energy at all, and not even enthalpy, which is identified with the energy of an expanded system [40], but only the multiplicative component of Gibbs free energy, which does not include the released thermal energy, can be considered identical to gravitational energy, and therefore equivalent to the gravitational mass of matter. After all, many experiments have proven that heated bodies have a weight that is not at all greater, but, on the contrary, less than a weight of cold bodies [43 – 45].

3. Non-rigid frame of reference of time and spatial coordinates comoving with gas that gradually cools

If during the inertial motion of matter the main role is played by conserving Lagrangian of its ordinary rest energy and Hamiltonian of its inert free energy, then during the quasi-equilibrium (quasi-uniform) motion of matter that gradually cools the main role is played by gradually decreasing Lagrangians of its ordinary rest energy W_0 (identical to the multiplicative component of Gibbs free energy G_0) and of multiplicative component of its thermodynamic internal energy [41, 42]. And therefore, according to the Lagrangian construction[1] of the energy-momentum tensor of matter that gradually cools, not only the parameter $b = v_l^2 c^{-2}$ of the gravitational field equations, but also the relativistic shrinkage $\Gamma_m = (1 - v_m^2 v_l^{-2})^{-1/2}$ of the radial dimensions of the matter are hidden thermodynamic parameters. Namely, for the non-rigid FR which is comoving ($b_c = b\Gamma_m^2$) with the gas that gradually cools, we will have the following gravitational field equations (which correspond not to the metric space-time continuum (STC), but to the inseparable from matter its own physical STC, in which there is no radial motion of the molecules of matter, and time is counted by the clocks comoving with it):

$$b_c'/a_c b_c r - r^{-2}(1 - 1/a_c) + \Lambda = \kappa(p - ST/V) = (\kappa T_{00}/\sqrt{b}\Gamma_m V)(R_T - S) = \kappa T_{00}R_{TS}/\sqrt{b_c}V ,$$

$$a_c'/a_c^2 r + r^{-2}(1 - 1/a_c) - \Lambda = \kappa(\mu_{gr}c^2 - p) = \kappa(\mu_{00}c^2 - p_{00})/\sqrt{b}\Gamma_m = \kappa U_{00}/\sqrt{b_c}V ,$$

$$(\ln a_c b_c)'/a_c r = \kappa G_0/V = (U_{00}/V + p - ST/V) = (\kappa/\sqrt{b}\Gamma_m V)(U_{00} + T_{00}R_{TS}) = \kappa G_{00}/\sqrt{b_c}V ,$$

where: $b_c = b\Gamma_m^2 = v_l^2 c^{-2}\Gamma_m^2 = v_{lc}^2 c^{-2}$, $U_{00} = (m_{00}c^2 - p_{00}V) = U_0\sqrt{b_c} = (U - U_{ad})v_l\Gamma_m/c = \textbf{const}(r)$,

$G_{00} = G_0\sqrt{b_c} = (G_T - U_{ad})v_l\Gamma_m/c = \textbf{const}(r)$, $\qquad S = \textbf{const}(r)^2$, $\qquad R_T = pV/T = \textbf{const}(r)$,

$R_{TS} = R_T - S = \textbf{const}(r)$, $T_{00} = T\Gamma_m v_l/c = \textbf{const}(r,t)$, $A_\rho = TS/R_T = T^2 S/pV = \textbf{const}(t)$.

Moreover, the thermodynamic processes in matter confront the intranuclear evolutionary and gravitational processes in it. While in mechanics the main role is played by the inert free energy

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[1] Obviously, instead of the Hamiltonian construction, the hidden Lagrangian construction of the energy-momentum tensor of matter should be used in the comoving with expanding Universe frame of references of spatial coordinates and time (CFREU). After all, the evolutionary self-contraction of matter in it can be caused by the evolutionary decrease of the coordinate velocity of light in it, which is a hidden thermodynamic parameter of matter.

[2] Therefore, entropy is spatially homogeneous not only in a maximally cooled matter, but also in a gradually cooling matter that has a non-rigid FR.

$\mathrm{H} \equiv E = m_{in0}c^2\Gamma_m = m_{00}cv_l\Gamma_m$ (equivalent to inertial mass $m_{in} = m_{00}v_l\Gamma_m/c$), in thermodynamics the main role is given not only to the internal energy U, but also enthalpy and Gibbs free energy $G = G_0 + U_{ad} = G_{00}/v_l\Gamma_m + U_{ad} = \mathrm{L}_m + U_{ad}$, the main part of which is an ordinary rest energy $W_0 \equiv G_0 = U + pV - pS - U_{ad} \equiv \mathrm{L}_m = m_{gr0}c^2/\Gamma_m = m_{00}c^3/v_l\Gamma_m = W_{00}/\sqrt{b_c}$ (equivalent to gravitational mass $m_{gr} = m_{00}c/v_l\Gamma_m$) of matter that gradually cools. Here: $U_{ad} = U_T - U_0 = \mathbf{const}(r) > 0$ is spatially homogeneous additive compensation of multiplicative representation U_0 of internal energy U of matter and, consequently, does not depend on the strength of gravitational field.

Therefore, in relativistic gravithermodynamics (RGTD) [41, 42] the frequency of intranuclear interaction $f_G = q_N N_{RE} = q_N N_E/\Gamma_E = \eta_m v_l\Gamma_m/c = v_{lc}/v_{lc/cr} \le \eta_m$ corresponds to inversely proportional to it frequency of electromagnetic interaction of matter molecules:

$f_I = \chi_m/f_G = q_M N_I = (v_{cm}/c)N_I = \psi_m c/v_l = \psi_{m0}c/v_{lc} = \chi_m v_{lc/cr}/v_{lc} \ge \psi_m$ ($f_{Gcr} = 1$, $f_{Icr} = 1$ when conditionally $\chi_m = \chi_{m0} = 1$).

This frequency is changing together with the change of velocity of light $v_{cm} = cq_M \le v_l$ in matter (that corresponds to radiation refractive index n_m at the wavelength of maximum of energy of thermal radiation) and with the change of internal scale factor[3] $N_I = \delta l_{cr}/\delta l \le 1$ of matter [41, 42, 46 – 49]. Here: $\psi_{m0} = \chi_m/\eta_m = \chi_m v_{lc/cr}/c = \chi_m v_{lcr}\Gamma_{mcr}/c$, $\psi_m = \psi_{m0}/\Gamma_m = \chi_m/\eta_m\Gamma_m = \chi_m v_{lcr}\Gamma_{mcr}/c\Gamma_m$, $\eta_m = c/v_{lc/cr} = c/\Gamma_{mcr}v_{lcr}$ and $\chi_m = \chi_{m0}$ are the constants of matter that cools to the limit (χ_{m0} , ψ_{m0}) and of matter that gradually cools (χ_m , ψ_m)[4], which is not identical for different matters and for their various phase or aggregate states and not dependent both on strength of gravitational field and on matter thermodynamic parameters; v_l and v_{lcr} are maximum possible (limit) velocities of matter individual (separate) motion (which are equivalent to the pseudo-vacuum coordinate velocities of light of GR) in any point and on the phase boundary of the same matter (or on the boundary of different matters) correspondingly; $v_{lc} = v_l\Gamma_m > v_l$ and $v_{lc/cr} = v_{lcr}\Gamma_{mcr}$ are the limit velocity of individual (separate) motion of matter that gradually cools in the comoving with it the non-rigid FR; Γ_{mcr} is the Lorentz shrinkage of dimensions of matter (that moves in the process of quasi-equilibrium cooling down) on the phase boundary of the same matter or with another matter; δl_{cr} is minimal possible distance of electromagnetic interaction between molecules of certain matter or its critical value; c is constant of the velocity of light.

In contrast to used in cosmology spatially inhomogeneous external scaling factor N_E, which is the cause of the curvature of matter intrinsic space, internal scaling factor N_I takes nonsimilar values for different matters and depends on thermodynamic state of matter. This factor characterizes the distinction between average statistic value of interaction distance δl in the atoms of concrete matter and the value of this distance δl_{cr} that corresponds to critical equilibrium values of internal

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[3] It is obvious, the every matter forms in gravitational field its own thermodynamic STC, the curvature of which partially compensates the curvature of STC of the whole gravithermodynamically bonded matter.

[4] These constants unambiguously correspond only to the homogeneous matter of not layered astronomical body that does not possesses any matter outside its borders. In any other case it is only gaugely changed since due to logarithmicity of gravitational potential (that is formed based on correspondent thermodynamic potential) it does not directly influence the strength of gravitational field. Changes of the strength of gravitational field in it take place under the influence of other matters on the formation of its spatially inhomogeneous thermodynamic state [41, 42, 46 – 49]. When there is a violation of thermodynamic equilibrium with environment they can substantially influence on the magnitude of limit velocity of individual motion of matter and, thus, on the magnitude of ordinary internal energy of matter and equivalent to it gravitational mass. So, for example, in spite of the increasing of thermal energy of matter during its heating its ordinary internal energy and, therefore, gravitational mass are ostensibly decreasing [43 – 45]. And this decrease in the gravitational mass of heated bodies is due to its equivalence not to thermodynamic internal (free thermal) energy at all, but actually to multiplicative component of Gibbs free energy.

energy multiplicative component U_{cr}, Gibbs free energy G_{cr}, temperature T_{cr}, pressure p_{cr}. And if parameter $q_M = v_{cm}/c = 1/n_m < 1$ characterizes the difference of real velocity of electromagnetic interaction propagation in matter from the constant of velocity of light c, then N_I is responsible for compensation of the influence of increase of propagation velocity of electromagnetic wave on the frequency of electromagnetic interaction f_I of matter microobjects. If for gases and simplest liquids the dependencies of instantaneous values of their thermodynamic parameters and potentials on q_M and $N_I = \sigma_m / N_{RE} = \sigma_m / \Gamma_{mE} N_E$ ($\sigma_m = \mathbf{const}(r)$) allow to separate these variables, then instantaneous value of their Gibbs free energy (that corresponds to their instantaneous thermodynamic microstates) can be expressed via these two parameters and via their function R_T in the following way:

$$\breve{G}(q_M, N_I, \breve{R}_T) = \breve{U}(q_M, N_I, \breve{R}_T) - \breve{S}(q_M, N_I, \breve{R}_T)\breve{T}(q_M, N_I, \breve{R}_T) + \breve{V}(q_M, N_I)\breve{p}(q_M, N_I).$$

4. Multiplicative representation of thermodynamic parameters and potentials with additive compensation that is invariant in space

Methods of thermodynamics allow us to analyze equilibrium states of matter even when there is no analytic dependence of thermal energy of matter on its thermodynamic parameters. In order to identify some features, let us consider the simplest analytical dependencies for gases and simple liquids. According to them the instantaneous values of main thermodynamic parameters and potentials can be represented in the following way [41, 42, 49]:

$$\breve{S} = -\left(\frac{\partial \breve{G}}{\partial \breve{T}}\right)_{\breve{p}} = -\left(\frac{\partial \breve{F}_T}{\partial \breve{T}}\right)_{\breve{V}} = \frac{\beta_{ST}\breve{R}_T}{\beta_{pV}}(\hat{S}), \quad \breve{T} = \left(\frac{\partial \breve{H}_T}{\partial \breve{S}}\right)_{\breve{p}} = \left(\frac{\partial \breve{U}}{\partial \breve{S}}\right)_{\breve{V}} = \frac{\beta_{pV}U_{cr}}{\breve{R}_T}(\hat{T}) = \frac{c\psi_{m0}\beta_{pV}U_{cr}}{\Gamma_m v_l \breve{R}_T} = \frac{\beta_{pV}\breve{U}_0}{\breve{R}_T},$$

$$\breve{V} = \left(\frac{\partial \breve{H}_T}{\partial \breve{p}}\right)_{\breve{S}} = \left(\frac{\partial \breve{G}}{\partial \breve{p}}\right)_{\breve{T}} = \frac{U_{cr}}{p_l}(\hat{V}), \qquad \breve{p} = -\left(\frac{\partial \breve{U}}{\partial \breve{V}}\right)_{\breve{S}} = -\left(\frac{\partial \breve{F}_T}{\partial \breve{V}}\right)_{\breve{T}} = (\hat{p})\beta_{pV}p_l,$$

$$\breve{U} = \breve{U}_0 + \breve{U}_{ad} = \frac{R_{UT}\breve{Z}\breve{T}}{\beta_{pV}} + \breve{U}_{ad} = \frac{\breve{R}_T\breve{T}}{\beta_{pV}} + \int\limits_{\breve{R}_{T0}}^{\breve{R}_T} \frac{\breve{T}\breve{S}}{\breve{R}_T}d\breve{R}_T = \frac{\breve{p}\breve{V}}{\beta_{pV}} + \beta_{ST}\int\limits_{\breve{R}_{T0}}^{\breve{R}_T}(\hat{T})(\hat{S})\frac{d\breve{R}_T}{\breve{R}_T} =$$

$$= U_{cr}\left[q_M N_I + \beta_{ST}\int\limits_{\breve{R}_{T0}}^{\breve{R}_T}(q_M N_I/\breve{R}_T)\ln(q_M^1 N_I)d\breve{R}_T\right] = \breve{a}_\rho\breve{T}/\beta_{pV} + \int\limits_{\breve{a}_{\rho0}}^{\breve{a}_\rho}\breve{A}_\rho d\breve{a}_\rho =$$

$$= U_{cr}\left[\left(\frac{p_l\breve{V}}{U_{cr}}\right)^{-\beta_{pV}}\exp\left(\frac{\beta_{pV}\breve{S}}{\breve{R}_T}\right) + \beta_{pV}\int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\left(\frac{p_l\breve{V}}{U_{cr}}\right)^{-\beta_{pV}}\exp\left(\frac{\beta_{pV}\breve{S}}{\breve{R}_T}\right)\frac{\breve{S}}{\breve{R}_T^2}d\breve{R}_T\right] =$$

$$= U_{cr}\left[\left(\frac{\breve{p}}{\beta_{pV}p_l}\right)^{\frac{\beta_{pV}}{\beta_H}}\exp\left(\frac{\beta_{pV}\breve{S}}{\beta_H\breve{R}_T}\right) + \beta_{pV}\int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\left(\frac{\breve{p}}{\beta_{pV}p_l}\right)^{\frac{\beta_{pV}}{\beta_H}}\exp\left(\frac{\beta_{pV}\breve{S}}{\beta_H\breve{R}_T}\right)\frac{\breve{S}}{\breve{R}_T^2}d\breve{R}_T\right] =$$

$$= \left\{\breve{R}_T\breve{T} + \int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\left[\beta_H\ln\left(\frac{\breve{R}_T\breve{T}}{U_{cr}\beta_{pV}}\right) - \beta_{pV}\ln\left(\frac{\breve{p}}{p_l\beta_{pV}}\right)\right]\breve{T}d\breve{R}_T\right\}\frac{1}{\beta_{pV}} =$$

$$= U_{cr}\left[(\hat{T}) + \beta_{ST}\int\limits_{\breve{R}_{T0}}^{\breve{R}_T}(\hat{T})(\hat{S})\frac{d\breve{R}_T}{\breve{R}_T}\right] = \frac{1}{\beta_{pV}}\left\{\breve{R}_T\breve{T} + \int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\left[\ln\left(\frac{\breve{R}_T\breve{T}}{\beta_{pV}U_{cr}}\right) + \beta_{pV}\ln\left(\frac{p_l\breve{V}}{U_{cr}}\right)\right]\breve{T}d\breve{R}_T\right\},$$

$$\breve{F}_T = \breve{U} - \breve{S}\breve{T} = U_{cr}\left[(\hat{T}) - \beta_{ST}\int\limits_{[(\hat{S})(\hat{T})/\breve{R}_T]_0}^{(\hat{T})(\hat{S})/\breve{R}_T}\breve{R}_T d\left(\frac{(\hat{T})(\hat{S})}{\breve{R}_T}\right)\right] = \breve{F}_{T0} + \int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\frac{\breve{S}\breve{T}}{\breve{R}_T}d\breve{R}_T =$$

$$= U_{cr}\left\{[1 - \ln(\hat{T}) - \beta_{pV}\ln(\hat{V})](\hat{T}) + \int\limits_{\breve{R}_{T0}}^{\breve{R}_T}[\ln(\hat{T}) + \beta_{pV}\ln(\hat{V})](\hat{T})/\breve{R}_T]d\breve{R}_T\right\} =$$

$$= \breve{U}_0(1 - \beta_{pV}\breve{S}/\breve{R}_T + \breve{U}_{ad} = \breve{T}(R_{UT}\breve{Z}/\beta_{pV} - \breve{S}) + \breve{U}_{ad} =$$

$$= \frac{1}{\beta_{pV}}\left\{\breve{R}_T\breve{T}\left[1 - \ln\left(\frac{\breve{R}_T\breve{T}}{\beta_{pV}U_{cr}}\right) - \beta_{pV}\ln\left(\frac{p_l\breve{V}}{U_{cr}}\right)\right] + \int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\left[\ln\left(\frac{\breve{R}_T\breve{T}}{\beta_{pV}}\right) + \beta_{pV}\ln\left(\frac{p_l\breve{V}}{U_{cr}}\right)\right]\breve{T}d\breve{R}_T\right\},$$

$$\breve{H}_T = \breve{U} + \breve{p}\breve{V} = \breve{U} + \breve{R}_T\breve{T} = \breve{U}_0(1 + \beta_{pV}) + \breve{U}_{ad} = R_{UT}\breve{Z}\breve{T}(1/\beta_{pV} + 1) + \breve{U}_{ad} =$$

$$= \breve{H}_{T0} + \int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\frac{\breve{T}\breve{S}}{\breve{R}_T}d\breve{R}_T = \frac{\beta_H\breve{R}_T\breve{T}}{\beta_{pV}} + \int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\frac{\breve{T}\breve{S}}{\breve{R}_T}d\breve{R}_T = U_{cr}\left[\beta_H(\hat{T}) + \beta_{ST}\int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\frac{(\hat{T})(\hat{S})}{\breve{R}_T}d\breve{R}_T\right] =$$

$$= U_{cr}\left[\beta_H\left(\frac{\breve{p}}{\beta_{pV}p_l}\right)^{\frac{\beta_{pV}}{\beta_H}}\exp\left(\frac{\beta_{pV}\breve{S}}{\beta_H\breve{R}_T}\right) + \beta_{pV}\int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\left(\frac{\breve{p}}{\beta_{pV}p_l}\right)^{\frac{\beta_{pV}}{\beta_H}}\exp\left(\frac{\beta_{pV}\breve{S}}{\beta_H\breve{R}_T}\right)\frac{\breve{S}}{\breve{R}_T^2}d\breve{R}_T\right] =$$

$$= \frac{\beta_H}{\beta_{pV}}\left\{\breve{R}_T\breve{T} + \int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\left[\ln\left(\frac{\breve{R}_T\breve{T}}{\beta_{pV}U_{cr}}\right) - \frac{\beta_{pV}}{\beta_H}\ln\left(\frac{\breve{p}}{\beta_{pV}p_l}\right)\right]\breve{T}d\breve{R}_T\right\} =$$

$$= \frac{1}{\beta_{pV}}\left\{\beta_H\breve{R}_T\breve{T} + \int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\left[\ln\left(\frac{\breve{R}_T\breve{T}}{\beta_{pV}U_{cr}}\right) + \beta_{pV}\ln\left(\frac{p_l\breve{V}}{U_{cr}}\right)\right]\breve{T}d\breve{R}_T\right\} =$$

$$= c\psi_{m0}U_{cr}\left\{\frac{\beta_H}{\Gamma_m v_l} + \beta_{ST}\int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\left[\ln\left(\frac{c\psi_{m0}}{\Gamma_m v_l}\right) - (1-1)\ln n_m\right]\frac{d\breve{R}_T}{\breve{R}_T\Gamma_m v_l}\right\},$$

$$\breve{G} = U_{cr}\left[\beta_H(\hat{T}) - \beta_{ST}\int\limits_{[(\hat{S})(\hat{T})/\breve{R}_T]_0}^{(\hat{S})(\hat{T})/\breve{R}_T}\breve{R}_Td\left(\frac{(\hat{S})(\hat{T})}{\breve{R}_T}\right)\right] = \breve{U}_0\left[1 + \beta_{pV}\left(1 - \frac{\breve{S}}{\breve{R}_T}\right)\right] + \breve{U}_{ad} = \breve{T}\left[R_{UT}\breve{Z}\left(\frac{1}{\beta_{pV}} + 1\right) - \breve{S}\right] + \breve{U}_{ad} =$$

$$= \breve{H}_T - \breve{S}\breve{T} = \breve{G}_0 + \int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\frac{\breve{S}\breve{T}}{\breve{R}_T}d\breve{R}_T = \beta_H U_{cr}\left\{\left[1 - \ln(\hat{T}) + \frac{\beta_{pV}}{\beta_H}\ln(\hat{p})\right](\hat{T}) + \int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\left[\ln(\hat{T}) - \frac{\beta_{pV}}{\beta_H}\ln(\hat{p})\right]\frac{(\hat{T})}{\breve{R}_T}d\breve{R}_T\right\} =$$

$$= \breve{H}_{T0} - \int\limits_{\tilde{A}_{\rho0}}^{\tilde{A}_\rho}\breve{R}_T d\breve{A}_\rho = \frac{\beta_H\breve{a}_\rho\breve{T}}{\beta_{pV}} - \int\limits_{\tilde{A}_{\rho0}}^{\tilde{A}_\rho}\breve{a}_\rho d\breve{A}_\rho = \frac{\beta_H\breve{R}_T\breve{T}}{\beta_{pV}}\left[1 - \ln\left(\frac{\breve{R}_T\breve{T}}{\beta_{pv}U_{cr}}\right) + \frac{\beta_{pV}}{\beta_H}\ln\left(\frac{\breve{p}}{\beta_{pV}p_l}\right)\right] +$$

$$+ \int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\left[\frac{\beta_H}{\beta_{pV}}\ln\left(\frac{\breve{R}_T\breve{T}}{\beta_{pV}U_{cr}}\right) - \ln\left(\frac{\breve{p}}{\beta_{pV}p_l}\right)\right]\breve{T}d\breve{R}_T = \frac{\beta_G R_{UT}\breve{Z}\breve{T}}{\beta_{pV}} + \breve{U}_{ad} = U_{cr}f_I[1 + \beta_{pV} - \beta_{ST}\ln(f_I^1 N_I^{1-1})] + \breve{U}_{ad} =$$

$$= c\psi_{m0}U_{cr}\left\{\frac{\beta_H}{\Gamma_m v_l} - \int\limits_{\tilde{A}_{\rho0}}^{\tilde{A}_\rho}\breve{R}_T d\left(\frac{\beta_{ST}}{\breve{R}_T\Gamma_m v_l}\left[\ln\left(\frac{c\psi_{m0}}{\Gamma_m v_l}\right) - (1-1)\ln n_m\right]\right)\right\} =$$

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$$= \frac{c\psi_{m0}U_{cr}\beta_{ST}}{\Gamma_m v_l}\left\{\left[\frac{\beta_H}{\beta_{ST}} - \ln\left(\frac{c\psi_{m0}}{\Gamma_m v_l}\right) + (1-1)\ln n_m\right] + \Gamma_m v_l \int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\left[\ln\left(\frac{c\psi_{m0}}{\Gamma_m v_l}\right) - (1-1)\ln n_m\right]\frac{d\breve{R}_T}{\breve{R}_T\Gamma_m v_l}\right\},$$

where: $\breve{U}_0 = (\hat{T})U_{cr}$, $\breve{H}_{T0} = \beta_H\breve{U}_0$, $\breve{G}_0 = \beta_G\breve{U}_0$, $\breve{F}_{T0}$ are multiplicatively dependent on $q_M = 1/n_m$ and N_I components of instantaneous values of internal energy, enthalpy, Gibbs free energy and Helmholtz free energy of instantaneous Gibbs microstate of matter correspondingly;

$$\breve{U}_{ad} = \sum_{i=2}^{n}\int\limits_{\breve{a}_{i0}}^{\breve{a}_i}\breve{A}_i d\breve{a}_i = \int\limits_{\breve{a}_{\rho 0}}^{\breve{a}_\rho}\breve{A}_\rho d\breve{a}_\rho = \int\limits_{\breve{R}_{T0}}^{\breve{R}_T}(\breve{T}\breve{S}/\breve{R}_T)d\breve{R}_T > 0$$ is instantaneous value of realized via negative feedback

partial additive compensation of multiplicative representation of thermodynamic potentials of microstate of matter (a multiplicative decrease in its free energies over time);

$\breve{a}_\rho \equiv \breve{R}_T = \breve{p}\breve{V}/\breve{T}$ and $\breve{A}_\rho = \breve{T}\breve{S}/\breve{R}_T = \breve{T}^2\breve{S}/\breve{p}\breve{V}$ are respectively, extensive and intensive resulting thermodynamic parameters of the real gas;

$(\hat{S}) = \ln(q_M^1 N_I) = \ln(f_I^1 N^{1-1})$, $(\hat{V}) = q_M^{-1/k}N_I^{-m} = q_M^{m-1/k}f_I^{-m} = f_I^{-1/k}N_I^{1/k-m} = (\Gamma_m v_l/c\psi_{m0})^{1/\beta_{pV}}\exp(\breve{S}/\breve{R}_T)$,

$(\hat{p}) = q_M^{1+1/k}N_I^{1+m} = q_M^{1/k-m}f_I^{1+m} = f_I^{1+1/k}N_I^{m-1/k} = (\Gamma_m v_l/c\psi_{m0})^{-\beta_H/\beta_{pV}}\exp(-\breve{S}/\breve{R}_T)$,

$(\hat{T}) = (\hat{p})(\hat{V}) = q_M N_I = f_I = \chi_m/f_G = \psi_m c/v_{cv} \equiv \psi_{m0}c/\Gamma_m v_l = \psi_{m0}b^{-1/2}/\Gamma_m$ are normalized values of thermodynamic parameters (entropy, molar volume, pressure and temperature) of Gibbs microstates of matter;

$$\beta_{pV} = \frac{\breve{p}\breve{V}}{\breve{U}_0} = \frac{k(1-1)}{k1m-1} > 0, \quad \beta_H = \frac{\breve{H}_{T0}}{\breve{U}_0} = 1 + \beta_{pV} = \frac{k(1m+1-1)-1}{k1m-1}, \quad \beta_{ST} = \frac{\breve{S}\breve{T}}{\breve{U}_0\ln(q_M^1 N_I)} = \frac{km-1}{k1m-1} > 0,$$

$$\beta_{pT} = \beta_{pV}\left(1 - \frac{\breve{S}}{\breve{R}_T}\right), \quad \beta_G = \frac{\breve{G}_0}{\breve{U}_0} = 1 + \beta_{pT} = 1 + \beta_{pV} - \beta_{ST}\ln(q_M^1 N_I) = \beta_H - \frac{1(km-1)}{k1m-1}\ln q_M - \frac{km-1}{k1m-1}\ln N_I =$$

$$= \frac{k(1m+1-1)-1}{k1m-1} - \frac{1(km-1)}{k1m-1}\ln f_I + \frac{(km-1)(1-1)}{k1m-1}\ln N_I,$$

$$\beta_{GR} = \frac{\breve{G}}{\breve{U}_0} = 1 + \beta_{pV} - \frac{\beta_{ST}}{(\hat{T})}\int\limits_{\breve{A}_{\rho 0}}^{\breve{A}_\rho}\breve{R}_T d\breve{A}_\rho = \frac{k(1m+1-1)-1}{k1m-1} - \frac{km-1}{(k1m-1)q_M N_I}\int\limits_{[(\hat{S}\}(\hat{T})/\breve{R}_T]_0}^{(\hat{S})(\hat{T})/\breve{R}_T}\breve{R}_T d\left[\frac{q_M N_I\ln(q_M^1 N_I)}{\breve{R}_T}\right];$$

$p_l(p_{lE},a) = n\,p_{cr} = \mathbf{const}(r)$, $p_{lE} = n_E\,p_{cr} \neq \mathbf{const}(r)$, while: $n = \mathbf{const}(r)$ and $n_E \neq \mathbf{const}(r)$ are the hidden variables that are the indicatorsi of the magnitudes of instantaneous microfluctuations of values of pressure and molar volume when $\breve{p}\breve{V} = \mathbf{const}$ and during not absolutely rigid retention of occupied by gas constant volume in the intrinsic space of matter and in Euclidean space correspondingly [50]; $k = \mathbf{const}(r)$, $1 = \mathbf{const}(r)$, $m = \mathbf{const}(r)$ are spatially homogeneous hidden variables that are indicators of the influence of parameters $q_M = 1/n_m$ and $N_I = \sigma_m/N_{RE}$ on the parameters of thermodynamic microstates of latently coherent matter[5].

Variables k, 1, m and n characterize instantaneous collective microstates of the whole gravithermodynamically bonded matter and similarly to the wave functions of quantum mechanics can take with certain probability any arbitrary instantaneous values. The probability that Gibbs microstate of matter have instantaneous energy, the corresponding certain composition of values of these variables, obviously, is represented by canonic Gibbs distribution. The concrete mathematical expectations $\tilde{k}(R_T)$, $\tilde{l}(R_T)$, $\tilde{m}(R_T)$, $\tilde{n}(R_T)$, $\tilde{n}_E(R_T,a)$ of those variables (that depend on the

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[5] It is possible that latent coherence of matter is brought on (together with the new moment of its proper time) by the next turn of spiral wave of space-time modulation of dielectric and magnetic permeabilities of physical vacuum [46, 51].

parameter R_T) correspond to parameters of c a thermodynamic macrostate of matter. It is exactly the dependence of the mathematical expectation $\tilde{n}_E(R_T, a)$ [6] of the hidden parameter n_E also on the index of curvature a of the intrinsic space of matter that is responsible for its curvature.

Normalized values of thermodynamic parameters of instantaneous microstates of matter are mutually related via the following dependencies:

$$(\hat{S}) = 1\ln q_M + \ln N_I = 1\ln\left(\frac{c\psi_{m0}}{\Gamma_m v_l}\right) - (1-1)\ln\left(\frac{\sigma_m}{N_{RE}}\right) = \frac{\beta_H}{\beta_{ST}}\ln(\hat{T}) - \frac{\beta_{pV}}{\beta_{ST}}\ln(\hat{p}) = \frac{\ln(\hat{T})}{\beta_{ST}} + \frac{\beta_{pV}}{\beta_{ST}}\ln(\hat{V}) = \frac{\beta_H}{\beta_{ST}}\ln(\hat{V}) + \frac{\ln(\hat{p})}{\beta_{ST}},$$

$$\ln(\hat{T}) = \ln q_M + \ln N_I = \ln\left(\frac{c\psi_{m0}}{\Gamma_m v_l}\right) = \frac{\beta_{ST}}{\beta_H}(\hat{S}) + \frac{\beta_{pV}}{\beta_H}\ln(\hat{p}) = \beta_{ST}(\hat{S}) - \beta_{pV}\ln(\hat{V}) = \ln(\hat{p}) + \ln(\hat{V}), \quad (1)$$

$$\ln(\hat{V}) = -\frac{\ln q_M}{k} - m\ln N_I = -m\ln\left(\frac{c\psi_{m0}}{\Gamma_m v_l}\right) + \frac{1-km}{k}\ln n_m = \frac{\beta_{ST}}{\beta_H}(\hat{S}) - \frac{\ln(\hat{p})}{\beta_H} = \frac{\beta_{ST}}{\beta_{pV}}(\hat{S}) - \frac{\ln(\hat{T})}{\beta_{pV}} = \ln(\hat{T}) - \ln(\hat{p}), \quad (2)$$

$$\ln(\hat{p}) = \frac{k+1}{k}\ln q_M + (m+1)\ln N_I = (m+1)\ln\left(\frac{c\psi_{m0}}{\Gamma_m v_l}\right) + \frac{km-1}{k}\ln n_m =$$

$$= \beta_{ST}(\hat{S}) - \beta_H\ln(\hat{V}) = -\frac{\beta_{ST}}{\beta_{pV}}(\hat{S}) + \frac{\beta_H}{\beta_{pV}}\ln(\hat{T}) = \ln(\hat{T}) - \ln(\hat{V}). \quad (3)$$

As it was expected, all instantaneous thermodynamic potentials reach their minimum independently both on the values of variables k, 1, m, n, and on the value of spatial gas-related (liquid-related) parameter $\breve{R}_T$:

$$\left(\frac{\partial \breve{U}}{\partial \breve{R}_T}\right)_{\bar{S},\bar{V}} = 0, \qquad \left(\frac{\partial \breve{H}_T}{\partial \breve{R}_T}\right)_{\bar{S},\bar{p}} = 0, \qquad \left(\frac{\partial \breve{F}_T}{\partial \breve{R}_T}\right)_{\bar{T},\bar{V}} = 0, \qquad \left(\frac{\partial \breve{G}}{\partial \breve{R}_T}\right)_{\bar{T},\bar{p}} = 0.$$

And, moreover, the change in space of available thermodynamic parameters of matter that gradually cools is inevitably accompanied by the change of its hidden thermodynamic parameters Γ_m and v_l:

$$\left(\frac{\partial U}{\partial \hat{r}}\right)_t = T\left(\frac{\partial S}{\partial \hat{r}}\right)_t - p\left(\frac{\partial V}{\partial \hat{r}}\right)_t = -U_0\left[\left(\frac{\partial \ln \Gamma_m}{\partial \hat{r}}\right)_t + \left(\frac{\partial \ln v_l}{\partial \hat{r}}\right)_t\right],$$

$$\left(\frac{\partial H_T}{\partial \hat{r}}\right)_t = T\left(\frac{\partial S}{\partial \hat{r}}\right)_t + V\left(\frac{\partial p}{\partial \hat{r}}\right)_t = -H_{T0}\left[\left(\frac{\partial \ln \Gamma_m}{\partial \hat{r}}\right)_t + \left(\frac{\partial \ln v_l}{\partial \hat{r}}\right)_t\right],$$

$$\mathbf{F}_{gr} = \left(\frac{\partial G}{\partial \hat{r}}\right)_t = -S\left(\frac{\partial T}{\partial \hat{r}}\right)_t + V\left(\frac{\partial p}{\partial \hat{r}}\right)_t = -G_0\left[\left(\frac{\partial \ln \Gamma_m}{\partial \hat{r}}\right)_t + \left(\frac{\partial \ln v_l}{\partial \hat{r}}\right)_t\right],$$

where: $\partial \hat{r}$ is the increment of metric radial distance.

And the bigger the distance from matter to the gravitational attraction center the smaller is its internal energy. That is why in contrast to inert free energy (which is the greater the greater the distance from the substance to the gravitational attraction center) the thermal energy behaves like a negative mass. And this is confirmed by numerous investigations of the influence of heating of matter on its weight [43 – 45].

Precisely the condition of spatial homogeneity of the compressibility coefficient of RGTD-bonded matter $Z(t) = R_T(t)/R_{UT} = \mathbf{const}(r)$ [7] determines the spatial distribution of the set of main thermodynamic parameters of this matter that gradually cools.

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[6] Differential equations of the gravitational field specify only the radial gradients of the parameters a and b, and not their absolute values. Therefore, at the same point in space, the values of not only the real velocity of radiation propagation v_{cm}, but also of the parameter $b = v_l^2 c^{-2}$ may differ for different substances that border each other in the same space. But the value of the parameter a is taken to be the same for them both in GR and in RGTD. However, the possibility of its change together with the change in pressure in the gas cylinder is not excluded.

Of course, every matter has its values of gravitational potentials, since the common for the whole RGTD-bonded matter gravitational field forms only its gradients in the space. However, in order to make all thermodynamic parameters of all individual thermodynamic STC of these matters conformed with the parameters a and b of Schwarzschild solution of common for them gravitational field the appropriate conditions should be fulfilled.

5. Multiplicative representation of thermodynamic parameters and potentials with additive compensation that is invariant in time

Since parameter:

$$R_T = \frac{TS}{A_\rho} = \frac{U_{cr}\widetilde{\beta}_{ST}q_M N_I \ln(q_M^{\tilde{l}} N_I)}{A_\rho} = \frac{U_{cr}\widetilde{\beta}_{ST}c\psi_{m0}[\ln\psi_{m0} - \ln\Gamma_m - \ln(v_l/c) - (1-\tilde{l})\ln(v_{cm}/c)]}{A_\rho\Gamma_m v_l} \neq R_{T0}$$

expressed not only via constants (including also $A_\rho = \textbf{const}(t)$ which characterizes the quasi-equilibrium of the process of cooling down of matter throughout the whole time), but also via velocity of the light in matter v_{cm}, limit velocity v_l of individual (separate) motion and Lorentz shrinkage of dimensions $\Gamma_m \neq \textbf{const}(r)$ ($\psi_m = \psi_{m0}/\Gamma_m \neq \textbf{const}(r)$) of the matter that moves in the process of quasi-equilibriumly cooling down, then only via them we can in temporal form (via A_ρ) or in spatial form (via $a_\rho \equiv R_T$) express instantaneous values of all main thermodynamic parameters and potentials of RGTD-bonded matter:

$$\breve{T} = \frac{\breve{A}_\rho\beta_{pV}}{\beta_{ST}(\mathbb{1}\ln q_M + \ln N_I)} = \frac{\breve{A}_\rho\beta_{pV}}{\beta_{ST}[\ln\psi_{m0} - \ln\Gamma_m - \ln(v_l/c) + (\mathbb{1}-1)\ln(v_{cm}/c)]} =$$

$$= \frac{U_{cr}\beta_{pV}q_M N_I}{\breve{R}_T} = \frac{U_{cr}\beta_{pV}c\psi_{m0}}{\breve{R}_T\Gamma_m v_l} = \frac{\breve{p}\breve{V}}{\breve{R}_T} = \frac{U_{cr}\beta_{pV}}{\breve{R}_T}\left[\frac{U_{cr}}{p_l\breve{V}}\exp\left(\frac{\breve{S}}{\breve{R}_T}\right)\right]^{\beta_{pV}} =$$

$$= \beta_{pV}\breve{A}_\rho\left[\ln\left(\frac{\breve{p}}{p_l\beta_{pV}}\right) + (1+\beta_{pV})\ln\left(\frac{p_l\breve{V}}{U_{cr}}\right)\right]^{-1} = \frac{U_{cr}\beta_{pV}}{\breve{R}_T}\left[\frac{\breve{p}}{p_l\beta_{pV}}\exp\left(\frac{\breve{S}}{\breve{R}_T}\right)\right]^{\frac{\beta_{pV}}{1+\beta_{pV}}},$$

$$\breve{S} = \frac{U_{cr}\beta_{ST}^2 q_M N_I(\mathbb{1}\ln q_M + \ln N_I)^2}{\beta_{pV}\breve{A}_\rho} = \frac{\beta_{ST}\breve{R}_T(\mathbb{1}\ln q_M + \ln N_I)}{\beta_{pV}} =$$

$$= \frac{U_{cr}c\psi_{m0}\beta_{ST}^2[\ln\psi_{m0} - \ln(v_{lc}/c) + (\mathbb{1}-1)\ln(v_{cm}/c)]^2}{\beta_{pV}\breve{A}_\rho v_{lc}} =$$

$$= (\beta_{ST}\breve{R}_T/\beta_{pV})[\ln\psi_{m0} - \ln\Gamma_m - \ln(v_l/c) + (\mathbb{1}-1)\ln(v_{cm}/c)] =$$

$$= \frac{U_{cr}\beta_{pV}\breve{A}_\rho}{\breve{T}^2}\left[\frac{\breve{p}}{p_l\beta_{pV}}\exp\left(\frac{\breve{A}_\rho}{\breve{T}}\right)\right]^{\frac{\beta_{pV}}{\beta_H}} = \frac{U_{cr}\beta_{pV}\breve{A}_\rho}{\breve{T}^2}\left[\frac{U_{cr}}{p_l\breve{V}}\exp\left(\frac{\breve{A}_\rho}{\breve{T}}\right)\right]^{\beta_{pV}} =$$

$$= (\beta_{pV}^{-2}\breve{p}\breve{V}/\breve{A}_\rho)[\ln(\breve{p}/p_l\beta_{pV}) + \beta_H\ln(\breve{V}p_l/U_{cr})]^2,$$

$$\breve{p} = p_l\beta_{pV}q_M^{1+1/k}N_I^{1+m} = p_l\beta_{pV}\left(\frac{c\psi_{m0}}{\Gamma_m v_l}\right)^{1+m}\left(\frac{c}{v_{cm}}\right)^{m-1/k} = \frac{\breve{R}_T\breve{T}}{\breve{V}} =$$

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[7] This is nothing more than the expression of the tendency to align the magnitudes of any intensive parameters of matters in the whole filled with them space. Only such main (field) intensive thermodynamic parameters as temperature and pressure in principle cannot be (or become) absolutely spatially homogenous in quasi-equilibriumly cooling down matter. Some other the fielded intensive thermodynamic parameters, which are related to the possibility of appearance of not only gravitational but also magnetic and electric fields in the RGTD-bonded matter, also cannot become absolutely spatially homogenous.

$$=p_l\beta_{pV}\left(\frac{U_{cr}}{p_l\breve{V}}\right)^{1+\beta_{pV}}\exp\left(\frac{\beta_{pV}\breve{A}_\rho}{\breve{T}}\right)=p_l\beta_{pV}\left(\frac{\breve{R}_T\breve{T}}{U_{cr}\beta_{pV}}\right)^{1+1/\beta_{pV}}\exp\left(-\frac{\breve{S}}{\breve{R}_T}\right)=$$

$$=p_l\beta_{pV}\left(\frac{\breve{S}\breve{T}^2}{U_{cr}\beta_{pV}\breve{A}_\rho}\right)^{1+\frac{1}{\beta_{pV}}}\exp\left(-\frac{\breve{A}_\rho}{\breve{T}}\right)=p_l\beta_{pV}\left(\frac{U_{cr}}{p_l\breve{V}}\right)^{1+\beta_{pV}}\exp\left(\frac{\beta_{pV}\breve{S}}{\breve{R}_T}\right),$$

$$\breve{V}=\frac{U_{cr}}{p_lq_M^{1/k}N_I^{m}}=\frac{U_{cr}}{p_l}\left(\frac{\Gamma_m v_l}{c\psi_{m0}}\right)^{m}\left(\frac{v_{cm}}{c}\right)^{m-\frac{1}{k}}=\frac{\breve{R}_T\breve{T}}{\breve{p}}=$$

$$=\frac{U_{cr}}{p_l}\left[\left(\frac{p_l\beta_{pV}}{\breve{p}}\right)\exp\left(\frac{\beta_{pV}\breve{A}_\rho}{\breve{T}}\right)\right]^{\frac{1}{1+\beta_{pV}}}=\frac{U_{cr}}{p_l}\left(\frac{U_{cr}\beta_{pV}}{\breve{R}_T\breve{T}}\right)^{\frac{1}{\beta_{pV}}}\exp\left(\frac{\breve{S}}{\breve{R}_T}\right)=$$

$$=\frac{U_{cr}}{p_l}\left(\frac{U_{cr}\beta_{pV}\breve{A}_\rho}{\breve{S}\breve{T}^2}\right)^{\frac{1}{\beta_{pV}}}\exp\left(\frac{\breve{A}_\rho}{\breve{T}}\right)=\frac{U_{cr}}{p_l}\left[\left(\frac{p_l\beta_{pV}}{\breve{p}}\right)\exp\left(\frac{\beta_{pV}\breve{S}}{\breve{R}_T}\right)\right]^{\frac{1}{1+\beta_{pV}}},$$

$$\breve{U}=\breve{U}_0+\breve{U}_{ad}=U_{cr}q_MN_I+\int_{\breve{R}_{T0}}^{\breve{R}_T}\breve{S}\breve{T}\frac{d\breve{R}_T}{\breve{R}_T}=\breve{U}_0+\breve{A}_\rho\breve{R}_T-U_{ad}^*=\breve{U}_0+\breve{S}\breve{T}-\int_{\breve{A}_{\rho 0}}^{\breve{A}_\rho}\breve{S}\breve{T}\frac{d\breve{A}_\rho}{\breve{A}_\rho}=$$

$$=U_{cr}[1+\beta_{ST}\ln(q_M^1 N_I)]q_MN_I-\int_{\breve{A}_{\rho 0}}^{\breve{A}_\rho}\breve{R}_Td\breve{A}_\rho=\frac{U_{cr}\psi_{m0}c}{\Gamma_m v_l}+\int_{\breve{R}_{T0}}^{\breve{R}_T}\breve{S}\breve{T}\frac{d\breve{R}_T}{\breve{R}_T}=\breve{S}\breve{T}\left(1+\frac{\breve{T}}{\beta_{pV}\breve{A}_\rho}\right)-\int_{\breve{A}_{\rho 0}}^{\breve{A}_\rho}\breve{S}\breve{T}\frac{d\breve{A}_\rho}{\breve{A}_\rho}=$$

$$=U_{cr}\left(1+\frac{\beta_{pV}\breve{A}_\rho}{\breve{T}}\right)\left[\frac{\breve{p}}{p_l\beta_{pv}}\exp\left(\frac{\breve{A}_\rho}{\breve{T}}\right)\right]^{\frac{\beta_{pV}}{1+\beta_{pV}}}-\int_{\breve{A}_{\rho 0}}^{\breve{A}_\rho}\breve{R}_Td\breve{A}_\rho=$$

$$=U_{cr}\left(1+\frac{\beta_{pV}\breve{A}_\rho}{\breve{T}}\right)\left[\frac{U_{cr}}{p_l\breve{V}}\exp\left(\frac{\breve{A}_\rho}{\breve{T}}\right)\right]^{\beta_{pV}}-\int_{\breve{A}_{\rho 0}}^{\breve{A}_\rho}\breve{R}_Ta\breve{A}_\rho=\frac{\breve{p}\breve{V}}{\beta_{pV}}\left[1+\ln\left(\frac{\breve{p}}{p_l\beta_{pV}}\right)+(1+\beta_{pV})\ln\left(\frac{p_l\breve{V}}{U_{cr}}\right)\right]-\int_{\breve{A}_{\rho 0}}^{\breve{A}_\rho}\breve{R}_Td\breve{A}_\rho,$$

$$\breve{F}_T=\breve{U}_0-\breve{U}_{ad}^*=U_{cr}q_MN_I-\int_{\breve{A}_{\rho 0}}^{\breve{A}_\rho}\breve{R}_Td\breve{A}_\rho=\frac{U_{cr}c\psi_{m0}}{v_{lc}}-\int_{\breve{A}_{\rho 0}}^{\breve{A}_\rho}\breve{R}_Td\breve{A}_\rho=\breve{U}_0-\int_{\breve{A}_{\rho 0}}^{\breve{A}_\rho}\breve{S}\breve{T}\frac{d\breve{A}_\rho}{\breve{A}_\rho}=$$

$$=\frac{\breve{p}\breve{V}}{\beta_{pV}}-\int_{\breve{A}_{\rho 0}}^{\breve{A}_\rho}\frac{\breve{p}\breve{V}}{\beta_{pV}}\left[\ln\left(\frac{\breve{p}}{p_l\beta_{pV}}\right)+(1+\beta_{pV})\ln\left(\frac{p_l\breve{V}}{U_{cr}}\right)\right]\frac{d\breve{A}_\rho}{\breve{A}_\rho}=\frac{\breve{S}\breve{T}^2}{\beta_{pV}\breve{A}_\rho}-\int_{\breve{A}_{\rho 0}}^{\breve{A}_\rho}\breve{S}\breve{T}\frac{d\breve{A}_\rho}{\breve{A}_\rho}=$$

$$=U_{cr}\left\{\left[\frac{\breve{p}}{p_l\beta_{pv}}\exp\left(\frac{\breve{A}_\rho}{\breve{T}}\right)\right]^{\frac{\beta_{pV}}{1+\beta_{pV}}}-\int_{\breve{A}_{\rho 0}}^{\breve{A}_\rho}\frac{\beta_{pV}}{\breve{T}}\left[\frac{\breve{p}}{p_l\beta_{pV}}\exp\left(\frac{\breve{A}_\rho}{\breve{T}}\right)\right]^{\frac{\beta_{pV}}{1+\beta_{pV}}}d\breve{A}_\rho\right\}=$$

$$=U_{cr}\left\{\left[\frac{U_{cr}}{p_l\breve{V}}\exp\left(\frac{\breve{A}_\rho}{\breve{T}}\right)\right]^{\beta_{pV}}-\int_{\breve{A}_{\rho 0}}^{\breve{A}_\rho}\frac{\beta_{pV}}{\breve{T}}\left[\frac{U_{cr}}{p_l\breve{V}}\exp\left(\frac{\breve{A}_\rho}{\breve{T}}\right)\right]^{\beta_{pV}}d\breve{A}_\rho\right\},$$

$$\breve{H}_T=\breve{H}_{T0}+\breve{a}_\rho\breve{A}_\rho-\breve{U}_{ad}^*=\breve{H}_{T0}+\breve{S}\breve{T}-\int_{\breve{A}_{\rho 0}}^{\breve{A}_\rho}\breve{S}\breve{T}\frac{d\breve{A}_\rho}{\breve{A}_\rho}=U_{cr}(1+\beta_{pV})q_MN_I+\int_{\breve{R}_{T0}}^{\breve{R}_T}\breve{A}_\rho d\breve{R}_T=$$

$$=\breve{H}_{T0}+\breve{U}_{ad}=U_{cr}[(1+\beta_{pV})+\beta_{ST}\ln(q_M^1 N_I)]q_M N_I - \int\limits_{\breve{A}_{\rho0}}^{\breve{A}_\rho}\breve{R}_T d\breve{A}_\rho=$$

$$=\frac{U_{cr}(1+\beta_{pV})\psi_{m0}c}{\Gamma_m v_l}+\int\limits_{\breve{R}_{T0}}^{\breve{R}_T}\breve{S}\breve{T}\frac{d\breve{R}_T}{\breve{R}_T}=\breve{S}\breve{T}\left(1+\frac{(1+\beta_{pV})\breve{T}}{\beta_{pV}\breve{A}_\rho}\right)-\int\limits_{\breve{A}_{\rho0}}^{\breve{A}_\rho}\breve{S}\breve{T}\frac{d\breve{A}_\rho}{\breve{A}_\rho}=$$

$$=U_{cr}\left\{\left(1+\beta_{pV}+\frac{\beta_{pV}\breve{A}_\rho}{\breve{T}}\right)\left[\frac{U_{cr}}{p_l\breve{V}}\exp\left(\frac{\breve{A}_\rho}{\breve{T}}\right)\right]^{\beta_{pV}}-\int\limits_{\breve{A}_{\rho0}}^{\breve{A}_\rho}\frac{\beta_{pV}}{\breve{T}}\left[\frac{U_{cr}}{p_l\breve{V}}\exp\left(\frac{\breve{A}_\rho}{\breve{T}}\right)\right]^{\beta_{pV}}d\breve{A}_\rho\right\}=$$

$$=U_{cr}\left(1+\beta_{pV}+\frac{\beta_{pV}\breve{A}_\rho}{\breve{T}}\right)\left[\frac{\breve{p}}{p_l\beta_{pV}}\exp\frac{\breve{A}_\rho}{\breve{T}}\right]^{\frac{\beta_{pV}}{1+\beta_{pV}}}-U_{cr}\int\limits_{\breve{A}_{\rho0}}^{\breve{A}_\rho}\frac{\beta_{pV}}{\breve{T}}\left[\frac{\breve{p}}{p_l\beta_{pV}}\exp\frac{\breve{A}_\rho}{\breve{T}}\right]^{\frac{\beta_{pV}}{1+\beta_{pV}}}d\breve{A}_\rho=$$

$$=\frac{\breve{p}\breve{V}(1+\beta_{pV})}{\beta_{pV}}\left[1+\frac{1}{1+\beta_{pV}}\ln\left(\frac{\breve{p}}{p_l\beta_{pV}}\right)+\ln\left(\frac{p_l\breve{V}}{U_{cr}}\right)\right]-\frac{1}{\beta_{pV}}\int\limits_{\breve{A}_{\rho0}}^{\breve{A}_\rho}\breve{p}\breve{V}\left[\ln\left(\frac{\breve{p}}{p_l\beta_{pV}}\right)+(1+\beta_{pV})\ln\left(\frac{p_l\breve{V}}{U_{cr}}\right)\right]\frac{d\breve{A}_\rho}{\breve{A}_\rho},$$

$$\breve{G}=\breve{H}_{T0}-\breve{U}_{ad}^*=\frac{U_{cr}(1+\beta_{pV})\psi_{m0}c}{v_{lc}}-\breve{U}_{ad}^*=\frac{(1+\beta_{pV})\breve{R}_T\breve{T}}{\beta_{pV}}-\int\limits_{\breve{A}_{\rho0}}^{\breve{A}_\rho}\breve{R}_T d\breve{A}_\rho=\frac{(1+\beta_{pV})\breve{S}\breve{T}^2}{\beta_{pV}\breve{A}_\rho}-\int\limits_{\breve{A}_{\rho0}}^{\breve{A}_\rho}\breve{S}\breve{T}\frac{d\breve{A}_\rho}{\breve{A}_\rho}=$$

$$=\frac{(1+\beta_{pV})\breve{p}\breve{V}}{\beta_{pV}}-\int\limits_{\breve{A}_{\rho0}}^{\breve{A}_\rho}\frac{\breve{p}\breve{V}}{\beta_{pV}}\left[\ln\left(\frac{\breve{p}}{p_l\beta_{pV}}\right)+(1+\beta_{pV})\ln\left(\frac{p_l\breve{V}}{U_{cr}}\right)\right]\frac{d\breve{A}_\rho}{\breve{A}_\rho}=$$

$$=U_{cr}\left\{(1+\beta_{pV})\left[\frac{\breve{p}}{p_l\beta_{pv}}\exp\left(\frac{\breve{A}_\rho}{\breve{T}}\right)\right]^{\frac{\beta_{pV}}{1+\beta_{pV}}}-\int\limits_{\breve{A}_{\rho0}}^{\breve{A}_\rho}\frac{\beta_{pV}}{\breve{T}}\left[\frac{\breve{p}}{p_l\beta_{pV}}\exp\left(\frac{\breve{A}_\rho}{\breve{T}}\right)\right]^{\frac{\beta_{pV}}{1+\beta_{pV}}}d\breve{A}_\rho\right\}=$$

$$=U_{cr}\left\{(1+\beta_{pV})\left[\frac{U_{cr}}{p_l\breve{V}}\exp\left(\frac{\breve{A}_\rho}{\breve{T}}\right)\right]^{\beta_{pV}}-\int\limits_{\breve{A}_{\rho0}}^{\breve{A}_\rho}\frac{\beta_{pV}}{\breve{T}}\left[\frac{U_{cr}}{p_l\breve{V}}\exp\left(\frac{\breve{A}_\rho}{\breve{T}}\right)\right]^{\beta_{pV}}d\breve{A}_\rho\right\},$$

where: $\quad\left(\dfrac{\partial\breve{U}}{\partial\breve{A}_\rho}\right)_{\breve{S},\breve{V}}=0$, $\quad\left(\dfrac{\partial\breve{H}_T}{\partial\breve{A}_\rho}\right)_{\breve{S},\breve{p}}=0$, $\quad\left(\dfrac{\partial\breve{F}_T}{\partial\breve{A}_\rho}\right)_{\breve{T},\breve{V}}=0$, $\quad\left(\dfrac{\partial\breve{G}}{\partial\breve{A}_\rho}\right)_{\breve{T},\breve{p}}=0$;

$v_{lc}=\Gamma_m v_l$ is the limit velocity of individual (separate) motion of matter that gradually cools in comoving with it FR (in its own space-time continuum (STC), in which the radial motion of molecules of matter that gradually cools is absent); $\breve{U}_{ad}^*=\int\limits_{\breve{A}_{\rho0}}^{\breve{A}_\rho}\breve{R}_T d\breve{A}_\rho=\int\limits_{\breve{A}_{\rho0}}^{\breve{A}_\rho}(\breve{S}\breve{T}/\breve{A}_\rho)d\breve{A}_\rho\geq0$ is instantaneous value of partial additive compensation of multiplicative representation of thermodynamic potentials of matter microstate (multiplicative increase of bound energy as we approach the gravitational attraction center); $\breve{G}$ is the instantaneous value of Gibbs energy G (that is similar to the Lagrangian, since it constantly tends to its minimum too).

As we can see, due to $A_\rho(r)=\mathbf{const}(t)$ when homogeneous matter is quasi-equilibrium cooling down the gravitational changes in time of its Gibbs free energy and Helmholtz free energy take

place similarly to the changes in space of multiplicative component of internal energy U_0 and enthalpy H_{T0} correspondingly[8]. Precisely, if:

$$\left(\frac{\partial U}{\partial \hat{t}}\right)_r = -(U_0 + U_{ad}^*)\left\{1 + \frac{\tilde{\beta}_{ST}}{1 + \tilde{\beta}_{ST}\left[\ln(c\psi_{m0}/v_{lc}) + (\tilde{l}-1)\ln n_m\right]}\right\}\left[\left(\frac{\partial \ln v_l}{\partial \hat{t}}\right)_r + \left(\frac{\partial \ln \Gamma_m}{\partial \hat{t}}\right)_r\right] +$$

$$+ \frac{(U_0 + U_{ad}^*)\tilde{\beta}_{ST}(\tilde{l}-1)}{1 + \tilde{\beta}_{ST}[\ln(c\psi_{m0}/\Gamma_m v_l) + (\tilde{l}-1)\ln n_m]}\left(\frac{\partial \ln n_m}{\partial \hat{t}}\right)_r , \text{ then:}$$

$$\left(\frac{\partial F_T}{\partial \hat{t}}\right)_r = -S\left(\frac{\partial T}{\partial \hat{t}}\right)_r - p\left(\frac{\partial V}{\partial \hat{t}}\right)_r = -U_0\left(\frac{\partial \ln v_{lc}}{\partial \hat{t}}\right)_r = -U_0\left[\left(\frac{\partial \ln \Gamma_m}{\partial \hat{t}}\right)_r + \left(\frac{\partial \ln v_l}{\partial \hat{t}}\right)_r\right],$$

$$\left(\frac{\partial G}{\partial \hat{t}}\right)_r = -S\left(\frac{\partial T}{\partial \hat{t}}\right)_r + V\left(\frac{\partial p}{\partial \hat{t}}\right)_r = -H_{T0}\left(\frac{\partial \ln v_{lc}}{\partial \hat{t}}\right)_r = -H_{T0}\left[\left(\frac{\partial \ln \Gamma_m}{\partial \hat{t}}\right)_r + \left(\frac{\partial \ln v_l}{\partial \hat{t}}\right)_r\right],$$

where: $\partial \hat{t}$ is the increment of metric time of matter that gradually cools in a comoving with it FR.

Moreover, the multiplicative component of only the Gibbs free energy is proportional to the absolute temperature of matter not only in space but also in time. It is smaller the lower the temperature and, consequently, the higher the limit velocity v_{lc} of individual motion of the matter molecules (and the corresponding coordinate pseudo-vacuum velocity of light v_{cv} of GR).

The equations of thermodynamic state of gas for a predominantly hydrogen Universe would be as follows:

$$^U U = U_0 + A_\rho R_T , \qquad ^U F = U_0 , \qquad ^U H_T = H_{T0} + A_\rho R_T , \qquad A_\rho = T^2 S / pV = \textbf{const}(t) ,$$

$$^U G = {}^U H_{T0} = \frac{\tilde{\beta}_H R_T T}{\tilde{\beta}_{pV}} = \frac{\tilde{\beta}_H pV}{\tilde{\beta}_{pV}} = \frac{\tilde{\beta}_H S T^2}{\tilde{\beta}_{pV} A_\rho} = U_{cr}\tilde{\beta}_H\left(\frac{p}{\tilde{\beta}_{pV} p_l}\right)^{\frac{\tilde{\beta}_{pV}}{\tilde{\beta}_H}} \exp\left(\frac{\tilde{\beta}_{pV} S}{\tilde{\beta}_H R_T}\right),$$

$$d^U U = TdS - pdV + A_\rho dR_T , \quad d^U F = TdS - pdV , \quad d^U H_T = TdS + Vdp + A_\rho dR_T , \quad d^U G = TdS + Vdp$$

In the process of free fall of matter in gravitational field the Helmholtz and Gibbs thermodynamic free energies, as well as the Hamiltonian of inert free energy of matter, are conserved not only due to the presence of weightlessness in its FR ($v_{lc} = v_l \Gamma_m = \textbf{const}(t)$), but also due to the total compensation of the influence of gravitation on its thermodynamic state by the motion. However it is possible only in hypothetical absolutely empty space. But when there is a resistance to motion these energies will be gradually increasing due to the matter cannot reach the required for their conservation value Γ_m and thus also due to accommodation of the matter of falling body to the new thermodynamic state of matter of the environment.

Before the appearance of spatial inhomogeneity of limit velocity of individual motion of matter v_l the only thing that could interfere its distancing from the future gravitational attraction center (due to tending of its Gibbs thermodynamic energy to its minimum) was the electromagnetic interaction of its molecules. That is why the hypothetic ideal gas and ideal liquid in principle cannot create their gravitational field.

As we see, here we have a dependency of spatial distribution of intrinsic values of these thermodynamic parameters and potentials (not the dependency of spatial distribution of other their values observed by other clocks and by other length standards) on v_{lc} (and, so, also on Γ_m and v_l). It would be non-logical if Γ_m and v_l would not influence on spatial distribution of the set of intrinsic values of main thermodynamic parameters of matter. So this does not contradict to invariance of thermodynamic parameters and potentials of matter relatively to the space-time

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[8] The authors of GR, obviously, intuitively understood this fact. That is why GR is the genial creation, despite the fact that it ignores the principal invariance of thermodynamic parameters and potentials relatively to space-time transformations.

transformations [46]. On the contrary, it only confirms the fact that limit velocity v_l of individual motion of defined substance, as well as Γ_m, is the internal hidden RGTD-parameter of substance and not the non-dependent on certain RGTD-state of external gravitational parameter of substance.

6. Physical and other thermodynamic characteristics of matter

Obviously, the stability of magnitude of extensive parameter $A_\rho = T^2 S / pV$ takes place in the process of quasi-equilibrium cooling down of matter. If we experimentally find its averaged value for researched matter at the beginning of the research or if we measure the increments of thermodynamic parameters:

$$\left(\frac{\partial \ln S}{\partial \hat{t}}\right)_r = \left(\frac{\partial \ln p}{\partial \hat{t}}\right)_r + \left(\frac{\partial \ln V}{\partial \hat{t}}\right)_r - 2\left(\frac{\partial \ln T}{\partial \hat{t}}\right)_r,$$

we can determine its entropy:

$$S = \frac{U_{cr}\widetilde{\beta}_{pV}A_\rho}{T^2}\left[\frac{p}{p_l\widetilde{\beta}_{pv}}\exp\left(\frac{A_\rho}{T}\right)\right]^{\frac{\widetilde{\beta}_{pV}}{\widetilde{\beta}_H}} = \frac{U_{cr}\widetilde{\beta}_{pV}A_\rho}{T^2}\left[\frac{U_{cr}}{p_lV}\exp\left(\frac{A_\rho}{T}\right)\right]^{\widetilde{\beta}_{pV}} = \frac{pV}{\widetilde{\beta}_{pV}^2 A_\rho}\left[\ln\left(\frac{p}{\widetilde{\beta}_{pV}p_l}\right) + \widetilde{\beta}_H\ln\left(\frac{p_lV}{U_{cr}}\right)\right]^2 =$$

$$= \frac{A_\rho R_T}{T} = R_T\left[\frac{1}{\widetilde{\beta}_{pV}}\ln\left(\frac{R_TT}{\widetilde{\beta}_{pV}U_{cr}}\right) + \ln\left(\frac{p_{cr}V}{U_{cr}}\right)\right] = R_T\left[\widetilde{\varepsilon} + \frac{1}{\widetilde{\beta}_{pV}}(\ln R_T + \ln T) + \ln V\right] =$$

$$= R_T\left[\left(1 + \frac{1}{\widetilde{\beta}_{pV}}\right)\ln\left(\frac{R_TT}{\widetilde{\beta}_{pV}U_{cr}}\right) - \ln\left(\frac{p}{\widetilde{\beta}_{pV}\widetilde{p}_l}\right)\right] = R_T\left[\widetilde{\varepsilon} + \left(1 + \frac{1}{\widetilde{\beta}_{pV}}\right)(\ln R_T + \ln T) - \ln p\right] =$$

$$= R_T\left[\left(1 + \frac{1}{\widetilde{\beta}_{pV}}\right)\ln\left(\frac{\widetilde{p}_lV}{U_{cr}}\right) + \frac{1}{\widetilde{\beta}_{pV}}\ln\left(\frac{p}{\widetilde{\beta}_{pV}\widetilde{p}_l}\right)\right] = R_T\left[\widetilde{\varepsilon} + \left(1 + \frac{1}{\widetilde{\beta}_{pV}}\right)\ln V + \frac{\ln p}{\widetilde{\beta}_{pV}}\right],$$

where: $\widetilde{\varepsilon} = \ln\widetilde{p}_l - (1 + 1/\widetilde{\beta}_{pV})\ln(m_{00}c^3/v_{lcr}) - (\ln\widetilde{\beta}_{pV})/\widetilde{\beta}_{pV} = \mathbf{const}(\Delta t)$ and $\widetilde{\beta}_{pV} = \mathbf{const}(\Delta t)$ are mathematical expectations of the values of functions of arbitrary changing hidden variables k, l , m, n, which are the strictly constant magnitudes during the whole not very long time of the existence of any Gibbs thermodynamic microstate.

However, if we know $\widetilde{\varepsilon}$ and $\widetilde{\beta}_{pV}$ and if we determine only the molar volume of gas that gradually cools and the pressure in it, then we can determine only its bond energy:

$$W_{bnd} = ST = Vp\left[\widetilde{\varepsilon} + \left(1 + \frac{1}{\widetilde{\beta}_{pV}}\right)\ln V + \frac{\ln p}{\widetilde{\beta}_{pV}}\right]. \tag{6}$$

In order to determine the entropy and, thus, the value of parameter A_ρ we should additionally measure the temperature of the gas. It is obvious that parameters $\widetilde{\varepsilon}$ and $\widetilde{\beta}_{pV}$ of the equation of the state of gas (6) can be determined also experimentally in the process of controlled change of its bond energy as well as of all its thermodynamic parameters.

The research of thermodynamic properties of matter should be performed only in its equilibrium states or using the dependencies of its thermodynamic potentials on thermodynamic parameters that take into account the variability of parameter R_T in the process of this research ($R_T \neq \mathbf{const}(t)$). In order to determine both the thermal expansion coefficient α and pressure γ and the elastic modulus K_T of gas or liquid it is enough to know only the thermal equation of the state (i.e. the gas compressibility coefficient $Z = R_T / R_{UT}$, which is determined by the parameter $R_T = pV / R_T$):

$$\alpha=\frac{1}{V_0}\left(\frac{\partial V}{\partial T}\right)_p=\frac{1}{V_0 p}\left[R_T+T\left(\frac{\partial R_T}{\partial T}\right)_p\right], \qquad \gamma=\frac{1}{p_0}\left(\frac{\partial p}{\partial T}\right)_V=\frac{1}{p_0 V}\left[R_T-T\left(\frac{\partial R_T}{\partial T}\right)_V\right],$$

$$K_T=-V_0\left(\frac{\partial p}{\partial V}\right)_T=\frac{R_T T}{V_0}-T\left(\frac{\partial R_T}{\partial V}\right)_T.$$

In order to determine their thermal capacity when volume and pressure are invariant[9] and, thus, all their thermodynamic potentials we should know not only R_T and critical phasic values of pressure p_{cr} and of internal energy multiplicative component $U_{cr}=m_{00}c^3/v_{lcr}$, but also the mathematical expectation $\widetilde{\beta}_{pV}$ of the value of hidden variable $\beta_{pV}=\breve{p}\breve{V}/\breve{U}_0$:

$$C_V=\left(\frac{\partial U}{\partial T}\right)_V=\frac{1}{\widetilde{\beta}_{pV}}\left[R_T+T\left(1+\frac{\widetilde{\beta}_{pV}S}{R_T}\right)\left(\frac{\partial R_T}{\partial T}\right)_V\right]=$$

$$=\frac{1}{\widetilde{\beta}_{pv}}\left\{R_T+T\left[1+(1+\widetilde{\beta}_{pV})\ln\left(\frac{R_T T}{\widetilde{\beta}_{pV}U_{cr}}\right)-\widetilde{\beta}_{pV}\ln\left(\frac{p}{\widetilde{\beta}_{pV}p_l}\right)\right]\left(\frac{\partial R_T}{\partial T}\right)_V\right\}=$$

$$=T\left(\frac{\partial S}{\partial T}\right)_V=\frac{1}{\widetilde{\beta}_{pv}}\left\{R_T+T\left[1+\ln\left(\frac{R_T T}{\widetilde{\beta}_{pV}U_{cr}}\right)+\widetilde{\beta}_{pV}\ln\left(\frac{p_l V}{U_{cr}}\right)\right]\left(\frac{\partial R_T}{\partial T}\right)_V\right\},$$

$$C_p=T\left(\frac{\partial S}{\partial T}\right)_p=\left(\frac{\partial U}{\partial T}\right)_p+R_T+T\left(\frac{\partial R_T}{\partial T}\right)_p=\frac{1+\widetilde{\beta}_{pV}}{\widetilde{\beta}_{pV}}\left[R_T+T\left(1+\frac{\widetilde{\beta}_{pV}S}{(1+\widetilde{\beta}_{pV})R_T}\right)\left(\frac{\partial R_T}{\partial T}\right)_p\right],$$

$$C_p-C_V=R_T+T\left[\left(\frac{1+\widetilde{\beta}_{pV}}{\widetilde{\beta}_{pV}}+\frac{S}{R_T}\right)\left(\frac{\partial R_T}{\partial T}\right)_p-\left(\frac{1}{\widetilde{\beta}_{pv}}+\frac{S}{R_T}\right)\left(\frac{\partial R_T}{\partial T}\right)_V\right], \qquad U=\frac{R_T T}{\widetilde{\beta}_{pV}}+\int_{R_{T0}}^{R_T}\frac{TS}{R_T}dR_T.$$

Based on thermodynamic dependencies of thermal capacities when volume and pressure are invariant we can determine the mathematical expectation of dependencies of these functions on individual parameters R_T and θ, and, consequently, on any pair of main thermodynamic parameters:

$$\widetilde{\beta}_{pV}=\frac{R_T T}{U_0}=\frac{R_T\left[\left(\frac{\partial R_T}{\partial T}\right)_p-\left(\frac{\partial R_T}{\partial T}\right)_V\right]}{C_V\left(\frac{\partial R_T}{\partial T}\right)_p-(C_p-R_T)\left(\frac{\partial R_T}{\partial T}\right)_V+T\left(\frac{\partial R_T}{\partial T}\right)_p\left(\frac{\partial R_T}{\partial T}\right)_V}.$$

The following correspond to the thermal Van der Waals equation [1] of the state of real gases:

$$R_T=\frac{pV}{T}=\frac{R_{UT}}{(1+a_m/pV^2)(1-b_m/V)}=\frac{R_{UT}V}{V-b_m}-\frac{a_m}{TV}=R_{UT}\left[\frac{V}{V-b_m}-\theta\right], \qquad \theta=\frac{a_m}{R_{UT}TV},$$

$$dV=\frac{V^2[R_{UT}dT-(V-b_m)dp]}{pV^2-a_m(1-2b_m/V)}, \qquad \left(\frac{\partial R_T}{\partial T}\right)_V=\frac{a_m}{T^2V}=\frac{R_{UT}\theta}{T},$$

$$\left(\frac{dR_T}{\partial T}\right)_p=\frac{p}{T}\left[\left(\frac{\partial V}{\partial T}\right)_p-\frac{V}{T}\right]=\frac{a_m(1-2b_m/V)/(TV)+R_{UT}-R_T}{T[1-a_m(1-2b_m/V)/(pV^2)]}=\frac{R_T\{R_{UT}[1+\theta(1-2b_m/V]-R_T\}}{T[R_T-R_{UT}\theta(1-2b_m/V)]},$$

$$\widetilde{\beta}_{pV}=\frac{R_T\{R_T[R_{UT}(1-2\theta b_m/V)-R_T]+R_{UT}^2\theta^2(1-2b_m/V)\}}{R_T(C_V+R_{UT}\theta)\{R_{UT}[1+\theta(1-2b_m/V)]-R_T\}-R_{UT}\theta(C_p-R_T)[R_T-R_{UT}\theta(1-2b_m/V)]},$$

where: a_m and b_m are individual constants of certain matter.

According to this we receive the simple expression for the entropy of hypothetical ideal gas:

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[9] Heat capacity when pressure is invariant is determined not by internal energy of the matter itself, but by equivalent to its enthalpy the energy of extended system that consists of this matter and the load that supports the needed pressure.

$$S=C_{V0}\ln\left(\frac{C_{V0}T}{U_{cr}}\right)+R_{UT}\ln\left(\frac{C_{V0}p_{cr}V}{R_{UT}U_{cr}}\right)=S_{cr}+C_{V0}\ln\left(\frac{T}{T_{cr}}\right)+R_{UT}\ln\left(\frac{V}{V_{cr}}\right)=S_k+C_{V0}\ln\left(\frac{T}{T_k}\right)+R_{UT}\ln\left(\frac{V}{V_k}\right),$$

where: $S_{cr}=(C_{V0}+R_{UT})\ln(C_{V0}T_{cr}/U_{cr})$, $C_{V0}=R_{UT}/\widetilde{\beta}_{pV}$; $V_{cr}=R_{UT}T_{cr}/p_{cr}$ and T_{cr} are critical phasic values of molar volume and temperature of ideal gas; V_k and T_k are their another arbitrary values.

The following expressions correspond to more precise Dieterici [2] first thermal equation that uses an exponent with the same parameter $\theta=a_m/(R_{UT}TV)$:

$$R_T=\frac{R_{UT}}{1-b_m/V}\exp(-\theta), \qquad\qquad dV=\frac{R_{UT}(1+\theta)\exp(-\theta)dT-(V-b_m)dp}{p-a_mV^{-2}\exp(-\theta)},$$

$$\left(\frac{dR_T}{\partial T}\right)_p=\frac{R_{UT}(1+2\theta)\exp(-\theta)-R_T}{T\left[1-a_m\exp(-\theta)/(pV^2)\right]}=\frac{R_T\left[R_{UT}(1+2\theta)\exp(-\theta)-R_T\right]}{T\left[R_T-R_{UT}\theta\exp(-\theta)\right]}=\frac{R_T\Psi}{T},$$

$$\left(\frac{\partial R_T}{\partial T}\right)_V=\frac{R_T\theta}{T}, \qquad C_V=(1+\theta)\frac{R_T}{\widetilde{\beta}_{pV}}+\theta\,S, \qquad C_p=(1+\Psi)\frac{(1+\widetilde{\beta}_{pV})R_T}{\widetilde{\beta}_{pV}}+\Psi S,$$

$$C_p-C_V=[(1+\Psi)(1+1/\widetilde{\beta}_{pV})-(1+\theta)/\widetilde{\beta}_{pV}]+(\Psi-\theta)S,$$

$$\widetilde{\beta}_{pV}=\frac{(\Psi-\theta)R_T}{\Psi C_p-\theta C_p+\theta(1+\Psi)R_T}=\frac{R_T(1+\theta)[R_{UT}(1+\theta)\exp(-\theta)-R_T]}{R_{UT}\left[(1+2\theta)C_V+\theta^2C_p+\theta(1+\theta)R_T\right]\exp(-\theta)-R_T(C_V+\theta C_p)}.$$

Obviously, experimentally found heat capacities of gases can be represented as functions of only R_T and θ parameters.

Hidden variables β_{ST} and β_{pV} are invariant magnitudes in any moment of time that corresponds to the certain Gibbs collective microstate of the whole RGTD-bonded matter. And, thus, their derivatives by any thermodynamic parameter are equal to zero. The same can be told regarding mathematical expectations of those hidden variables $\widetilde{\beta}_{ST}$ and $\widetilde{\beta}_{pV}$, despite the dependence of their values on other thermodynamic parameters of matter.

Conclusion

Now we know for sure about the four hidden thermodynamic parameters (k, l , m, and n) and functions on them (β_{pV}, β_H, β_{ST}, β_{pT}, β_G, β_{GR}), and functions of the explicit thermodynamic parameters (A_ρ, $a_\rho\equiv R_T$, Z, U_{ad}, U_{ad}^*, C_p, C_V, K_T, α, γ and etc), as well as critical (U_{cr}, S_{cr}, p_{cr}, T_{cr}) and spatially homogeneous (T_{00}, U_{00}, H_{00}, G_{00}) thermodynamic parameters. That is why it is now possible to experimentally determine at the points of phase transitions the critical values, and at the control points the standard values of the mathematical expectations of the hidden parameters, as well as of the corresponding explicit thermodynamic parameters. And on the basis of all this it is possible to obtain exact equations of thermodynamic state of real gases and liquids without using corrections. And this is facilitated by the spatial homogeneity of the hidden thermodynamic parameters, which correspond to the spatially inhomogeneous states of real matter. Of course, this will not deny the expediency of using also approximate equations of thermodynamic state of matter based on the use of corrections.

Enthalpy, which consists of the Lagrangian of its own multiplicative component and additive compensation of its multiplicative representation, is de facto the total energy of matter since it includes even the released thermal energy and the released kinetic energy of its motion. Enthalpy of matter (as well as Gibbs free energy, which multiplicative component is identical to the ordinary rest energy of matter and is equivalent to its gravitational mass) is equal in all FRs of bodies that move inertially relatively to it. And exactly this is the guarantee of Lorentz-invariance of all thermodynamic potentials and parameters of matter. Since matter motion is accompanied by the all-sided conformally-gaugely self-contraction of its size in background Euclidean space of the Universe the rate of the intrinsic time of inertially moving body is not dilated but, quite the

contrary, remains invariant, despite the presence of gravitational decreasing of the rate of intrinsic time for nearby static objects. De facto the motion of the matter as well as its gravitational self-contraction in background Euclidean space of the Universe leads to its advance over unobservable in people's world evolutionary self-contraction of the conventionally motionless matter in the Universe. That is why the release of kinetic energy is always accompanied by the decreasing of limit velocity of matter individual (separate) motion (that is equivalent to coordinate velocity of light in GR) and the decreasing of its inert free energy.

The ordinary rest energy of matter is bonded in a different ways in different physical processes. That is why we have various free energies in different processes. Both the change of the inert free energy of matter (caused by its inertial motion) and its evolutionary decrease in CFREU do not directly influence the thermodynamic parameters of matter that are changed only in thermodynamic processes. That is why it is fundamentally unobservable in intrinsic FRs of matter in the similar way as evolutionary and caused by motion reduction of molar volume of matter is unobservable in comoving with expanding Universe FR. The gravitational reduction of molar volume of matter when approaching the gravitational attraction center is also unobservable directly in intrinsic FRs of matter. However, we still can say about its presence in Euclidean space of CFREU due to the presence of gravitational curvature of intrinsic space of matter.

The hidden thermodynamic parameters discovered here (which, similarly to wave functions, can take on any values with a certain probability) confirmed Gibbs's idea of the presence of a multitude of instantaneous thermodynamic microstates in matter that is in an invariable quasi-equilibrial state. The another important thing is the substantiation of the fact that the limit velocity of individual (separate) motion of matter and the indicators of the relativistic-gravitational decrease in the molar volume of matter in the background Euclidean space do belong to the hidden internal thermodynamic parameters [50]. Therefore, an external relativistic interpretation of thermodynamics is not needed. And therefore, all thermodynamic parameters and potentials are invariant both gravitationally and relativistically. After all, all gravitational and relativistic influences on them are already contained in their formation as hidden parameters.

The acceptance of the fact of existence of an extensive parameter (that is spatially homogeneous in the gravitational field), characterizing the compressibility of gases, and a spatially inhomogeneous intensive thermodynamic parameter (that is invariant during the gas leisurely cooling process) allows us to consider the gravitational field as a consequence of a spatially inhomogeneous gravithermodynamic state of both any continuous matter and arbitrarily rarefied gas-dust matter of the cosmic vacuum.

The proved here equivalence of the Hamiltonian of the inert free energy to the inertial mass of matter, and the equivalence of Lagrangian of the ordinary rest energy (and the multiplicative component of the Gibbs free energy identical to it) of matter to a much larger gravitational mass [41, 42] have solved the problem of the shortage in baryonic mass in distant galaxies of the Universe. And it is this, together with the logarithmic gravitational potential [52, 53], that allows us to abandon the need for non-baryonic dark matter in the Universe.

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