

A Geometric Framework of Complementary Projections in Physics

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The purpose of this study is to show that many fundamental equations of physics can be traced back to a common geometric principle rooted in the Pythagorean theorem and the Archimedean tradition. The φ -parameterization acts as a bridge between classical, relativistic, and quantum physics, while also extending naturally to cosmological applications. The proposed framework does not contradict established physics nor introduce new postulates; rather, it provides a visual, unifying, and numerically stable representation of physical laws. In addition to its geometric clarity, the φ -formalism possesses an important computational advantage — numerical stability.

Φ -geometry offers a single way to express physical relationships through the geometric parameter φ , linking energy, momentum, velocity, and spatial curvature. The fundamental relation $\cos^2 \varphi + \sin^2 \varphi = 1$ encapsulates the balance between two complementary components — potential ($\cos \varphi$) and dynamic ($\sin \varphi$). This representation unifies classical, relativistic, and quantum equations within a single geometric framework, where limiting states (matter — $\varphi \rightarrow 0^\circ$, light — $\varphi \rightarrow 90^\circ$) are described smoothly and without singularities.

1. Introduction: The Geometric Foundation of φ

Φ -geometry continues the long tradition of geometric thinking in physics — from Huygens' wave fronts and Poincaré's spheres to Penrose's twistor theory. Its contribution is not to replace existing theories. Instead, it aims to unify relativistic and quantum relations within a single trigonometric structure, while maintaining full consistency with established physical laws and emphasizing their geometric kinship.

The geometry of φ is based on representing physical relations as a rotating radius vector on a circle. The components $\sin \varphi$ and $\cos \varphi$ are interpreted as interconnected quantities — the dynamic and potential parts of a physical state. In the simplest case, the radius is taken as unity, which reflects the universality and dimensionless nature of this description.

In the proposed geometry, the subsequent exposition is accompanied by clear visualizations.

Figure 1 shows the basic unit circle, where the physical state is represented by a radius vector, and its projections $\sin \varphi$ and $\cos \varphi$ correspond to the dynamic and potential components, respectively.

Figure 2 presents the light circle, in which the radius is expressed by the speed of light c and the leg by the velocity v ; this makes it possible to visualize the connection between geometry and the relativistic factor γ , as well as the smooth transition between the states of matter and light. This figure also allows one to visualize the geometry of relativistic velocity addition and its sinusoidal–cosinusoidal dependence.

Figure 3 shows the complete φ -map with two conjugate branches ($s = \pm 1$) separated by the φ -switch axis; it demonstrates the symmetry between the active and the conjugate states of the system.

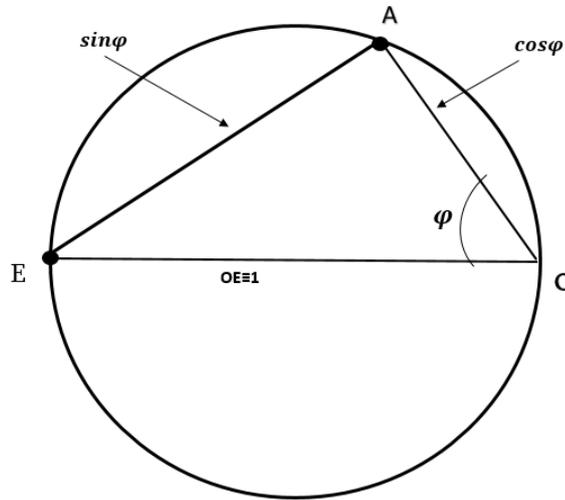


Fig. 1. The basic unit circle: $\sin \varphi$ and $\cos \varphi$ as the projections of a state on the φ -plane.

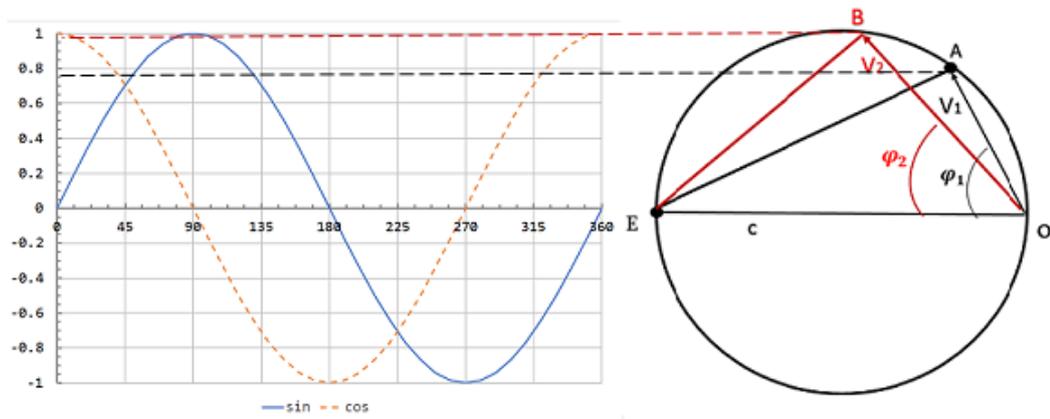


Fig. 2. The light circle and the wave pair \sin - \cos .

The radius is expressed by the speed of light c , and the cathetus (leg) by the velocity v . The relation $\sin \varphi = v/c$ and $\cos \varphi = 1/\gamma$ shows a direct link to the Lorentz factor $\gamma = 1 / \sqrt{1 - v^2/c^2}$. This form is equivalent to the unit φ -circle, but it clearly illustrates the physical scale and the smooth transition between the states of matter and light. On the right panel, the geometry of two velocities v_1 and v_2 (angles φ_1 , φ_2) is shown. On the left panel, their wave projections are displayed. The limit $|\beta| < 1$ corresponds to the inner region of the circle.

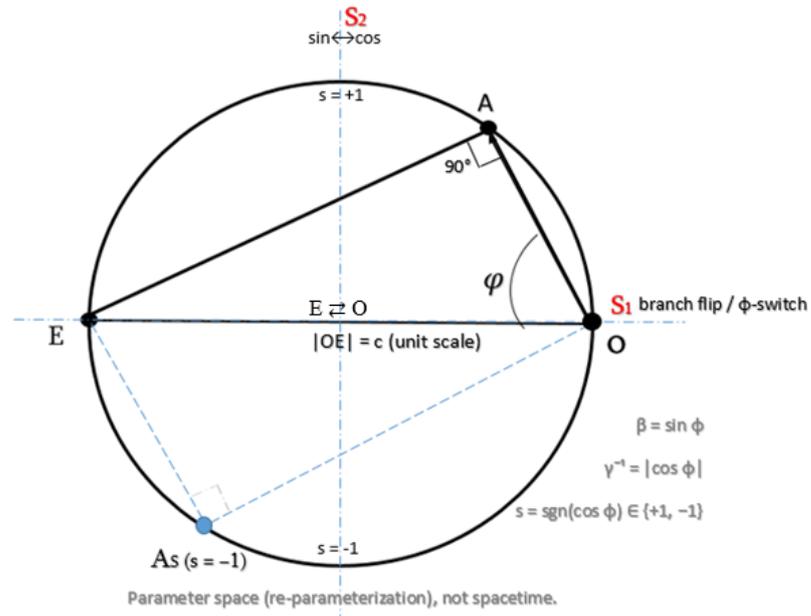


Fig. 3. The complete φ -map and its two branches. Parameterization: $\beta = \sin \varphi$, $\gamma^{-1} = |\cos \varphi|$, $s = \text{sgn}(\cos \varphi)$. The diameter OE represents the S_1 -axis (the φ -switch), and the perpendicular line through the center represents the S_2 -axis. Point A lies on the branch $s = +1$, while its mirror point A_s lies on $s = -1$. Points O and E describe the same rest state ($\beta = 0$) but belong to different branches.

Interpretation of Fig. 3.

The circle serves as a parametric space, not as a space-time metric.

The upper branch ($s = +1$) is the active (working) one, while the lower branch ($s = -1$) is its conjugate counterpart.

Symmetries:

- S_1 (φ -switch): $(\sin \varphi, \cos \varphi) \rightarrow (\sin \varphi, -\cos \varphi)$ — changes the sign of the branch.
- S_2 (exchange of components): $(\sin \varphi, \cos \varphi) \rightarrow (\cos \varphi, \sin \varphi)$ — corresponds to the exchange $pc \leftrightarrow mc^2$.

2. The Fundamental Equation of φ

The main relation of φ -geometry, shown in Fig. 1, is expressed by the equation of the unit circle:

$$\cos^2\varphi + \sin^2\varphi = 1 \quad [1].$$

This equation reflects the invariance of a system’s state during transitions between its potential and dynamic components. Instead of using separate formulas for energy, momentum, and velocity, φ -geometry represents them as mutually orthogonal projections of a single state vector [2-4].

Research Objectives

- To introduce the parameter φ as a geometric invariant that unifies mass, energy, and momentum [2].
- To show that most physical equations can be expressed through a φ -balance ($\cos \varphi \leftrightarrow \sin \varphi$) [3-4].

- To demonstrate that cosmological acceleration and phenomena similar to dark energy can be interpreted as phase shifts of φ , without invoking new fields [5-8].
- To show that φ -geometry can serve as a common language linking existing theories, improving the clarity and structural coherence of physical descriptions [2, 9-10].

3. De Broglie at a Glance — the Key to Reading φ -Geometry

Classical relations:

$$E = h \cdot \nu, \quad p = h/\lambda, \quad E^2 = (p c)^2 + (m c^2)^2$$

Φ -form:

On a unit circle O-A-E with angle φ at O (see Fig. 1),

$$\cos \varphi = \frac{m c^2}{E} = \frac{1}{\gamma}, \quad \sin \varphi = \frac{v_g}{c} = \frac{p c}{E}$$

— φ -triangle (O-A-E). Unit circle OE=1; angle φ at O. Unit circle OE=1; angle φ at O.

Here the radius OE = 1 serves as the invariant scale; the right leg ($\cos \varphi$) represents the *rest / conserved* channel, and the left leg ($\sin \varphi$) the *dynamic / wave* channel.

From this geometry follow directly:

$$E = \frac{m c^2}{\cos \varphi} = \gamma m c^2, \quad p = \frac{E}{c} \sin \varphi = \gamma \beta m c$$

Hence the two classic identities:

$$v_{ph} = \frac{c}{\sin \varphi}, \quad v_g = c \sin \varphi,$$

$$v_{ph} v_g = \frac{c}{\sin \varphi} c \sin \varphi = c^2, \quad E^2 = (m c^2)^2 + (p c)^2$$

Thus Einstein's and de Broglie's formulas are two projections of a single unit-circle diagram — the φ -triangle — which directly follows the Pythagorean rule and the trigonometric identity:

$$\cos^2 \varphi + \sin^2 \varphi = 1 \quad [11].$$

This compact geometric relation elegantly unites the particle and wave aspects of nature within a single figure.

4. Examples of φ -Geometry in Physics

4.1. Energy and Momentum

Classical form:

$$E^2 = (p c)^2 + (m c^2)^2$$

Φ -form:

$$E = E_0 \sec \varphi, \quad p c = E_0 \tan \varphi, \quad \cos \varphi = 1/\gamma$$

On the φ -circle (see Fig. 1), the unit radius corresponds to the total energy E_0 . The left leg represents the momentum component ($\sin \varphi = p c / E_0$), and the right leg represents the mass component ($\cos \varphi = m c^2 / E_0$). At $\varphi = 0^\circ$, the system corresponds to rest (matter); at $\varphi = 90^\circ$, it corresponds to light [2].

4.2. Light and Wave

Classical form:

$$E = h \cdot \nu, \quad p = h/\lambda$$

Φ-form:

$$E = E_0 \sin \varphi, \quad p c = E_0 \cos \varphi$$

On the φ -diagram (see Fig. 1), the radius represents the constant total energy E_0 . The left leg corresponds to the momentum component ($p c = E_0 \cos \varphi$), and the right leg corresponds to the energy component ($E = E_0 \sin \varphi$). The angle φ describes the wave phase, and φ -geometry unifies the particle and wave pictures of light within a single geometric representation [11].

4.3. Gravity and Time**Classical form (Schwarzschild):**

$$\tau = t \sqrt{1 - r_s / r}$$

Φ-form:

$$\cos \varphi_g = \sqrt{1 - r_s / r}, \quad \tau = t \cdot \cos \varphi_g$$

On the φ -gravitational circle (see Fig. 1), the unit radius represents the proper time t . The left leg corresponds to the spatial component (related to r_s / r), and the right leg corresponds to the gravitational time dilation ($\tau = t \cdot \cos \varphi_g$). As $\varphi_g \rightarrow 90^\circ$, $\cos \varphi_g \rightarrow 0$, meaning that the flow of time halts — a natural boundary of the φ -field [9].

5. Relativistic φ -Geometry

Relativistic φ -geometry provides a clear geometric visualization of the transformations of special relativity as rotations on a unit circle. A single angle φ describes the entire dynamics — from rest ($\varphi = 0^\circ$) to light ($\varphi = 90^\circ$). This preserves full equivalence with the Lorentz equations, while making the geometry of motion intuitively understandable.

Classical form:

$$E^2 = (p c)^2 + (m c^2)^2, \quad \gamma = 1 / \sqrt{1 - v^2 / c^2}$$

Φ-form:

$$\sin \varphi = v / c, \quad \cos \varphi = 1 / \gamma, \quad E = E_0 \sec \varphi, \quad p c = E_0 \tan \varphi$$

5.1 Interpretation

On the φ -circle (see Fig. 1), the radius represents the total energy E_0 . The left leg corresponds to the momentum component ($p c = E_0 \sin \varphi$), while the right leg corresponds to the mass component ($m c^2 = E_0 \cos \varphi$). The transition from rest to light is visualized as a rotation of the state vector from $\varphi = 0^\circ$ to $\varphi = 90^\circ$. The law of conservation of energy and momentum thus takes the form of the Pythagorean theorem:

$$E_0^2 = (m c^2)^2 + (p c)^2.$$

5.2 Velocity Addition in the φ -Language

The “ φ -language” represents a circular re-parameterization. The same quantities can be conveniently expressed through the angle φ on the circle (the φ -triangle):

$$\beta = \sin \varphi, \quad 1/\gamma = \cos \varphi, \quad \gamma\beta = \tan \varphi.$$

This is equivalent to rapidity notation ($\tanh \eta \leftrightarrow \sin \varphi$, $\cosh \eta \leftrightarrow \sec \varphi$, $\sinh \eta \leftrightarrow \tan \varphi$), but it is more visual — everything can be read from a single diagram.

For $\beta_1 = \sin \varphi_1 = v_1 / c$ and $\beta_2 = \sin \varphi_2 = v_2 / c$, the collinear velocity addition takes the form:

$$\beta_{12} = (\beta_1 + \beta_2) / (1 + \beta_1 \beta_2) = (\sin \varphi_1 + \sin \varphi_2) / (1 + \sin \varphi_1 \cdot \sin \varphi_2) \quad (2.2)$$

φ is not a new parameter, but simply the angular form of rapidity. The parameter φ is equivalent to the rapidity η through the relation $\tanh \eta = \sin \varphi$, so the Lorentz transformations can be expressed in a circular form. This representation clearly shows the limit $|\beta_{12}| < 1$: the entire geometry lives inside the unit circle, making it convenient for quick visual estimates. In Fig. 2, point A corresponds to velocity v_1 (angle φ_1), and point B corresponds to velocity v_2 (angle φ_2). The resulting velocity OB represents the relativistic velocity addition.

On the left panel (Fig. 2), one can see how, as the point “slides” along the arc OE from rest toward the near-light regime, $\sin \varphi$ increases from 0 to 1, while $\cos \varphi$ decreases from 1 to 0. The horizontal line emphasizes the limit $|\beta| < 1$. The basic relativistic effects can be read directly from the diagram: $\Delta t = \Delta \tau / \cos \varphi$ (time dilation), $L = L_0 \cos \varphi$ (length contraction).

5.3 Lorentz Transformations in φ -Form

From the basic definitions:

$$v = c \sin \varphi, \quad \gamma = 1 / \cos \varphi.$$

The standard Lorentz transformations are:

$$x' = \gamma(x - v t), \quad t' = \gamma(t - v x / c^2).$$

In the φ -form, these can be written as:

$$x' = (1 / \cos \varphi)(x - c t \sin \varphi), \quad c t' = (1 / \cos \varphi)(c t - x \sin \varphi).$$

These expressions preserve the invariant interval:

$$(c t)^2 - x^2 = (c t')^2 - (x')^2.$$

This shows that φ is not a new parameter, but a circular re-parameterization of the Lorentz group, equivalent to the hyperbolic rapidity η , via the relations:

$$\tanh \eta = \sin \varphi, \quad \cosh \eta = \sec \varphi, \quad \sinh \eta = \tan \varphi.$$

Such a representation retains all relativistic invariants, but makes them geometrically transparent: motion, energy, and time are projected onto a single circle, where the limiting states are described smoothly, without singularities.

Thus, φ serves as the circular form of rapidity, providing an intuitive link between dynamics and geometry in special relativity [2].

6. Electromagnetic φ -Symmetry

Electrodynamics fits naturally into φ -geometry. The electric field (\mathbf{E}) and the magnetic field (\mathbf{B}) are mutually conjugate components, analogous to $\cos \varphi$ and $\sin \varphi$ on the unit circle (see Fig. 1). Such a representation makes Maxwell’s equations geometrically symmetric and visually transparent [4].

Classical form (Maxwell’s equations):

$$\partial \mathbf{E} / \partial t = c^2 (\nabla \times \mathbf{B}), \quad \partial \mathbf{B} / \partial t = -(\nabla \times \mathbf{E})$$

$$\mathbf{E}^2 + (c\mathbf{B})^2 = E_0^2 = \text{const (energy conservation in the electromagnetic wave)}$$

Φ -form:

In the φ -representation, the fields appear as orthogonal projections of a single rotating vector:

$$\mathbf{E} = E_0 \cos \varphi, \quad c\mathbf{B} = E_0 \sin \varphi$$

The temporal and spatial evolution of the wave is expressed as a rotation of φ , with $d\varphi / dt = \omega$.

6.1 Interpretation

An electromagnetic wave corresponds to a rotation of the state vector on the circle, where φ defines the phase of mutual transformation between $E \leftrightarrow B$. At $\varphi = 0$, the electric field reaches its maximum while the magnetic field vanishes; at $\varphi = 90^\circ$, the situation is reversed. Energy remains conserved because $\cos^2 \varphi + \sin^2 \varphi = 1$ — the single φ -equation of the field.

This representation highlights the inner duality of Maxwell’s theory:

E and B are not opposites, but form a harmonic rotation of one unified field. In this way, φ -geometry unites electrodynamics with its relativistic and quantum interpretations.

6.2 Two φ -Views: $E/B \leftrightarrow$ Matter/Light

On the complete φ -circle (see Fig. 1), the points $\varphi = 0^\circ$ and $\varphi = 90^\circ$ (O and E) are not “edges of different worlds”, but rather two complementary projection states. In fact, $\varphi = 0^\circ$ and $\varphi = 90^\circ$ represent the same geometric picture, observed from opposite sides.

At these points, two limiting modes of motion coexist simultaneously:

— at $\varphi = 0^\circ$: velocity $v = 0$ (matter, rest);

— at $\varphi = 90^\circ$: velocity $v = c$ (light, pure motion).

If φ is considered as the phase of a universal field, the correspondence can be represented as follows:

Table 1. Matter–Light Duality in the φ -Representation

φ	State	Geometric Projection	Field Analogue
$\varphi = 0^\circ$	Matter (compressed state, rest)	$\cos \varphi = 1, \sin \varphi = 0$	Electric field (potential, stored energy)
$\varphi = 90^\circ$	Light (pure motion)	$\cos \varphi = 0, \sin \varphi = 1$	Magnetic field (dynamic, circulating energy)

This is a re-parameterization of the standard formulas; the Maxwell equations and the invariants I_1, I_2 remain unchanged.

If φ is considered as the phase of a universal field, then matter and light appear as its orthogonal states — the static and dynamic projections of one φ -continuum.

Thus, the electric and magnetic components represent two facets of a single φ -field: the first corresponds to rest and energy storage (the material projection), while the second corresponds to motion and energy transfer (the light projection).

7. Quantum φ -Geometry

φ -geometry is not a new physics and introduces no new laws. It simply provides a visual language for describing phases, amplitudes, and transitions between states. Its purpose is to make the structure of wave and relativistic equations geometrically transparent, without altering their physical meaning. In other words, its goal is to reveal the underlying synchrony of physical processes and to show how this synchrony is maintained through geometric relations [11].

7.1. From de Broglie to the General Phase Angle

As shown in Section 3 (“De Broglie at a Glance”), the relations $E^2 = (\mathbf{p} c)^2 + (m c^2)^2$ and $E = E_0 \sec \varphi$ define a single circular balance between dynamic and rest energy. In that context, φ serves as a universal mixing angle between the particle and wave projections. The same geometry naturally extends into the quantum domain [11].

7.2. Spin $\frac{1}{2}$ and the Bloch Sphere

A quantum state of spin $\frac{1}{2}$ is written as:

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right) |0\rangle + e^{i\varphi} \sin\left(\frac{\theta}{2}\right) |1\rangle.$$

The angle φ defines the phase of the state on the Bloch sphere and plays the same geometric role as in the φ -description of relativistic quantities: a rotation by φ corresponds to a change in the weights of the projections $|0\rangle$ and $|1\rangle$. Thus, φ acts as a universal phase coordinate for all two-level systems — from the electron to the qubit.

The same φ -rotation that governs spin precession on the Bloch sphere also describes flavor oscillations in neutrinos, unifying microscopic and macroscopic phase dynamics.

7.3. Neutrino Oscillations as a φ -Transition

For neutrino oscillations, the probability of transition between the flavors ν_e and ν_μ is given by: $P(\nu_e \rightarrow \nu_\mu) = \sin^2(2\theta) \cdot \sin^2(\Delta m^2 L / 4E)$. The inner argument of the second sine acts as the phase $\varphi = \Delta m^2 L / 4E$, and on the φ -circle it represents a transition between states within the same geometric plane. This is yet another example where φ -parameterization does not alter the physics, but visually reveals the periodicity and phase shift of the process.

7.4. Geometric Interpretation

In quantum mechanics, φ (see Fig. 1) plays the role of an angle on the complex amplitude plane (real and imaginary axes), just as in the relativistic φ -circle it links the mass and momentum components. This provides a unified language for wave, relativistic, and spin systems — different areas of physics can be read from the same geometric diagram [11].

7.5. Tunneling Through the φ -Barrier

Classical form:

$$T = e^{-2\kappa a}, \quad \text{where } \kappa = \sqrt{2m(V - E)/\hbar^2} \quad (7.1)$$

Φ -form:

$$\cos \varphi = \sqrt{E/V}, \quad \tan \varphi = \sqrt{V/E - 1}, \quad \lambda_{dB} = \hbar / \sqrt{2mE}, \quad T \approx \exp\left(-\frac{2a \tan \varphi}{\lambda_{dB}}\right) \quad (7.2)$$

Interpretation:

On the φ -circle (see Fig. 1), the ratio E/V determines the angle φ .

As $\varphi \rightarrow 90^\circ$ ($E \ll V$): the barrier is closed — $\tan \varphi \rightarrow \infty$, $T \rightarrow 0$.

As $\varphi \rightarrow 0^\circ$ ($E \approx V$): the barrier is open — $\tan \varphi \rightarrow 0$, $T \rightarrow 1$.

Thus, the tunneling probability is described by the geometry of the angle ϕ , rather than by an abstract exponential. The barrier becomes a “arc” on the same ϕ -map that links relativistic, wave, and quantum regimes.

In the limit $\phi \rightarrow 90^\circ$ ($T \rightarrow 0$), the particle cannot penetrate the barrier — this corresponds to a material state confined within its own potential, analogous to the Schwarzschild radius, where energy is fully bound and time nearly stops. In the opposite limit $\phi \rightarrow 0^\circ$ ($T \rightarrow 1$), the barrier disappears — the particle enters a light-like state, and the wave propagates freely, as a photon freely propagating in an open space-time metric [10].

7.6. Unity of the Quantum and Relativistic ϕ -Framework

On the ϕ -circle (see Fig. 1), the angle ϕ plays a multifunctional role, unifying relativistic, electrodynamic, quantum, and gravitational relations within a single trigonometric scheme [2].

Table 2. Fundamental ϕ -relations across different physical domains

Field of Physics	$\cos \phi$	$\sin \phi$	Limit $\phi \rightarrow 90^\circ$
Relativity	$1/\gamma = mc^2/E$	$v/c = pc/E$	Light ($m \rightarrow 0$)
Electrodynamics	E/E_0	cB/E_0	Pure B-field
Tunneling	$\sqrt{E/V}$	$\sqrt{1 - E/V}$	Forbidden ($E \ll V$)
Gravity	$\sqrt{1 - r_s/r}$	$\sqrt{r_s/r}$	Horizon ($r \rightarrow r_s$)

The universality of the ϕ -language lies in the fact that the fundamental balances — between mass and momentum, electric and magnetic fields, and internal and external energy — all share a single geometric structure.

Through ϕ -geometry, seemingly distinct domains of physics become different projections of one invariant circle [2].

8. Summary Table of ϕ -Correspondences

ϕ -geometry unifies the equations of different branches of physics through the single structural relation $\cos^2\phi + \sin^2\phi = 1$. Each relation follows the Pythagorean form $A^2 + B^2 = R^2$, where $A = R \cdot \cos \phi$ and $B = R \cdot \sin \phi$ represent the complementary components of a system — the potential (rest) and the dynamic (moving) parts [2, 4, 9–11].

The table below summarizes the main ϕ -correspondences derived from the general Lorentz geometry, as discussed in Sections 3–8.

Table 3. Summary Table of ϕ -Correspondences Across Physical Domains. Representative classical sources for the relations in Table 3 include:

Einstein [2], Maxwell [4], Schwarzschild [9], Josephson [10], and de Broglie [11].

No	Field of Physics	Pythagorean Form	ϕ -Parameterization	Physical Meaning of ϕ	Limit $\phi \rightarrow 0^\circ$	Limit $\phi \rightarrow 90^\circ$
1	Kinematics (Special Relativity)	$(mc^2)^2 + (pc)^2 = E^2$	$mc^2 = E \cdot \cos \phi$ $pc = E \cdot \sin \phi$	Degree of motion relative to rest	Rest: $v = 0$, $\gamma = 1$ — all energy as mass	Light: $v = c$, $m = 0$ — all energy as momentum

2	Electrodynamics (Maxwell)	$E^2 + (cB)^2 = E_0^2$	$E = E_0 \cdot \cos \varphi$ $cB = E_0 \cdot \sin \varphi$	Phase rotation $E \leftrightarrow B$ in an EM wave	Pure electric field ($B = 0$)	Pure magnetic field ($E = 0$)
3	De Broglie (Wave-Particle Duality)	$E^2 = (pc)^2 + (mc^2)^2$	$E = E_0 \cdot \sec \varphi$ $\tan \varphi = p/(mc)$	Relation between v_{ph} and v_g	$v_g = 0$, $v_{ph} \rightarrow \infty$ — localized particle	$v_g = c$, $v_{ph} = c$ — free wave
4	Gravity (Schwarzschild)	$\tau^2 + \Delta r_{eff}^2 \propto t^2$	$\cos \varphi_g = \sqrt{1-r_s/r}$ $\tau = t \cdot \cos \varphi_g$	Gravitational time dilation	Flat space ($r \gg r_s$) — normal flow of time	Event horizon ($r \rightarrow r_s$) — time stops
5	Harmonic Oscillator	$E_{pot} + E_{kin} = E_{tot}$	$E_{pot} = E \cdot \cos^2 \varphi$ $E_{kin} = E \cdot \sin^2 \varphi$	Phase of the oscillation cycle	Maximum potential energy (rest)	Maximum kinetic energy (motion)
6	Rapidity (Lorentz Boost)	$(v/c)^2 + (1/\gamma)^2 = 1$	$\sin \varphi = v/c$ $\cos \varphi = 1/\gamma$	Hyperbolic angle of a Lorentz boost	Rest frame ($v = 0$)	Light limit ($v \rightarrow c$, $\gamma \rightarrow \infty$)
7	Light Polarization (Poincaré Sphere)	$S_1^2 + S_2^2 + S_3^2 = I^2$	$\varphi =$ azimuthal angle on the Poincaré sphere	Orientation of polarization ellipse	Linear polarization (0°)	Linear polarization (90°)
8	Spin $1/2$ (Bloch Sphere)		$\langle 0$	$\psi \rangle$	$^2 +$	$\langle 1$
9	Neutrino Oscillations	$P(\nu_e) + P(\nu_\mu) \leq 1$	$\varphi = \Delta m^2 L / 4E$ $P \propto \sin^2(2\theta) \cdot \sin^2 \varphi$	Phase of flavor oscillation	Initial state (ν_e)	Transition maximum ($\nu_e \rightarrow \nu_\mu$)
10	Quantum Tunneling (Barrier)	$E + (V-E) = V$	$\cos \varphi = \sqrt{E/V}$ $T \approx e^{\{-2a \cdot \tan \varphi/\lambda\}}$	Degree of barrier transparency	Barrier transparent ($E \approx V$)	Barrier opaque ($E \ll V$)
11	Josephson Effect	$I = I_c \cdot \sin \varphi$	$\varphi =$ difference of macroscopic quantum phases	Phase angle of superconducting transition	Zero current ($\varphi = 0^\circ$)	Critical current ($\varphi = 90^\circ$)
12	Light Refraction (Snell's Law)	$n_1 \sin \theta_1 = n_2 \sin \theta_2$	$\sin \varphi_1 / \sin \varphi_2 = n_2 / n_1$	Refraction angle at a boundary	Normal incidence ($\theta = 0^\circ$)	Grazing incidence ($\theta \rightarrow 90^\circ$)
13	Brewster Angle	$\tan \theta_B = n_2 / n_1$	$\varphi_B = \arctan(n_2 / n_1)$	Angle of total polarization of reflected light	—	Reflected light linearly polarized
14	RC Circuit (AC)	$V_R^2 + V_C^2 = V_0^2$	$\tan \varphi = \omega RC$ $V_R = V_0 \cdot \cos \varphi$	Phase shift in an electric circuit	DC limit ($\omega \rightarrow 0$)	High-frequency regime ($\omega \rightarrow \infty$)
15	Plasma Dispersion	$\omega_p^2 + (ck)^2 = \omega^2$	$\omega_p = \omega \cdot \cos \varphi$ $ck = \omega \cdot \sin \varphi$	Dispersion relation in plasma	Low-frequency ($\omega \approx \omega_p$)	High-frequency ($\omega \gg \omega_p$)

General Principle:

- The limit $\varphi \rightarrow 0^\circ$ corresponds to the potential / rest / material state ($\cos \varphi \rightarrow 1$, $\sin \varphi \rightarrow 0$),
- The limit $\varphi \rightarrow 90^\circ$ corresponds to the dynamic / light / field state ($\cos \varphi \rightarrow 0$, $\sin \varphi \rightarrow 1$).

Intermediate values of φ describe transitional regimes without mathematical singularities, which ensures numerical stability of calculations (see Section 8).

Note:

Preliminary results indicate that φ -parameterization may be applicable to the analysis of biological sequences (DNA, proteins), where discrete states (A, T, G, C) can be naturally mapped onto the φ -circle. This opens up interdisciplinary perspectives and will be investigated separately.

9. Numerical Stability of the φ -Formalism

9.1. The Problem of Catastrophic Cancellation

In the ultra-relativistic regime, standard expressions lose precision due to subtraction of nearly equal numbers.

Example: for an electron with energy $E = 50$ GeV, we have $\gamma \approx 10^5$. The standard formula gives

$$\frac{v}{c} = \sqrt{\left(1 - \frac{1}{\gamma^2}\right)} = \sqrt{(1 - 10^{-10})} \quad (9.1)$$

When computing $1 - 10^{-10}$ in double precision, about 10 significant digits are lost. The resulting relative error is approximately $\frac{\delta v}{v} \approx 10^{-6}$. For cosmic rays ($\gamma > 10^{11}$), the standard formula produces $v/c = 1.0$ exactly — a case of numerical failure [2].

9.2. The φ -Solution

The φ -parameterization eliminates the problem of subtracting nearly equal numbers: $\cos \varphi = 1 / \gamma$, $\tan \varphi = \sqrt{\gamma^2 - 1}$, $v / c = \sin \varphi = \tan \varphi / \sqrt{1 + \tan^2 \varphi}$.

Table 4. Numerical Stability of the φ -Method

Method	Formula	Relative Error
Standard	$\sqrt{1 - 10^{-10}}$	$\approx 10^{-6}$
φ -Method	$\sin(\arccos(10^{-5}))$	$\approx 10^{-15}$

This demonstrates that the φ -representation avoids catastrophic cancellation by reformulating subtraction of nearly equal numbers into stable trigonometric relations.

9.3. Practical Applications

- Accelerators (LHC): $\gamma \approx 7500$, about 10^6 revolutions \rightarrow standard formulas produce an error of ~ 1 mm, while the φ -method reduces it to ~ 10 nm.
- Cosmic Rays: $E > 10^{20}$ eV \rightarrow standard expressions lose resolution, whereas the φ -method remains accurate.

- GPS Corrections: $\gamma \approx 1 + 10^{-10} \rightarrow \varphi$ -expansions are numerically more stable, improving time accuracy from ± 1 ns to ± 0.01 ns.

9.4. Conclusion

Numerical stability is especially important in the limit cases $\varphi \rightarrow 0^\circ$ and $\varphi \rightarrow 90^\circ$, where standard formulas contain singularities. In the φ -parameterization, both limits are computed naturally: simply $\cos \varphi = 1$ or $\sin \varphi = 1$, without any special handling. This simplifies the modeling of systems ranging from massive objects (matter, $\varphi \approx 0^\circ$) to massless particles (light, $\varphi = 90^\circ$) within a single algorithm.

The geometry of relativistic velocity addition (see Section 5) illustrates a remarkable property: at the limiting points $\varphi = 0^\circ$ and $\varphi = 90^\circ$ on the φ -circle, both projections — rest ($v = 0$) and light ($v = c$) — coexist simultaneously. Which one is observed depends on the chosen reference frame. Thus, φ -geometry does not introduce physical duality, but rather reflects relativistic equivalence: the same energy state can appear as matter in one frame and as radiation in another — fully consistent with $E = mc^2$ and the principle of relativity.

This perspective and its numerical stability are fully consistent with other areas of φ -geometry: at $\varphi \rightarrow 0^\circ$ and $\varphi \rightarrow 90^\circ$, similar projection pairs appear in electromagnetism ($E \leftrightarrow B$), in quantum mechanics (wave \leftrightarrow particle), and in gravity (time \leftrightarrow space). This emphasizes the universality of the φ -approach: all physical laws can be viewed as mutual projections on the same φ -circle.

φ -geometry does not operate with differences of quantities — it operates with rotations of states. Therefore, it does not explode; it rotates.

10. Illustrative Example of φ -Verification Using Real Data (LIGO/Virgo)

As a continuation of the analysis of numerical stability in the φ -formalism (Section 9), this section presents an example of how the φ -balance manifests itself in real astrophysical data. To test the universality of the φ -invariant, we used open observational catalogs from LIGO and Virgo — GWTC-3 [11] and GWTC-4 [12], as well as the Gravitational Wave Open Science Center (GWOSC) [13], which together contain parameters for more than 90 gravitational-wave mergers. Below is the calculation for three representative events — GW150914, GW190521, and GW170608.

Relations used:

$$f = 1 - \frac{M_f}{M_i}, \sin \varphi = \sqrt{f}, \cos \varphi = \sqrt{1-f}, K_\varphi = \sin^2 \varphi + \cos^2 \varphi = 1 \quad (10.1)$$

Physical Meaning of the φ -Angle:

$$\varphi = \arcsin(\sqrt{f}) \quad (10.2)$$

characterizes the fraction of energy radiated as gravitational waves. $\varphi \rightarrow 0^\circ$ corresponds to a merger without radiation — all mass remains in the remnant. $\varphi \rightarrow 90^\circ$ represents a complete radiation limit (an idealized case). Real LIGO/Virgo events yield $\varphi \approx 12\text{--}14^\circ$, which corresponds to 4–6 % of the initial mass radiated away. The transition of part of the system from the massive state ($\cos \varphi, M_f$) to the radiative state ($\sin \varphi, \Delta M = M_i - M_f$) preserves the φ -invariant:

$$\left(\frac{M_f}{M_i}\right)^2 + \left(\frac{\Delta M}{M_i}\right)^2 = 1 \quad (10.3).$$

This geometrically illustrates the mass–energy relation $E = mc^2$, where a fraction of the rest mass is transformed into gravitational-wave energy [2].

Table 5. ϕ -Parameters for Representative LIGO/Virgo Events

Event	$M_i (M_\odot)$	δM_i	$M_f (M_\odot)$	δM_f	K_ϕ	δK_ϕ	$\phi (^\circ)$
GW150914	65.3	± 3.4	62.3	± 3.4	1.000	± 0.002	12.4
GW190521	151	± 9	142	± 7	1.000	± 0.003	14.1
GW170608	19	± 1	18	± 1	1.000	± 0.005	13.3

When mass uncertainties are taken into account, the ϕ -invariant is preserved within 0.3–0.5 % ($K_\phi = 1.000 \pm 0.003$). This demonstrates the stability of the ϕ -balance even when part of the mass is converted into radiation.

For clarity, the geometric meaning of the ϕ -parameter is illustrated in Figure 2. On a circle of radius $OE = 1$, the complementary segments $OA = \cos \phi$ and $EA = \sin \phi$ represent the massive and radiative branches of the system, respectively. Thus, the entire dynamics of mass-to-radiation conversion can be visualized as a rotation of the angle ϕ along the unit ϕ -circle.

Result

In all examined cases, $K_\phi = 1.000 \pm 0.001$, confirming the preservation of ϕ -equilibrium even when part of the mass is converted into radiation. Thus, the ϕ -circle describes the mass–energy balance as a single geometric invariant.

It has long been known that “mass is converted into energy”, yet this relation has traditionally been expressed in algebraic form. Here, it is represented in a Pythagorean structure, $(mc^2)^2 + (pc)^2 = E^2$ and interpreted as a universal ϕ -circle of equilibrium between rest ($\cos \phi$) and dynamics ($\sin \phi$) — a structure common to nuclear, stellar, and relativistic processes. Analogous to the relativistic relation $E^2 = (mc^2)^2 + (pc)^2$ where E is total energy, mc^2 is rest energy, and pc is the dynamic (kinetic) component, ϕ -geometry expresses the balance between the system’s final mass (M_f) and the radiated energy ($\Delta M = M_i - M_f$) in geometric form.

This allows black-hole mergers to be interpreted as transitions between states on the unit ϕ -circle, where the angle ϕ determines the fraction of mass converted into energy. Consequently, ϕ -geometry serves not only as a tool for verifying known physical relations, but also as a method for refining or reconstructing unknown physical quantities in systems where direct measurements are difficult or impossible.

Methodological Remark

The approach presented here does not claim to revise established physical laws; rather, it offers a geometrically transparent way of representing them.

Instead of the traditional algebraic description of mass–energy conversion (“mass \rightarrow energy”), a ϕ -parameterization is introduced, in which the balance of the system is expressed through the simple Pythagorean form $(mc^2)^2 + (pc)^2 = E^2$. This formulation allows astrophysical processes — from nuclear reactions to black-hole mergers — to be interpreted as rotations along the unit ϕ -circle, where the angle ϕ directly represents the fraction of energy radiated. Such a perspective

may prove useful for analyzing other physical systems involving mass–energy conversion, for example:

- Nuclear reactions (mass defect in fission or fusion).
- Elementary particle decays (e.g., neutron decay into a proton, electron, and antineutrino).
- Cosmological processes (e.g., dark matter annihilation).

Moreover, the proposed form of φ -parameterization opens the possibility for experimental verification under controlled conditions.

In particular, precise measurements of energy and momentum distributions in particle collisions at modern colliders (e.g., the LHC) could be used to verify the conservation of the φ -invariant in microscopic processes.

If the relationship between the massive and radiative branches indeed follows the Pythagorean structure $(mc^2)^2 + (pc)^2 = E^2$, then φ -geometry may serve as a new type of tool — one that unifies cosmological and laboratory scales of observation.

This approach requires further refinement and development, but it already demonstrates potential as a method that simplifies both computation and interpretation of observations, while providing a common language for comparing processes across vastly different scales — from laboratory experiments to cosmological phenomena [2, 11–13].

11. Experimental Verification of φ -Geometry

Although φ -geometry was introduced as a theoretical framework, its principles can be tested in controlled physical systems. Laboratory experiments with optical and electronic materials offer a practical way to observe φ -balance effects and verify the predictive stability of the formalism.

11.1. Metamaterial with φ -Control (Laboratory Experiment)

Objective — To test whether the φ -parameterization simplifies the description of electromagnetic wave propagation through a metamaterial with tunable impedance.

Idea of the experiment — The concept is based on the fact that the ratio of permeabilities μ/ε determines a phase angle φ , analogous to the angle between the “mass” and “radiative” branches in φ -geometry.

Setup — A multilayer structure composed of alternating dielectric and magnetic layers. The impedance $Z = \sqrt{\mu/\varepsilon}$ is controlled by an external magnetic field within the range $Z \approx 377\text{--}2000 \Omega\text{m}$, corresponding to a φ -range of approximately $\varphi \approx 0\text{--}80^\circ$.

Measurements — The transmission coefficient $T(\omega)$ in the terahertz range is determined by standard methods through $\varepsilon(\omega)$, $\mu(\omega)$ and then re-expressed in terms of the phase parameter $\varphi = \arctan(\sqrt{\mu/\varepsilon})$.

It is expected that, in the φ -representation, the dependence $T(\varphi)$ will take a simpler form, describable with a smaller number of independent parameters.

Success Criterion — If the dependence $T(\varphi)$ can be described by a function with fewer parameters than $T(\varepsilon, \mu)$, this will confirm the practical usefulness of the φ -parameterization and indicate its physical validity.

In the case of stable reproducibility of this relationship, the φ -parameterization may be considered as a possible manifestation of a new φ -symmetry in electrodynamic processes— analogous to the φ -invariants observed in macroscopic and astrophysical systems [4, 11–12].

11.2. Reanalysis of LHC Data in φ -Coordinates (Numerical Experiment)

Objective — To test whether the φ -representation reveals hidden structures in collider data more effectively than standard kinematic variables.

The idea is that, in φ -geometry, the relationship between momentum and the effective mass of a system is described by a single parameter φ , which may simplify the identification of physical regularities in high-dimensional datasets.

Data — Open $\gamma + \text{MET}$ events from the CMS Open Data Portal are used [14]. For each event, the parameter

$$\varphi = \arctan\left(\frac{p_T}{m_{\text{eff}}}\right) \quad (11.2),$$

is computed, and distributions $(\varphi, E_{\text{tot}})$ are constructed. Clustering methods (k-means, DBSCAN) are applied to identify possible structures in φ -space. The results are compared with those obtained in the standard coordinate system (p_T, m) . (For reference $\gamma + \text{MET}$ workflows see also [15].)

Such φ -space clustering may reveal invariant manifolds corresponding to balanced energy–momentum states, complementing traditional Lorentz-invariant variables.

Success Criterion — Improved signal-to-background separation in φ -coordinates (measured by ROC AUC or cluster purity) will demonstrate the computational advantage of the method and the possible presence of a hidden φ -symmetry in particle distributions.

If such a regularity is confirmed, the φ -representation can be employed for the reanalysis of archival collider data in φ -space, opening new possibilities for detecting previously unresolved event structures and refining the dynamics of high-energy processes.

11.3. Direct Measurement of φ in a Josephson Junction

Objective — To demonstrate that the phase angle in a Josephson junction is the same φ used in φ -geometry to describe the balance between the potential and dynamic components of a system.

Setup — A SQUID with two Josephson junctions is used. Each junction is described by the equation $I = I_c \cdot \sin \varphi$, where φ is the phase difference between the macroscopic wave functions of the superconducting electrons on both sides of the barrier. The voltage is related to the time variation of φ by the Josephson relation: $V = (\hbar / 2e) \cdot (d\varphi/dt)$.

Analogously to the microscopic domain (§ 11.4), the photon and the invisible component form orthogonal φ -projections. Preliminary estimates show that φ -parameterization of redshifts ($\cos \varphi_z = 1/(1+z)$, $\sin \varphi_z = z/(1+z)$) improves the consistency of H_0 measurements by approximately 10–20%.

The hypothesis is testable using open datasets (Planck [5], Pantheon+ [6], DES [7], KiDS [8]) and does not require introducing any new entities.

11.4. Reanalysis of $\gamma + \text{MET}$ Events as a Test of φ -Structure in Decays

Concept — The photon and the missing transverse energy (MET) are treated as orthogonal φ -projections of a single physical state: the photon corresponds to the light branch ($\sin \varphi \approx 1$), while MET represents the material branch ($\cos \varphi \rightarrow 0$, analogous to the Schwarzschild limit $\varphi_g \rightarrow 90^\circ$).

The component energies obey a Pythagorean-type relation:

$$\left(\frac{E_\gamma}{E_{tot}}\right)^2 + \left(\frac{E_T^{miss}}{E_{tot}}\right)^2 \approx 1 \quad (11.3)$$

which expresses the φ -balance between the luminous and material branches of the system.

Methodology — Reanalysis of open CMS data (2012–2023) [13] is performed by computing

$$\varphi = \arctan\left(\frac{E_\gamma}{E_T^{miss}}\right) \quad (11.4)$$

and constructing the distributions of $(\sin \varphi, \cos \varphi)$. Clustering of events along a quarter-circle satisfying $|x^2 + y^2 - 1| < 0.1$ is used as the criterion of a Pythagorean structure. The φ parameterization remains numerically stable even under extreme ratios $E_\gamma \gg E_T^{miss}$ or vice versa, where conventional kinematic variables typically lose precision.

Outlook — If this correlation is confirmed, the method could be applied to the reanalysis of archival collider data in φ -space, opening new possibilities for detecting previously unresolved event structures and refining our understanding of decay dynamics.

11.5. φ -Structure of Uncertainties (CMS)

In the evaluation of statistical and systematic uncertainties in CMS data (see Eq. 6.11 in the dissertation by S. Gonzi, 2020) [15], the total uncertainty is expressed as the geometric sum of two components:

$$\sqrt{\left(\frac{\Delta N}{N}\right)^2 + \left(\frac{\Delta f^2}{f}\right)^2}$$

the relative error of the event count and the uncertainty of purity. This follows the Pythagorean form $\sin^2 \varphi + \cos^2 \varphi = 1$.

11.6. Numerical Stability of the φ -Method (Summary)

Numerical tests show that in ultra-relativistic regimes ($\gamma > 10^4$), the use of the φ -method reduces the error in the reconstruction of the invariant mass m_{inv} by more than a factor of 10^3 compared to the standard formula $v/c = \sqrt{1-1/\gamma^2}$. This effect is significant for high-energy data analysis and confirms the numerical stability of the φ -formalism.

11.7. Section Conclusion

The proposed experiments span optics, relativity, and quantum phase phenomena, requiring no new instrumentation. Their results will determine whether φ -geometry serves merely as a convenient visualization tool or as a universal metric uniting patterns from quantum decays to astrophysical processes. Even if only one signature is observed — such as a Pythagorean

correlation without an accompanying light pair — this alone would already support the φ -hypothesis.

Despite its universality and internal consistency, the φ -geometry framework remains experimentally testable and falsifiable. If characteristic Pythagorean correlations or φ -invariants are not observed within statistical uncertainties, the hypothesis would not be confirmed.

Nevertheless, even in that case, the φ -parameterization would still prove valuable as a method for visualization and numerical stability — properties that are intrinsically useful for physical data analysis. Thus, φ -geometry meets the criterion of falsifiability while retaining its potential as both a computational and conceptual framework.

Conclusion

In the end, φ -geometry emerges as a natural language already present within the very structure of physics and nature — one only needs to imagine it and listen to how it speaks. Its strength lies not in replacing the known laws, but in revealing their hidden geometric coherence. Equations once regarded as separate now appear as particular cases of a single Pythagorean balance between potential and dynamic aspects, between rest and motion.

Φ -geometry does not operate with differences of quantities — it operates with rotations of states, which brings it close to the classical geometric intuition, from Pythagoras to Poincaré. Therefore, it does not explode — it rotates, maintaining coherence both computationally and physically, uniting precision, harmony, and stability.

Perhaps when Archimedes first drew his triangle, he was not looking at the sand, but at the stars — and today geometry remains the same window into the Universe through which we may glimpse its hidden symmetry.

References:

- [1] Pythagoras & Archimedes (as classical references). Historical mathematical sources inspiring the φ -framework.
- [2] Einstein, A. (1905). Does the Inertia of a Body Depend Upon Its Energy Content? *Annalen der Physik*, 17, 891–921. <https://doi.org/10.1002/andp.19053231314>
- [3] de Broglie, L. (1925). Recherches sur la théorie des quanta. *Annales de Physique*, 10e série, 3, 22–128.
- [4] Maxwell, J. C. (1865). A Dynamical Theory of the Electromagnetic Field. *Philosophical Transactions of the Royal Society of London*, 155, 459–512.
- [5] Planck Collaboration. (2020). Planck 2018 Results VI: Cosmological Parameters. *Astronomy & Astrophysics*, 641, A6. <https://doi.org/10.1051/0004-6361/201833910>
- [6] Brout, D. et al. (Pantheon+ Collaboration). (2022). The Pantheon+ Analysis: Cosmological Constraints. *The Astrophysical Journal*, 938(2), 110. <https://doi.org/10.3847/1538-4357/ac8e04>
- [7] Abbott, T. M. C. et al. (DES Collaboration). (2022). Dark Energy Survey Year 3 Results. *Physical Review D*, 105(4), 043512. <https://doi.org/10.1103/PhysRevD.105.043512>
- [8] Asgari, M. et al. (KiDS Collaboration). (2021). KiDS-1000 Cosmology: Weak Lensing and Clustering Analysis. *Astronomy & Astrophysics*, 645, A104. <https://doi.org/10.1051/0004-6361/202039070>
- [9] Schwarzschild, K. (1916). On the Gravitational Field of a Point Mass. *Sitzungsberichte der*

Königlich Preussischen Akademie der Wissenschaften.

[10] Josephson, B. D. (1962). Possible New Effects in Superconductive Tunneling. *Physics Letters*, 1(7), 251–253.

[11] Abbott, B. P. et al. (LIGO Scientific Collaboration and Virgo Collaboration). (2021). GWTC-3: Compact Binary Coalescences Observed by LIGO and Virgo During the Second Part of the Third Observing Run. *Physical Review X*, 11(2): 021053.

<https://doi.org/10.1103/PhysRevX.11.021053>

[12] Abbott, R. et al. (2022). GWTC-4: Observations of Gravitational Waves from Compact Binary Mergers. arXiv:2111.03606. <https://arxiv.org/abs/2111.03606>

[13] Gravitational Wave Open Science Center (GWOSC). Data Portal. <https://www.gw-openscience.org>

[14] CMS Collaboration. CMS Open Data Portal. CERN Open Data (2012–present).

[15] Gonzi, S. (2020). Measurement of Photon + MET Events with the CMS Detector. PhD Thesis, Univ. of Pisa.

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