Higgs Mechanism from Primordial Dimensional Fluctuations

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Abstract

According to [1-3], there are compelling reasons for connecting the continuous and evolving dimensionality of spacetime above the Standard Model (SM) scale to the onset of complex dynamics in the deep ultraviolet sector of field theory. A key observation inspired by this viewpoint is that dimensional fluctuations unfolding in far-from-equilibrium conditions lead to the *complex Ginzburg-Landau equation* (CGLE), a universal model of nonlinear phenomena. Framed as sequel to [4-5], this work advances a novel interpretation of the Higgs mechanism as emerging manifestation of CGLE. Our interpretation focuses on the reduction of CGLE to the behavior of an overdamped scalar field and the formation of *supercritical Hopf bifurcations*. We find that, in this classical picture, Hopf bifurcations provide an *analog description* of the Higgs condensate and the standard mechanism of electroweak symmetry breaking. Built entirely outside

Lagrangian field theory, our approach suggests unconventional explanations of the SM structure and the emergence of broken discrete symmetries in electroweak interactions.

Key words: complex Ginzburg Landau equation, Hopf bifurcations, Higgs condensate, spontaneous symmetry breaking, Standard Model of particle physics, chirality.

1. Introduction

The origin of mass generation in SM is conventionally attributed to the Higgs mechanism, where a scalar field acquires a nonzero vacuum expectation value, spontaneously breaking the electroweak symmetry. While phenomenologically successful, this paradigm leaves open a host of long-standing questions [7-8], for instance,

1) What underlying physical processes give rise to the Higgs condensate?
2) What is the root cause of discrete symmetry breaking in electroweak interactions?

At the same time, the last three decades have seen remarkable progress in the field of *nonlinear science and complexity*. In particular, CGLE has emerged as a universal description of weakly nonlinear oscillatory systems near criticality, capturing the essence of amplitude dynamics in a wide range of physical contexts—from fluid instabilities, plasma physics, to superconducting condensates.

Refs. [15-16] have unveiled the deep connection between bifurcation theory and gauge structures, pointing out that particle masses and chiral symmetry breaking appear to be direct consequences of the underlying nonlinear dynamics. Building on this insight, we explore here the hypothesis that the Higgs condensate and chiral symmetry breaking reflect the dynamic behavior of CGLE.

The paper is organized in the following way: the emergence of supercritical Hopf bifurcations from CGLE and their analog condensates in SM is developed in section two. Unfolding of the full flavor structure of SM through sequential bifurcations of the scalar condensate forms the topic of section three. The last couple of sections deal with the emergence of broken time reversal and parity symmetries from the internal dynamics of the condensate. Summary and conclusions are presented in the last section.

Several key observations are now in order:

1) the parallel between CGLE and complex dynamics and SM must be understood as a helpful *analogy*, not an *identity*. Although one can argue that memory effects inherent in complex dynamics may carry over to the low-energy physics of SM, complex dynamics is a classical framework defined by *nonlocality* and *dissipation*, whereas SM is a quantum field theory built on a *relativistic* and *conservative* basis.

2) There are robust reasons to suspect that, sufficiently far above the SM scale, Lagrangian field theory breaks down. Primordial cosmology and deep ultraviolet physics are likely to favor phenomena that are neither steady nor uniform but instead mix the ultraviolet and infrared limits of field theory, exhibit long-range correlations, non-local couplings, and scale-dependent fluctuations in dimensionality. Clearly, these manifestations of complexity

stand at odds with the premises underlying the use of variational principles [12-13].

3) the derivation detailed herein is neither rigorous nor complete. In the interest of clarity and accessibility, many technical details are left out for follow-up studies, rebuttals or further clarifications.

2. Emergence of scalar condensates from CGLE

The canonical CGLE in dimensionless form reads

$$\partial_t \Phi = \mu\Phi + (\eta + i\alpha)\nabla^2\Phi - (\nu + i\beta)|\Phi|^2\Phi \qquad (1)$$

where $\Phi(\vec{x},t)$ is a complex amplitude, $\mu,\eta,\alpha,\nu,\beta$ are coefficients controlling the gain, dispersion and nonlinearity, respectively. Consider the limit of an *overdamped* scalar field, where damping is produced by rapid cooling and expansion of the early Universe. Dispersion can be reasonably assumed to be negligible in this scenario ($\alpha,\eta \to 0$), and (1) reduces to,

$$\partial_t \Phi = \mu\Phi - (\nu + i\beta)|\Phi|^2\Phi \qquad (2)$$

Separating the amplitude and phase $\Phi = \varphi e^{i\vartheta}$, the amplitude dynamics (given by the real part of Φ) satisfies the *Stuart-Landau equation* [9],

$$\boxed{\partial_t \varphi = \mu\varphi - \nu\varphi^3} \tag{3}$$

To streamline the ensuing derivation, we assume in what follows $\mu = \nu$. Equation (3) describes a *supercritical Hopf bifurcation* when making the transition from a stable fixed point at $\varphi = 0$ (for $\mu < 0$) to a stable fixed point $\varphi \neq 0$ (for $\mu > 0$). In the latter case, the fixed point $\varphi = 0$ becomes unstable and all phase-space trajectories of the system converge to a circle of unit radius $\varphi_0 = 1$ called a *stable limit cycle* (Fig. 1) [6, 9].

The onset of a stable oscillatory state embodied in the limit cycle represents the analogue of a *spontaneously chosen vacuum expectation value* in the Higgs mechanism. Mapping the amplitude φ to the magnitude of the Higgs field, the system underlying (3) approaches a nonzero equilibrium $\varphi_0 = 1$ defining the limit cycle and corresponding to the vacuum expectation value of the scalar condensate.

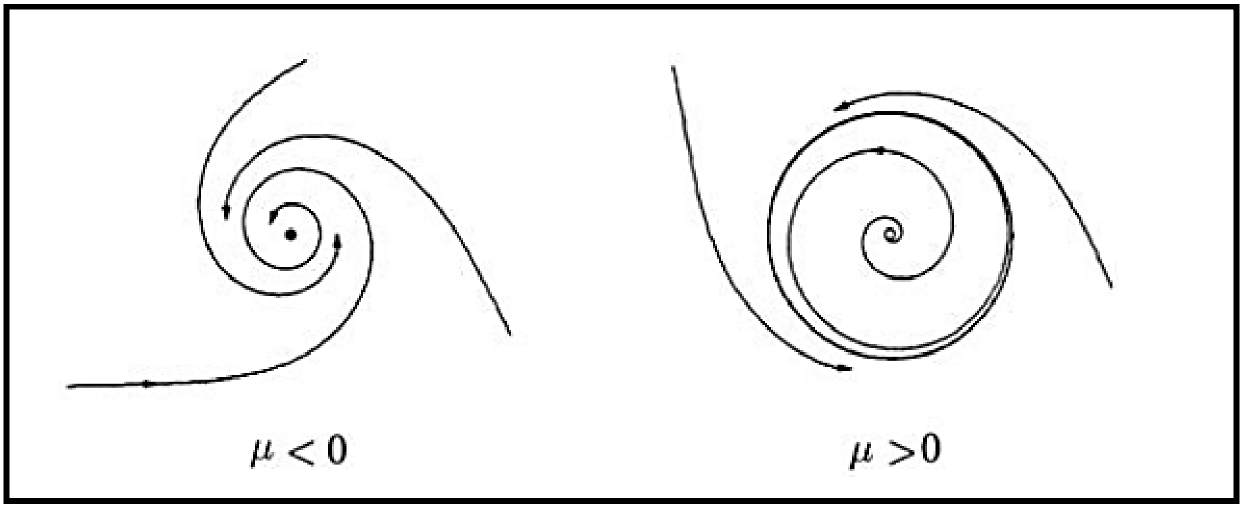

Fig. 1: Supercritical Hopf bifurcation in the phase plane (φ, ϑ) [6, 9]

Note that the overdamped nature of (2) and (3) ensures that the system relaxes monotonically, avoiding oscillatory overshoots, which is consistent with the gradient equation (4) and the classical scalar field graciously settling into its vacuum expectation value.

Note also that this analogy provides a natural interpretation of symmetry breaking: under variations of the control parameter μ, the zero-amplitude state becomes unstable and the system bifurcates to a finite-amplitude solution, spontaneously choosing a vacuum in a manner entirely dictated by the bifurcation dynamics. The phase ϑ can then be interpreted as a *Goldstone* *mode*, echoing the conventional formulation of the Higgs mechanism in field theory.

3. SM structure from sequential condensate bifurcations

According to [6, 16], the evolution of an overdamped scalar field is appropriately described by the gradient equation

$$\dot{\varphi} = -\frac{\partial V(\varphi)}{\partial \varphi} \tag{4}$$

where the potential function assumes the conventional Higgs-like form

$$V(\varphi) = \lambda(\varphi^2 - \frac{1}{2}v^2)^2 \tag{5}$$

and v denotes the vacuum energy The flow of the scalar field amplitude φ with the normalized scale κ/κ_0 is given by,

$$\dot{\varphi} = \kappa\frac{d\varphi}{d\kappa} = \frac{d\varphi}{d[\log(\kappa/\kappa_0)]} = \frac{d\varphi}{d\tau} \tag{6}$$

in which κ_0 is an arbitrary reference scale. One obtains,

$$\frac{d\varphi}{d\tau} = 2\lambda\varphi(v^2 - 2\varphi^2) \tag{7}$$

To cast (7) in a dimensionless form, we use the substitution

$$y = \frac{\sqrt{2}}{v}\varphi \tag{8}$$

Since in four dimensional spacetime φ and v have mass dimension $[\varphi] = [v] = M$, dimensional consistency of (7) under (8) requires passing to the normalized parameter

$$\mu(\kappa) = \frac{2\lambda(\kappa)v^2}{m_0^2} \tag{9}$$

where m_0 is an arbitrary reference mass. One finds that (7) turns into the Stuart-Landau equation

$$\boxed{\frac{dy}{d\tau} = \mu y(1 - y^2)} \tag{10}$$

It is apparent that (10) is a duplicate of (3) under the previously made assumption $\mu = \nu$. As pointed out in [15-16], upon casting (10) as a *cubic map*, sequential bifurcations induced by the flow of the observation scale κ in $\mu = \mu(\kappa)$ reproduce the entire architecture of SM.

4. Breaking of discrete symmetries via CGLE

The emergence of a supercritical Hopf bifurcation in the Stuart-Landau equation (3) means that

$$\Phi(t) = \varphi e^{i\vartheta(t)} = \varphi e^{\pm i\omega t} \tag{11}$$

and the condensate spontaneously picks one of the *two possible orientations* of rotation along the limit cycle ($+\omega$ counterclockwise CCW, $-\omega$ clockwise CW). Besides being spontaneous (not imposed), this choice is *dynamic* (a persistent phase rotation, not just a *static* phase value or a static set of degenerate vacua – see section 5).

Further assuming that the angular velocity ω is a scalar constant, picking an orientation of rotation in (11) amounts to a manifest breaking of temporal symmetry T (either $+t$ or $-t$).

The immediate question is then: How does T- symmetry breaking in the condensate space translates into chiral symmetry breaking in the physical space of electroweak channels?

The answer appears to be deceptively simple: assuming that the *conjugation-parity-time reversal* (CPT) invariance holds as exact symmetry of Nature up to ultrahigh energies, violation of $T-$symmetry necessarily implies a broken CP symmetry, consistent with Sakharov's conditions for baryogenesis [14]. Moreover, if the $C-$symmetry is taken to be *nearly exact* (minimal breaking between matter and antimatter in the primordial Universe), violation of CP symmetry implies broken parity P. This, in turn, suggests that electroweak interactions are sensitive to *chirality*, that is, they distinguish between "left" and "right" spatial orientations. This distinction emerges as natural outcome of Hopf bifurcations, unlike the postulated assumption that chirality is apriori built in the $SU(2)_L \times U(1)_Y$ gauge group of the electroweak model.

5. A word of caution

A potential objection to these findings may sound as follows: in the standard Higgs mechanism, the Mexican-hat potential $V(\Phi_{SM})$ has a continuous circle of degenerate minima (Fig.2). Choosing a particular point on that circle gives a static vacuum expectation value of the form $\langle \Phi_{SM} \rangle = ve^{i\theta}$. Small perturbations along the circle are the *Goldstone modes*, massless states corresponding to infinitesimal changes of the phase θ. The textbook interpretation of the Higgs mechanism correctly emphasize a *static* set of degenerate vacua and the existence of left/right phase orientations around the circle.

The counterpoint to this objection is that we are dealing with two different situations that look similar *geometrically* (a field moving on a circle) but are fundamentally different *dynamically*. Specifically,

1) In the standard Higgs scenario, one has a static degeneracy of the broken symmetry vacuum**:** the field sits at a fixed point on the circle. Phase

excitations are Goldstone mode fluctuations, and not autonomous oscillations. That is to say that there is no *persistent time-periodic motion* around the circle.

2) In the limit-cycle / Hopf bifurcation scenario, the field undergoes a sustained, time-periodic motion around the circle, and not just a small perturbation. The rotation angle is therefore locked into a *persistent rotation*.

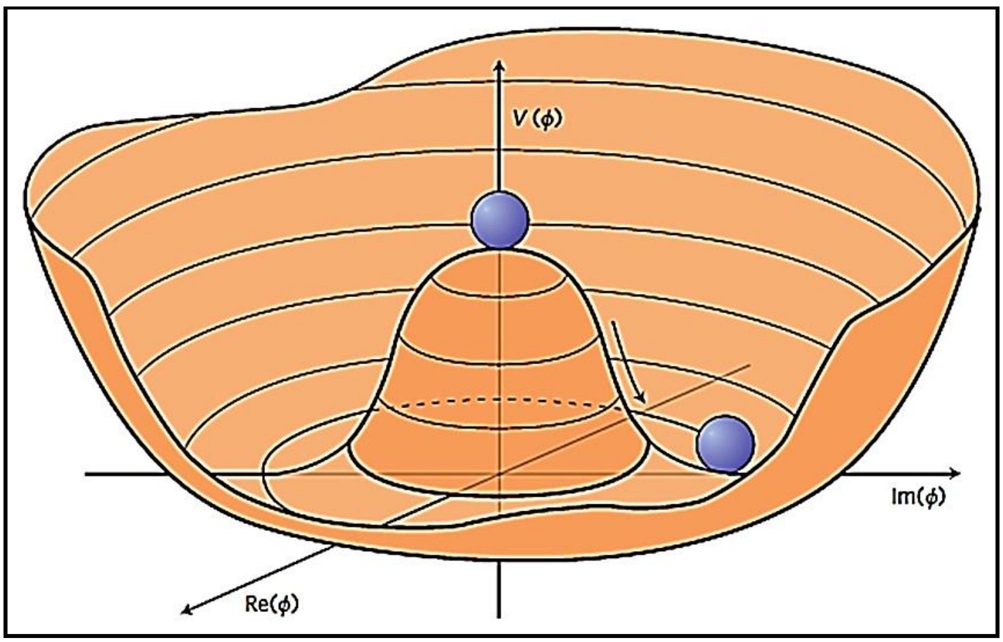

Fig 2: Symmetry breaking in the double well potential [10]

6. Conclusions

We point out here that the formation of the Higgs condensate can be interpreted as an emergent manifestation of CGLE in the overdamped limit. As outcome of a reduced CGLE model, the Stuart-Landau equation and it's supercritical Hopf bifurcation provide an analogue of the Higgs mechanism for spontaneous symmetry breaking in the SM. In summary, our work suggests an unconventional mechanism for chirality and mass generation in particle physics, as it bridges the divide between bifurcation theory/complex dynamics and standard gauge theory. On this note, it is worth mentioning that the Stuart-Landau equation may be used to explain the basis of spin-statistics theorem in Quantum Field Theory [11].

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