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What is G? A predictive Toy Model of Newtonian Gravity in the spirit of Principia Mathematica

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November 10, 2025

ABSTRACT

Based on a toy model that postulates space as a dynamic 3D grid of nodes and stems arranged as a repeating pattern of a unit cell of unknown geometry, an equivalence and interchangeability of space and mass is proposed, the latter being a collapsed state of the former. Masses would originate from the collapse of a certain amount of unit cells proportional to the mass. From the model and under spherical symmetry, Newtons' Law is reproduced and the value of G is shown to be independently derivable from a Cavendish experiment. The locally collapsed grid would then generate a dynamical system of force lines, akin to Faraday's lines in electromagnetism, which surface density decays as $\frac{1}{r^2}$ and determines the force on the test mass. A correction is added to the classical formulation for the case of non-baryonic gravity and gravity of unknown origin that reduces to the standard when there is none. Finally, an argument is made about the range of the gravitational force, which in this model would be extremely large but finite.

1 INTRODUCTION

When in a pop-sci debate in 2017 theoretical physicist and Nobel laureate David Gross was asked point blank «What is gravity?», his answer shocked the audience: "I wish we'd really know" Gross (2017).

Newtonian gravity (NG) is a correct mathematical description that has served us well and remains unchallenged after more than 300 years. While its original formulation is limited to the non-relativistic case and corrections are needed in the high acceleration, high speed regimes, it is still valid and used for most practical purposes today and is thought to work at large scales as well. However, the internal mechanism of the theory is unknown to us, as it was to Newton.

Two questions remain unanswered: (1) Can the theory be made local and not depend on the knowledge of the actual separation between the bodies? In other words, how do masses communicate with each other through vacuum? And (2) What exactly causes the attractive force on the masses? Newton was aware of these problems and thought they needed an answer, as he conferred in letters to his friend Richard Bentley in 1693, but no answer was found and none is available today.

Important refinements of the theory were achieved in the following decades, namely the introduction of the concept of a gravitational field analogous to the electromagnetic field, and the modern formulation embodied in the Poisson equation, which improved its applicability to arbitrary mass distributions without changing its essential character nor solving any of the aforementioned problems.

At the start of the XX century, Einstein's General Relativity (GR) brought us closer to the 'secret of the Old One' by showing that gravity is mediated by deformations of spacetime, and yet none of those questions received a satisfactory answer. Although many argue to this day that gravity is the apparent force inferred from curved paths in spacetime, Einstein himself seemed to know better and keenly advised not to take the geometric analogy too seriously. He insisted that it is all about the metric and that GR is not just a geometrization of gravity. While this may be true, the subtle hint implied in his observations was never made clear.

Furthermore, if gravity is all about curved spacetime, how can Newton's equation, based on flat Euclidean space, be so successful without any reference to space curvature? A geometric interpretation for NG is not available and no further progress has been made in this direction. The so-called geometric Newton-Cartan approach is a remarkable exception of high complexity that goes far beyond the level of Newtonian mechanics.

In recent years, most new ideas have developed while attempting to solve the Quantum Gravity problem, i.e, how to make GR consistent with quantum mechanics, and the field of debate has been twofold: On the one hand the study of black holes and the conceptual problems they present with the singularities at their center where all mathematical models break down. On the other, several ongoing theoretical attempts to model spacetime as a real entity, with geometric and dynamical properties that could serve as the substrate for gravity while at the same time embracing the fertile ideas of QM. None of these paths has so far led to significant progress, though the work is still ongoing. But one thing has come up from it: Space (or spacetime) is today widely accepted to be not an empty framework, but rather a physical entity of unknown structure that is able to deform and react to masses and energy in ways we can not imagine.

Based on the idea that space and matter are equivalent and interchangeable between one another, a simple geometric-mechanistic toy model of gravity is proposed that reproduces Newton's Universal Law.

We also provide an estimate of the equivalence mass-space as related by G and check its agreement with Newtonian predictions in all but the strongest gravitational fields. A discussion on some implications of the model follows, as well as ideas for future research.

2 MASS-SPACE EQUIVALENCE AT A GLANCE

We start from the assumption that mass and space are different states of the same entity. Space can collapse into mass from the input of a large amount of energy, as described in the equation $E = mc^2$. Likewise, mass can be converted into space with the release of the same amount of energy. The validity of Einstein's equation is not questioned here, but rather the interpretation might be incomplete. Einstein's famed equation, according to this model, would correctly describe the energy balance of the interconversion, but falls short of a complete description of the physical processes going on. Neither Einstein nor anyone else can be blamed for this. After all, vacuum is a very difficult object to measure, perhaps even impossible, but its physical reality is today beyond doubt. Since the advent of GR and quantum mechanics, space can bend and warp, can fluctuate, and is the substrate and carrier of the electromagnetic field (the photon), as well as other fields like the Higgs. And is also the substrate of the gravitational field. Space is where gravity lives.

The precise structure and dynamics of space (or spacetime, the distinction will be irrelevant to our discussion), is unknown, but its seems clear that there must be one, and it must be the same everywhere. It can be argued that space might consist of a repeating pattern of elementary space units with definite geometry. In this case, one can invoke the so-called Platonic solids as candidates for these units. Or it could be a combination of a finite number of them, the only requirement being that it must 'fill the space'. It is also assumed to be constantly changing, forming and reforming, both spontaneously and under stress from mass and other forms of energy. The precise geometry of space will not be discussed here, although it might play a role when the question comes up later as to how exactly the gravitational forces are generated. An obvious difficulty that has hindered research in this direction is the lack of evidence of frictional forces between a putative spacetime grid and the masses that move freely in it obeying the law of inertia. If moving masses do feel any resistance in their paths, all hypotheses about the nature and structure of the space grid contradict inertia and become untenable. To this, we argue that such frictional forces between moving masses and an underlying grid might no longer be expected if the masses are of the same nature as the grid. Under the assumption of a fundamental lattice that includes masses and can change constantly by forming and reforming its elementary units, frictional forces become a question of how fast the grid can reform itself.

3 A TOY MODEL OF GRAVITY

Spherical symmetry will be assumed. We shall also assume that every unit of mass contains a certain amount of space units following a linear relation. The number of these units will determine the space volume that we measure.

$$V = KM \tag{1}$$

We now determine the value of K. In the process of formation of the mass, a certain volume of space ΔV 'disappeared'. It collapsed into the mass located at the center O, occupying now a smaller volume. For simplicity, we shall assume that this volume is finite and much smaller than the original. We now consider a virtual surface at distance R from the center. Obviously, the original virtual R-sphere of space with volume V around O has contracted to a smaller r-sphere with volume V', and the difference is proportional to the mass that was generated,

$$\begin{aligned}\Delta V &= V - V' \\ &= \frac{4\pi R^3}{3} - \frac{4\pi r^3}{3} \\ &= \frac{4}{3}\pi(R^3 - r^3) \\ &= KM\end{aligned} \tag{2}$$

Calling the (negative) increment in radius $\delta(r)$, we have

$$\begin{aligned}\Delta V &= (\frac{4}{3})\pi[(r+\delta)^3 - r3] \\ &= (\frac{4}{3})\pi[r^3(1+\frac{\delta}{r})^3 - r^3] \\ &\approx (\frac{4}{3})\pi[r^3 + \frac{3\delta r^3}{r} - r^3] \\ &= (\frac{4}{3})\pi 3\delta(r) r^2\end{aligned} \tag{3}$$

The binomial approximation was used here for the cubed expression since $\frac{\delta}{r}$ is much smaller than 1. In the case of the earth, it will be shown to be of the order of 10^{-6}.

As stated, this increment in volume is proportional to the mass M,

$$\Delta V = 4\pi r^2 \delta(r) = KM \tag{4}$$

$$\delta(r) = \frac{KM}{4\pi r^2} \tag{5}$$

If we now make K equal to $4\pi G$ (G: gravitational constant), it turns out that the radial displacement that results from the collapse of space is numerically equal to the gravitational acceleration in Newton's equation,

$$\delta = \frac{GM}{r^2} \quad (For\ G = \frac{K}{4\pi}) \tag{6}$$

Of course, we could have derived the same result much faster by taking the derivative of the volume with respect to the radius,

$$\frac{dV}{dr} = 4\pi r^2 \tag{7}$$

$$dV = 4\pi r^2 dr \tag{8}$$

But this differential expression is only accurate for a small radial increment, which we see more clearly in the direct, non-differential notation, where $\frac{\delta}{r} \ll 1$ was a requirement. That δ be comparable to r would mean a collapse of a large fraction of the virtual sphere, leaving its radius small compared to the original. This would be expected only in strong gravitational fields like those described for black holes and neutron stars. For the earth, the acceleration at its surface corresponds to a radial displacement of $9.82\,m$. The value of $\frac{\delta}{r}$ for the earth is $\frac{9.82\,m}{6.37\ 10^6\,m} \approx 1.54\ 10^{-6}$.

The equations work numerically. The model obtains -under spherical symmetry- a radial displacement that is numerically equal to Newton's acceleration. There is a dimensional discrepancy between K, in units of [m^3Kg^{-1}], and G, which units are [$m^3Kg^{-1}s^{-2}$] which will be addressed later.

We should point out that in picking Newton's constant G (multiplied by 4π) as the value of K we are not making the model dependent on Newton's Law. Indeed, by realizing that δ is a radial displacement and a change in curvature, we could have predicted that it corresponds to an acceleration. From Newton's second law it then becomes a force after multiplication by the mass of the test particle. Therefore, starting from equation (5) we could have put

$$\delta m = F = \frac{KMm}{4\pi r^2} \tag{9}$$

From this equation, by means of careful torsion balance (Cavendish) experiments with different masses and distances, we could have in principle derived the same law and the value of the constant ($\frac{K}{4\pi} = G$). Of course, it would be a waste of time to perform the experiment today.

The ratio of compression in volume is given by $4\pi G$. Its current value of $8.38710^{-10} m^3 Kg^{-1} s^{-2}$ means that every kilogram of mass 'contains' about 10^{-6} liters (one cubic millimeter) of space. This is the space that would be released, along with a huge amount of energy according to $E = mc^2$, if all the mass was completely transformed into space through a nuclear reaction. While this might seem an incredibly small volume, let's remind that virtually all matter is contained in the atomic nucleus, which size is about ten thousand times smaller than the atom ($1/10^{12}$ in volume). This would mean that the actual generation of new space would represent an expansion of about $10^{-9}/10^{-12} = 10^3$ times in volume. Possibly more if instead of protons, the ultimate destiny of compressed space is in smaller particles like quarks.

4 THE ISSUE OF DIMENSIONS AND AN INTERNAL INCONSISTENCY

When we place a mass M at some location O we obtain a radial displacement $\delta(r)$ for all virtual r-spheres according to equation (5). When we do the same by half-steps, first placing $\frac{M}{2}$ at O, obtaining $\delta 1$, then placing the other half $\frac{M}{2}$ at the same location giving $\delta 2$, we should obtain the same final radial displacement as before. We should get

$$\delta 1 + \delta 2 = \delta \ \ and \ \ \delta 1 = \delta 2 \tag{10}$$

But we don't. And the reason is that when we place the second half-mass, space has already been contracted by the first, and the radius when calculating the radial displacement of the second half was smaller and therefore the radial displacement for the second half is wrong and too large. To obtain the correct value of radial displacement for the second half we must first recover the original radius and use it for the calculation. When we do that we obtain a corrected expression for radial displacement:

$$\delta(r) = \frac{GM}{(r + \delta_0)^2} \tag{11}$$

The expression in the above denominator can be simplified using binomial approximation:

$$\begin{aligned}(r + \delta_0)^2 &\approx r^2 + 2\delta_0 r \\ &= r^2(1 + \frac{2\delta_0}{r})\end{aligned} \tag{12}$$

Where δ_0 is the radial displacement previous to the placement of the second half-mass.

The correction allows us to calculate radial displacements or accelerations for masses located in background gravitational fields. Since radial displacement and Newtonian acceleration are equivalent (5) (6), we should consider the same correction for Newton's formula. Indeed, although the problem never arises when placing successive masses as before because Newton's acceleration is based on the total central mass, it would be helpful when a non-baryonic acceleration or an acceleration of unknown origin (Acc_0) is present. The corrected formula for Newtonian acceleration would be:

$$\begin{aligned}Acc &= \frac{GM}{(r + Acc_0)^2} \\ &\approx \frac{GM}{r^2(1 + \frac{2Acc_0}{r})}\end{aligned} \tag{13}$$

Which reduces to Newton's original formula when there is no background acceleration of unknown origin to be accounted for. Furthermore, we can see that in the corrected formula in the geometrical model (11) (12), the units of G become the expected [$m^3 Kg^{-1} s^{-2}$] by changing δ_0 to units of acceleration, since dimensions of $[Acc/Distance] = Time^{-2}$. Likewise, in the corrected Newtonian formula (13) the conventional units of G are recovered if we take Acc_0 in units of radial displacement, since then the whole correcting factor is adimensional. And this would fix the dimensional discrepancies between K an G.

5 THE ACTUAL ORIGIN OF THE GRAVITATIONAL FORCE

The space grid provides a basis for the generation of lines of force, that will bind and attract the masses living in that space. For any two masses A and B, the force lines from A to B are defined as the n (finite) non-overlapping trajectories in the grid that originate from A and reach B, for which the sum-total of the path lengths is minimal. Their number is linearly related to the number of space cells collapsed at the center, i.e., to the central mass. In undeformed flat space, far from masses, lines in the grid are uniformly spaced and distributed and would generate no forces. A particle located in such flat space is the origin of many force lines, but their density is evenly distributed across all concentric virtual surfaces centered on the particle and no force is exerted on it. We don't experience frictional forces between masses and the grid because both share a common nature; they are both space. There can be no longer a frictional force between a substance and itself. The question about a frictional resistance becomes an issue about how fast the grid changes and reforms, which at first glance might be very fast, of the order of light's speed c.

In the presence of gravity, n space units have collapsed into the central mass. A finite number of force lines emerge from it proportional to n and and spread uniformly out of it. For point-like masses and spherical symmetry, the density of the lines decays towards the periphery following an inverse square law of radial distance. The gravitational force on a test particle away from the center is given by the number of force lines emerging from the mass and reaching the particle. Their density would give the acceleration. Moreover, the same number of force lines originating from the mass that are responsible for gravity would also oppose any change in its state of rest or constant velocity, 'fixing' the mass to the surrounding space grid, which would suggest a possible a rationale for the equality between inertial and gravitational mass. This hypothesis and the whole discussion on the lines of force that follows is speculative and only offered as a complement and a hint for further consideration about the toy model itself. A lot of reflection, discussion and mathematical modeling must be done before we can include the concept of force lines in our theories.

From the model, the number of cells contained in a mass M is

$$\#Cells \propto \Delta Volume = KM = 4\pi GM \tag{14}$$

The density of force lines at the surface of the virtual sphere at radial distance r is then

$$\phi(r) = \frac{4\pi GM}{4\pi r^2} = \frac{GM}{r^2} \quad (\phi(r) : \text{density of force lines}) \tag{15}$$

Which is precisely the expression for the gravitational force on each test particle per unit mass. While the constant relating space and matter is $4\pi G$, the constant relating force and mass is simply G.

A test particle is also a mass and would generate its own force lines, but we are interested here in the simplest case and think of a large central mass M and a small test particle which mass is negligible in comparison. Intuitively, force acting on a mass composed of a number of particles bound by strong electromagnetic forces would be linearly related to the number of particles, i.e., to its mass. Therefore, the force acting on mass m at distance r from central mass M becomes the familiar

$$F = \frac{GMm}{r^2} \tag{16}$$

In summary, contraction of space is the origin of and a necessary condition for the appearance of gravitational forces, which might ultimately depend on the density of force lines. The distribution of force lines and the collapse of space based on the mass-space equivalence are intimately related and can be though of as the same process. Under non-spherical symmetry conditions, the analysis is analogous but more complex and will not be done here. For an infinite flat distribution of mass, for instance, force lines would also be generated by the mass, with a larger density than in flat space without gravity. They would depart from the plane of the mass parallel to each other and perpendicular to the mass. The resulting space would be flat for obvious symmetry reasons, and yet a clear gravitational field would arise in which force lines would remain parallel and their surface density, acceleration and δ would be constant, a well-known but often overlooked fact in Newtonian gravity Alonso (1967). This is highlighted in the toy model, casting doubts on the essential nature of space curvature for gravity. Of course, an infinite flat distribution of mass is not found in the real world, where all masses are finite and any mass can be thought of as a point-mass at sufficiently large scales. Therefore, in practice, the inverse-square law holds universally.

What would be the range of gravity?

In standard Newtonian gravity, the range of accelerations and forces is infinite. Based on the equations, there is no theoretical limit for either acceleration or force.

In the toy model, we posit that a linear contraction of radius for all concentric virtual surfaces is an essential part of the phenomenon of gravity. Virtual spheres are sets of nodes from the grid that we choose arbitrarily, but once we choose one, we are referring to a real physical entity: those nodes in the space grid that are equidistant from the center. Without space contraction and radial displacement of those nodes, there can be no force of gravity.

Under this assumption, and given that linear distances smaller than Planck's length cannot be imagined as physical, it is easy to determine at what distance this radial displacement or acceleration becomes smaller than l_P. From Newton's law and the value of the gravitational field at a known distance taken as a boundary, we calculated this distance for the earth, the sun and a spherical 1-Kg mass of density 1. Table 1.

Table 1. Calculated maximum range of gravity for various objects

Object	Mass (Kg)	Range (m)
1-Kg mass	1	$2.0321 10^{12} (\approx 13.59 AU)$
Earth	$5.972 10^{24}$	$4.9661 10^{24}$
Sun	$1.988 10^{30}$	$2.8652 10^{27}$

6 DISCUSSION

The toy model outlines a possible new way to derive Newton's Universal Law of Gravity from first principles by assuming a generic 3-D dynamic grid of undetermined geometry that collapses to mass following a linear relation between mass and space governed by G. Its consistency with Newtonian gravity is ensured from the equations. Considering that radial displacement is a measure of curvature in Euclidean space, it highlights that Newtonian gravity contains a simplified version of space curvature, albeit in disguised form. However, the model suggests also that space deformation, with or without curvature, might be the essential feature of gravity. It hints at an intrinsic limitation of the classical formulation in the presence of strong gravitational fields and modifies the expression for gravitational acceleration to include non-baryonic gravity or gravity of unknown origin. Unbeknownst to NG and GR is the suggestion that gravity might be finite, and the range can be estimated if the minimal distance in space is indeed Planck's length.

But when facing a new theoretical model our main concern is whether and how it can be tested. In this model and according to many physicists including Einstein himself Einstein (1920), space might be undetectable directly. Generation of space from nuclear reactions in particle accelerators might one day be either ruled out or confirmed. However, the tiny amount of space generated per unit mass and the extreme energies that are released make this unlikely to be feasible any time soon. As for the properties of the space grid itself, the most likely size of its unit cells -Planck's scale- puts them also beyond our current technologies. Furthermore, all measurements are based on some kind of energy exchange between the object and the detector, be it light, heat, momentum, or mass. For the postulated space grid, no such exchanges seem possible, at least directly. The law of inertia ensures this for purely mechanical processes. We measure masses because we interact with collapsed forms of space using other collapsed units in our detectors, but this is far from measuring space itself in its pristine, non-collapsed state. As for other forms of energy like light, we can indeed measure light as it impinges on our detectors, but these photons and masses are also excited or collapsed states of space, not space itself. However, even if we do not achieve a direct detection, clues might accumulate so overwhelmingly that a common grounds for a working answer should be reached in the future. As a matter of fact, QM and GR have already reached such conclusion: Space does fluctuate, it bends, and carries light and fields. It should be just a matter of time that its properties are better understood. Based on the toy model, deviations from Newton's equations in the presence of dark matter and other unknown sources of gravity at the center of the Milky Way, or the anomalous distribution of dark matter in galaxies described by the NFW model that depart from expectations -the core-cusp problem- might be tested. But there are further questions raised, such as the tension of the spatial grid, its geometry, how it changes and reforms over time, or how many force lines are there. These might eventually be partially answered from an observational macroscopic-statistical approach. And that might suffice.

Finally, on a more philosophical note, a point can be made for the need of a fundamental equivalence between mass and space. Objects can only communicate when they share a common ground. Strictly speaking, it can even be required that they are of one and the same nature. Historically, the rejection to accept this hypothesis has been a roadblock for progress in the so-called mind-body problem of philosophy. In our case, both Newtonian gravity and GR make it clear that matter and space do communicate, the former bending the latter in the precise manner at the right distance; the latter causing the former to change paths exactly as predicted from its curvature. It should come as no surprise that maybe both are states of the same thing. If confirmed, this might bring us closer to the right answers on several of the standing problems related to gravity in physics and astrophysics.

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