An Enlightening Perspective on Quantum Time

Shan Qiuyu†

(college of science, Lanzhou University of Technology, Lanzhou 730000, China)

Abstract: This paper identifies a long-overlooked issue in foundational physics and proposes a tentative solution: Einstein's clock synchronization scheme cannot be realized at the quantum scale, yet Quantum Field Theory (QFT) employs this unexamined scheme and relies on continuous spacetime coordinates as its foundation. This paper presents a new scheme for defining simultaneity at the quantum scale. Within this scheme, interesting conclusions such as discrete spacetime can be derived, illustrating how it can provide a foundation for the ultraviolet cutoff in renormalization methods, and also offer a possible operational basis for the background spacetime of QFT. It further discusses how to recover traditional Lorentz transformations and the Schrödinger equation by stipulating good clocks, and presents some currently testable corollaries.

Keywords: Quantum time; Simultaneity; Foundations of quantum field theory; Discrete spacetime; quantum reference frame

PACS:03.65.Ta

† Corresponding author.E-mail: shanqiuyu@outlook.com

Consider a reference frame K with a clock A at its origin. We can associate an event P occurring in frame K with a set of numbers (t, x, y, z), i.e., a set of coordinates for P in spacetime. Einstein provided an operational definition for measuring these coordinates[1], but this definition fails at the quantum scale, compelling us to examine quantum reference frames to obtain a complete theory unifying Special Relativity and Quantum Mechanics. As an attempt to unify relativity and quantum mechanics, QFT has achieved great experimental success[2,3], yet this success remains limited because it assumes we can measure the spacetime coordinates of fields in a classical manner, an assumption that does not actually hold. Regarding the concept of time at the quantum scale, the Page-Wootters mechanism provides significant insight[4-6]. They attempt to define a quantum state within the system as a clock, and the evolution of this quantum state $|T_{\mathrm{A}}\rangle$ is defined as coordinate time. However, the Page-Wootters mechanism does not provide a synchronization mechanism like in relativity; its correspondence between an event P in the quantum system and the clock being in state $|T_{\mathrm{A0}}\rangle$ is ambiguous, lacking an effective definition of simultaneity. This paper attempts to provide a new class of definitions for coordinate time and simultaneity at the quantum scale, thereby offering an operational basis for the background spacetime used in QFT.

1. Introduction

1.1 Classical Simultaneity Protocol (Radar Method)

Einstein[1] gave the classical definition of simultaneity in his "*On the Electrodynamics of Moving Bodies*". Consider a clock A and an object B. Clock A emits a beam of light towards B at time t_1. The light is reflected at B and returns to A at time t_2. Then A can define $t_b = \frac{t_1+t_2}{2}$ as the time when the light reached B, and the spatial distance from B to A at time t_b is $x_b = ct_b$, where c is the speed of light. For simplicity, natural units ($\hbar = c = 1$) are used hereafter. Setting A's position as the origin and the line AB as the x-axis direction, the spacetime coordinates $(t_b, 0, 0, x_b)$ can be assigned to "Event P: light reaches B". If B is also a clock, the two clocks can be synchronized, but we discuss the more general case. It is particularly important to note that all times t mentioned above are measured by clock A (coordinate time), not the proper time of object B.

This synchronization method requires that A must be able to distinguish at time t_2 that the returning light is the one emitted at t_1 (and not one emitted earlier from a more distant B). This is straightforward at the macroscopic scale, as we can directly observe the light's trajectory. However, at the quantum scale, this is clearly impossible, especially for a single photon propagating in vacuum, where there is no distinguishable trajectory. We must provide a new way to define simultaneity.

1.2 Difference Between Measuring Coordinate Time and Coordinate Position

Measurement can be described as the entanglement between the observer and the observed object. Taking Schrödinger's cat as an example, the initial state of the observer is $|A_0\rangle = |not\ measured\rangle$, and the initial state of the cat is $|B_0\rangle = |alive\rangle + |dead\rangle$. The state of the entire system can be represented as $|A_0\rangle|B_0\rangle$. Measurement means the two become entangled, i.e., the final state of the system is $|sees\ cat\ alive\rangle|alive\rangle + |sees\ cat\ dead\rangle|dead\rangle$.

If the observer (clock) A measures the coordinate time (note: not proper time) and coordinate position of particle B, it can also be seen as A entangling with B, i.e., $|sees\ B\ located\ at\ x_b\ in\ t_b\rangle|located\ at\ x_b\ in\ t_b\rangle$. If A continuously measures B's coordinate position, B will be found "frozen" at position x_b, known as the "quantum Zeno effect", which has been confirmed by numerous experiments[7]. This spatial coordinate entangled state requires decoherence to dissolve. However, if A continuously measures B's coordinate time, a similar "freezing" obviously does not occur. It is crucial to emphasize that coordinate time is a label assigned by A to B, not something inherent to B itself. The dissolution of the entangled state corresponds to the ticking of clock A. Considering only the results observed by A: one can try to write the state of B $|\text{located at } x_b \text{ in } t_b\rangle$ as two parts, $|t_b\rangle$ and $|x_b\rangle$. $|x_b\rangle$ belongs to the coordinate Hilbert space, while $|t_b\rangle$ corresponds to the quantum state representing B's coordinate time, which cannot be frozen by the quantum Zeno effect.

1.3 Limitations of the Coordinate Time Operator

Assume the existence of a time operator $\hat{T}$. This operator has a canonical commutation relation with the Hamiltonian operator $\hat{H}$:

$$[\hat{T}, \hat{H}] = -i \tag{1}$$

Pauli proved that this necessarily implies that the operator $\hat{H}$ has negative infinite eigenvalues, which does not occur in reality. Thus, a time operator satisfying the canonical commutation relation (1) is impossible. However, the validity of (1) requires the following special properties:

a. The eigenvalue spectrum of the time operator $\hat{T}$ is continuous.

The canonical commutation relation (1) corresponds to a differential relation. It naturally requires $\hat{T}$ to have a continuous eigenvalue spectrum, and allows the construction of the time translation operator based on (1):

$$\hat{U}(\tau) = e^{i\hat{H}\tau} \tag{2}$$

If and only if τ is a continuous parameter, $\hat{U}(\tau)$ forms a strongly continuous one-parameter unitary group, enabling $\hat{H}$ to be defined as the generator of this unitary group.

b. The eigenvalue spectrum of the time operator $\hat{T}$ is unbounded.

The time translation operator (2) requires the following relation:

$$\hat{U}(\tau)\hat{T}\hat{U}^{\dagger}(\tau) = \hat{T} + \tau\hat{I} \tag{3}$$

where $\hat{I}$ is the identity operator. Equation (3) requires that the eigenvalue spectrum of $\hat{T}$ must cover the entire real axis; otherwise, for some parameters τ, equation (3) would exceed the operator's domain.

c. The eigenvalue spectrum of the time operator $\hat{T}$ cannot be both bounded and discrete.

This is a corollary of the above two properties and a direct consequence of (1). Using proof by contradiction, if the eigenvalue spectrum of $\hat{T}$ is both bounded and discrete, then the eigenvectors of $\hat{T}$ form a finite-dimensional Hilbert space $\mathcal{H}$. Representing equation (1) in matrix form within $\mathcal{H}$ and taking the trace yields:

$$\mathrm{Tr}([\hat{T}, \hat{H}]) = \mathrm{Tr}(\hat{T}\hat{H}) - \mathrm{Tr}(\hat{H}\hat{T}) = 0 \neq -i \tag{4}$$

Therefore, Pauli's argument shows that defining a time operator with a continuous and unbounded eigenvalue spectrum is impossible. However, we can operationally define a time operator with a bounded and discrete eigenvalue spectrum. The concept of continuous, unbounded coordinate time is indefinable at the quantum scale; we can only define such discrete, bounded coordinate time. Indeed, at the microscopic scale, there might be no coordinate time in the classical sense at all.

2. Definition of Coordinate Time

To define a coordinate time operator, three aspects must be included: first, the definition of the temporal sequence of coordinate times; second, the definition of time intervals; third, the definition of simultaneity. Our core goal is to establish the

correspondence between an event P occurring at B and a certain time label of the clock. Before specifying the simultaneity between clock A and the vicinity of object B, one cannot simply say that B's position is measured at a specific time.

2.1 Natural Definition of Temporal Sequence and Time Interval

Special Relativity does not stipulate temporal sequence and time intervals; it assumes the existence of naturally ticking clocks without explaining the internal mechanism of this ticking. However, the quantum nature of light suggests we can give a natural definition for temporal sequence and time intervals.

Returning to Einstein's clock A and object B, now clock A does not emit a continuous, trackable beam of light, but instead continuously emits individual photons. This photon is scattered by B and returns to A. Suppose clock A emits one photon per second, or conversely, every time clock A emits a photon, we say clock A has ticked by 1 second (i.e., coordinate time has advanced by one second).

We can define the temporal sequence as the number of photons emitted by A, and the time interval as the difference in the number of photons emitted. The state of clock A can be described as $|\Phi_{An}\rangle$, where n represents the number of photons already emitted by clock A. The mechanism for exciting photons clearly has:

$$\langle \Phi_{Am} | \Phi_{An} \rangle = \delta_{mn} \tag{5}$$

The states of clock A at different times are orthogonal, which is the automatic disentanglement of coordinate time we anticipated in section 1.2. Quantum mechanics and thermodynamics together require that clock A cannot excite photons continuously without intervals, as this would violate the basic principles of quantum mechanics and cause clock A to rapidly deplete its energy and stop.

2.2 Definition of Simultaneity at the Quantum Scale

Combining the discussion in 2.1, we generalize the classical definition of simultaneity from 1.1 and attempt to express it in a more general form.

Consider a type of quantum memory $|n\rangle$. Clock A can emit this quantum memory, where n indicates the number of memory units already emitted by clock A. $|n\rangle$ stores the data n, corresponding to the time information of its emission by clock A in the classical case. $|n\rangle$ interacts with object B ($|\Psi_B\rangle$), acquires information about B, and changes its state to $|n, \Psi_B\rangle$. Finally, when clock A becomes $|\Phi_{Am}\rangle$, the memory $|n, \Psi_B\rangle$ is received and read by A. Based on the information m and n, the coordinate time and spatial position of $|\Psi_B\rangle$ are defined.

For the clock, timing is synchronizing, and synchronizing is timing. The so-called coordinate time interval is the number of synchronizations performed by the

clock. These are not two processes but two aspects of the same process. This is the core assumption of the time protocol discussed in this paper.

3. Field Theory Time Protocol

3.1 Protocol Overview

Section 2.1 left the problem of photons themselves being indistinguishable, while the problem in section 2.2 is that the quantum memory is abstract and vague. Therefore, we attempt to concretize the quantum memory as excitations of a field, simultaneously solving the problems in both 2.1 and 2.2.

Consider a system composed of the clock field Φ_A, the probe field ϕ, and the matter field Ψ_B. Initially, the three fields are independent. The clock field Φ_A can spontaneously interact with the probe field ϕ. Φ_A undergoes the n-th local interaction g_1 with ϕ, causing ϕ to exhibit an excitation mode $|\phi_n\rangle$ that can serve as quantum memory. This mode records the excitation ordinal n and changes the excitation state of Φ_A itself from $|\Phi_{A(n-1)}\rangle$ to $|\Phi_{An}\rangle$. The property of $|\phi_n\rangle$ recording the excitation ordinal requires $\langle\phi_m|\phi_n\rangle = \delta_{mn}$; for example, the photon's orbital angular momentum can be related to n. The probe field $|\phi_n\rangle$ propagates to the region where B is located, interacts with the matter field Ψ_B via interaction g_2. g_2 modulates the probe field, changing its state to $|\phi_n, \Psi_B\rangle$, thus containing information about B. However, g_2 must not destroy the orthogonality of $|\phi_n\rangle$. Therefore, the information of n and Ψ_B can reside in two different degrees of freedom of ϕ, e.g., encoded in angular momentum and polarization, respectively. The modulated probe field $|\phi_n, \Psi_B\rangle$ propagates back to the region of clock A. When the clock field is in state $|\Phi_{Am}\rangle$, the reception interaction g_3 occurs. A can then define the coordinate time of the occurrence of g_2 as (n+m)/2. We can view this as an operational definition of the coordinate time operator $\hat{T}$ from section 1.2. It is the sum of the measurement operators for $|\Phi_{Am}\rangle$ and $|\phi_n, \Psi_B\rangle$, ultimately yielding the result (n+m)/2.

For mathematical simplicity, we describe the probe field ϕ as a massless scalar field to discuss the photons used for synchronization. Real physical situations require generalization to vector fields. Since we can always find orthogonal states for photons, this simplification is valid.

3.2 Scalar Field Synchronization Process

a. Initial System State

$$|\Gamma_{initial}\rangle = |0\rangle_A \otimes |vac\rangle_\phi \otimes |\Psi_B\rangle_B \quad (6)$$

Clock A is in the "zero" state $|0\rangle_A$, the photon field $\phi(x)$ is in the vacuum state $|vac\rangle_\phi$, and the matter field B is in a general state $|\Psi_B\rangle$.

b. Emission Event: "Spontaneous" Local Excitation

From the perspective of the reference frame where clock A defines coordinate time, this process is not driven by a parameter time; it is itself a step that generates time. It can be expressed as the action of a local operator $\widehat{O_A^{emit}}$:

$$\widehat{O_A^{emit}}|n_0\rangle_A \otimes |vac\rangle_\phi = |n_0 + 1\rangle_A \otimes |\gamma_n\rangle_\phi \quad (7)$$

where $|\gamma_n\rangle_\phi = a^\dagger(\gamma_n)|vac\rangle_\phi$ is a photon state of a specific mode produced near A, which records the excitation count n (e.g., via orbital angular momentum) and satisfies $\langle \gamma_m | \gamma_n \rangle = \delta_{mn}$.

From the system's internal perspective, this is a "spontaneous" process that cannot be described as driven by time, because the time interval has not yet been defined. We need to postulate, as a protocol axiom, that the emission event is considered spontaneous from the internal perspective.

c. Detection Event (P): Local Interaction between Photon and Matter Field B

The photon field ϕ propagates according to the Klein-Gordon equation and interacts locally with the matter field operator Ψ_B at coordinate time t_b. This interaction causes another degree of freedom of the photon (e.g., polarization or another) to carry information related to B. The Hamiltonian form can be given as:

$$H_p(t_b) = \int dx \; g_2(x, t_b)\phi(x) \otimes O_B + h.c. \quad (8)$$

where $g_2(x, t_b)$ is a function describing the coupling interaction strength at different positions, and O_B is a modulation operator related to B. It can transform the photon $|\gamma_n\rangle_\phi$ into $|\gamma_n, \Psi_B\rangle_\phi$. Note that t_b is yet to be defined, so here we only provide the form of its Hamiltonian.

d. Reception Event: The modulated photon wave packet returns to A, is detected and absorbed by A.

Since clock A is continuously ticking, it is now in state $|m\rangle_A$. The reception action can also be described mathematically by a local operator $\widehat{M_A}$:

$$\widehat{M_A}|m\rangle_A \otimes |\gamma_n, \Psi_B\rangle_\phi = \frac{n_0+m}{2}|m\rangle_A \otimes |vac\rangle_\phi \quad (9)$$

$\widehat{M_A}$ simultaneously reads the current excitation count m of clock A and the excitation ordinal n of the photon, annihilates the photon, and can be written as:

$$\widehat{M_A} = \frac{n_0+m}{2}\eta \sum_{\gamma_n} |vac\rangle_\phi\langle\gamma_n, \Psi_B| \otimes \sum_{m>n_0} |m\rangle\langle m|_A \quad (10)$$

The eigenvalue of $\widehat{M_A}$ is defined as the coordinate time $t_b = \frac{n_0+m}{2}$ of the detection event P. $\widehat{M_A}$ is a constructed post-processing algorithm, the concrete implementation of the protocol rule. It represents a fundamental process within the system that maps two discrete quantum numbers (n_0, m) into a classical coordinate time reading t_b. Coordinate time t_b is classical information after measurement, our definition of coordinate time within the protocol, a convention, a post-processing function. It naturally returns to Einstein's result in the limit, rather than there being an externally measurable coordinate time. $\widehat{M_A}$ gives an expectation value of a quantum operator; it is merely a tool we use to describe the reception event; maintaining a linear form is not its duty.

It can be seen that coordinate time becomes discrete; it can only be half-integers, and so are time intervals. However, specifically, if the velocity of object B is less than the speed of light, $\Delta t_b \neq 0.5$. Since spatial distance is defined as $x_b = t_b$, the spatial resolution also has a corresponding minimum precision of 0.5. Time and space are grid-like, or rather, built from "spacetime bricks". The spatial grid points can be defined based on the state $|m_1\rangle$ of A when the photon with ordinal n_0 returns: $x_{m_1} = \frac{n_0+m_1}{2}$. This provides an operational basis for the ultraviolet cutoff in the renormalization methods of QFT.

3.3 Description Introducing an External Clock

We introduce an external classical clock (with a continuous time spectrum) to view the process in 3.2, i.e., using the coordinate time of QFT. It is crucial to note that this is only for convenience of description using field theory language. It does not imply the existence of such a continuous spectrum clock, nor that the process in 3.2 is driven by an external time parameter, nor that an external clock is necessary. The events in 3.2 are assumed spontaneous by the protocol; they define time rather than being driven by external time. The external time parameter is denoted by τ.

a. Emission Event: Similar to a controlled version of atomic spontaneous emission. When clock A transitions from state $|n_0\rangle_A$ to $|n_0+1\rangle_A$, it produces a photon excitation at its location x_A. Its Hamiltonian is given as:

$$H^n_{emit}(\tau) = g_1(\tau)[a^\dagger_{\gamma_n}(x_A) \otimes \Theta^+_A] + h.c. \quad (11)$$

where $g_1(\tau)$ represents a coupling function peaked around $\tau = \tau_n$, where τ_n denotes the emission time of the n-th particle in the view of 3.2. Within the system, this is seen as a spontaneous excitation process, described by the property of

the function $g_1(\tau)$ for an external observer. But it must be emphasized that $g_1(\tau)$ is merely a descriptive method, indicating that from the external observer's view, the emission event can be described as driven by an external time parameter τ. The premise for such a description is having a clock external to the system. Within the system, as described in section 3.2, no such parameter description for the emission event can be found. H^n_{emit} can be experimentally carefully chosen so that the excited photons carry information about the excitation count. *h. c.* ensures its Hermiticity.

From the external perspective, the state $\Psi_A(\tau)$ of clock A undergoes complete unitary evolution between two excitations, described by the Hamiltonian of clock A and the Schrödinger equation. Therefore, the states of A between two excitations are indistinguishable, or can be seen as unchanged, and cannot be used to measure time. Thus, the expectation of emitting only one photon and waiting for its return before emitting the second photon for synchronization is unattainable. Between two excitations, clock A simply does not tick. The only thing that can be used to measure time is the excitation count. No matter when (external time) the photon is received by A between the *m*-th and (*m+1*)-th excitation, A can only record it as time m. If the unobservable unitary evolution $\Psi_A(\tau)$ is used as the clock standard, it essentially introduces an external clock. This is why time must be discrete: because the non-unitary evolution of the clock cannot be continuous.

b. Detection Event (P): The photon propagates to B and scatters or is absorbed and re-emitted with the matter field Ψ_B.

This is an interaction between the photon and the matter field, but it needs to modulate the photon. The general form of the Hamiltonian is given:

$$H_p(\tau) = \int dx\, g_2(x,\tau)[\phi(x) \otimes O_B] + h.c. \quad (12)$$

The key to this interaction is to modulate the photon and entangle it with the state of B.

$$H_p:\ |\gamma_n\rangle_\phi \otimes |\Psi_B\rangle_B \rightarrow c_1|\gamma_n^1\rangle_\phi \otimes |\Psi_B^1\rangle_B + c_2|\gamma_n^2\rangle_\phi \otimes |\Psi_B^2\rangle_B + \dots \quad (13)$$

Thus, information about B is recorded on the photon.

c. Reception Event: The modulated photon wave packet returns to clock A, is detected and absorbed by A. We introduce a simple two-level probe S, its states denoted as $|\text{ready}\rangle$ and $|finish\rangle$. The Hamiltonian form is given:

$$H^m_{rece}(\tau) = g_3(\tau)[\sum_{n<m} a(\gamma_n) \otimes \sum_{m>n} |m\rangle\langle m|_A \otimes ((|finish\rangle\langle ready|)\,] + h.c. \quad (14)$$

It can describe:

α. Annihilation of a photon (operator $a(\gamma_n)$, summation indicates any photon with time ordinal n < m can be annihilated).

β. Clock A being in any state $|m\rangle_A$ with excitation count greater than n can undergo this process.

γ. The probe S transitions from the ready state $|\text{ready}\rangle$ to the finished state $|\text{finish}\rangle$, indicating a measurement has been completed.

When probe S shows $|\text{finish}\rangle$, we know a photon has been absorbed. We simultaneously (simultaneous in external time τ) record the quantum number m of clock A and the quantum number n of the photon. Finally, through a classical post-processing algorithm, we calculate the coordinate time of the detection event P as measured by clock A (not the external time):

$$t_b = \frac{n_0+m}{2} \tag{15}$$

Section 3.2 is an internal description, independent of any external observer. Section 3.3 is an attempt, within an external classical reference frame using QFT language, to describe the internal simultaneity protocol of the system. The operational protocol is fundamental, rather than assuming spacetime as the fundamental stage. QFT, which relies on spacetime coordinates, is built upon the protocol and cannot be used to argue for the protocol (e.g., arguing that "spontaneous" processes like atomic spontaneous radiation are caused by interaction with vacuum fluctuations and can still be described by corresponding Hamiltonians and continuous time parameters). At most, as in 3.3, an external clock can be introduced to describe the emission event using an external parameter. Within the system, "spontaneous emission" must be taken as a protocol postulate; externally, it can be described with parameters. In other words, whether a process is "spontaneous" or parameter-dependent depends on the clock used. The external clock itself also relies on its own protocol postulates.

3.4 Protocol Postulates

Postulate I: Existence of a Quantum Clock

There exists a physical system that can serve as a quantum clock A. Its Hilbert space is spanned by a set of discrete, orthogonal states $|n\rangle_A$ (n is an integer, practically the excitation count described in Postulate II, which inherently carries a sequential order), satisfying $\langle m|n\rangle = \delta_{mn}$. The "ticking" of the clock manifests as a series of spontaneous quantum transitions of the system state among these orthogonal states: $|n\rangle_A \rightarrow |n+1\rangle_A$. Each transition marks a fundamental discrete unit of coordinate time. Not any object can serve as a clock; only those carefully selected and designed for synchronizing simultaneity can. Proper time is not the object's own time, but the time recorded by a clock moving with the object near it.

Postulate II: Generation and Orthogonality of Quantum Memory

The clock interacts with a probe field, thereby producing a local excited state $|\gamma_n\rangle$ in that field, and causes clock A to undergo one transition ($|n\rangle_A \rightarrow |n+1\rangle_A$). $|\gamma_n\rangle$ serves as a quantum memory, recording the excitation ordinal n through some

physical degree of freedom (e.g., angular momentum). All memory states are mutually orthogonal: $\langle \gamma_m | \gamma_n \rangle = \delta_{mn}$.

Postulate III: Operational Definition of Simultaneity

The coordinate time t_b of an event P occurring at a distant object B (e.g., the interaction between the probe field and B) is operationally defined by the following protocol:

a. Clock A emits memory $|n_0\rangle_A$ while in state $|\gamma_{n0}\rangle$.

b. $|\gamma_{n0}\rangle$ interacts with B, its state becomes $|\gamma_{n0}, \Psi_B\rangle$, encoding information about B.

c. This memory propagates back to A and is received when clock A is in state $|m\rangle_A$.

d. The coordinate time of event P is defined as $t_b = (n_0 + m)/2$, and the coordinate position is defined as $(n_0 + m)/2$. This definition is a fundamental convention, not a measurement result.

Postulate IV: Completeness of Internal Description

The above protocol (Postulates I-III) provides a complete physical description for the system internally. The entire process is spontaneous and does not rely on any external, continuous classical time parameter for driving. This set of postulates is the prerequisite for the system's internal description of its dynamics. It is fundamentally impossible to express this using dynamical mechanisms described internally. We can introduce an external clock for auxiliary description; the spontaneous excitation of the clock, which is a postulate from the internal perspective, is described as a controlled excitation under the field $g_1(\tau)$ from the external perspective. However, the external clock itself cannot describe its own protocol. The internal perspective is fundamental, but utilizing the external perspective can provide more familiar approximate calculation methods.

4. **Evolution Operators**

4.1 Evolution Operator for Clock A

In the internal perspective, clock A itself is stipulated to undergo spontaneous excitation, so its evolution cannot be described by dynamical mechanisms or a Hamiltonian operator. We can write the evolution operator for clock A as:

$$\widehat{U_A}(n+1, n) = \sum_{n>0} |n+1\rangle\langle n|_A \tag{16}$$

We can use a form similar to 3.3 to introduce an external clock and then study clock A itself as a new observed object. Using the external clock as a standard, the time interval between the n-th and (n+1)-th excitation of clock A is denoted $\delta\tau_n$.

Via the external clock, we can also describe the dynamics of clock A using a Hamiltonian including (11). But this merely shifts the problem of clock A to the

external clock. No matter what, a process that cannot be described by dynamical mechanisms will always appear. In contrast, we are more concerned with the evolution operator of object B (whether it is another clock or not).

4.2 Evolution Operator for Object B

We give the evolution operator for object B in the following form:

$$\widehat{U_B}(n+1,n)|\Psi_B\rangle_B = |\Psi'_B\rangle_B \tag{17}$$

$$\widehat{U_B}(n+1,n) = e^{i\hat{S}_B(n+1,n)} \tag{18}$$

The unitary operator $\widehat{U_B}(n+1,n)$ gives the evolution of object B between the n-th and $(n+1)$-th detection events P_n and P_{n+1} by clock A (note n is not B's coordinate time). The two detection events occur at coordinate times t_n and $t_n + \frac{k}{2}$, where k is a positive integer. Since B may be moving, k is not always equal to 2. $\hat{S}_B(n+1,n)$ is a Hermitian operator with the dimension of action. Introducing the external clock from 3.3, we can give:

$$\hat{S}_{\text{Bout}}(n+1,n) = \int_{\tau_n}^{\tau_{n+1}} d\tau \int dx\, \mathcal{H}_{Bout}(x) \tag{19}$$

where $\mathcal{H}_B(x)$ is the Hamiltonian density of object B from the external perspective. However, from the internal perspective, we can only obtain:

$$\hat{S}_{\text{Bin}}(n+1,n) = \sum \mathcal{H}_{Bin}^{eff}(x_{m_1})_n \tag{20}$$

The summation Σ is over all spatial grid points, multiplied by 1 in time, i.e., one discrete time unit. $\mathcal{H}_{Bin}^{eff}(x_{m_1})_n$ can be defined as the effective Hamiltonian density from the internal perspective. Its specific form requires further analysis. The label n indicating the n-th measurement is important; it actually signifies that it is no longer a three-dimensional density of the Hamiltonian distributed in space, but a four-dimensional density (although the dimension hasn't changed) of $\hat{S}_{\text{Bin}}$ distributed over spacetime, representing its value on a spacetime grid. Further, a more general evolution operator is given:

$$\widehat{U_B}(m,n) = \prod_{i=n}^{m-1} e^{i\hat{S}_B(i+1,i)} = e^{i\sum_{i=n}^{m-1}\hat{S}_B(i+1,i)} \tag{21}$$

Due to the discrete time in the internal perspective, (21) cannot be written in integral form. But by introducing an external clock, B's evolution operator can be written as:

$$\widehat{U_B}(\tau_m,\tau_n) = e^{i\int_{\tau_n}^{\tau_m} d\tau \int dx\, \mathcal{H}_{Bout}(x)} \tag{22}$$

The photon field $\phi(x)$ is also a special object B. Of course, in the same protocol, the same object cannot simultaneously play the role of probe particle and object B.

5. Return to the Classical Limit

5.1 The Classical Good Clock (Ideal Clock)

Discussion of good clocks often appears in General Relativity[8], but this paper does not consider gravitational effects. A good clock is defined as one that measures

time uniformly when stationary, meaning the time interval between one second and the next is equal, and the ticking rate is consistent at different times, so its ticking is proportional to proper time. But directly comparing the lengths of two time intervals is impossible. For this, we introduce another criterion: Suppose you have a classical clock C. You use this clock to perform time measurements for all physical experiments. Based on the form of the evolution operator obtained from the experiments, you can derive the form of the Hamiltonian density operator for the measured object. If the eigenvalues corresponding to the eigenvectors of this Hamiltonian operator are conserved, then we call clock C a good clock. This is a reverse application of Noether's theorem: a clock from which the law of energy conservation can be obtained experimentally is a good clock.

Now we assume that the external clock introduced in 3.3 is such a classical good clock. First, use it as a stepping stone to judge whether clock A is a good clock: it requires that the time interval between each excitation of clock A is consistent, i.e., one discrete time unit can be expressed as:

$$\delta\tau_n = \delta\tau_m = \delta\tau, \quad \forall n, m = 0,1,2\ldots \tag{23}$$

Then, from the external clock's perspective, (20) can be written as:

$$\hat{S}_{\mathrm{Bin}}(n+1,n) = \delta\tau^4 \sum \mathcal{H}_{Bin}^{eff}(x_{m_1})_n \tag{24}$$

When $\delta\tau \to 0$, clock A itself becomes a classical clock, and (24) becomes:

$$\hat{S}_{\mathrm{Bin}}(n+1,n) = d\tau^4 \sum \mathcal{H}_{Bin}^{eff}(x_{m_1})_n \tag{25}$$

Comparing (19) and (25), we find that the Hamiltonian density defined by good clock A should, in the limit, return to the result of the classical good clock (reverse application of Noether's theorem and the external clock is, in fact, also a larger-scale internal clock):

$$\lim_{\delta\tau\to 0} \mathcal{H}_{Bin}^{eff}(x_{m_1})_n = \mathcal{H}_{Bout}(x) \tag{26}$$

And (21) should also approximately return to (22), i.e., the operator form of the Schrödinger equation. That is, the form of $\mathcal{H}_{Bin}^{eff}(x_{m_1})_n$ will change depending on clock A. For example, considering some radioactive atoms, using time t = N, where N is the number of decayed atoms, the form of $\mathcal{H}_{Bin}^{eff}(x_{m_1})_n$ we give would be completely different. But if clock A is a good clock, then the form of $\mathcal{H}_{Bin}^{eff}(x_{m_1})_n$ should return to the Hamiltonian given by QFT in the limit.

5.2 Return of Lorentz Transformation

a. Spacetime Bricks and Return of Classical Equations of Motion

Spacetime is not continuous but a discrete "grid". Assume clock A is a good clock. Construct the spacetime brick $\frac{\delta\tau^4}{16}$ brought by clock A, where $\delta\tau$ is the

synchronization time interval of A as seen by the external clock. The denominator 16 arises because coordinate time can be half-integer, so four-dimensional spacetime takes $\frac{1}{2^4}$. Events on the same spacetime brick are considered simultaneous and co-located by A. Label each spacetime brick $\frac{\delta\tau^4}{16}$ with a number (t, x, y, z) (using fixed vertex coordinates). Then we can use four discrete index numbers to label the spacetime bricks. Spacetime bricks are not actual bricks but our mathematical description of co-location and simultaneity. Each clock will give different spacetime bricks. We can assume these spacetime bricks from different clocks share the origin (0,0,0,0).

In the view of clock A, the motion of object B is not continuous progression but jumps between spacetime bricks. Velocity is the ratio of the number of bricks jumped in the x-direction to the number jumped in the t-direction. Motion is not from one point to another but from "lighting up" one spacetime brick to "lighting up" an adjacent one.

For simplicity, consider one-dimensional space and one-dimensional time (i, j), where i is the time index and j is the space index. The motion of object B manifests as "lighting up" the spacetime brick at space index j(i) at time index i. An object moving slower than light first lights up q spacetime bricks in the t-direction before lighting up p bricks in the x-direction. That is, it's not truly uniform motion but stationary-jump-stationary-jump. Since spacetime is discrete, the object's velocity v must be a rational number. We give the equation of motion for object B:

$$j(i) = int(\frac{p}{q}i) \tag{27}$$

where int denotes the floor function, and p and q are coprime integers. When i and j are large, the above formula naturally becomes $j(i) = \frac{p}{q}i$. Restoring dimensions via $\delta\tau$ yields $x = \frac{p}{q}t = vt$, the classical equation of motion. Specifically, the equation for light-speed motion is $j(i) = int(i)$.

b. Return of Lorentz Transformation

Consider object B being a new clock identical to A when at rest. Another object C moves with B, at a distance $x' = \frac{k\delta\tau}{2}$ from clock B (i.e., the spatial distance between the spacetime bricks of B and C as determined by clock A), where k is a positive integer. Let B perform a light signal synchronization with C. We want to know how A describes this synchronization process.

1. Emission event: Clock B emits a light signal at time index i_e (clock A's time). The space index of B at this moment is $j_B(i_e)$. B records its own proper time as n_e at the moment of emission.

2. Reflection event: The light signal reaches object C and is reflected at time index i_r. The space index of C at this moment is $j_C(i_r) = j_B(i_r) + k$. Light signal propagation requires:

$$i_r - i_e = j_B(i_r) - j_B(i_e) + k \tag{28}$$

3. Reception event: The light signal returns to the spacetime brick where clock B is located at time index i_s. The space index of B at this moment is $j_B(i_s)$, and B's proper time is n_s. Light signal propagation requires:

$$i_s - i_r = j_C(i_r) - j_B(i_s) \tag{29}$$

Substituting $j_C(i_r)$ gives:

$$i_s - i_r = j_B(i_r) + k - j_B(i_s) \tag{30}$$

Clock B can calculate the index of event C in B's own spacetime bricks: $i_r' = j_C'(i_r') = \frac{n_s+n_e}{2}$, but this is B's own time protocol, unrelated to A.

Equations (28) and (30) are Diophantine equations, solving them is mathematically difficult, but it is sufficient for discussing how to approximate the return to Lorentz transformation. When i and j are large, B satisfies the equation $j_B(i) = vi$. When $\delta\tau$ is small, (28) and (30) can be written as:

$$(1 - v)(t_r - t_e) = x' \tag{31}$$

$$(1 - v)(t_s - t_r) = x' \tag{32}$$

Where t represents the time measured by A, and B's proper time can be written as $\frac{\tau_s+\tau_e}{2}$.

These are the relations Einstein obtained in "*On the Electrodynamics of Moving Bodies*", thus returning our synchronization protocol to the classical case. Clearly, this can lead to the Lorentz transformation.[1]

Here we have completed the initial goal of this paper: to provide an operational spacetime measurement protocol for QFT. Next, we need to discuss what deviations this protocol brings to traditional QFT based on Einstein's synchronization.

5.3 Perfect Good Clocks Do Not Exist

The clock A cannot be described by a Hamiltonian internally. However, each excitation of a photon by A necessarily inputs energy into the observed object (including B). That is, the effective Hamiltonian density of B obtained by clock A in each measurement must be inconsistent:

$$\mathcal{H}_{Bin}^{eff}(x_{m_1})_n \neq \mathcal{H}_{Bin}^{eff}(x_{m_1})_m \tag{33}$$

$$\mathcal{H}_{Bin}^{eff}(x_{m_1})_{n+1} = \mathcal{H}_{Bin}^{eff}(x_{m_1})_n + \mathcal{H}_{g_{2n}}^{B}(x_{m_1})_n \tag{34}$$

where $\mathcal{H}_{g_{2n}}^{B}(x_{m_1})_n$ represents the influence of the g_2 interaction during the n-th measurement on B's Hamiltonian. This quantity is small but non-zero. This indicates that clock A can at most obtain an approximate energy conservation law. Therefore, distinguishing between good and bad clocks at the quantum scale is

meaningless; good and bad clocks are equal; every clock can consider its own ticking uniform. Clock A itself will eventually deplete its energy and cease operation. This is evident from the external clock's perspective, but clock A itself cannot define this "termination time". From clock A's own perspective, it is still working, but from the external clock's perspective, the time interval $\delta\tau$ for clock A to execute the protocol becomes infinitely long.

Therefore, for an observer not introducing an external clock, (26) only holds approximately. Given enough time, the evolution of object B will necessarily deviate from the Schrödinger equation and can only use (21). Introducing an external clock is reasonable, but any external clock is just a larger-scale internal clock. The classical returns in 5.1 and 5.2 can hold because classical theory and the Schrödinger equation all implicitly assume measurements by ideal good clocks. It is not that nature favors such clocks, but that at the classical scale, we choose such clocks for timing.

5.4 Influence of the g_2 Interaction

For single-photon synchronization, if the g_2 interaction can disturb the excitation ordinal information (e.g., using orbital angular momentum to label the excitation ordinal and ensuring photon state orthogonality, while g_2 can change the photon's orbital angular momentum), causing the orthogonality of the photon states to fail, the coordinate time information we obtain for state B will deviate or even be lost. Coordinate time is not only discrete but also deviant. However, this does not mean the protocol in this paper cannot be executed; it only indicates that such errors are unavoidable. The protocol cannot be executed without disturbance or error.

A g_2 that perfectly executes the protocol needs to be carefully selected, which is impossible for arbitrary object B. Moreover, even with careful selection, it cannot be guaranteed that B's evolution won't cause deviation from the initially chosen g_2. Therefore, g_2 will almost certainly cause our measurement results to deviate. Suppose we label the excitation count n with orbital angular momentum l, i.e., set n=l. This is theoretically feasible. But if g_2 perturbs the angular momentum, causing $\Delta l = +1$, then the corresponding coordinate time will also deviate. Clock A cannot distinguish whether the photon was released at time n or n+1, leading to a change in coordinate time of $\Delta n = 0.5$. This is negligible at the classical scale but not ignorable at the quantum scale. Larger Δl will cause greater deviations.

Single-photon synchronization experiments will necessarily exhibit deviations related to g_2. But it's important to note that we are not trying to measure this deviation, because g_2 itself is an unobservable process from the internal perspective. If B is a quantum object (where classical synchronization cannot be realized), this deviation is immeasurable. However, we can see that this deviation leads to results different from QFT predictions. That is, multiple single-photon synchronizations will yield experimental results that deviate from the predictions of QFT.

Here we emphasize a point: the form given by (15) is precisely B's coordinate time, regardless of whether g_2 perturbs the photon or not. What we obtain is B's coordinate time because coordinate time is not B's own but a label we assign to B. What is illustrated here is not that g_2 disturbs our measurement, but that it changes B's coordinate time itself.

Creating quantum clocks with current experimental technology is challenging. Most clocks quickly interact with the environment and decohere due to synchronization, completely losing their quantum clock properties. We can consider a semi-classical clock, like the radioactive atomic clock mentioned in 5.1, plus a single-photon excitation device D. The role of D is to excite a single photon with orbital angular momentum $l = n$ upon receiving signals from n decayed atoms. The entire device uses only the number of decayed atoms for timing (more precisely, the number of times D receives decay signals, to avoid time error from decay to reception),making no reference to an external clock(in a completely intrinsic framework), thus approximately simulating some effects of a quantum clock and having it execute the protocol in 3.4. Needless to say, this clock is only an approximation - a compromise imposed by current experimental limitations.

6. **Observable Phenomena**

1. The deviation in predicted results due to coordinate time deviation caused by single-photon synchronization mentioned in 5.4, which differs from QFT, is observable experimentally. We can design experiments for the same object B (which can be macroscopic) using the semi-classical synchronization mentioned in 5.4 and classical synchronization (where the light trajectory can be observed). This deviation can then be observed, and such experiments are feasible in the short term. However, changing the timing of the semi-classical clock from decay count to the logarithm of the decay count can approximate a good clock, thereby reducing the difference from the classical clock and more clearly demonstrating that the deviation originates from g_2 rather than different uniformity of the two clocks.

2. Spacetime is not continuous but discrete. There exists a minimum resolvable spatial length and a maximum momentum $\frac{2}{\delta\tau}$ (similar to quasi-momentum in solid-state physics) definable by this clock. The momentum measured by this clock observer is also discrete and bounded.

3. Clock A cannot measure permanently; it will inevitably deplete its energy and be unable to excite more photons. Therefore, not all objects can serve as clocks. Proper time is not the object's own time but the time of a clock moving with it nearby. Every object changes with time, but not every object can serve as a clock capable of timing and synchronizing simultaneity. An object serving as a clock must guarantee the execution of the protocol.

4. The Hamiltonian of object B is perturbed by the photon during measurement, making it non-conserved in each observation. Therefore, continuous observation will

cause the energy eigenstates to deviate from the description of the Schrödinger equation, and the quantum Zeno effect also cannot "freeze" the energy eigenstates.

7. **Directions for Future Research**

1. This paper only considers the one-dimensional case. How to generalize it to three dimensions or how to consider angles will be an important future direction, possibly requiring the introduction of more labels like angular quantum numbers.

2. How to reformulate existing QFT theories to conform to this protocol will be a significant challenge and a necessary task for a qualified theory. Furthermore, the protocol itself requires a more effective mathematical formulation.

3. The protocol given in this paper does not consider gravity. QFT is very successful without gravity. This paper can be seen as providing a possible operational basis for its assumed background spacetime. But the true boundary between this paper and QFT lies in experiments with extremely high precision (e.g., high enough to measure quantum effects of macroscopic clocks like watches), which would inevitably introduce gravity. This paper actually discusses situations where the quantum effects of macroscopic objects are strong enough while gravitational effects can be ignored. Will the protocol need modification after introducing gravity? Will it naturally lead to a theory of quantum gravity? This requires further exploration.

4. Experimentally, one can attempt to use photon orbital angular momentum to encode the excitation ordinal (naturally ensuring orthogonality) to verify the conclusions of this paper.

5. From the internal perspective, the reception process means the photon's energy and information disappear from the world observed by A. From A's view, this is a non-unitary, irreversible process, which should lead to entropy increase. This might provide a microscopic perspective on the irreversibility of thermodynamics, but the specific connection needs further study.

6. Many specific mathematical forms in the paper need refinement, such as the form of the Hamiltonian density for a general clock, the explicit expression for discrete Lorentz transformations, and discussing the possible impact of g_2 on the form of Lorentz transformations. This paper proposes a discrete spacetime theory based on an operational protocol, which naturally returns to Lorentz transformations in the continuous limit. However, the core mathematical structure, the precise transformation relations between discrete spacetime bricks, requires solving Diophantine equations, which is mathematically very complex. We speculate that this indicates the need for a new set of mathematical tools to naturally describe this discrete, operational spacetime structure. Exploring this mathematical framework will be a highly attractive future direction.

References

[1] Einstein A .On the Electrodynamics of Moving Bodies [J] .Annalen der Physik, 1970, 17(3):160-183.DOI:10.1016/B978-0-08-006995-1.50014-4.
[2] Peskin M E , Schroeder D V .An Introduction to Quantum Field Theory[M].BeiJing:World Publishing Corporation ,2020
[3] Weinberg S .The Quantum Theory of Fields[M].BeiJing:World Publishing Corporation, 2014.
[4] Page D N , Wootters W K . Evolution without evolution: Dynamics described by stationary observables [J].Physical Review D, 1983, 27(12):2885-2892. DOI:10.1103/PhysRevD.27.2885.
[5] Loc N P D .Time-system entanglement and special relativity[J].Modern Physics Letters, A, 2024, 39(1). DOI:10.1142/S0217732323501833
[6] Wei Zixia .Observers and Timekeepers: From the Page-Wootters Mechanism to the Gravitational Path Integral 2025. arXiv:2506.21489 [hep-th]
[7] Itano W M , Heinzen D J , Bollinger J J ,et al.*Quantum Zeno effect*[J].*Physical Review A*, 1990, 41(5):2295.DOI:10.1103/PhysRevA.41.2295.
[8] Zhao zheng .Fundamentals of General Relativity[M].BeiJing: Tsinghua University Press, 2010: pp226-pp231.