Four suares represented as sum of three out of four variables

Author: Oliver Couto

Email: matt345@celebrating-mathematics.com

In volume 2 of L. Dickson book (history of theory of numbers) the below system of equation, has been considered in. Ajai Choudry in his latest paper (ref # 1), has considered three simultaneous equation with one of them having a maximum degree of three. This paper has considered a system of four equations with a maximum degree of two. The author has provided two methods to arrive at parameterization.

Consider the below system of equation with variables (a,b,c,d):

$$a+b+c=x^2$$

$$a+b+d=y^2$$

$$c+d+a=z^2$$

$$b+c+d=w^2$$

1st method:

we take:

$$a=3(2n^2+4n-m^2-2m-15)$$

$$b = 3(2n^2 + 4n - m^2 + 10m - 15)$$

$$c = 3(2m^2 + 4m - n^2 + 10n - 15)$$

$$d = 3(2m^2 + 4m - n^2 - 2n - 15)$$

Condition is: $(m + n = 4)$

And we get:

$$(x, y, z, w) = [(3n + 3), (3n - 3), (3m - 3), (3m + 3)]$$

where, (m+n)=4

Taking (m,n)=(9,-5) we get:

(p,q,r,s)=[(-252),(72),(324),(504)]

& (x,y,z,w)=(12,18,24,30)

2nd method:

We have:

$$a + b + c = x^2$$

$$a + b + d = y^2$$

$$c + d + a = z^2$$

$$b + c + d = w^2$$

we take:

$$a = 3(m^2 - 8m - 6)$$

$$b = 3(m^2 - 2m + 3)$$

$$c = 3(m^2 + 4m + 6)$$

$$d = 3(m^2 + 10m + 3)$$

& we get:

$$(x, y, z, w) = [(3m - 3), (3m), (3m + 3), (3m + 6)]$$

For m=13 we get,

(a,b,c,d)=[177,438,681,906]

(x,y,z,w)=(36,39,42,45)

Conclusion:

There is a system of six equations in four variables which can be considered by viewers for parametrization & is shown below:

$[(ab + 1), (ac + 1), (bc + 1), (ad + 1), (bd + 1), (cd + 1)]$ are all to be made squares & represented as, (p, q, r, s, t, u)

The numerical solution to the above is:

$$(a, b, c, d) = (1,3,8,120) \ \& \ (p, q, r, s, t, u) = (2,3,5,11,19,31)$$

References

1) Choudhry Ajai, Arxiv 2505.19088, dated 25 may 2025,

Three integers whose sum, product and the sum of the products of the integers, taken two at a time, are perfect squares.

2) Wolfram Mathworld: website: mathworld.wolfram.com, section Number-

-theory (Diophantine equations, degree six)

3) Website, (author) Oliver Couto, Number theory is:

[http://www.celebrating-mathematics.com]

4) Published paper, Oliver Couto,

Vixra.org, # 2502.0196,dtd 28 Feb, 2025, on Sixth powers

5) Ajai Choudhry, Number theory web site,

https://sites.google.com/view/ajaichoudhry/publications

6) Jaroslaw Wroblewski , Tables of Numerical solutions for various degrees,

Website,   www.math.uni.wroc.pl/~jwr/eslp

7) Collection of Algebraic identities (Book), Tito Piezas,

http://sites.google.com/site/tpiezas/

8) Website: Seiji Tomita, computation number theory:

Website: http://www.maroon.dti.ne.jp/fermat/eindex.html
