Classical Field Theory from Primordial Dimensional Fluctuations

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Abstract

Complex Ginzburg-Landau equation (CGLE) is a universal amplitude equation governing the dynamics of phenomena unfolding in far-from-equilibrium conditions. It was recently argued that CGLE emerges from primordial dimensional fluctuations acting in the far ultraviolet sector of field theory and primordial cosmology. Here we show that classical Maxwell, Dirac and non-Abelian field theories can be derived directly from a generalized version of CGLE without invoking a Lagrangian or variational principle. Demanding that CGLE preserves *local coherence* under continuous internal transformations, we introduce a natural covariant derivative whose connection acts as a gauge field. The commutator of these covariant derivatives defines a curvature tensor that reproduces the familiar structure of Maxwell and Yang–Mills field strengths, while a first-order, spinor generalization of CGLE yields Dirac-type dynamics. In a nutshell,

classical field theories naturally emerge from demanding *local coherence invariance* of the generalized CGLE.

Key words: dimensional fluctuations, complex Ginzburg-Landau equation, complex dynamics, classical field theory, local coherence.

1. Introduction

CGLE is a key dynamic model of modern nonlinear science. Originally developed to describe superconductivity, superfluidity, and pattern formation, it has since been recognized as a *universal equation* governing the evolution of slowly varying order parameters near criticality. Its appeal lies in its generality: with only a few phenomenological parameters, CGLE captures self-organization, oscillatory instabilities, and dissipative structures across fields as diverse as condensed matter, plasma physics, and biological systems [1-5].

In its simplest scalar form, the CGLE describes the diffusion and self-interaction of a complex field $W(\vec{x},t)$. Here we find that demanding CGLE to maintain *local coherence* - that is, to remain form-invariant under local transformations of amplitude or phase orientation - leads naturally to the concept of a *covariant derivative*. The auxiliary fields introduced to maintain local coherence acquire the transformation properties of gauge connections, and their commutators generate curvature tensors identical to those of electromagnetic and Yang–Mills fields.

This observation suggests a profound change of perspective: *gauge fields emerge as geometric responses to local variations in coherence*. The standard gauge-covariant formalism of field theory arises not as a postulate of Particle Physics, but from demanding local phase and amplitude consistency in a complex, dissipative medium.

Building on these ideas, we develop a non-variational derivation of the Maxwell, Dirac, and Yang–Mills equations from CGLE. The derivation proceeds algebraically, using only the requirements of local coherence, covariance, and operator consistency. For scalar coherence fields, the resulting curvature corresponds to the electromagnetic tensor $F_{\mu\nu}$; for two-component (spinor) fields, the demand of linear dispersion yields a Dirac-type equation; and for multi-component fields transforming under a non-Abelian group, the curvature satisfies the Yang–Mills structure.

This unified viewpoint resonates with several contemporary themes in theoretical physics: emergent spacetime from quantum information, analog gravity, symmetry breaking in condensed matter [24] and non-unitary extensions of field theory. Yet, our approach differs fundamentally from these in that it *does not rely* on either Lagrangian formulation, quantization constraints, or apriori background constraints. As explained below, in our approach, all classical field structures arise from the dynamics of coherence encoded in the generalized CGLE.

The paper is organized in the following way: after introducing CGLE in section 2, next section defines the concepts of "coherence" and "coherence field" in the CGLE context; section 4 details the construction of generalized CGLE. The remaining sections derive the classical field equations for

Maxwell, Dirac, and Yang–Mills theories. Concluding remarks are presented in the last section.

2. CGLE as universal equation of complex dynamics

It has been known for a while that CGLE,

$$\partial_t W = \mu W + (1+i\alpha)\nabla^2 W - (1+i\beta)\left|W\right|^2 W \tag{1.1}$$

is not just a model of superconductivity and superfluidity – it is the *standard form* for any weakly nonlinear oscillatory medium near a Hopf bifurcation or near criticality [1-5]. It represents a universal amplitude equation describing how the macroscopic order parameter $W(\vec{x},t)$ evolves. In particular, the complex field

$$W(\vec{x},t) = R(\vec{x},t)\, e^{i\varphi(\vec{x},t)} \tag{1.2}$$

describes the phase-synchronized state of a large collection of microscopic degrees of freedom, such as atoms, oscillators, spins, and the like.

Near a Hopf bifurcation or a critical point, the system described by CGLE becomes unstable and breaks a continuous symmetry, usually a $U(1)$ phase symmetry. It follows that W does not merely represent the analogue of a wavefunction – it is the coarse-grained amplitude of a spontaneously broken phase field.

There is a deep motivation for generalizing CGLE coming from the following observation: while (1.1) has a global $U(1)$ phase invariance $W \to e^{i\theta} W$, with $\theta = const.$, CGLE must remain valid when θ varies in space and time. Thus, the generalization of CGLE is dictated by *symmetry restoration* – the same logic that leads from global to local gauge fields in Quantum Field Theory. Note that this requirement *does not follow* from writing down a Lagrangian; it simply says that the evolution of a complex system obeying CGLE must be unaffected by a *local change* in the phase function. Stated differently, the need for a generalization of CGLE stems from the fact that invariance under a local change in phase has a two-fold meaning, namely,

a) (1.2) *is invariant* under a local shift in phase $\varphi(\vec{x},t) \to \varphi(\vec{x},t) + \theta(\vec{x},t)$, as the choice of phase is entirely arbitrary and does not impact (1.2),

b) (1.1) *is not invariant* under a local shift in phase. This is because CGLE contains derivative terms like $\nabla^2 W$ and $\partial_t W$, which are not invariant under local phase transformations as derivatives introduce extra terms having the generic form $\partial_\mu \xi(\vec{x},t)$.

The generalization of CGLE to ensure compliance with local phase invariance is covered in section 4.

It is important to note that there are other well-motivated reasons for generalizing CGLE. In its standard form, CGLE arises as a universal amplitude equation near a Hopf bifurcation and describes the local evolution of a complex order parameter under *weak nonlinearity, spatial homogeneity, and short-range interactions.* However, these assumptions break down in the regime relevant to primordial cosmology, where the underlying phenomena are neither static nor uniform, but instead exhibit long-range correlations, nonlocal couplings, and scale-dependent fluctuations in dimensionality. In such far-from-equilibrium conditions, both the phase and amplitude of the order parameter acquire *dynamical roles*: the phase encodes flow and transport, while the amplitude encodes fluctuating local coherence and effective dimensionality. This motivates the generalization of the CGLE to enable spatial and temporal dependence in its coefficients, inclusion of hydrodynamic phase dynamics, and treatment of correlated, multiplicative fluctuations.

In this generalized form, CGLE naturally accommodates emergent geometrical structures, where phase gradients define a velocity field and correlation lengths define an emergent metric. Under these conditions, curvature arises not from an assumed geometric background, but as a measure of the *anisotropic and scale-dependent coherence* of the evolving order parameter field. This provides the basis for viewing gravitational physics as an effective late-time hydrodynamic description of a primitive dynamical medium shaped by dimensional fluctuations in the early Universe [23].

3. "Coherence" and "coherence field" concepts in the CGLE context

In the common use of the term, phase coherence refers to the spatial and temporal correlation of the phase $\varphi(\vec{x},t)$ of (1.2). If φ varies smoothly across space, the system exhibits *long-range order* – that is, coherent behavior.

However, in the CGLE context, "coherence" is not an exclusive property of the phase because the system is *dissipative* and *nonlinear*. The amplitude R measures how strongly local oscillators participate in the coherent state, while the phase φ measures how aligned they are. Both are essential to describe the *complete spatiotemporal coherence* of the field. Hence, the "coherence field" (1.2) naturally includes both amplitude and phase as complementary components of local order. This is to say that the nonlinear and dissipative nature of CGLE make the amplitude R and phase φ *dynamically coupled*.

The two parameters decouple in the conservative limit of CGLE, where the amplitude "freezes" and coherence becomes a property of the phase alone.

This transition parallels the relaxation of a nonequilibrium system toward a fully ordered, lossless state.

As the next sections show, in this final state, all classical fields can be interpreted as manifestations of pure phase coherence, where:

$$|W| = const., \; W \propto e^{i\varphi}, \;\; A_\mu \to D_\mu \varphi = (\partial_\mu - iA_\mu)\varphi \tag{1.3}$$

What (1.3) tells us is that amplitude uniformity ensures energy conservation, while local phase curvature encodes the dynamics of the field.

4. Building the generalized CGLE

The most straightforward generalization of (1.1) may be cast in the form

$$\boxed{\partial_t W = L(\nabla)W + N(W, W^\dagger)} \tag{1.4}$$

in which $W(\vec{x}, t)$ is a multicomponent complex field having n components ψ_a $(a = 1, 2, ..., n)$, L is a linear operator containing spatial derivatives ∇ and constant linear couplings and N is a nonlinear term, such as a cubic term or a matrix-type nonlinearity.

A critical observation is in order: While the derivation of Maxwell equations can be carried out by starting directly from the scalar CGLE (1.1), recovering either Dirac-like or Yang-Mills theory requires starting from (1.4) instead. This is because standard CGLE naturally carries a built-in $U(1)$ structure and its phase invariance is already present at the scalar level. In contrast, the Dirac and Yang–Mills equations require richer internal symmetry structures that are *not available* in the single-field (scalar) CGLE.

5. From scalar CGLE to Maxwell electrodynamics

Appealing to Madelung transformation, write [8–9]

$$W(\vec{x},t) = \sqrt{\rho(\vec{x},t)}\,e^{i\varphi(\vec{x},t)} \tag{1.5}$$

Compute the time derivative $\partial_t W$ and Laplacian $\nabla^2 W$, insert into (1.1) and separate the real and imaginary parts. The real part represents the *amplitude* *equation*, which can be shown to reduce to the continuity equation (after several algebraic manipulations),

$$\boxed{\partial_t \rho + \nabla \cdot \vec{J} = S(\rho, \varphi)} \tag{1.6}$$

Here, $\vec{J}$ is a probability/density current coming from gradient terms,

$$\vec{J} \equiv 2(\alpha \rho \nabla \varphi - \sqrt{\rho} \nabla \sqrt{\rho}) \tag{1.7}$$

and the source S is defined by,

$$S(\rho, \varphi) = 2\rho(\mu - \rho) + (\text{higher derivative/amplitude gradient terms}) \tag{1.8}$$

The most straightforward identification of the emergent electromagnetic potentials consistent with (1.5) is given by,

$$\boxed{A_i(\vec{x}, t) \equiv \kappa \partial_i \varphi(\vec{x}, t), \quad \Phi(\vec{x}, t) \equiv \kappa \partial_t \varphi(\vec{x}, t)} \tag{1.9}$$

where κ is a dimensional constant, whose role is to map the units of φ to the physical units for potentials. Using these definitions leads to the homogeneous Maxwell identities in the following way,

- Magnetic field:

$$\vec{B} = \nabla \times \vec{A} = \kappa \nabla \times \nabla \varphi = 0 \tag{1.10}$$

- Electric field:

$$\vec{E} = -\nabla \Phi - \partial_t \vec{A} = -\kappa (\nabla \partial_t \varphi + \partial_t \nabla \varphi) = -2\kappa \partial_t \nabla \varphi \tag{1.11}$$

Thus, the homogeneous Maxwell equations

$$\boxed{\nabla \cdot \vec{B} = 0, \quad \nabla \times \vec{E} = -\partial_t \vec{B}} \tag{1.12}$$

are identically satisfied. Additional explanations on the derivation of (1.10) - (1.12) are given in Appendix B.

Next, define the emergent charge density proportional to the amplitude,

$$\rho_{em}(\vec{x}, t) \equiv q \rho(\vec{x}, t) \tag{1.13}$$

where q stands for an effective charge determined by dimensional consistency requirements (see Appendix A); also define an emergent current

$$\vec{J}_{em}(\vec{x},t) \equiv q J(\vec{x},t) \tag{1.14}$$

in which $\vec{J}(\vec{x},t)$ is given by (1.7). By the amplitude equation (1.6) we get

$$\boxed{\partial_t \rho_{em} + \nabla \cdot \vec{J}_{em} = qS} \tag{1.15}$$

The usual charge conservation matches a regime where the source vanishes away such as a steady oscillatory state or a state with saturated amplitude.

To recover the inhomogeneous Maxwell equations (Gauss and Ampère) we need to eliminate $\varphi-$ and $\rho-$ time derivatives between the phase and amplitude equations. These operations are going to lead to Maxwell's equations containing effective permittivity and permeability parameters.

We begin with recovering the Gauss law for $\nabla \cdot \vec{E}$. Let's compute the dot product in terms of φ. After some lengthy algebra, we obtain

$$\nabla \cdot \vec{E} = 2\partial_t(\Phi - \kappa F) \tag{1.16}$$

in which

$$F=\alpha[\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}-(\nabla\varphi)^2+2\frac{\nabla\sqrt{\rho}}{\sqrt{\rho}}\cdot\nabla\varphi-\beta\rho] \quad (1.17)$$

Now express $\partial_t\Phi$ and $\partial_t F$ in terms of the emergent charge density ρ_{em} and emergent current $\vec{J}_{em}$ using the amplitude/continuity relations. Keeping the lowest order in gradients and assuming a slowly varying amplitude, the leading-order contribution is proportional to ρ. Collecting terms and grouping constants, we find the Gauss-like law in the form

$$\boxed{\nabla\cdot\vec{E}\approx\frac{1}{\varepsilon_{eff}}\rho_{em}} \quad (1.18)$$

where the effective permittivity is given by

$$\varepsilon_{eff}\propto\frac{1}{2\kappa q} \quad (1.19)$$

Similarly, to get the Ampère law, compute $\nabla\times\vec{B}$ and relate to $\vec{J}_{em}$ and $\partial_t\vec{E}$. Using $\vec{B}=\nabla\times\vec{A}=\kappa\nabla\times\nabla\varphi$ and making the same substitutions yields

$$\boxed{\nabla\times\vec{B}\approx\mu_{eff}\vec{J}_{em}+\mu_{eff}\varepsilon_{eff}\partial_t\vec{E}} \quad (1.20)$$

with the effective permeability determined by the CGLE coefficients and κ. The $\partial_t \vec{E}$ term shows up because time derivatives of φ enter the phase equation and thus produce a second order in time contribution when reactive terms dominate. In these regimes (α, β much larger than unity) one obtains approximately lossless Maxwell equations supporting wave propagation; when the opposite case is true, strong dissipation dominates, and one gets diffusive and lossy electrodynamics.

It is also instructive to note that the vacuum value of $\mu_{eff}\varepsilon_{eff}$ must reproduce the speed of light in vacuum ($c = 1/\sqrt{\mu_{0,eff}\,\varepsilon_{0,eff}}$), which sets up a constraint relationship between the CGLE coefficients and κ. Finally, one needs to keep in mind that, since Maxwell equations in free space are conservative, the above derivation recovers classical electrodynamics in free space only in the limit of vanishing nonlinearity and dissipation.

6. From generalized CGLE to spinor equations

It is well known that the Dirac equation [12-13],

$$(i\gamma^{\mu}\partial_{\mu}-m)\psi=0 \tag{1.21}$$

is linear in derivatives and acts on a spinor field, not a scalar. By contrast, the standard CGLE is a nonlinear scalar equation with a Laplacian, which is, by definition, a quadratic derivative operator. So, to recover the Dirac form, one must factorize the Laplacian operator – a procedure conceptually analogous to how Dirac factorized the Klein-Gordon equation.

To further motivate the use of a linear gradient operator when connecting CGLE (1.1) to the Dirac equation, note that in the standard CGLE the Laplacian ∇^2 encodes the *diffusive spreading* of the complex coherence field W, which implies that the amplitude and phase evolve irreversibly. This corresponds to a *scalar coherence transport* with no internal degrees of freedom (such as chirality, spin or handedness). Stated differently, the Laplacian assumes that the underlying coherence embodied in W is directionally isotropic and nonchiral. But as soon as the coherence field becomes *locally oriented* – for instance, if it supports rotational or helical phase patterns – this scalar description breaks down [10–11].

Let's suppose now that the order parameter describes the propagation of coherent modes with a preferred direction or spin structure, such as helical phase vortices or rotating wave packets. Then the gradient of the phase $\nabla\varphi$ (or $\nabla\theta$) acquires a vectorial meaning: it is not only the magnitude of the gradient that matters, but also the *orientation* and *sense of rotation* (left or right). The Laplacian, being a scalar operator, erases all this directional information: $\nabla^2 = \nabla \cdot \nabla$ is rotationally invariant and cannot distinguish "left-handed" from "right-handed" coherence orientations. Thus, to describe directional coherence – i.e. propagating phase fronts that can have chirality – one must resolve the Laplacian into its directional components.

To linearize a second-order isotropic operator while preserving directionality and still recovering it when squared, one introduces matrix-valued coefficients satisfying a Clifford algebra:

$$\sigma_i \sigma_j + \sigma_j \sigma_i = 2\delta_{ij} \tag{1.22}$$

This is precisely what Dirac did to express the relativistic dispersion relation $E^2 = p^2 + m^2$ in a first-order form. In the CGLE context, factorizing

$$\nabla^2 = (\vec{\sigma} \cdot \nabla)^2 \tag{1.23}$$

means that the coherence flux is no longer a scalar but has internal rotation structure represented by the Pauli matrices $\vec{\sigma}$. In a nutshell, while ∇^2 represents *diffusive transport* (with scalar, isotropic and dissipative properties), the linear operator $\vec{\sigma} \cdot \nabla$ describes *propagation-type transport* (with chiral, directional and conservative properties).

Now define an operator D such that

$$\nabla^2 = D^\dagger D \tag{1.24}$$

The minimal nontrivial realization is obtained when D acts on a two-component object

$$D = \sigma^i \partial_i \tag{1.25}$$

Then

$$D^{\dagger}D = (\sigma^{i}\partial_{i})(\sigma^{j}\partial_{j}) = \delta^{ij}\partial_{i}\partial_{j} = \nabla^{2} \tag{1.26}$$

It follows that the Pauli matrices provide precisely the algebraic structure needed to linearize the Laplacian.

We next proceed by promoting W to a two-component complex field,

$$W = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{1.27}$$

The most straightforward generalized of CGLE (1.4), considering (1.24) and (1.25), reads:

$$\partial_t W = \mu W + (1+i\alpha)D^2 W - (1+i\beta)|W|^2 W \tag{1.28}$$

where – to preserve local coherence invariance - we define

$$D = \sigma^{i}(\partial_{i} - iqA_{i}), \tag{1.29}$$

and perform the Dirac factorization trick to obtain, in the conservative limit,

$$(i\partial_t - \alpha D)W = 0 \tag{1.30}$$

That is, the first order, linearized to a Dirac-type equation:

$$\boxed{i\partial_t W = \alpha \sigma^i D_i W} \tag{1.31}$$

This is formally identical to the (2+1)-dimensional Weyl/Dirac equation, with

$$\gamma^0 = 1, \quad \gamma^i = \alpha \sigma^i \tag{1.32}$$

7. Mass generation mechanism through the emergent Dirac equation

The generalized CGLE for (1.27), written in a form convenient for the conservative Dirac limit, can be presented as

$$i\partial_t W = \tau_1 (\vec{\sigma} \cdot \vec{D}) W + \tau_2 W - \tau_3 (W^\dagger W) W \tag{1.33}$$

where,

- $\vec{D} = \nabla - iq\vec{A}$ is the spatial covariant derivative,
- τ_1 is a real coefficient,
- $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are Pauli matrices,
- $\tau_2 = \tau_{2,R} + i\tau_{2,I}$ is a complex coefficient,

- $\tau_3 = \tau_{3,R} + i\tau_{3,I}$ is another a complex coefficient,
- $W^{\dagger}W \equiv R^2(\vec{x},t)$ is the local coherence density (the sum of squared moduli of components).

Equation (1.33) is a CGLE that can be interpreted as a Dirac-type equation when the Laplacian has been factorized, as discussed earlier. The linear term plays the role of a growth/decay parameter, and the cubic term the role of a self-interaction parameter.

Now, let's look for spatially uniform, time-independent (up to a global phase) solutions $W(\vec{x},t) = W_0 \exp(-i\omega t)$ with constant spinor W_0. Inserting into (1.33) and dropping covariant derivatives (on account of no spatial dependence) yields

$$\omega W_0 = [-\tau_2 + \tau_3 (W_0^{\dagger} W_0)] W_0 \tag{1.34}$$

Let's multiply by $W_0^\dagger$, take the scalar equation $R_0^2 = W_0^\dagger W_0$ and separate the real parts (the imaginary parts set the oscillation frequency ω). We obtain the amplitude equation,

$$-\tau_{2,R} R_0 + \tau_{3,R} R_0^3 = 0 \Rightarrow R_0^2 = \frac{\tau_{2,R}}{\tau_{3,R}} \tag{1.35}$$

provided $\tau_{2,R}, \tau_{3,R} > 0$. This result shows that there is a *non-vanishing stable amplitude* R_0 of the coherence field. The symmetry $W \mapsto e^{i\varphi} W$ remains a symmetry of the equations, but the solution picks a particular phase: this is the *spontaneous symmetry breaking* of a global $U(1)$ symmetry by the nonzero expectation value $\langle W \rangle = W_0$.

Assume next that there are small fluctuations around the steady state that can be written as,

$$W(\vec{x}, t) = [R_0 + \eta(\vec{x}, t)] \exp[i\,\varphi(x, t)]\, u \tag{1.36}$$

where u is a fixed normalized spinor direction of constant internal orientation, $\eta \ll R_0$ and φ small. For simplicity, also assume $R_0 = R_0 u$ and choose a gauge where,

$$u = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.37}$$

Alternatively, expand directly into components,

$$R = R_0 + \delta R \tag{1.38}$$

Plug into (1.33) and keep only terms linear in δW. Expanding the cubic term, collecting linear terms proportional to δW and mass-like terms proportional to W_0 and focusing on the homogeneous (zero derivative) linear terms, yields an expression looking like,

$$i\partial_t \delta W \propto \tau_2 \delta W - \tau_3 R_0^2 \delta W - \tau_3 [2R_0 \operatorname{Re}(u^\dagger) \delta W] W_0 \tag{1.39}$$

It is seen that the mean-field nonlinear term of (1.39) produces a contribution proportional to R_0^2 multiplying δW. This contribution defines an *effective mass* coefficient,

$$\boxed{m_{eff} \equiv \tau_{3,R} R_0^2 = \tau_{3,R} \frac{\tau_{2,R}}{\tau_{3,R}} = \tau_{2,R}} \tag{1.40}$$

It follows that, at leading order, the linearized equation (1.39) contains a mass term, that is,

$$i\partial_t \delta W \approx \tau_1 \vec{\sigma} \cdot \vec{D} \delta W + m_{eff} \delta W + \ldots \tag{1.41}$$

Thus, the cubic self-interaction generates a mass term proportional to the square of the background amplitude. The imaginary part $\tau_{3,I} R_0^2$ produces an imaginary component in m_{eff} responsible for growth or damping. The interpretation of this result goes as follows: the nonzero condensed background R_0 shifts the energy of small fluctuations producing a mass term in an analogous way the Higgs condensate gives mass to fermions via

Yukawa coupling $y\phi\bar{\psi}\psi$ - here, the cubic self-interaction plays that role at the level of classical amplitude equation [12–13].

Note that the effective mass term is expected to vanish in the low-energy regime of the Standard Model of Particle Physics, where fermions do not self-interact. However, one must bear in mind that, as CGLE is a coarse-grained description of primordial dimensional fluctuations of the form $\delta\varepsilon = O(\varepsilon = 4 - d << 1)$ [16-17], R_0^2 carries the memory of how primordial dimensional fluctuations interact. This means that, ultimately, the mass term stems from dimensional fluctuations $m_{eff} \propto m_{eff}(\varepsilon)$ - a conclusion in full alignment with our previous investigations [see e.g. 18]. From this standpoint, standard Dirac theory represents an emergent embodiment of (1.33) in the limit of vanishing self-interaction.

8. From generalized CGLE to the Yang-Mills theory

Here we extend section 5 with a stepwise, algebraic derivation of non-Abelian field theory outside the standard Lagrangian formalism [12 – 15]. To simplify the presentation, in what follows, we employ the notation $x \equiv x^{\mu} \equiv (\vec{x}, t)$.

Let $W(x) \in C^N$ be a multicomponent complex field which transforms under a representation of a compact Lie group G, e.g. $SU(N)$. The dynamics is specified by the generalized CGLE (1.4),

$$\partial_t W = L(\nabla)W + N(W, W^{\dagger}) \tag{1.42}$$

We ask that (1.42) stay invariant under *global rotations* of the form $W \mapsto UW$ with L, N commuting with U. More generally, we also demand (1.42) to stay invariant to *local rotations* defined by $U = U(x)$. Since ordinary derivatives spoil covariance, to comply with this requirement we introduce a Lie-algebra valued connection as in

$$A_{\mu}(x) = A_{\mu}^{a}(x) T^{a} \tag{1.43}$$

such that

$$D_{\mu} W = \partial_{\mu} W - igA_{\mu} W \tag{1.44}$$

where g is the self-coupling constant of A_μ. Under $U(x)$,

$$D_\mu W \mapsto U D_\mu W \tag{1.45}$$

provided A_μ transforms as

$$A_\mu \mapsto U A_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1} \tag{1.46}$$

Next, define the *curvature (field strength)* by the commutator acting on W:

$$[D_\mu, D_\nu]W = -igF_{\mu\nu}W \tag{1.47}$$

Hence,

$$\boxed{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]} \tag{1.48}$$

Note again that the derivation of (1.48) is purely algebraic and follows directly from the definition of D_μ.

Conclusions

We have shown that CGLE and its generalized version include, as limiting cases, the equations governing classical electromagnetic, spinor, and non-Abelian gauge fields—without resorting to Lagrangians or postulated geometric principles. The key insight is that the requirement of invariance to *local coherence* in a complex, dissipative system forces the introduction of a covariant derivative, whose connection and curvature reproduce the familiar mathematical structure of gauge theories. It is important to note that the UV regime of field theory discards quantum properties of the fields on account of *environmental and gravitational decoherence*.

In this framework:

- The electromagnetic field emerges from the curvature of a $U(1)$ connection ensuring local phase invariance of CGLE,

- The Dirac equation arises when the coherence field possesses an intrinsic two-component structure and linear dispersion, leading naturally to a first-order, covariant evolution law.
- The Yang–Mills equations follow from demanding local invariance under non-Abelian internal symmetries, with the curvature and covariant current obtained algebraically from commutator identities.
- The physics of the Higgs boson is encoded in the CGLE. This is because the dynamics of the Higgs scalar is a particular embodiment of the *real Ginzburg-Landau equation* for superconductivity. Note that, besides Hopf bifurcations, the dynamics of the CGLE contains a rich variety of other bifurcation scenarios, including period-doubling and pitchfork bifurcations, as different parameter ranges are explored [19-22].
- Chirality, spin and mass appear to be properties encoded in CGLE; The family structure of Particle Physics is rooted in the ability of CGLE to sustain sequential cascades of bifurcations [19-22].

The hierarchy of emerging gauge fields reflects the progressive enrichment of coherence symmetry, in the following sense:

1. **Scalar coherence** (*U(1)* symmetry): phase rotations of a single order parameter → **Maxwell.**
2. **Spinorial coherence** (*SU(2)* symmetry): local rotations leading to the mixing of the two phase components → **Dirac.**
3. **Multiplet coherence** (*SU(N)* symmetry): local rotations among the N components → **Yang–Mills.**

Each level in the hierarchy corresponds to a deeper and more structured form of local coherence invariance — the CGLE provides the dynamical backbone, but the algebraic complexity of the order parameter determines which field theory emerges. From this standpoint, primordial dimensional fluctuations and their associated CGLE provide a common foundation from which classical fields emerge as different facets of primordial complex dynamics.

Recall the key observation that, at the dissipative level of CGLE, amplitude and phase are *dynamically coupled;* at the conservative level, amplitude freezes, and coherence becomes a property of the phase alone. This amounts to saying that, in the conservative regime of classical field theory, amplitude uniformity ensures energy conservation, while local phase curvature encodes the dynamics of the field.

In the final conservative regime, all macroscopic fields—electromagnetic, spinor, or Yang–Mills—can be interpreted as manifestations of *pure phase coherence*, where:

$$|W| = const., \quad W \propto e^{i\varphi}, \quad \partial_\mu \varphi \to (\partial_\mu - iA_\mu)\varphi \tag{1.49}$$

and where, under a local coherence symmetry, the connection field transforms as,

$$A'_\mu = A_\mu + \partial_\mu \theta \tag{1.50}$$

Finally, it is instructive to note that, according to (1.50), classical gauge fields are tied to the spacetime gradient of the coherent phase, which is entirely analogous to the emergence of GR metric from flow velocities – as gradients of the coherent phase [23]. Bottom line is that, while GR emerges from demanding stationarity of the coherence field, classical nongravitational fields arise from the local conservation of coherence.

APPENDIX A: Effective charge from dimensional analysis

Start from the CGLE written in a manifestly gauge-covariant form (1.28). Dimensional analysis in natural units gives,

$$\left[W\right] \propto L^{-3/2}, \quad \left[\nabla\right] \propto L^{-1}, \quad \left[t\right] \propto L^{2}/\Gamma \tag{1.51}$$

in which Γ is the diffusion or transport coefficient. Now, since the covariant derivative is $D_\mu = \partial_\mu - iqA_\mu$, the combination qA_μ must have the same dimensions as ∂_μ, i.e. L^{-1}. Thus,

$$\left[q\right]\left[A_\mu\right] = L^{-1} \tag{1.52}$$

(1.52) provides the dimensional reason for introducing q: it converts the phase connection A_μ (which is dimensionless in pure coherence space) into a quantity with the units of a gradient – just like the derivative operator.

One can alternatively use the coupling between phase and the coherence current to motivate the physical origin of q. The reasoning goes as follows: when one separates the amplitude and phase of the order parameter $W = R\exp(i\varphi)$ and substitutes into the CGLE, one finds cross-terms coupling of the phase gradient $\nabla\varphi$ to the amplitude R. Specifically, one gets the coherence current

$$\vec{J}_{coh} = R^2(\nabla\varphi - q\vec{A}) \qquad (1.53)$$

and the conservation law

$$\partial_t R^2 + \nabla\cdot\vec{J}_{coh} = 0 \qquad (1.54)$$

The coefficient q here measures how strongly the coherence current (1.53) couples to the connection field $\vec{A}$. While in ordinary electromagnetism this is the electric charge, by (1.53), in the CGLE context q represents the *coherence coupling strength* – the response of the phase gradient to changes in the local coherence connection.

APPENDIX B: Pure gauge vs. physical gauge field

Recall the gauge potential introduced from phase gradients (for the choice $q=1$)

$$A_\mu = \partial_\mu \theta \tag{1.55}$$

which corresponds to a *pure gauge* configuration. The associated field strength is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{1.56}$$

On account of (1.55),

$$F_{\mu\nu} = \partial_\mu \partial_\nu \theta - \partial_\nu \partial_\mu \theta = 0 \tag{1.57}$$

So, in the initial step of deriving the local symmetry structure, the magnetic field cancels out (per (1.10))

$$\vec{B} = \nabla \times \vec{A} = 0 \tag{1.58}$$

Once we no longer assume A_μ is purely a gradient, i.e. once coherence is locally broken (singularities such as defects, vortices, topological structures or spatial inhomogeneities [24-25]), then $A_\mu \neq \partial_\mu \theta$ and the magnetic field is nonzero.

Thus, when coherence is globally intact, electromagnetic fields are described by the pure gauge condition (1.55) and no actual forces appear. Only when coherence is locally broken, fields are in a non-pure gauge setting and electromagnetism emerges dynamically. In different words, the initial zero - $\vec{B}$ condition of (1.10) is only used to obtain the gauge structure but not the dynamics of the actual (physical) field.

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