ERRATA TO CHAPTER 7 (HYPERGEOMETRIC CONSTANTS) IN VOL. 3 "INTEGRALS AND SERIES — MORE SPECIAL FUNCTIONS" (1990) BY PRUDNIKOV, BRYCHKOV AND MARICHEV

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ABSTRACT. The manuscript contains one corrigendum to the evaluation of the generalized hypergeometric function ${}_3F_2(-1)$ in chapter 7.4.5 in volume 3 of the Prudnikov-Brychkov-Marichev book series, four corrigenda to the evaluation of ${}_4F_3(1)$ in chapter 7.5.3, sixteen corrigenda to the evaluation of ${}_4F_3(-1)$ in chapter 7.5.4, and six corrigenda to the evaluation of ${}_5F_4(1)$ in chapter 7.7.2.

1. Corrigenda for Hypergeometric Constants

Notational conventions: A † in the last column of our tables indicates an error in the printed version, reprinted here; the followup line in our tables shows the corrected evaluation. The checkmark ✓in the last column flags a correct value. G is the Catalan constant. The square root symbol $\surd$ with no upper bar means the square root only covers the next symbol (integer number or π), not any further symbols or functions.

1.1. Corrigendum: Table in Section 7.4.5.

An error in [10, §7.4.5] is corrected in Table 1.

1.2. Corrigenda: Table in Section 7.5.3.

Four errors in [10, §7.5.3] are corrected in Table 2.

Date: October 28, 2025.
2020 *Mathematics Subject Classification.* Primary 00A20 Secondary 33C20.

	a_1	a_2	a_3	b_1	b_2	${}_3F_2(a_1,a_2,a_3;b_1,b_2;-1)$	
27	2	$1+\frac{i}{2}$	$1-\frac{i}{2}$	$2+\frac{i}{2}$	$2-\frac{i}{2}$	$0.645159647\ldots$	✓
						$\frac{8}{5}\Re[\psi(1+\frac{i}{4})-\psi(\frac{1}{2}+\frac{i}{4})]$	✓
89	1	1	$\frac{7}{4}$	2	$\frac{11}{4}$	$\frac{7\surd 2}{18}[3\pi+\surd 2(3\ln 2-4)-6\ln(1+\surd 2))]$	✓
90	1	1	$\frac{7}{4}$	$\frac{11}{4}$	3	$-\frac{14}{3}[13+2\ln 2+\surd 2\ln(1+\surd 2)]$	†
						$-\frac{14}{3}[\frac{7}{3}-2\surd 2\pi+2\ln 2+4\surd 2\log(1+\surd 2)]$	✓

TABLE 1. Values of ${}_3F_2(a_1,a_2,a_3;b_1,b_1;-1)$. Entry [27] is a superposition of digamma functions, see (7).

	a_1	a_2	a_3	a_4	b_1	b_2	b_3	${}_4F_3(a_1,a_2,a_3,a_4;b_1,b_2,b_3;1)$	
111	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	$\frac{11}{4}$	$\frac{11}{4}$	$\frac{11}{4}$	$\frac{3^3\cdot 7^3}{256_3}(128-12\pi-6\pi^2+\pi^3)$	†
								$\frac{7^3}{2048}(212-81\pi^2+648G)$	✓
112	$\frac{3}{4}$	1	1	$\frac{5}{4}$	$\frac{7}{4}$	2	$\frac{9}{4}$	$15(3\ln 2-2)$	✓
113	$\frac{3}{4}$	1	1	$\frac{3}{2}$	$\frac{7}{4}$	2	$\frac{5}{2}$	$24\ln 2-3\pi-6$	✓
118	1	1	$\frac{4}{3}$	$\frac{3}{2}$	2	$\frac{7}{3}$	$\frac{5}{2}$	$24-6\sqrt{3}\pi+48\ln 2-54\ln 3$	†
								$60-6\sqrt{3}\pi+48\ln 2-54\ln 3$	✓
119	1	1	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{7}{3}$	$\frac{8}{3}$	3	$\frac{20}{9}(1-\sqrt{3}\pi)$	†
								$110-20\sqrt{3}\pi$	✓
121	1	1	$\frac{3}{2}$	$\frac{5}{3}$	2	$\frac{5}{2}$	$\frac{8}{3}$	$\frac{15}{4}(7+\sqrt{3}\pi-16\ln 2+9\ln 3)$	†
								$\frac{15}{4}(7-\sqrt{3}\pi-16\ln 2+9\ln 3)$	✓

TABLE 2. Values of ${}_4F_3(a_1,a_2,a_3,a_4;b_1,b_1,b_3;1)$.

1.3. **Corrigenda: Table in Section 7.5.4.** A list of the hypergeometric series in [10, §7.5.4] stretching from 7.5.4.18 to 7.5.4.50 is reproduced in Tables 3 and 4. In the Russian version of 1986 the table starts at page 474, in the English version of 1990 at page 561; the contents is the same. In all cases, the excess of the function (sum of the lower parameters b_i minus sum of the upper parameters a_i) is a positive integer.

	a_1	a_2	a_3	a_4	b_1	b_2	b_3	${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; -1)$	
18	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	$\frac{3}{4}(\sqrt{2}-1)\pi$	✓
19	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{5}{2}$	$\frac{3}{20}[(8\sqrt{2}-7)\pi - 2 - 30\sqrt{2}\ln(1+\sqrt{2})]$	†
								$\frac{3}{20}[(8\sqrt{2}-7)\pi - 2 - 4\sqrt{2}\ln(1+\sqrt{2})]$	✓
20	$\frac{1}{4}$	$\frac{1}{2}$	1	1	$\frac{5}{4}$	$\frac{3}{2}$	2	$\frac{1}{6}[(2\sqrt{2}-3)\pi + 2\ln 2 + 4\sqrt{2}\ln(1+\sqrt{2})]$	✓
21	$\frac{1}{4}$	$\frac{3}{4}$	1	1	$\frac{5}{4}$	$\frac{7}{4}$	2	$\frac{\sqrt{2}}{4}[2\sqrt{2}\ln 2 - \pi + 6\ln(1+\sqrt{2})]$	†
								$\frac{1}{4}[2\sqrt{2}\pi - \ln 2 - 2\sqrt{2}\ln(1+\sqrt{2})]$	✓
22	$\frac{1}{4}$	$\frac{1}{2}$	1	1	$\frac{5}{4}$	2	$\frac{5}{2}$	$\frac{1}{5}[5\ln 2 + (2\sqrt{2}-3)\pi + 4\sqrt{2}\ln(1+\sqrt{2}) - 3]$	✓
23	$\frac{1}{4}$	$\frac{3}{4}$	1	$\frac{3}{2}$	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{5}{2}$	$\frac{3}{20}[4 - (1+\sqrt{2})\pi + 16\ln(1+\sqrt{2})]$	†
								$\frac{3}{20}[4 - (1+\sqrt{2})\pi + 8\sqrt{2}\ln(1+\sqrt{2})]$	✓
24	$\frac{1}{4}$	1	1	$\frac{3}{2}$	$\frac{5}{4}$	2	$\frac{5}{2}$	$\frac{1}{10}[12 - (3-2\sqrt{2})\pi - 10\ln 2 + 4\sqrt{2}\ln(1+\sqrt{2})]$	✓
25	$\frac{1}{4}$	1	$\frac{3}{2}$	$\frac{7}{4}$	$\frac{5}{4}$	$\frac{5}{2}$	$\frac{11}{4}$	$\frac{7}{60}[3(3-2\sqrt{2})\pi - 16 + 18\sqrt{2}\ln(1+\sqrt{2})]$	✓
26	$\frac{1}{3}$	$\frac{1}{2}$	1	1	$\frac{4}{3}$	$\frac{3}{2}$	2	$2\ln 2 - (1 - \sqrt{3}/2)\pi$	✓
27	$\frac{1}{3}$	$\frac{2}{3}$	1	1	$\frac{4}{3}$	$\frac{5}{3}$	2	$4\ln 2 - \pi/\sqrt{3}$	✓
28	$\frac{1}{3}$	$\frac{2}{3}$	1	1	$\frac{5}{3}$	2	$\frac{7}{3}$	$\frac{4}{3}(3 - \sqrt{3}\pi + 2\ln 2)$	†
								$\frac{4}{3}(-3 - \pi/\sqrt{3} + 8\ln 2)$	✓
29	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{7}{3}$	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{10}{3}$	$\frac{7}{60}(16\ln 2 - 3)$	✓
30	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{27}{128}(12\pi - \pi^3 - 64)$	†
								$\frac{27}{128}(12\pi + \pi^3 - 64)$	✓
31	$\frac{1}{2}$	$\frac{2}{3}$	1	1	$\frac{3}{2}$	$\frac{5}{3}$	2	$8\ln 2 + 2(1-\sqrt{3})\pi$	✓
32	$\frac{1}{2}$	$\frac{3}{4}$	1	1	$\frac{3}{2}$	$\frac{7}{4}$	2	$\frac{3}{2}[(1-2\sqrt{2})\pi + 2\ln 2 + 4\sqrt{2}\ln(1+\sqrt{2})]$	✓
33	$\frac{1}{2}$	$\frac{3}{4}$	1	1	$\frac{7}{4}$	2	$\frac{5}{2}$	$3(1-2\sqrt{2})\pi + 12\sqrt{2}\ln(1+\sqrt{2}) - 3$	†
								$3(1-2\sqrt{2})\pi + 9\ln 2 + 12\sqrt{2}\ln(1+\sqrt{2}) - 3$	✓
34	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	$\frac{9}{4}$	$\frac{5}{4}[4 - (1-2\sqrt{2})\pi + 2\sqrt{2}\ln(1+\sqrt{2})]$	†
								$\frac{5}{4}[4 + (1-2\sqrt{2})\pi + 2\sqrt{2}\ln(1+\sqrt{2})]$	✓

TABLE 3. Values of ${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_1, b_3; -1)$.

	a_1	a_2	a_3	a_4	b_1	b_2	b_3	${}_4F_3(a_1,a_2,a_3,a_4;b_1,b_2,b_3;-1)$	
35	$\frac{1}{2}$	1	1	1	2	2	$\frac{5}{2}$	$\frac{1}{8}(6\pi-\pi^2-12)$	†
								$3\pi-\pi^2/4-6$	✓
36	$\frac{1}{2}$	1	1	$\frac{5}{4}$	$\frac{3}{2}$	2	$\frac{9}{4}$	$\frac{5}{6}[16+(1-6\sqrt{2})\pi-6\ln 2-12\sqrt{2}\ln(1+\sqrt{2})]$	†
								$\frac{5}{6}[16+(1-2\sqrt{2})\pi-6\ln 2-4\sqrt{2}\ln(1+\sqrt{2})]$	✓
37	$\frac{1}{2}$	1	1	$\frac{7}{4}$	$\frac{3}{2}$	2	$\frac{11}{4}$	$\frac{7}{45}[8+3(3-\sqrt{2})\pi-15\ln 2+6\sqrt{2}\ln(1+\sqrt{2})]$	†
								$\frac{7}{45}[8+\frac{3}{2}(3-2\sqrt{2})\pi-15\ln 2+6\sqrt{2}\ln(1+\sqrt{2})]$	✓
38	$\frac{1}{2}$	1	1	$\frac{5}{2}$	$\frac{3}{2}$	2	$\frac{7}{2}$	$\frac{10}{9}(5-6\ln 2)$	✓
39	$\frac{2}{3}$	1	1	$\frac{4}{3}$	$\frac{5}{3}$	2	$\frac{7}{3}$	$12-16\ln 2$	✓
40	$\frac{3}{4}$	1	1	$\frac{5}{4}$	$\frac{7}{4}$	2	$\frac{9}{4}$	$15[2-\ln 2-\sqrt{2}\ln(1+\sqrt{2})]$	✓
41	$\frac{3}{4}$	1	1	$\frac{3}{2}$	$\frac{7}{4}$	2	$\frac{5}{2}$	$6+3(\sqrt{2}-1)\pi-9\ln 2+6\sqrt{2}\ln(1+\sqrt{2})$	†
								$6-\frac{3}{2}(1-2\sqrt{2})\pi-9\ln 2-6\sqrt{2}\log(1+\sqrt{2})$	✓
42	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	$\frac{9}{4}$	$\frac{5}{2}$	$\frac{15}{4}[(2\sqrt{2}-1)\pi-8+4\sqrt{2}\ln(1+\sqrt{2})]$	†
								$\frac{15}{4}[-(1-2\sqrt{2})\pi-8+2\sqrt{2}\ln(1+\sqrt{2})]$	✓
43	5/6	1	1	7/6	$\frac{11}{6}$	2	$\frac{13}{6}$	$\frac{35\sqrt{3}}{3}[\sqrt{3}(3-\ln 2)-\ln(2+\sqrt{3})]$	†
								$35[3-2\sqrt{3}\ln(1+\sqrt{3})+(\sqrt{3}-1)\ln 2]$	✓
44	1	1	1	1	3	3	3	$4(3\zeta(3)+24\ln 2-20)$	✓
45	1	1	$\frac{5}{4}$	$\frac{3}{2}$	2	$\frac{9}{4}$	$\frac{5}{2}$	$\frac{15}{2}[(2\sqrt{2}-1)\pi+4\sqrt{2}\ln(1+\sqrt{2})-12]$	†
								$\frac{15}{2}[-(1-2\sqrt{2})\pi+2\ln 2+4\sqrt{2}\log(1+\sqrt{2})-12]$	✓
46	1	1	$\frac{5}{4}$	$\frac{7}{4}$	2	$\frac{9}{4}$	$\frac{11}{4}$	$\frac{35\sqrt{2}}{18}[3\pi-16\sqrt{2}-6\ln(1+\sqrt{2})]$	†
								$\frac{35}{18}[3\sqrt{2}\pi-32+6\ln 2+12\sqrt{2}\log(1+\sqrt{2})]$	✓
47	1	1	$\frac{4}{3}$	$\frac{3}{2}$	2	$\frac{7}{3}$	$\frac{5}{2}$	$12[4\ln 2-(1-\sqrt{3})\pi-5]$	✓
48	1	1	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{7}{3}$	$\frac{8}{3}$	3	$160\ln 2-110$	✓
49	1	1	$\frac{3}{2}$	$\frac{5}{3}$	2	$\frac{5}{2}$	$\frac{8}{3}$	$\frac{15}{4}[2(2-\sqrt{3})\pi+8\ln 2-7]$	✓
50	1	1	$\frac{3}{2}$	$\frac{7}{4}$	2	$\frac{5}{4}$	$\frac{11}{4}$	$\frac{7}{2}[2\ln 2-3(\sqrt{2}-1)\pi-4+6\sqrt{2}\ln(1+\sqrt{2})]$	†
								$\frac{7}{2}[2\ln 2+(3-2\sqrt{2})\pi-\frac{20}{3}+4\sqrt{2}\log(1+\sqrt{2})]$	✓

TABLE 4. Values of ${}_4F_3(a_1,a_2,a_3,a_4;b_1,b_1,b_3;-1)$.

	a_1	a_2	a_3	a_4	a_5	b_1	b_2	b_3	b_4	${}_5F_4(a_1,a_2,a_3,a_4,a_5;b_1,b_2,b_3,b_4;1)$	
22	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{\sqrt{\pi}}{512\sqrt{2}}\Gamma^2(1/4)[3\pi^2+\psi'(\frac{1}{4})-\psi'(\frac{3}{4})]$	†
										$\frac{\sqrt{\pi}}{3\cdot 512\sqrt{2}}\Gamma^2(\frac{1}{4})[48G+5\pi^2]$	✓
23	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{7}{4}$	$\frac{9\pi}{32}(\pi-2)$	✓
25	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	$\frac{7}{4}$	1.014678	✓
										$\frac{9}{16}(-8G-2\pi+\pi^2+8\ln 2)$	✓
48	$\frac{1}{2}$	$\frac{3}{4}$	1	1	$\frac{5}{4}$	$\frac{7}{4}$	2	$\frac{9}{4}$	$\frac{5}{2}$	$135-150\ln 2$	†
										$105-150\ln 2$	✓
52	$\frac{2}{3}$	1	1	1	$\frac{4}{3}$	$\frac{5}{3}$	2	2	$\frac{7}{3}$	$\frac{4}{27}(27-3\sqrt{3}\pi-\pi^2)$	†
										$\frac{4}{3}(27-3\sqrt{3}\pi-\pi^2)$	✓
54	$\frac{3}{4}$	1	1	$\frac{5}{4}$	$\frac{3}{2}$	2	$\frac{9}{4}$	$\frac{5}{2}$	$\frac{11}{4}$	$\frac{35}{2}(3\pi-24\ln 2-26)$	†
										$\frac{35}{2}(3\pi+24\ln 2-26)$	✓
55	1	1	1	1	1	3	3	3	3	$\frac{16}{45}(\pi^4+150\pi^2-525)$	†
										$\frac{16}{45}(\pi^4+150\pi^2-1575)$	✓
56	1	1	1	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	3	3	$72\pi^4+12\pi^2-828$	†
										$84\pi^2-828$	✓

TABLE 5. Values of ${}_5F_4(a_1,a_2,a_3,a_4,a_5;b_1,b_1,b_3,b_4;1)$. In entry [22] one can rewrite $\psi'(\frac{1}{4})-\psi'(\frac{3}{4})=16G$; the correction appeared already in [8].

	a_1	a_2	a_3	a_4	a_5	a_6	b_1	b_2	b_3	b_4	b_5	${}_6F_5(a_1,\ldots,a_6;b_1,\ldots,b_5;1)$	
10	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{7}{4}$	$1.030461\ldots=9G/8$	✓
11	$\frac{1}{2}$	1	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{9}{4}$	$\frac{5}{4}$	2	2	2	3	$\frac{32}{5}(1-\frac{8}{\pi^2})$	✓

TABLE 6. Values of ${}_6F_5(a_1,a_2,a_3,a_4,a_5,a_6;b_1,b_1,b_3,b_4,b_5;1)$.

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	b_1	b_2	b_3	b_4	b_5	b_6	${}_7F_6(a_1,\ldots;b_1,\ldots;-1)$	
6	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{7}{4}$	0.961977	†
														0.949477...	✓

TABLE 7. Values of ${}_7F_6(a_1,a_2,a_3,a_4,a_5,a_6,a_7;b_1,b_1,b_3,b_4,b_5,b_6;-1)$. The corrected numerical value is a superposition of trigamma functions, see (48).

1.4. **Corrigenda: Table in Section 7.6.2.** Six corrigenda of the hypergeometric series in [10, §7.6.2] are reproduced in Table 5. The numerical constant [25] is recognized as a mix of other basic constants.

1.5. **Verification Table in Section 7.7.2.** Two hypergeometric series ${}_6F_5(1)$ in [10, §7.7.2] are reproduced in Table 6. Entry [10] is recognized as a rational multiple of the Catalan constant.

1.6. **Numerical error in Table of Section 7.8.2.** The hypergeometric series ${}_7F_6(-1)$ in [10, §7.8.2] is considered in Table 7.

2. Examples of Contiguously Linked Reductions

The integer excess in the cases of the table indicates that the contiguous relations of the hypergeometric series are a versatile tool to search for simplifications. This section presents pen-and-paper examples of the approach.

2.1. **Exercise 7.4.5.27.** The fundamental reduction is [11, 4][10, 7.2.3.26]

$$(a_i - b_j + 1)\,{}_pF_q(...a_i..;...,b_j...;z) = a_i\,{}_pF_q(...a_i+1..;...,b_j...;z) - (b_j-1)\,{}_pF_q(...a_i..;...,b_j-1...;z). \quad (1)$$

For $a_i = 1 + i/2$, $b_j = 2 - i/2$ in entry [27] in Table 1 this is

$$ {}_3F_2(2,1+\frac{i}{2},1-\frac{i}{2};2+\frac{i}{2},2-\frac{i}{2};-1) = (\frac{1}{2}-i)\,{}_3F_2(2,2+\frac{i}{2},1-\frac{i}{2};2+\frac{i}{2},2-\frac{i}{2};-1) + (\frac{1}{2}+i)\,{}_3F_2(2,1+\frac{i}{2},1-\frac{i}{2};2+\frac{i}{2},1-\frac{i}{2};-1) \quad (2)$$

where pairs of same numbers in the upper and lower parameters cancel [10, 7.2.3.6]:

$$\ldots = (\frac{1}{2}-i)\,{}_2F_1(2,1-\frac{i}{2};2-\frac{i}{2};-1) + (\frac{1}{2}+i)\,{}_2F_1(2,1+\frac{i}{2};2+\frac{i}{2};-1) = 2\Re\left[(\frac{1}{2}-i)\,{}_2F_1(2,1-\frac{i}{2};2-\frac{i}{2};-1)\right]. \quad (3)$$

In (1) insert $a_i = 1$, $b_j = 2 - i/2$

$$\frac{i}{2}\,{}_2F_1(1,1-\frac{i}{2};2-\frac{i}{2};-1) = {}_2F_1(2,1-\frac{i}{2};2-\frac{i}{2};-1) - (1-\frac{i}{2})\,{}_2F_1(1,1-\frac{i}{2};1-\frac{i}{2};-1); \quad (4)$$

The left hand side is a case of polygamma differences: [4]

$$ {}_pF_{p-1}(1,a,a,\ldots;a+1,a+1,\ldots;-1) = \frac{(-a/2)^{p-1}}{(p-2)!}[\psi^{(p-2)}(a/2) - \psi^{(p-2)}(a/2+\frac{1}{2})]. \quad (5)$$

$$\rightsquigarrow \frac{i}{2}(\frac{1}{2}-\frac{i}{4})[\psi(1-i/4) - \psi(1/2-i/4)] = {}_2F_1(2,1-\frac{i}{2};2-\frac{i}{2};-1) - (1-\frac{i}{2})\frac{1}{2}. \quad (6)$$

Isolating the ${}_2F_1$ term on the right hand side inserting it into (3) gives essentially real parts of digamma functions:

$$ {}_3F_2(2,1+\frac{i}{2},1-\frac{i}{2};2+\frac{i}{2},2-\frac{i}{2};-1) = 2\Re\left\{(\frac{1}{2}-i)[\frac{i}{2}(\frac{1}{2}-\frac{i}{4})[\psi(1-i/4) - \psi(1/2-i/4)] + (1-\frac{i}{2})\frac{1}{2}]\right\} $$
$$ = 2\Re[\frac{5}{16}[\psi(1-i/4) - \psi(1/2-i/4)] - \frac{5i}{8}] = \frac{5}{8}\Re[\psi(1-i/4) - \psi(1/2-i/4)] $$
$$ = \frac{5}{8}\times[-0.505905819869\ldots - (-1.538161255709235\ldots)] = 0.64515964739978\ldots \quad (7)$$

The digamma functions are for example evaluated as series [1, 6.3.9,6.3.16]:

$$(8)\quad \Re[\psi(1+i/4)-\psi(1/2+i/4)] = \Re[\sum_{n\ge 1}\frac{i/4}{n(n+i/4)} - \frac{-1/2+i/4}{n(n-1/2+i/4)}]$$
$$= 8\sum_{n\ge 1}\frac{16n^2-8n-1}{(16n^2+1)(16n^2-16n+5)} = 1.03225543583966\ldots$$

2.2. **Exercise 7.4.5.90.** For $a_i = \frac{7}{4}$, $b_j = 3$, Eq. (1) is

$$(9)$$
$$-\frac{1}{4}{}_3F_2(1,1,\frac{7}{4};\frac{11}{4},3;-1) = \frac{7}{4}{}_3F_2(1,1,\frac{11}{4};\frac{11}{4},3;-1) - 2{}_3F_2(1,1,\frac{7}{4};\frac{11}{4},2;-1)$$
$$= \frac{7}{4}{}_2F_1(1,1;3;-1) - 2{}_3F_2(1,1,\frac{7}{4};\frac{11}{4},2;-1)$$
$$= \frac{7}{4}(-2+4\ln 2) - 2{}_3F_2(1,1,\frac{7}{4};\frac{11}{4},2;-1)$$

In the right hand side insert [10, 7.4.5.89]—which is found by a recursive application of this method—

$${}_3F_2(1,1,\frac{7}{4};2,\frac{11}{4};-1) = \frac{7\sqrt{2}}{18}[3\pi + \sqrt{2}(3\ln 2 - 4) - 6\ln(1+\sqrt{2})]$$

and this gives the checked entry [90] in Table 1.

2.3. **Exercise 7.5.4.35.** To simplify the value of ${}_4F_3(\frac{1}{2},1,1,1;2,2,\frac{5}{2};-1)$ of [10, 7.5.4.35] we observe that the upper value of $\frac{1}{2}$ and the lower value of $\frac{5}{2}$ and the upper value of 1 and the lower value of 2 are contiguously related. The excess is 3. For $a_i = \frac{1}{2}$, $b_j = \frac{5}{2}$ Equation (1) is

$$(10)\quad -{}_4F_3(\frac{1}{2},1,1,1;2,2,\frac{5}{2};-1)$$
$$= \frac{1}{2}{}_4F_3(\frac{3}{2},1,1,1;2,2,\frac{5}{2};-1) - \frac{3}{2}{}_4F_3(\frac{1}{2},1,1,1;2,2,\frac{3}{2};-1);$$

$$(11)\quad \rightsquigarrow {}_4F_3(\frac{1}{2},1,1,1;2,2,\frac{5}{2};-1)$$
$$= -\frac{1}{2}{}_4F_3(\frac{3}{2},1,1,1;2,2,\frac{5}{2};-1) + \frac{3}{2}{}_4F_3(\frac{1}{2},1,1,1;2,2,\frac{3}{2};-1).$$

$a_i = \frac{3}{2}$ and $b_j = 2$ in (1) give a difference of generalized hypergeometric functions where upper and lower parameters cancel:

$$(12)$$
$$\frac{1}{2}{}_4F_3(\frac{3}{2},1,1,1;2,2,\frac{5}{2};-1) = \frac{3}{2}{}_4F_3(\frac{5}{2},1,1,1;2,2,\frac{5}{2};-1) - {}_4F_3(\frac{3}{2},1,1,1;2,1,\frac{5}{2};-1)$$
$$= \frac{3}{2}{}_3F_2(1,1,1;2,2;-1) - {}_3F_2(\frac{3}{2},1,1;2,\frac{5}{2};-1);$$

$$(13)\quad \rightsquigarrow {}_4F_3(\frac{3}{2},1,1,1;2,2,\frac{5}{2};-1) = 3{}_3F_2(1,1,1;2,2;-1) - 2{}_3F_2(\frac{3}{2},1,1;2,\frac{5}{2};-1).$$

Furthermore (5) is at $a=1$, $p=3$ [1, 6.3.2,6.3.3][10, 7.4.5.75,7.10.2.28],

$$ {}_3F_2(1,1,1;2,2;-1) = (-\frac{1}{2})^2[\psi'(\frac{1}{2})-\psi'(1)] = \frac{1}{4}\frac{\pi^2}{3} = \frac{\pi^2}{12}. \tag{14}$$

(1) is at $a=\frac{3}{2}$, $b=2$,

$$\begin{aligned} \frac{1}{2}{}_3F_2(\frac{3}{2},1,1;2,\frac{5}{2};-1) &= \frac{3}{2}{}_3F_2(\frac{5}{2},1,1;2,\frac{5}{2};-1) - {}_3F_2(\frac{3}{2},1,1;1,\frac{5}{2};-1) \\ &= \frac{3}{2}{}_2F_1(1,1;2;-1) - {}_2F_1(\frac{3}{2},1;\frac{5}{2};-1). \end{aligned} \tag{15}$$

This reduces to a standard Gaussian Hypergeometric Series [1, 15.1.3,15.1.22][10, 7.4.5.83]

$$\rightsquigarrow {}_3F_2(\frac{3}{2},1,1;2,\frac{5}{2};-1) = 3{}_2F_1(1,1;2;-1) - 2{}_2F_1(\frac{3}{2},1;\frac{5}{2};-1) = 3\ln 2 - 6 + 3\frac{\pi}{2}. \tag{16}$$

Insertion of this and (14) in (13) yields

$$ {}_4F_3(\frac{3}{2},1,1,1;2,2,\frac{5}{2};-1) = 3\frac{\pi^2}{12} - 2(3\ln 2 - 6 + 3\frac{\pi}{2}). \tag{17}$$

The case of $a_i=\frac{1}{2}$, $b_j=2$ in (1) is with (14)

$$\begin{aligned} -\frac{1}{2}{}_4F_3(\frac{1}{2},1,1,1;2,2,\frac{3}{2};-1) &\\ = \frac{1}{2}{}_4F_3(\frac{3}{2},1,1,1;2,2,\frac{3}{2};-1) - {}_4F_3(\frac{1}{2},1,1,1;1,2,\frac{3}{2};-1) &\\ = \frac{1}{2}{}_3F_2(1,1,1;2,2;-1) - {}_3F_2(\frac{1}{2},1,1;2,\frac{3}{2};-1) = \frac{1}{2}\frac{\pi^2}{12} - {}_3F_2(\frac{1}{2},1,1;2,\frac{3}{2};-1). & \end{aligned} \tag{18}$$

$$\rightsquigarrow {}_4F_3(\frac{1}{2},1,1,1;2,2,\frac{3}{2};-1) = -\frac{\pi^2}{12} + 2{}_3F_2(\frac{1}{2},1,1;2,\frac{3}{2};-1). \tag{19}$$

The last term is reduced again by (1) with $a_i=\frac{1}{2}$, $b_j=2$, or looked up in [10, 7.4.5.57]

$$ {}_3F_2(\frac{1}{2},1,1;2,\frac{3}{2};-1) = -\ln 2 + \frac{\pi}{2}. \tag{20}$$

Insertion in (19) gives

$$ {}_4F_3(\frac{1}{2},1,1,1;2,2,\frac{3}{2};-1) = -\frac{\pi^2}{12} - 2\ln 2 + \pi. \tag{21}$$

Insert (19) and this in (11)

$$\begin{aligned} {}_4F_3(\frac{1}{2},1,1,1;2,2,\frac{5}{2};-1) &= -\frac{1}{2}(\frac{\pi^2}{4} - 6\ln 2 + 12 - 3\pi) + \frac{3}{2}(-\frac{\pi^2}{12} - 2\ln 2 + \pi) \\ &= -\frac{\pi^2}{4} - 6 + 3\pi \end{aligned} \tag{22}$$

to obtain the corrected entry [35] in Table 4.

2.4. **Exercise 7.6.3.7.** To uncover the numerical constant 0.993395... in formula [10, 7.6.3.7] insert $a_i = \frac{1}{4}$, $b_j = \frac{7}{4}$ in (1)

$$(23)\quad {}_5F_4(\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{5}{4},\frac{7}{4},\frac{7}{4};-1)$$
$$= -\frac{1}{2}{}_5F_4(\frac{5}{4},\frac{1}{4},\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{5}{4},\frac{7}{4},\frac{7}{4};-1) + \frac{3}{2}{}_5F_4(\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{5}{4},\frac{3}{4},\frac{7}{4};-1)$$
$$= -\frac{1}{2}{}_4F_3(\frac{1}{4},\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4},\frac{7}{4};-1) + \frac{3}{2}{}_4F_3(\frac{1}{4},\frac{1}{4},\frac{3}{4},1;\frac{5}{4},\frac{5}{4},\frac{7}{4};-1)$$

and the same $a_i = \frac{1}{4}$, $b_j = \frac{7}{4}$ again for both terms

$$(24)$$
$$= -\frac{1}{2}[-\frac{1}{2}{}_4F_3(\frac{5}{4},\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4},\frac{7}{4};-1) + \frac{3}{2}{}_4F_3(\frac{1}{4},\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{3}{4},\frac{7}{4};-1)]$$
$$+ \frac{3}{2}[-\frac{1}{2}{}_4F_3(\frac{5}{4},\frac{1}{4},\frac{3}{4},1;\frac{5}{4},\frac{5}{4},\frac{7}{4};-1) + \frac{3}{2}{}_4F_3(\frac{1}{4},\frac{1}{4},\frac{3}{4},1;\frac{5}{4},\frac{5}{4},\frac{3}{4};-1)]$$
$$= -\frac{1}{2}[-\frac{1}{2}{}_3F_2(\frac{3}{4},\frac{3}{4},1;\frac{7}{4},\frac{7}{4};-1) + \frac{3}{2}{}_3F_2(\frac{1}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};-1)]$$
$$+ \frac{3}{2}[-\frac{1}{2}{}_3F_2(\frac{1}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};-1) + \frac{3}{2}{}_3F_2(\frac{1}{4},\frac{1}{4},1;\frac{5}{4},\frac{5}{4};-1)]$$
$$= \frac{1}{4}{}_3F_2(\frac{3}{4},\frac{3}{4},1;\frac{7}{4},\frac{7}{4};-1) - \frac{3}{2}{}_3F_2(\frac{1}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};-1) + \frac{9}{4}{}_3F_2(\frac{1}{4},\frac{1}{4},1;\frac{5}{4},\frac{5}{4};-1).$$

The first and third term are a case of (5), the second is read off [10, 7.4.5.34]:

$$(25)\quad {}_5F_4(\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{5}{4},\frac{7}{4},\frac{7}{4};-1)$$
$$= \frac{1}{4}(\frac{3}{8})^2[\psi'(3/8) - \psi'(7/8)] - \frac{3}{2}\frac{3}{2\sqrt{2}}\ln(1+\sqrt{2}) + \frac{9}{4}(\frac{1}{8})^2[\psi'(1/8) - \psi'(5/8)].$$

Remark 1. *The trigamma reference values are Hurwitz zeta-functions* [9, II.2.]

$$(26)\qquad \psi'(z) = \sum_{n\ge 0}\frac{1}{(z+n)^2} = \zeta(z,2) = \frac{1}{z^2}{}_3F_2(z,z,1;z+1,z+1;1),$$

$$(27)\quad \psi'(1/8) = 8^2{}_3F_2(\frac{1}{8},\frac{1}{8},1;\frac{9}{8},\frac{9}{8};1) \approx 65.38813344498803...;$$
$$(28)\quad \psi'(3/8) = (\frac{8}{3})^2{}_3F_2(\frac{3}{8},\frac{3}{8},1;\frac{11}{8},\frac{11}{8};1) \approx 8.16177763276150785...;$$
$$(29)\quad \psi'(5/8) = (\frac{8}{5})^2{}_3F_2(\frac{5}{8},\frac{5}{8},1;\frac{13}{8},\frac{13}{8};1) \approx 3.40118317304040848...;$$
$$(30)\quad \psi'(7/8) = (\frac{8}{7})^2{}_3F_2(\frac{7}{8},\frac{7}{8},1;\frac{15}{8},\frac{15}{8};1) \approx 2.0057409579249181388...$$

The duplication formula of the Γ*-function leads to* $\psi'(\frac{z}{2}) + \psi'(\frac{z-1}{2}) = 4\psi'(z-1)$ [2, (5.8)]*:*

$$(31)\qquad \psi'(1/8) + \psi'(5/8) = 4\psi'(1/4) = 4(\pi^2 + 8G);$$

$$(32)\qquad \psi'(3/8) + \psi'(7/8) = 4\psi'(3/4) = 4(\pi^2 - 8G).$$

The derivative of [5, 8.365.9] *is* $\psi'(1/2+z)+\psi'(1/2-z)=[\pi/\cos(\pi z)]^2$*, which leads with* $z=1/8$ *or* $3/8$ *to more sum rules:*

$$\psi'(5/8)+\psi'(3/8)=4\pi^2/(2+\sqrt{2}); \tag{33}$$

$$\psi'(1/8)+\psi'(7/8)=4\pi^2/(2-\sqrt{2}). \tag{34}$$

The representation of the individual $\psi'(1/8)-\psi'(5/8)$ *and* $\psi'(3/8)-\psi'(7/8)$ *as hypergeometric series* ${}_6F_5(+1)$ *do not provide much insight:*

$$\psi'(z)-\psi'(z+1/2)=\sum_{n\ge 0}\left[\frac{1}{(z+n)^2}-\frac{1}{(z+1/2+n)^2}\right] \tag{35}$$
$$=\frac{z+1/4}{z^2(z+1/2)^2}{}_6F_5(z,z,z+\frac{1}{2},z+\frac{1}{2},z+\frac{5}{4},1;z+\frac{1}{4},z+1,z+1,z+\frac{3}{2},z+\frac{3}{2};1).$$

The representation as a ${}_3F_2(-1)$ *in* (5) *is simpler.*

2.5. **Exercise** ${}_5F_4(\frac{3}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{7}{4},\frac{7}{4};-1)$**.** Another generic contiguous relation is [4]

$$b_j\,{}_pF_q(...a_i....;..b_j...;z) \tag{36}$$
$$=a_i\,{}_pF_q(...a_i+1....;..b_j+1...;z)+(b_j-a_i){}_pF_q(...a_i....;..b_j+1...;z).$$

Remark 2. *A strategic difference to* (1) *is that the two terms on the right hand side have the same and one higher excess compared to the left hand side, whereas in* (1) *the two terms on the right hand side have excesses that are one lower.*

We apply it with $a_i=\frac{3}{4}$, $b_j=\frac{1}{2}$ to

$${}_5F_4(\frac{3}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{7}{4},\frac{7}{4};-1) \tag{37}$$
$$=\frac{3}{2}{}_5F_4(\frac{7}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{3}{2},\frac{7}{4},\frac{7}{4};-1)-\frac{1}{2}{}_5F_4(\frac{3}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{3}{2},\frac{1}{2},\frac{7}{4},\frac{7}{4};-1)$$
$$=\frac{3}{2}{}_3F_2(\frac{3}{4},1,\frac{3}{2};\frac{1}{2},\frac{7}{4};-1)-\frac{1}{2}{}_4F_3(\frac{3}{4},\frac{3}{4},1,\frac{3}{2};\frac{1}{2},\frac{7}{4},\frac{7}{4};-1)$$

and again with $a_i=\frac{3}{4}$, $b_j=\frac{1}{2}$ for both these terms

$$\cdots=\frac{3}{2}[\frac{3}{2}{}_3F_2(\frac{7}{4},1,\frac{3}{2};\frac{3}{2},\frac{7}{4};-1)-\frac{1}{2}{}_3F_2(\frac{3}{4},1,\frac{3}{2};\frac{3}{2},\frac{7}{4};-1)] \tag{38}$$
$$-\frac{1}{2}[\frac{3}{2}{}_4F_3(\frac{7}{4},\frac{3}{4},1,\frac{3}{2};\frac{3}{2},\frac{7}{4},\frac{7}{4};-1)-\frac{1}{2}{}_4F_3(\frac{3}{4},\frac{3}{4},1,\frac{3}{2};\frac{3}{2},\frac{7}{4},\frac{7}{4};-1)]$$
$$=\frac{3}{2}[\frac{3}{2}{}_1F_0(1;;-1)-\frac{1}{2}{}_2F_1(\frac{3}{4},1;\frac{7}{4};-1)]$$
$$-\frac{1}{2}[\frac{3}{2}{}_2F_1(\frac{3}{4},1;\frac{7}{4};-1)-\frac{1}{2}{}_3F_2(\frac{3}{4},\frac{3}{4},1;\frac{7}{4},\frac{7}{4};-1)]$$
$$=\frac{9}{4}{}_1F_0(1;;-1)-\frac{3}{2}{}_2F_1(\frac{3}{4},1;\frac{7}{4};-1)+\frac{1}{4}{}_3F_2(\frac{3}{4},\frac{3}{4},1;\frac{7}{4},\frac{7}{4};-1).$$

Inserting for these three hypergeometric series [10, 7.3.1.1,7.3.6.35] and (5) gives

$${}_5F_4(\frac{3}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{7}{4},\frac{7}{4};-1) \tag{39}$$
$$=\frac{9}{4}\frac{1}{2}-\frac{3}{2}[\frac{3\sqrt{2}}{8}(\pi-2\ln(1+\sqrt{2}))]+\frac{1}{4}(\frac{3}{8})^2[\psi'(3/8)-\psi'(7/8)]\approx 0.24455830195.$$

Note that this ${}_5F_4$, excess -1, does not converge at $z=-1$: this evaluation is the limit of the argument $z\to -1^+$.

2.6. **Exercise** ${}_5F_4(\frac14,\frac34,1,\frac32,\frac32;\frac12,\frac12,\frac54,\frac74;-1)$**.** (36) with $a_i=\frac34$, $b_j=\frac12$ facilitates

$$
\begin{aligned}
(40)\quad & {}_5F_4(\frac14,\frac34,1,\frac32,\frac32;\frac12,\frac12,\frac54,\frac74;-1)\\
&=\frac32{}_5F_4(\frac14,\frac74,1,\frac32,\frac32;\frac12,\frac32,\frac54,\frac74;-1)-\frac12{}_5F_4(\frac14,\frac34,1,\frac32,\frac32;\frac32,\frac12,\frac54,\frac74;-1)\\
&=\frac32{}_3F_2(\frac14,1,\frac32;\frac12,\frac54;-1)-\frac12{}_4F_3(\frac14,\frac34,1,\frac32;\frac12,\frac54,\frac74;-1)
\end{aligned}
$$

and again with $a_i=\frac14$, $b_j=\frac12$ for both these terms

$$
\begin{aligned}
(41)\quad \cdots &=\frac32[\frac12{}_3F_2(\frac54,1,\frac32;\frac32,\frac54;-1)+\frac12{}_3F_2(\frac14,1,\frac32;\frac32,\frac54;-1)]\\
&-\frac12[\frac12{}_4F_3(\frac54,\frac34,1,\frac32;\frac32,\frac54,\frac74;-1)+\frac12{}_4F_3(\frac14,\frac34,1,\frac32;\frac32,\frac54,\frac74;-1)]\\
&=\frac32[\frac12{}_1F_0(1;;-1)+\frac12{}_2F_1(\frac14,1;\frac54;-1)]\\
&-\frac12[\frac12{}_2F_1(\frac34,1;\frac74;-1)+\frac12{}_3F_2(\frac14,\frac34,1;\frac54,\frac74;-1)]
\end{aligned}
$$

Inserting for these 4 hypergeometric series [10, 7.3.1.1, 7.3.6.24, 7.3.6.35, 7.4.5.34] gives

$$
\begin{aligned}
(42)\quad & {}_5F_4(\frac14,\frac34,1,\frac32,\frac32;\frac12,\frac12,\frac54,\frac74;-1)\\
&=\frac34\frac12+\frac34\frac{\sqrt2}{8}(\pi+2\ln(1+\sqrt2))-\frac14\frac{3\sqrt2}{8}(\pi-2\ln(1+\sqrt2))-\frac14\frac{3}{2\sqrt2}\ln(1+\sqrt2)\\
&=\frac38[1+\frac{1}{\sqrt2}\ln(1+\sqrt2)]\approx 0.608709465\ldots
\end{aligned}
$$

Note that this ${}_5F_4$, excess -1, does not converge at $z=-1$: this evaluation is the limit of the argument $z\to -1^+$.

2.7. **Exercise** ${}_5F_4(\frac14,\frac14,1,\frac32,\frac32;\frac12,\frac12,\frac54,\frac44;-1)$**.** We use (36) with $a_i=\frac14$, $b_j=\frac12$ for

$$
\begin{aligned}
(43)\quad & {}_5F_4(\frac14,\frac14,1,\frac32,\frac32;\frac12,\frac12,\frac54,\frac54;-1)\\
&=\frac12{}_5F_4(\frac54,\frac14,1,\frac32,\frac32;\frac32,\frac12,\frac54,\frac54;-1)+\frac12{}_5F_4(\frac14,\frac14,1,\frac32,\frac32;\frac32,\frac12,\frac54,\frac54;-1)\\
&=\frac12{}_3F_2(\frac14,1,\frac32;\frac12,\frac54;-1)+\frac12{}_4F_3(\frac14,\frac14,1,\frac32;\frac12,\frac54,\frac54;-1)
\end{aligned}
$$

and again with $a_i = \frac{1}{4}$, $b_j = \frac{1}{2}$

$$(44)\quad = \frac{1}{2}[\frac{1}{2}{}_3F_2(\frac{5}{4},1,\frac{3}{2};\frac{3}{2},\frac{5}{4};-1) + \frac{1}{2}{}_3F_2(\frac{1}{4},1,\frac{3}{2};\frac{3}{2},\frac{5}{4};-1)] + \frac{1}{2}[\frac{1}{2}{}_4F_3(\frac{5}{4},\frac{1}{4},1,\frac{3}{2};\frac{3}{2},\frac{5}{4},\frac{5}{4};-1) + \frac{1}{2}{}_4F_3(\frac{1}{4},\frac{1}{4},1,\frac{3}{2};\frac{3}{2},\frac{5}{4},\frac{5}{4};-1)]$$
$$= \frac{1}{2}[\frac{1}{2}{}_1F_0(1;;-1) + \frac{1}{2}{}_2F_1(\frac{1}{4},1;\frac{5}{4};-1)] + \frac{1}{2}[\frac{1}{2}{}_2F_1(\frac{1}{4},1;\frac{5}{4};-1) + \frac{1}{2}{}_3F_2(\frac{1}{4},\frac{1}{4},1;\frac{5}{4},\frac{5}{4};-1)]$$
$$= \frac{1}{4}{}_1F_0(1;;-1) + \frac{1}{2}{}_2F_1(\frac{1}{4},1;\frac{5}{4};-1)] + \frac{1}{4}{}_3F_2(\frac{1}{4},\frac{1}{4},1;\frac{5}{4},\frac{5}{4};-1)$$

Inserting for these 3 hypergeometric series [10, 7.3.1.1, 7.3.6.24] and (5) gives

$$(45)\quad {}_5F_4(\frac{1}{4},\frac{1}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{5}{4},\frac{5}{4};-1)$$
$$= \frac{1}{4}\frac{1}{2} + \frac{1}{2}\frac{\sqrt{2}}{8}[\pi + 2\ln(1+\sqrt{2})]] + \frac{1}{4}(\frac{1}{8})^2[\psi'(1/8) - \psi'(5/8)] \approx 0.800623018\ldots$$

Note that this ${}_5F_4$, excess -1, does not converge at $z=-1$: this evaluation is the limit of the argument $z \to -1^+$.

2.8. **Exercise 7.8.2.6.** To simplify the value of ${}_7F_6(\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{5}{4},\frac{5}{4},\frac{7}{4},\frac{7}{4};-1)$ of [10, 7.8.2.6], excess 1, insert $a_i = \frac{1}{4}$, $b_j = \frac{7}{4}$ in (1)

$$(46)\quad {}_7F_6(\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{5}{4},\frac{5}{4},\frac{7}{4},\frac{7}{4};-1)$$
$$= -\frac{1}{2}{}_7F_6(\frac{1}{4},\frac{5}{4},\frac{3}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{5}{4},\frac{5}{4},\frac{7}{4},\frac{7}{4};-1) + \frac{3}{2}{}_7F_6(\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{5}{4},\frac{5}{4},\frac{3}{4},\frac{7}{4};-1)$$
$$= -\frac{1}{2}{}_6F_5(\frac{1}{4},\frac{3}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{5}{4},\frac{7}{4},\frac{7}{4};-1) + \frac{3}{2}{}_6F_5(\frac{1}{4},\frac{1}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{5}{4},\frac{5}{4},\frac{7}{4};-1)$$

and again $a_i = \frac{1}{4}$, $b_j = \frac{7}{4}$ in (1) for both terms

$$(47)\quad \cdots = [(-\frac{1}{2}){}_6F_5(\frac{1}{4},\frac{3}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{5}{4},\frac{7}{4},\frac{7}{4};-1)] - 3[(-\frac{1}{2}){}_6F_5(\frac{1}{4},\frac{1}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{5}{4},\frac{5}{4},\frac{7}{4};-1)]$$
$$= [\frac{1}{4}{}_5F_4(\frac{3}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{7}{4},\frac{7}{4};-1) - \frac{3}{4}{}_5F_4(\frac{1}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{5}{4},\frac{7}{4};-1)] - 3[\frac{1}{4}{}_5F_4(\frac{1}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{5}{4},\frac{7}{4};-1) - \frac{3}{4}{}_5F_4(\frac{1}{4},\frac{1}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{5}{4},\frac{5}{4};-1)]$$
$$= \frac{1}{4}{}_5F_4(\frac{3}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{7}{4},\frac{7}{4};-1) - \frac{3}{2}{}_5F_4(\frac{1}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{5}{4},\frac{7}{4};-1) + \frac{9}{4}{}_5F_4(\frac{1}{4},\frac{1}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{5}{4},\frac{5}{4};-1).$$

The three ${}_5F_4$ on the previous line are collected from (39), (42) and (45) to obtain

$$(48)\quad {}_7F_6(\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{3}{4},1,\frac{3}{2},\frac{3}{2};\frac{1}{2},\frac{1}{2},\frac{5}{4},\frac{5}{4},\frac{7}{4},\frac{7}{4};-1)$$
$$=\frac{9}{1024}[32\sqrt{2}\ln(1+\sqrt{2})+\psi'(1/8)+\psi'(3/8)-\psi'(5/8)-\psi'(7/8)]\approx 0.9494771687,$$

which is the corrected value in Table 7.

Appendix A. Mathematica Results

Mathematica 14.1 is capable of reducing all values in our tables to some "standard" functions. The Mathematica script `Prud.m` scans the cases relevant to our hypergeometric constants; the output and numerical constants are in the file `Prud.out`. In a postprocessing step one can condensate the Mathematica formulae via

$$\sqrt{3+2\sqrt{2}}+\sqrt{3-2\sqrt{2}}=2^{3/2}; \tag{49}$$

$$\ln(2-\sqrt{2})-\ln(2+\sqrt{2})=-2\ln(1+\sqrt{2}) \tag{50}$$

to lead to notations closer to the printed equations.

A.1. Prud.m.

```
(* Mathematica program associated with the Prudnikov errata.
 The argument e is a list of 5 elements:
   - the equation number of the printed table
   - the list of the p upper parameters of pFq
   - the list of the q lower parameters of pFq
   - the argument of pFq
   - the expression found in Prudnikov vol 3
   Richard J. Mathar, 2025-10-21
*)
prud[e_] := Module[{f,f2,v,vP,err},
        Print[""];
        Print["idx ", e[[1]] ];
        (* the excess is the sum of the lower minus the sum of the upper parameters *)
        Print["excess ", Total[ e[[3]] ]-Total[ e[[2]] ] ];
        Print[ "claim ", e[[5]] ];
        (* value claimed by Prudnikov, 17 digits *)
        vP = N[e[[5]],17] ;
        Print[ "claim ", vP ];
        (* attempt to let Mma reduce the pFq *)
        f = HypergeometricPFQ[e[[2]],e[[3]],e[[4]] ] //FunctionExpand;
        f2 = Simplify[f] ;
        Print[f2] ;
        v = N[f2,17] ;
        (* relative error of the claimed expression *)
        err = N[Abs[(vP-v)/v],10] ;
        (* if the relative deviation between Mma and Prudnikov is larger than 10^-3,
        * print the Mma with an eclamation mark
        *)
        If [ err > 0.001,
                Print[ "!correct ", v ]
        ] ;
] ;

(* list of 3F2(a_i,b_i,-1) of Prudnikov vol3 section 7.4.5 *)
sect745 = {
{"7.4.5.89", {1,1,7/4},{2,11/4},-1, 7*Sqrt[2]/18*(3*Pi-4*Sqrt[2]+3*Sqrt[2]*Log[2]-6*Log[1+Sqrt[2]])
},
{"7.4.5.90", {1,1,7/4},{11/4,3},-1, -14/3*(13+2*Log[2]+Sqrt[2]*Log[1+Sqrt[2]]) }
} ;

(* list of 4F3(a_i,b_i,1) of Prudnikov vol3 section 7.5.3 *)
sect753 = {
{"7.5.3.111", {3/4,3/4,3/4,1},{11/4,11/4,11/4},1, 3^3*7^3/256*(128-12*Pi-6*Pi^2+Pi^3) },
{"7.5.3.112", {3/4,1,1,5/4},{7/4,2,9/4},1, 15*(3*Log[2]-2) },
{"7.5.3.113", {3/4,1,1,3/2},{7/4,2,5/2},1, 24*Log[2]-3*Pi-6 },
```

```
{"7.5.3.118", {1,1,4/3,3/2},{2,7/3,5/2},1, 24-6*Sqrt[3]+48*Log[2] -54*Log[3]},
{"7.5.3.119", {1,1,4/3,5/3},{7/3,8/3,3},1, 20/9*(1-Sqrt[3]*Pi)},
{"7.5.3.121", {1,1,3/2,5/3},{2,5/2,8/3},1, 15/4*(7+Sqrt[3]*Pi-16*Log[2]+9*Log[3])}
} ;

(* list of 4F3(a_i,b_i,-1) of Prudnikov vol3 section 7.5.4 *)
sect754 = {
{"7.5.4.18", {1/4,1/2,3/4,1},{5/4,3/2,7/4},-1, 3/4*(Sqrt[2]-1)*Pi },
{"7.5.4.19", {1/4,1/2,3/4,1},{5/4,7/4,5/2},-1, 3/20*((8*Sqrt[2]-7)*Pi-2-30*Sqrt[2]*Log[1+Sqrt[2]])
},
{"7.5.4.20", {1/4,1/2,1,1},{5/4,3/2,2},-1, 1/6*((2*Sqrt[2]-3)*Pi+2*Log[2]+4*Sqrt[2]*Log[1+Sqrt[2]])
},
{"7.5.4.21", {1/4,3/4,1,1},{5/4,7/4,2},-1, Sqrt[2]/4*(2*Sqrt[2]*Log[2]-Pi+6*Log[1+Sqrt[2]]) },
{"7.5.4.22", {1/4,1/2,1,1},{5/4,2,5/2},-1, 1/5*(
5*Log[2]+(2*Sqrt[2]-3)*Pi+4*Sqrt[2]*Log[1+Sqrt[2]]-3) },
{"7.5.4.23", {1/4,3/4,1,3/2},{5/4,7/4,5/2},-1, 3/20*(4-(1+Sqrt[2])*Pi+16*Log[1+Sqrt[2]]) },
{"7.5.4.24",{1/4 , 1 , 1 , 3/2},{ 5/4 , 2 , 5/2},-1, 1/10*(12-(3-2*Sqrt[2])*Pi-10*Log[
2]+4*Sqrt[2]*Log[1+Sqrt[2]])  },
{"7.5.4.25",{1/4 , 1 , 3/2 , 7/4},{ 5/4 , 5/2 , 11/4},-1, 7/60*(3*(3-2*Sqrt[2])*Pi
-16+18*Sqrt[2]*Log[1+Sqrt[2]])  },
{"7.5.4.26",{1/3 , 1/2 , 1 , 1},{ 4/3 , 3/2 , 2},-1, 2*Log[2]-(1-Sqrt[3]/2)*Pi  },
{"7.5.4.27",{1/3 , 2/3 , 1 , 1},{ 4/3 , 5/3 , 2},-1, 4*Log[ 2]-Pi/Sqrt[3] },
{"7.5.4.28",{1/3 , 2/3 , 1 , 1},{ 5/3 , 2 , 7/3},-1, 4/3*(3-Sqrt[ 3]*Pi+2*Log[ 2])  },
{"7.5.4.29",{1/3 , 2/3 , 1 , 7/3},{ 4/3 , 5/3 , 10/3},-1, 7/60*(16*Log[ 2]-3) },
{"7.5.4.30",{1/2 , 1/2 , 1/2 , 1},{ 5/2 , 5/2 , 5/2 },-1, 27/128*(12*Pi-Pi^3-64)  },
{"7.5.4.31",{1/2 , 2/3 , 1 , 1},{ 3/2 , 5/3 , 2 },-1, 8*Log[ 2]+2*(1-Sqrt[3])*Pi  },
{"7.5.4.32",{1/2 , 3/4 , 1 , 1},{ 3/2 , 7/4 , 2 },-1, 3/2*((1-2*Sqrt[2])*Pi+2*Log[
2]+4*Sqrt[2]*Log[1+Sqrt[2]])  },
{"7.5.4.33",{1/2 , 3/4 , 1 , 1},{ 7/4 , 2 , 5/2 },-1,
3*(1-2*Sqrt[2])*Pi+12*Sqrt[2]*Log[1+Sqrt[2]]-3  },
{"7.5.4.34",{1/2 , 3/4 , 1 , 5/4},{ 3/2 , 7/4 , 9/4 },-1,
5/4*(4-(1-2*Sqrt[2])*Pi+2*Sqrt[2]*Log[1+Sqrt[2]])  },
{"7.5.4.35",{1/2 , 1 , 1 , 1},{ 2 , 2 , 5/2 },-1, 1/8*(6*Pi-Pi^2-12)  },
{"7.5.4.36",{1/2 , 1 , 1 , 5/4},{ 3/2 , 2 , 9/4} ,-1, 5/6*(16+(1-6*Sqrt[2])*Pi-6*Log[
2]-12*Sqrt[2]*Log[1+Sqrt[2]])  },
{"7.5.4.37",{1/2 , 1 , 1 , 7/4},{ 3/2 , 2 , 11/4} ,-1,
7/45*(8+3*(3-Sqrt[2])*Pi-15*Log[2]+6*Sqrt[2]*Log[1+Sqrt[2]]) },
{"7.5.4.38",{1/2 , 1 , 1 , 5/2},{ 3/2 , 3 , 7/2 },-1, 10/9*(5-6*Log[ 2])  },
{"7.5.4.39",{2/3 , 1 , 1 , 4/3},{ 5/3 , 2 , 7/3 },-1, 12-16*Log[ 2] },
{"7.5.4.40",{3/4 , 1 , 1 , 5/4},{ 7/4 , 2 , 9/4} ,-1, 15*(2-Log[ 2]-Sqrt[2]*Log[1+Sqrt[2]])  },
{"7.5.4.41",{3/4 , 1 , 1 , 3/2},{ 7/4 , 2 , 5/2 },-1, 6+3*(Sqrt[2]-1)*Pi-9*Log[
2]+6*Sqrt[2]*Log[1+Sqrt[2]]  },
{"7.5.4.42",{3/4 , 1 , 5/4 , 3/2},{ 7/4 , 9/4 , 5/2 },-1, 15/4*((2*Sqrt[2]-1)*Pi
-8+4*Sqrt[2]*Log[1+Sqrt[2]])  },
{"7.5.4.43",{5/6 , 1 ,1  , 7/6},{ 11/6 , 2 , 13/6 },-1, 35*Sqrt[3]/3*(Sqrt[ 3]*(3-Log[
2])-Log[2+Sqrt[3]])  },
{"7.5.4.44",{1 , 1 ,1  , 1},{ 3 , 3 , 3 },-1, 4*(3*Zeta[3]+24*Log[ 2]-20)  },
{"7.5.4.45",{1 , 1 , 5/4  , 3/2},{ 2 , 9/4 , 5/2} ,-1,
15/2*((2*Sqrt[2]-1)*Pi+4*Sqrt[2]*Log[1+Sqrt[2]]-12)  },
{"7.5.4.46",{1 , 1 , 5/4  , 7/4},{ 2 , 9/4 , 11/4 },-1,
35*Sqrt[2]/18*(3*Pi-16*Sqrt[2]-6*Log[1+Sqrt[2]])  },
{"7.5.4.47",{1 , 1 , 4/3  , 3/2},{ 2 , 7/3 , 5/2 },-1, 12*(4*Log[ 2]-(1-Sqrt[ 3])*Pi-5)  },
{"7.5.4.48",{1 , 1 , 4/3  , 5/3},{ 7/3 , 8/3 , 3 },-1, 160*Log[ 2]-110  },
{"7.5.4.49",{1 , 1 , 3/2  , 5/3},{ 2 , 5/2 , 8/3 },-1, 15/4*(2*(2-Sqrt[3])*Pi+8*Log[ 2]-7)  },
{"7.5.4.50",{1 , 1 , 3/2  , 7/4},{ 2 , 5/2 , 11/4} ,-1, 7/2*(2*Log[
2]-3*(Sqrt[2]-1)*Pi-4+6*Sqrt[2]*Log[1+Sqrt[2]]) }
} ;

(* list of 5F4(a_i,b_i,1) of Prudnikov vol3 section 7.6.2 *)
sect762 = {
{"7.6.2.22", {1/4,1/4,1/4,1/4,1/2},{5/4,5/4,5/4,5/4},1,
Sqrt[Pi/2]/512*Gamma[1/4]^2*(3*Pi^2+PolyGamma[1,1/4]-PolyGamma[1,3/4]) },
{"7.6.2.23", {1/4,1/4,3/4,3/4,1},{5/4,5/4,7/4,7/4},1, 9*Pi/32*(Pi-2) },
{"7.6.2.25", {1/4,1/2,3/4,3/4,1},{5/4,3/2,7/4,7/4},1, 1.014678 },
{"7.6.2.48", {1/2,3/4,1,1,5/4},{7/4,2,9/4,5/2},1,135-150*Log[2] },
{"7.6.2.52", {2/3,1,1,1,4/3},{5/3,2,2,7/3},1,4/27*(27-3*Sqrt[3]*Pi-Pi^2) },
{"7.6.2.54", {3/4,1,1,5/4,3/2},{2,9/4,5/2,11/4},1,35/2*(3*Pi-24*Log[2]-26) },
{"7.6.2.55", {1,1,1,1,1},{3,3,3,3},1,16/45*(Pi^4+150*Pi^2-525) },
{"7.6.2.56", {1,1,1,3/2,3/2},{5/2,5/2,3,3},1,72*Pi^4+12*Pi^2-828 }
} ;

(* list of 5F4(a_i,b_i,-1) of Prudnikov vol3 section 7.6.3 *)
```

```
sect763 = {
{"7.6.3.7", {1/4,1/4,3/4,3/4,1},{5/4,5/4,7/4,7/4},-1, 0.99339507 }
} ;

(* list of 6F5(a_i,b_i,1) of Prudnikov vol3 section 7.7.2 *)
sect772 = {
{"7.7.2.10", {1/4,1/4,3/4,3/4,1,3/2},{1/2,5/4,5/4,7/4,7/4},1, 1.030461 },
{"7.7.2.11", {1/2,1,3/2,3/2,3/2,9/4},{5/4,2,2,2,3},1, 32/5*(1-8/Pi^2) }
} ;

(* list of 7F6(a_i,b_i,1) of Prudnikov vol3 section 7.8.2 *)
sect782 = {
{"7.8.2.4",
{1,3/2,3/2,(1+I)/2,(1+I)/2,(1-I)/2,(1-I)/2},{1/2,1/2,(3+I)/2,(3+I)/2,(3-I)/2,(3-I)/2},-1, 0.738632
},
{"7.8.2.5",
{1,3/2,3/2,(2+I)/4,(2+I)/4,(2-I)/4,(2-I)/4},{1/2,1/2,(6+I)/4,(6+I)/4,(6-I)/4,(6-I)/4},-1, 0.876963
},
{"7.8.2.6", {1/4,1/4,3/4,3/4,1,3/2,3/2},{1/2,1/2,5/4,5/4,7/4,7/4},-1, 0.961977 }
} ;

allsect = Join[sect745, sect753, sect754, sect762, sect763, sect772, sect782 ] ;
(* for all entries in the table, run the classification program prud on it
*)
Table[ prud[allsect[[i]]], {i,Length[allsect]}
] ;
```

A.2. **Prud.out.**

```
idx 7.4.5.89
excess 1
claim (7*(-4*Sqrt[2] + 3*Pi + 3*Sqrt[2]*Log[2] - 6*Log[1 + Sqrt[2]]))/(9*Sqrt[2])
claim 0.7812112840591128365421979528594503082‘17.
(7*(-16 + 3*(Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi + 6*Log[4] + 6*Sqrt[2]*Log[2 - Sqrt[2]]
- 6*Sqrt[2]*Log[2 + Sqrt[2]]))/36

idx 7.4.5.90
excess 2
claim (-14*(13 + 2*Log[2] + Sqrt[2]*Log[1 + Sqrt[2]]))/3
claim -72.95280925986830767957168721594828266012‘17.
(14*(-7 + 3*(Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi - 6*Log[2] + 6*Sqrt[2]*Log[2 - Sqrt[2]]
- 6*Sqrt[2]*Log[2 + Sqrt[2]]))/9
!correct 0.8415692167944340286550842220467322519‘17.

idx 7.5.3.111
excess 5
claim (9261*(128 - 12*Pi - 6*Pi^2 + Pi^3))/256
claim 2246.1375601819704353675716502605081724758‘17.
(343*(212 + 648*Catalan - 81*Pi^2))/2048
!correct 1.0229285882518343042052294752959649500‘17.

idx 7.5.3.112
excess 2
claim 15*(-2 + 3*Log[2])
claim 1.1916231251975389237754454656179455634‘17.
(15*(-4 + Log[64]))/2

idx 7.5.3.113
excess 2
claim -6 - 3*Pi + 24*Log[2]
claim 1.2107543726693077106256407651577289763‘17.
-3*(2 + Pi - Log[256])

idx 7.5.3.118
excess 2
claim 24 - 6*Sqrt[3] + 48*Log[2] - 54*Log[3]
claim -12.446303766613812244480779012859146985‘17.
-6*(Sqrt[3]*Pi + 9*Log[3] - 2*(5 + Log[16]))
!correct 1.2976125225835302059904903986172840186‘17.

idx 7.5.3.119
excess 3
```

```
claim (20*(1 - Sqrt[3]*Pi))/9
claim -9.86977353933923011507163282872548266593`17.
110 - 20*Sqrt[3]*Pi
!correct 1.17203814594692896435530454147065600665`17.

idx 7.5.3.121
excess 2
claim (15*(7 + Sqrt[3]*Pi - 16*Log[2] + 9*Log[3]))/4
claim 42.14457675658693433873897985711890044615`17.
(-15*(-7 + Sqrt[3]*Pi - 9*Log[3] + 8*Log[4]))/4
!correct 1.33409106131703270037221906017039644855`17.

idx 7.5.4.18
excess 2
claim (3*(-1 + Sqrt[2])*Pi)/4
claim 0.97596771342642975641492820508589311131`17.
(3*(-2 + Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi)/8

idx 7.5.4.19
excess 3
claim (3*(-2 + (-7 + 8*Sqrt[2])*Pi - 30*Sqrt[2]*Log[1 + Sqrt[2]]))/20
claim -3.87623992174131802451289908662575761391`17.
(3*(-2 + (-7 + 4*Sqrt[3 - 2*Sqrt[2]] + 4*Sqrt[3 + 2*Sqrt[2]])*Pi + 2*Sqrt[2]*Log[2 - Sqrt[2]] -
2*Sqrt[2]*Log[2 + Sqrt[2]]))/20
!correct 0.98491695135247997996045753932867280674`17.

idx 7.5.4.20
excess 2
claim ((-3 + 2*Sqrt[2])*Pi + 2*Log[2] + 4*Sqrt[2]*Log[1 + Sqrt[2]])/6
claim 0.972180699631514584105076119200629317`17.
((-3 + Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi + Log[4] + 2*Sqrt[2]*(-Log[2 - Sqrt[2]] +
Log[2 + Sqrt[2]]))/6

idx 7.5.4.21
excess 2
claim (-Pi + 2*Sqrt[2]*Log[2] + 6*Log[1 + Sqrt[2]])/(2*Sqrt[2])
claim 1.45210216644104528784532211469469003321`17.
-1/4*((Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi) + Log[2] + Sqrt[2]*(-Log[2 - Sqrt[2]] + Log[2
+ Sqrt[2]])
!correct 0.96460667204168423948537194743010172937`17.

idx 7.5.4.22
excess 3
claim (-3 + (-3 + 2*Sqrt[2])*Pi + 5*Log[2] + 4*Sqrt[2]*Log[1 + Sqrt[2]])/5
claim 0.98250514789378468657643061591566112124`17.
(-3 + (-3 + Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi + Log[32] - 2*Sqrt[2]*Log[2 - Sqrt[2]] +
2*Sqrt[2]*Log[2 + Sqrt[2]])/5

idx 7.5.4.23
excess 2
claim (3*(4 - (1 + Sqrt[2])*Pi + 16*Log[1 + Sqrt[2]]))/20
claim 1.57762527008467933773648372395047205423`17.
(-3*(-8 + (2 + Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi + 8*Sqrt[2]*Log[2 - Sqrt[2]] -
8*Sqrt[2]*Log[2 + Sqrt[2]]))/40
!correct 0.95806923757432930932386953660033371893`17.

idx 7.5.4.24
excess 2
claim (12 - (3 - 2*Sqrt[2])*Pi - 10*Log[2] + 4*Sqrt[2]*Log[1 + Sqrt[2]])/10
claim 0.95153180310697437916236712577056570852`17.
(12 + (-3 + Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi - 5*Log[4] - 2*Sqrt[2]*Log[2 - Sqrt[2]] +
2*Sqrt[2]*Log[2 + Sqrt[2]])/10

idx 7.5.4.25
excess 2
claim (7*(-16 + 3*(3 - 2*Sqrt[2])*Pi + 18*Sqrt[2]*Log[1 + Sqrt[2]]))/60
claim 0.93953357148072801706287652978671138385`17.
(-7*(16 + 3*(-3 + Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi + 9*Sqrt[2]*Log[2 - Sqrt[2]] -
9*Sqrt[2]*Log[2 + Sqrt[2]]))/60

idx 7.5.4.26
excess 2
```

```
claim -((1 - Sqrt[3]/2)*Pi) + 2*Log[2]
claim 0.9654007538814241562629382461000838503‘17.
((-2 + Sqrt[3])*Pi)/2 + Log[4]

idx 7.5.4.27
excess 2
claim -(Pi/Sqrt[3]) + 4*Log[2]
claim 0.9587893580055633870748502281905996547‘17.
-(Pi/Sqrt[3]) + Log[16]

idx 7.5.4.28
excess 3
claim (4*(3 - Sqrt[3]*Pi + 2*Log[2]))/3
claim -1.406804975443683910597027373346818747‘17.
-4 - (4*Pi)/(3*Sqrt[3]) + (20*Log[2])/3 + Log[16]
!correct 0.9751707736604594996583716186976757497‘17.

idx 7.5.4.29
excess 2
claim (7*(-3 + 16*Log[2]))/60
claim 0.9438747370452312442454999600552629270‘17.
(7*(-3 + 16*Log[2]))/60

idx 7.5.4.30
excess 5
claim (27*(-64 + 12*Pi - Pi^3))/128
claim -12.08823008285157918340596914554045887485‘17.
(27*(-64 + 12*Pi + Pi^3))/128
!correct 0.9925428916499074531231012733929422261‘17.

idx 7.5.4.31
excess 2
claim 2*(1 - Sqrt[3])*Pi + 8*Log[2]
claim 0.9455665662538418486986741923714839192‘17.
-2*(-1 + Sqrt[3])*Pi + Log[256]

idx 7.5.4.32
excess 2
claim (3*((1 - 2*Sqrt[2])*Pi + 2*Log[2] + 4*Sqrt[2]*Log[1 + Sqrt[2]]))/2
claim 0.9418845892721932056262594321185189664‘17.
(3*(-((-1 + Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi) + Log[4] + 2*Sqrt[2]*(-Log[2 - Sqrt[2]]
+ Log[2 + Sqrt[2]])))/2

idx 7.5.4.33
excess 3
claim -3 + 3*(1 - 2*Sqrt[2])*Pi + 12*Sqrt[2]*Log[1 + Sqrt[2]]
claim -5.275113904815285445250873864512021475‘17.
-3 - 3*(-1 + Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi + Log[512] - 6*Sqrt[2]*Log[2 - Sqrt[2]]
+ 6*Sqrt[2]*Log[2 + Sqrt[2]]
!correct 0.9632107202242223395042152286115676372‘17.

idx 7.5.4.34
excess 2
claim (5*(4 - (1 - 2*Sqrt[2])*Pi + 2*Sqrt[2]*Log[1 + Sqrt[2]]))/4
claim 15.29634272910982663643149864730519565454‘17.
(-5*(-4 + (-1 + Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi + Sqrt[2]*Log[2 - Sqrt[2]] -
Sqrt[2]*Log[2 + Sqrt[2]]))/4
!correct 0.9359096722924784975087021552004843719‘17.

idx 7.5.4.35
excess 3
claim (-12 + 6*Pi - Pi^2)/8
claim -0.3775060599438248985073288375248917300‘17.
-6 + 3*Pi - Pi^2/4
!correct 0.9573768604970400606793073998694708671‘17.

idx 7.5.4.36
excess 2
claim (5*(16 + (1 - 6*Sqrt[2])*Pi - 6*Log[2] - 12*Sqrt[2]*Log[1 + Sqrt[2]]))/6
claim -22.19332818507134035132709767653945898294‘17.
(-5*(-16 + (-1 + Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi + Log[64] - 2*Sqrt[2]*Log[2 -
Sqrt[2]] + 2*Sqrt[2]*Log[2 + Sqrt[2]]))/6
```

```
!correct 0.92595147732628731731277336033709338108`17.

idx 7.5.4.37
excess 2
claim (7*(8 + 3*(3 - Sqrt[2])*Pi - 15*Log[2] + 6*Sqrt[2]*Log[1 + Sqrt[2]]))/45
claim 3.1153391486181419660466965854060730357 5`17.
(-7*(3*(-3 + Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi + 2*(-8 + 5*Log[8] + 3*Sqrt[2]*Log[2 -
Sqrt[2]] - 3*Sqrt[2]*Log[2 + Sqrt[2]]))))/90
!correct 0.91622429110528669912284621711042101 68`17.

idx 7.5.4.38
excess 3
claim (10*(5 - 6*Log[2]))/9
claim 0.9345743518225868261073414125010451017 2`17.
(-10*(-5 + Log[64]))/9

idx 7.5.4.39
excess 2
claim 12 - 16*Log[2]
claim 0.9096451110408750493242860566691749108`17.
-2*(-6 + Log[256])

idx 7.5.4.40
excess 2
claim 15*(2 - Log[2] - Sqrt[2]*Log[1 + Sqrt[2]])
claim 0.9060350873939049569209157706103113993 5`17.
(-15*(-4 + Log[4] - Sqrt[2]*Log[2 - Sqrt[2]] + Sqrt[2]*Log[2 + Sqrt[2]]))/2

idx 7.5.4.41
excess 2
claim 6 + 3*(-1 + Sqrt[2])*Pi - 9*Log[2] + 6*Sqrt[2]*Log[1 + Sqrt[2]]
claim 11.144249110348977401632864690226799362 38`17.
(3*(4 + (-1 + Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi - 3*Log[4] + 2*Sqrt[2]*Log[2 - Sqrt[2]]
- 2*Sqrt[2]*Log[2 + Sqrt[2]]))/2
!correct 0.89923232736813493787034783913242162503`17.

idx 7.5.4.42
excess 2
claim (15*(-8 + (-1 + 2*Sqrt[2])*Pi + 4*Sqrt[2]*Log[1 + Sqrt[2]]))/4
claim 10.237406789432937610204797145674107003 39`17.
(15*(-8 + (-1 + Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi - Sqrt[2]*Log[2 - Sqrt[2]] +
Sqrt[2]*Log[2 + Sqrt[2]]))/4
!correct 0.88902818732947990929449594191558696363`17.

idx 7.5.4.43
excess 2
claim (35*(Sqrt[3]*(3 - Log[2]) - Log[2 + Sqrt[3]]))/Sqrt[3]
claim 54.127738809868792006808576828149616270 29`17.
-35*(-3 + Log[2] - Sqrt[3]*Log[-1 + Sqrt[3]] + Sqrt[3]*Log[1 + Sqrt[3]])
!correct 0.90351906880254767963197898652120857583`17.

idx 7.5.4.44
excess 5
claim 4*(-20 + 24*Log[2] + 3*Zeta[3])
claim 0.9668121716698811288511415981174335969 7`17.
-80 + 96*Log[2] + 12*Zeta[3]

idx 7.5.4.45
excess 2
claim (15*(-12 + (-1 + 2*Sqrt[2])*Pi + 4*Sqrt[2]*Log[1 + Sqrt[2]]))/2
claim -9.525186421134124779590405708651785993 22`17.
(15*(-12 + (-1 + Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi + Log[4] - 2*Sqrt[2]*Log[2 -
Sqrt[2]] + 2*Sqrt[2]*Log[2 + Sqrt[2]]))/2
!correct 0.87202128726505486166807611322086252798`17.

idx 7.5.4.46
excess 2
claim (35*(-16*Sqrt[2] + 3*Pi - 6*Log[1 + Sqrt[2]]))/(9*Sqrt[2])
claim -50.847327352903797760490051652714162957 17`17.
(35*(3*(Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi + 4*(-16 + Log[8] - 3*Sqrt[2]*Log[2 -
Sqrt[2]] + 3*Sqrt[2]*Log[2 + Sqrt[2]])))/36
!correct 0.86515656344503345362572871517039111773`17.
```

```
idx 7.5.4.47
excess 2
claim 12*(-5 - (1 - Sqrt[3])*Pi + 4*Log[2])
claim 0.86872993623169861186223850575604701944`17.
6*(-10 + 2*(-1 + Sqrt[3])*Pi + Log[256])

idx 7.5.4.48
excess 3
claim -110 + 160*Log[2]
claim 0.90354888959124950675713943330825089196`17.
-110 + 160*Log[2]

idx 7.5.4.49
excess 2
claim (15*(-7 + 2*(2 - Sqrt[3])*Pi + 8*Log[2]))/4
claim 0.85781952537535622108985359598933632882`17.
(-15*(7 + 2*(-2 + Sqrt[3])*Pi - Log[256]))/4

idx 7.5.4.50
excess 2
claim (7*(-4 - 3*(-1 + Sqrt[2])*Pi + 2*Log[2] + 6*Sqrt[2]*Log[1 + Sqrt[2]]))/2
claim 3.36394236183928213866047334952858853649`17.
(-7*(20 + 3*(-3 + Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*Pi - 2*Log[8] + 6*Sqrt[2]*Log[2 -
Sqrt[2]] - 6*Sqrt[2]*Log[2 + Sqrt[2]]))/6
!correct 0.85554595009700348236644235789973111123`17.

idx 7.6.2.22
excess 7/2
claim (Sqrt[Pi/2]*Gamma[1/4]^2*(3*Pi^2 + PolyGamma[1, 1/4] - PolyGamma[1, 3/4]))/512
claim 1.42431275138042154010015666374255267173`17.
(Pi^(3/2)*(48*Catalan + 5*Pi^2)*Gamma[1/4])/(1536*Gamma[3/4])
!correct 1.00087399096348542477192195955206569496`17.

idx 7.6.2.23
excess 3
claim (9*(-2 + Pi)*Pi)/32
claim 1.00868037016212341491196369062044713444`17.
(9*(-2 + Pi)*Pi)/32

idx 7.6.2.25
excess 3
claim 1.014678
claim 1.014678
(9*(-8*Catalan - 2*Pi + Pi^2 + Log[256]))/16

idx 7.6.2.48
excess 4
claim 135 - 150*Log[2]
claim 31.02792291600820358741518178127351478879`17.
(-15*(-14 + Log[1048576]))/2
!correct 1.02792291600820358741518178127351478868`17.

idx 7.6.2.52
excess 3
claim (4*(27 - 3*Sqrt[3]*Pi - Pi^2))/27
claim 0.11943723271150825567685995279176996502`17.
(-4*(-27 + 3*Sqrt[3]*Pi + Pi^2))/3
!correct 1.07493509440357430109173957512592968522`17.

idx 7.6.2.54
excess 4
claim (35*(-26 + 3*Pi - 24*Log[2]))/2
claim -581.18820152171288493594871339026025715495`17.
(35*(-26 + 3*Pi + Log[16777216]))/2
!correct 1.05543014864117497452626863460806001211`17.

idx 7.6.2.55
excess 7
claim (16*(-525 + 150*Pi^2 + Pi^4))/45
claim 374.34657820352221513301830494937876731705`17.
(16*(-1575 + 150*Pi^2 + Pi^4))/45
```

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!correct 1.0132448701888817996849716160454334392 9`17.

idx 7.6.2.56
excess 5
claim -828 + 12*Pi^2 + 72*Pi^4
claim 6303.889807261247784449717845585281823614 63`17.
-828 + 84*Pi^2
!correct 1.046769691506123982097243989596695366 96`17.

idx 7.6.3.7
excess 3
claim 0.99339507
claim 0.99339507
(9*(16*Sqrt[2]*Log[2 - Sqrt[2]] - 16*Sqrt[2]*Log[2 + Sqrt[2]] + PolyGamma[1, 1/8] + PolyGamma[1,
3/8] - PolyGamma[1, 5/8] - PolyGamma[1, 7/8]))/256

idx 7.7.2.10
excess 2
claim 1.030461
claim 1.030461
(9*Catalan)/8

idx 7.7.2.11
excess 2
claim (32*(1 - 8/Pi^2))/5
claim 1.212355397512306102073371483661944888 17`17.
(32*(-8 + Pi^2))/(5*Pi^2)

idx 7.8.2.4
excess 1
claim 0.738632
claim 0.738632
(-1/2704*I)*((320 + 64*I)*HypergeometricPFQ[{5/2 - I/2, 5/2 - I/2, 5/2 + I/2, 3}, {7/2 - I/2, 7/2 -
I/2, 7/2 + I/2}, -1] - (320 - 64*I)*HypergeometricPFQ[{5/2 - I/2, 5/2 + I/2, 5/2 + I/2, 3}, {7/2 -
I/2, 7/2 + I/2, 7/2 + I/2}, -1] + 169*(12*PolyGamma[0, 1/4 - I/4] - 12*PolyGamma[0, 1/4 + I/4] -
12*PolyGamma[0, 3/4 - I/4] + 12*PolyGamma[0, 3/4 + I/4] + (4 + 3*I)*PolyGamma[1, 1/4 - I/4] - (4 -
3*I)*PolyGamma[1, 1/4 + I/4] - (4 + 3*I)*PolyGamma[1, 3/4 - I/4] + (4 - 3*I)*PolyGamma[1, 3/4 +
I/4]))

idx 7.8.2.5
excess 1
claim 0.876963
claim 0.876963
(-25*I)/2611456*((10240 + 1024*I)*HypergeometricPFQ[{5/2 - I/4, 5/2 - I/4, 5/2 + I/4, 3}, {7/2 -
I/4, 7/2 - I/4, 7/2 + I/4}, -1] - (10240 - 1024*I)*HypergeometricPFQ[{5/2 - I/4, 5/2 + I/4, 5/2 +
I/4, 3}, {7/2 - I/4, 7/2 + I/4, 7/2 + I/4}, -1] + 10201*(24*PolyGamma[0, 1/4 - I/8] -
24*PolyGamma[0, 1/4 + I/8] - 24*PolyGamma[0, 3/4 - I/8] + 24*PolyGamma[0, 3/4 + I/8] + (2 +
3*I)*PolyGamma[1, 1/4 - I/8] - (2 - 3*I)*PolyGamma[1, 1/4 + I/8] - (2 + 3*I)*PolyGamma[1, 3/4 -
I/8] + (2 - 3*I)*PolyGamma[1, 3/4 + I/8]))

idx 7.8.2.6
excess 1
claim 0.961977
claim 0.961977
(9*(32*Pi^2 - 16*Sqrt[2]*Log[2 - Sqrt[2]] + 16*Sqrt[2]*Log[2 + Sqrt[2]] - 3*PolyGamma[1, 1/8] -
3*PolyGamma[1, 3/8] - 5*PolyGamma[1, 5/8] - 5*PolyGamma[1, 7/8]))/1024
!correct 0.949477168790850309630105384987662864 45`17.
```

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