Remarks on Mie's Electrodynamics

Martin Kraus (kraus.martin@gmail.com)

October 28, 2025

Abstract

Mie's electrodynamics shares several features with a specific modified Born-Infeld field theory, including nonlinearity, lack of gauge invariance, and particle-like solutions of negative energy and mass. Unfortunately, Mie has never identified suitable field equations to completed his theory. The present remarks discuss potential reasons that might have prevented Mie and other physicists of his time from researching the kind of field equations underlying the mentioned modified Born-Infeld field theory.

1 Introduction

In Mie's electromagnetic worldview, all elementary particles can be represented by solutions of nonlinear generalizations of Maxwell-Heaviside equations [Mie12]. While Mie spent considerable efforts on determining suitable nonlinear field equations, he never successfully completed his theory of matter. However, his work inspired research by several well-known scientists, including Born [Bor14], Hilbert [Hil15], Weyl [Wey19], and Infeld [BIF34]. Moreover, when Mie published his research in 1912, Einstein had already worked on similar ideas for a few years without much success [Cor70] and had decided to focus on his theory of gravitation before searching for particle-like solutions of a unified field theory. Vizgin refers to this search as the "maximum problem" of unified field theories while the "minimum problem" is the unification of gravitation and electromagnetism [Viz94].

Today, Mie's theory of matter is mainly of historical interest due to its influence on the works of other researchers [Viz94]. For the modified Born-Infeld model of electrons [Kra23], however, Mie's generalization of electrodynamics is of particular interest, because it does not require gauge invariance and its field equations were never fixed; thus, it provides a unique perspective on the potential reasons why the field equations of the modified Born-Infeld model have not been researched by Mie nor by other physicists of the first half of the 20th century.

Section 2 provides a brief summary of Born's interpretation of Mie's electrodynamics [Bor14]. Features of Mie's electrodynamics that might have contributed to its lack of success are discussed in Section 3. Section 4 very briefly comments on the general historical context while Section 5 concludes the present remarks.

2 Born's Interpretation of Mie's Electrodynamics

Two years after Mie's first publication of his theory of matter [Mie12], Born published a summary of Mie's electrodynamics [Bor14] employing a formalism that resembles relativistic Lagrangian field theory. Here, the key equations of Born's summary are repeated in SI units using basic Ricci calculus and the Minkowski metric tensor η in the form $\mathrm{diag}(+1, -1, -1, -1)$. A point in space-time is denoted by the four-vector $(x^0, x^1, x^2, x^3) = (c\,t, x, y, z)$ with c denoting the speed of light, and the electromagnetic four-potential is denoted by $(A^0, A^1, A^2, A^3) = (\phi/c, A_x, A_y, A_z)$, where all components of the four-potential are twice-differentiable functions of the coordinates x, y, z, and t.

In general, the Lagrangian density $\mathscr{L}$ of Mie's electrodynamics is a function of $\partial_0 A_0$, $\partial_0 A_1$, $\partial_0 A_2$, $\partial_0 A_3$, $\partial_1 A_0$, $\ldots$, $\partial_3 A_3$, A^0, A^1, A^2, and A^3 [Bor14, Section 1]. This leads to four Euler-Lagrange equations [Bor14, Eq. (8)]:

$$\partial_\gamma \left(\frac{\partial \mathscr{L}}{\partial \left(\partial_\gamma A_\beta\right)} \right) - \frac{\partial \mathscr{L}}{\partial A_\beta} = 0 \qquad \text{for } \beta = 0, \ldots, 3. \tag{1}$$

However, Mie's electrodynamics further restricts the Lagrangian density $\mathscr{L}$ to depend only on the components of the four-potential itself and independent components of the electromagnetic tensor

$$F_{\beta\alpha} \stackrel{\text{def}}{=} \partial_\beta A_\alpha - \partial_\alpha A_\beta. \tag{2}$$

In terms of the electric field $\mathbf{E} = (E_x, E_y, E_z)$ and magnetic field $\mathbf{B} = (B_x, B_y, B_z)$, the components of the electromagnetic tensor $F_{\beta\alpha}$ are:

$$F_{\beta\alpha} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{pmatrix}. \tag{3}$$

With this notation, the Lagrangian density $\mathscr{L}$ should depend only on a set of 10 arguments, for example, $F_{32}(= B_x)$, $F_{13}(= B_y)$, $F_{21}(= B_z)$, $F_{01}(= E_x/c)$, $F_{02}(= E_y/c)$, $F_{03}(= E_z/c)$, $A^1(= A_x)$, $A^2(= A_y)$, $A^3(= A_z)$, and $A^0(= \phi/c)$ [Bor14, Eq. (6')].

In order to achieve Lorentz invariance, the Lagrangian density $\mathscr{L}$ is further restricted to depend only on Lorentz-invariant combinations of these 10 arguments, e.g., $A_\beta A^\beta$ or $F_{\beta\alpha}F^{\beta\alpha}$.

For the four-current $(J^0, J^1, J^2, J^3) = (c\rho, J_x, J_y, J_z)$, Mie and Born [Bor14, Eq. (13)] employ the definition

$$J^\beta \stackrel{\text{def}}{=} -\frac{\partial \mathscr{L}}{\partial A_\beta}, \tag{4}$$

which would be consistent with an interaction term $\mathscr{L}_{\text{int}} = -J^\beta A_\beta$ in the Lagrangian density for a given four-current (J^0, J^1, J^2, J^3). However, Born notes that the Lagrangian density $\mathscr{L}$ in Mie's electrodynamics should not depend on a four-current that is given as a function of the coordinates x, y, z, and t because $\mathscr{L}$ should not explicitly depend on these coordinates [Bor14, Section 3].

In fact, if $\mathscr{L}$ does not depend on these coordinates, there are four continuity equations corresponding to energy and momentum conservation according to Noether's (first) theorem [Noe18]. This theorem provides a "canonical" stress-energy tensor for any Lagrangian density $\mathscr{L}$ in Mie's electrodynamics, which may be contracted to produce the four continuity equations. If the canonical stress-energy tensor is not symmetric, the symmetric Belinfante-Rosenfeld tensor can be constructed such that the four continuity equations are satisfied [Bel40, Ros40]. Since these general results were unknown in 1914, Born had to derive them for the special case of Mie's electrodynamics [Bor14, Sections 4 and 5]. Since Born's work influenced Hilbert's research, it is likely that it also had some effect on Noether's research leading to the publication of her famous theorem [Noe18].

In any case, the objective of Mie's research program (according to Born [Bor14, page 24]) was to find a Lagrangian density $\mathscr{L}$ such that the corresponding Euler-Lagrange equations imply the existence of electrons (and atoms), as well as all their interactions (i.e., all forces between them). Therefore, these Euler-Lagrange equations have to be nonlinear; otherwise their particle-like solutions would not be able to interact with each other.

3 Limitations of Mie's Electrodynamics

As mentioned, Mie did not successfully complete his research program. The present remarks assume that he could have made more progress by considering the kind of field equations and solutions that are part of the modified Born-Infeld model of electrons [Kra23]. Therefore, this section attempts to identify the specific limitations of Mie's approach that might have prevented him and other researchers of his time from considering these field equations and solutions.

3.1 Limitations of Mie's Lagrangian Density

The Lagrangian density of the modified Born-Infeld model [Kra23] depends on the Lorentz invariant $(\partial_\beta A_\alpha)(\partial^\beta A^\alpha)$, which is outside the limits of Mie's electrodynamics even though dependencies on Lorentz invariants such as $A_\alpha A^\alpha$ or $F_{\beta\alpha}F^{\beta\alpha} = (\partial_\beta A_\alpha - \partial_\alpha A_\beta)(\partial^\beta A^\alpha - \partial^\alpha A^\beta)$ are allowed.

One reason to exclude a dependency of $\mathscr{L}$ on $(\partial_\beta A_\alpha)(\partial^\beta A^\alpha)$ is that it lacks gauge invariance. In fact, this might have been the most important reason for Born and Infeld not to consider $(\partial_\beta A_\alpha)(\partial^\beta A^\alpha)$

in their field theory [BIF34]. For Mie, however, gauge invariance was not a strict requirement, as the gauge-violating term $A_\alpha A^\alpha$ illustrates.

On the other hand, Mie had specific reasons to permit dependencies of $\mathscr{L}$ on the electric potential ϕ and the magnetic vector potential $\mathbf{A}$: One of the working hypothesis of his field theory was that the field's state is completely determined by the electric field, the magnetic field, the charge density, and the current density [Mie12, page 513]. A refined version of this hypothesis stated that the field's state is determined *either* by the 10 "extensive quantities" („Quantitätsgrößen“) electric displacement $\mathbf{D}$, magnetic field strength $\mathbf{H}$, charge density ρ, and current density $\mathbf{J}$, *or* by the 10 "intensive quantitites" („Intensitätsgrößen“) electric field strength $\mathbf{E}$, magnetic induction $\mathbf{B}$, electric potential ϕ, and magnetic vector potential $\mathbf{A}$. In the latter case, ρ and $\mathbf{J}$ are determined by Eq. (4), and analogous equations determine $\mathbf{D}$ and $\mathbf{H}$. Further analogous equations were provided by Mie for the former case [Mie12, page 523]. The symmetries of this system of related quantities probably had some aesthetic appeal to Mie; thus, he might have been reluctant to modify it in any way. A dependency of $\mathscr{L}$ on $(\partial_\beta A_\alpha)(\partial^\beta A^\alpha)$ would certainly not fit easily into Mie's system. (In fact, even the removal of the dependency of $\mathscr{L}$ on ϕ and $\mathbf{A}$ would be a problem for this system, since Eq. (4) would then imply that there are neither charges nor currents. However, Born and Infeld [BIF34] solved this problem by defining $J^\beta \stackrel{\text{def}}{=} \partial_\alpha F^{\alpha\beta}/\mu_0$ with the vacuum permeability μ_0 instead of the definition in Eq. (4).)

Thus, Mie's original publication [Mie12] reveals that Mie had specific reasons to exclude a dependency of $\mathscr{L}$ on $(\partial_\beta A_\alpha)(\partial^\beta A^\alpha)$, even though Born's interpretation of Mie's electrodynamics from 1914 [Bor14] might suggest that there are no fundamental reasons to exclude it—unless gauge invariance was considered a fundamental requirement. Twenty years later, Born and Infeld published a gauge-invariant version of Mie's electrodynamics [BIF34].

While these considerations might explain some of the reluctance of Mie and Born to consider terms such as $(\partial_\beta A_\alpha)(\partial^\beta A^\alpha)$ in the Lagrangian density, it is likely that there were additional contributing factors as discussed next.

3.2 Focus on Particle-Like Static Field Solutions

Mie assumed that electrons correspond to static spherically symmetric solutions of nonlinear field equations [Mie12]. In the modified Born-Infeld model, however, electrons are represented by field solutions that are neither static nor spherically symmetric. Thus, from this point of view, it is not surprising that Mie's search for field equations with particle-like static spherically symmetric solutions failed to produce satisfying results. While Mie's assumption might have been plausible in 1912, it is neither consistent with the electron's spin and magnetic moment [UG25] nor with de Broglie's internal clock hypothesis [dB25]. Nonetheless, Born and Infeld employed the same assumption in 1934 [BIF34]. Even Einstein tried to model elementary particles as static spherically symmetric solutions of nonlinear field equations in unified field theories [Viz94, page 310] for many years after the electron's spin and magnetic moment had been widely accepted.

It is plausible to assume that most researchers in this field were aware that static spherically symmetric field solutions could not represent an electron with spin and magnetic moment. Searching for this kind of solutions might have been motivated in part by necessity since many nonlinear field equations could not be solved without simplifying assumptions. These assumptions might have been justified either by further assuming that the field equations were only simplified versions of the actual field equations or by assuming that a static spherical symmetric solution did not correspond to the electron but to a different hypothetical particle, which might not exist in nature.

In any case, the assumption that field equations feature particle-like static solutions all but ruled out any model of electrons as rotating field solutions, including the modified Born-Infeld model of electrons. Thus, it might be very interesting to review early discussions of particle-like rotating field solutions; however, this task is beyond the scope of the present remarks.

3.3 Negative Energy and Mass of Particle-Like Field Solutions

Mie expressly allowed for and provided an example of a particle-like field solution with negative energy and mass [Mie13, pages 12–13]. While this feature is shared by the modified Born-Infeld model of electrons [Kra24], the "negative energy problem" has been considered a flaw of Mie's theory of matter [SM07]. Part of the problem has been that Mie never completed his theory and, therefore, could not know how a completed version of his theory would solve the issue. Here, the modified Born-Infeld model of electrons might provide an example of a particle-like field solution with negative mass that could be studied numerically.

3.4 Nonlinearity of Field Equations

As mentioned, field equations in Mie's electrodynamics have to be nonlinear in order to allow for interacting particles. Solving nonlinear partial differential equations is a particularly difficult subject, which is why simplifying assumptions are often employed. As discussed in Section 3.2, the specific assumptions proposed by Mie might have contributed to the lack of success of Mie's research program.

Furthermore, the mathematical difficulties probably had an adverse effect on the motivation and interest of many physicists—in general and in particular compared to linear quantum theories. There are, however, examples of very successful nonlinear theories in physics, e.g., general relativity, nonlinear fluid dynamics, quantum chromodynamics (QCD), etc. The history of QCD might serve as an example of how a research community may overcome such mathematical difficulties: the nature of the so-called strong force between protons and neutrons inside the atomic nucleus had been an unsolved problem for decades. QCD did not solve this problem overnight but required many years of research and technological progress because its nonlinear field equations required new numeric methods (in particular lattice QCD) and high-performance computers in order to compare QCD's predictions with experiments. A crucial factor for the success of research on the strong force (and QCD in particular) was the funding of and interest in research related to nuclear weapons and nuclear power. Mie's electrodynamics (and research on the internal structure of electrons in general) never offered hopes for applications of similar practical relevance; thus, it is not surprising that Mie's research program attracted far less research effort.

4 Historical Context

Some of the factors that negatively affected the development of Mie's electrodynamics were not related to the theory itself but to the wider historical context of the decades after its first publication in 1912: two world wars and a fascist German regime that attacked all independent research, killed many scientists, and forced many more scientists to flee Germany had a devastating effect on the scientific communities in Germany. The German physics community, in particular, never regained its former international role.

Correspondingly, the German language lost its role for international publications of physics research. Since almost all of the research on Mie's electrodynamics was originally published in German and only some of it was translated to English (in some cases decades after the original publication), the vast majority of physicists who have been active after World War II never took notice of Mie's electrodynamics. Research on Mie's quantum electrodynamics [Mie28, Lan29, Bol29] fared even worse and is all but unknown among today's physicists—including German-speaking physicists. (A notable exception is Hubert Goenner [Goe04, Section 7.1].)

The lack of interest in Mie's interpretation of quantum mechanics [Mie28] also illustrates the "shut up and calculate!"-attitude of many physicists after World War II. As David Kaiser wrote: "Anything that smacked of 'interpretation', or worse, 'philosophy', began to carry a taint for many scientists who had come through the wartime projects. Conceptual scrutiny of foundations struck many as a luxury. [...] Openly philosophical areas of physics, the intellectual roots of which stretched back before the war, became increasingly marginalized, such as grand questions about [...] the subtle foundations of quantum theory. Sometimes these were denigrated as not even being 'real physics' by influential physicists in the United States" [Kai14]. While Kaiser identified a more recent "era that reclaimed more openly speculative and philosophical approaches to the deep mysteries of nature" [Kai14], and, in fact, interest in minority interpretations of quantum mechanics has been growing since the 1990s, Mie's interpretation of quantum mechanics still waits to be recovered from the dustbin of history.

5 Conclusion

The original motivation for the present remarks was the question why the field equations of the modified Born-Infeld model had not been researched earlier within the framework of Mie's electrodynamics in spite of several shared features. Initially, the present author assumed that the lack of computing power in the early 20th century was the primary reason among several others. However, an informal review of the literature about Mie's electrodynamics revealed multiple contributing factors that might have been of similar importance. In particular, the example of the development of QCD suggests that the lack of computing power was not a sufficient reason for the lack of success of Mie's electrodynamics. The relative importance of the contributing factors discussed in Sections 3 and 4 remains unclear—not least because this might have been a case where "a chain is no stronger than its weakest link," i.e., the "weakest" factor might have been the most "important" one.

Furthermore, the informal literature review has provided additional insights: Born's publication about Mie's electrodynamics [Bor14] and the similarities between Born's nonlinear field theory and Mie's electrodynamics suggest that the latter had influenced Born's work about twenty years later more deeply than a superficial reading of some of Born's publications suggests [Bor33, BIF34]. (Whether Born was fully aware of this influence remains unclear.)

Moreover, Mie's interpretation of quantum mechanics [Mie28] was new to the present author and (if accurate) might provide a way to link the modified Born-Infeld model of electrons to quantum theories.

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A Revisions

- Original version submitted to vixra.org on October 28, 2025.