The Novel Whole Number Classification and the 3:2 Ratio

The four whole number subsets

Jean-Yves Boulay

National Education - FRANCE
jean-yvesboulay@orange.fr, https://www.researchgate.net/profile/Jean-Yves-Boulay .
ORCID: 0000-0001-5636-2375

Abstract: *Grounded in a novel mathematical framework, this study partitions the set of whole numbers ($\mathbb{N}_0$) into four distinct hierarchical classes. A key innovation is the definition of Ultimate Numbers—the union of the prime numbers with zero and one—which resolves classic conceptual limitations. Three further subsets, representing increasing degrees of numerical complexity, are subsequently defined by the initial distinction between ultimate and non-ultimate numbers within $\mathbb{N}_0$. The structural interaction among these four classes yields unique arithmetic arrangements in their initial distribution, most notably revealing an exact and recurring 3:2 ratio.*

1. Introduction

The classical theory of numbers rests on well-established foundations, notably the distinction between prime and composite numbers, as well as the notion of divisibility. However, these definitions, though functional, present conceptual limitations when applied to the extremities of the set of whole numbers, particularly in the cases of zero and one. These two numbers, often treated as exceptions or marginal cases, reveal implicit inconsistencies in the way divisibility is traditionally formulated.

In this study, we propose a radical reformulation of the structure of whole numbers through the introduction of two fundamental concepts: the ultimate number and the ultimate divisor. These notions are based on an innovative approach to divisibility, which integrates the relative value of the divisors that is, their position below or above the number in question. This perspective allows for the redefinition of the properties of all whole numbers, including those that elude classical classifications.

Building upon these definitions, we construct a new algebraic structure, termed the Ultimate Algebra, which organizes the whole numbers into four distinct categories based on their capacity to be decomposed into ultimate divisors: ultimate numbers, elevated numbers (raised), pure Composites, and mixed Composites. This classification excludes trivial forms of decomposition (such as ($n=n\times 1$)) and prioritizes meaningful representations founded on the internal structure of numbers.
The objective of this approach is twofold:

- To offer an alternative, more coherent, and conceptually robust vision of arithmetic.
- To rehabilitate marginal whole numbers 0 and 1 by assigning them a structured and logical place within the system.

This work is part of a process that is both mathematical and philosophical, aiming to question the very foundations of numeration and to propose a new grammar for contemplating numbers.

The concept of number ultimity was previously introduced in the article, *"The ultimate numbers and the 3/2 ratio, just two primary sets of whole number"* [1]. That work presented several unique arithmetic phenomena related to two primary sets: ultimate numbers and non-ultimate numbers. This new article provides a more comprehensive description of how the set $\mathbb{N}_0$ (of whole numbers) can be organized into subsets that possess both distinct and unique arithmetic properties, yet are simultaneously interactive and inclusive.

Building upon the concept of the ultimate number, we propose a novel classification of all whole numbers into four distinct sets. Accordingly, every whole number is uniquely classified into one of these four subsets of $\mathbb{N}_0$, based on the level of complexity of its arithmetic properties. This classification also unambiguously incorporates the sometimes-exotic numbers 0 (zero) and 1 (one).

This new classification also introduces several novel mathematical concepts: fundamental numbers, primordial numbers, and the extreme or median class of numbers. These concepts are extensively demonstrated through the study of numerous closed matrices, often of a size related to 5*x* entities. These matrices are consistently constructed using initial sequences of numbers from the set $\mathbb{N}_0$, as the singular arithmetic phenomena concerning the distribution of the number types are revealed in this set's initial constitution.

Based on multiple approaches and demonstrations, it is evident that within its initial segment, the various identified components of the set $\mathbb{N}_0$ are uniquely organized according to a 3:2 ratio (or its inverse) as a reciprocal and/or transcendent magnitude. These transcendences frequently integrate the concepts of remarkable identity and triangular numbers.

Finally, a glossary is included at the end of the article, listing the new mathematical concepts introduced in this paper.

In statements, when this is not specified, the term "number" always means "whole number". It is therefore agreed that the number zero (0) is well integrated into the set of whole numbers ($\mathbb{N}_0$) .

2. Depiction of the ultimity concept

We introduce the concept of ultimity possessed by a whole number using a definition that is unambiguous and self-evident. By first recalling the notion of a prime number, we then present a compact and absolute definition that divides the set $\mathbb{N}$ into two primary subsets. A subsequent explanation, utilizing the initial sequence of numbers as an example, will clarify this innovative concept, which, notably, is characterized by its strong simplicity.

2.1 Prime number definition

In mathematical literature the definition of primes looks like this:

> *“Prime numbers are numbers greater than 1. They only have two factors, 1 and the number itself. This means these numbers cannot be divided by any number other than 1 and the number itself without leaving a remainder.”*

This is often supplemented by:

> *“Numbers that have more than 2 factors are known as composite numbers.”*

These definitions, which are intended to describe and classify the whole numbers, immediately encounter two major ambiguities because they are complicated by the singular status of the numbers zero (0) and one (1).

2.2 New definition approach

The conventional definition used to determine whether a number is prime does not explicitly specify that its divisors must be inferior in value. While this criterion may appear obvious or trivial for most numbers, this notion is of paramount importance when considering the specific numbers zero (0) and one (1). Indeed, zero, the first whole number, possesses numerous divisors, but all of these divisors are **superior to it** in value. Similarly, the number one, the second whole number, has no divisor inferior to it, given that division by zero (the only whole number inferior to one) is undefined. This novel approach to the concept of divisibility, which incorporates the relative value of divisors (i.e., the notions of inferiority and superiority), makes it possible to create two unique sets to which all whole numbers can be uniquely referenced.

2.3 Absolute definitions

Considering the set of all whole numbers ($\mathbb{N}_0$), these are organized into two primary sets: ultimate numbers and non-ultimate numbers.

Ultimate number definition:

An ultimate number admits at most one divisor being inferior to it in value.

Non-ultimate number definition:

A non-ultimate number admits more than one divisor being inferior to it in value.

2.4 Conventional designations

Just as the term "primes" designates prime numbers, we agree that the appellations "Ultimates" and "non-Ultimates" will henceforth designate the ultimate and non-ultimate numbers, respectively. Consequently, the core concept introduced here is referred to as the ultimity of whole numbers. Furthermore, transcendent conventional appellations will be applied based on these two newly defined classes of whole numbers, particularly upon the introduction of the four comprehensive subsets. As noted previously, a glossary at the end of this paper lists all the main number concepts introduced concerning the whole number set.

2.5 Expended definitions

Let *n* be a whole number (belonging to $\mathbb{N}_0$), this one is Ultimate if **at most one divisor being inferior to it in value** divides it.

Let *n* be a whole number (belonging to $\mathbb{N}_0$), this one is non-Ultimate if **more than one divisor being inferior to it in value** divides it.

2.6 Abbreviated definition

It is therefore possible to classify all whole numbers very clearly and unequivocally according to the concept of ultimity. Figure 1 summarizes the process for identifying any whole number, which can belong exclusively to either the ultimate or the non-ultimate set.

Let *n* be a whole number

↙ ↘

if this one admits
at most one divisor
being inferior to it…

if this one admits
more than one divisor
being inferior to it…

↙ ↘

this one is an Ultimate

this one is a non-Ultimate

Figure 1: Process of identifying any whole number according to ultimity concept.

This ultimity or non-ultimity identification mechanism is universal for all the sequence of whole numbers starting with the number zero.

3. The four classes of whole numbers

The segregation of whole numbers into two entities, designated as Ultimate and non-Ultimate, represents only a preliminary step in the investigation of these numbers. This paper now provides a further exploration of the entire set, revealing its organization into four distinct subsets of mathematical entities, each possessing its own unique yet interactive properties.

3.1 Four unlike types of whole numbers

Building upon the definition of ultimate numbers introduced above, it's possible to differentiate the whole number set into four final classes. These classes are inferred from the two primary classes and are defined progressively based on the established criteria.

Whole numbers are subdivided into these two categories:

- **Ultimates**: an ultimate number admits at most one divisor being inferior to it in value.

- **non-Ultimates**: a non-ultimate number admits more than one divisor being inferior to it in value.

Non-Ultimate numbers are subdivided into these two categories:

- **Raiseds**: a raised number is a non-ultimate number, power* of an ultimate number.

- **Composites**: a composite number is a non-ultimate and non-raised number.

Composite numbers are subdivided into these two categories:

- **pure Composites**: a pure composite number is a non-ultimate and non-raised number not admitting raised number as divisor.

- **mixed Composites**: a mixed composite number is a non-ultimate and non-raised number admitting at least one raised number as divisor.

*****It is implied an integral power and greater than 1.

3.1 Degree of complexity of number classes

The table in Figure 2 summarizes these different definitions. It is more fully developed in Figure 8 in Chapter 6 where the interactions of the four classes of whole numbers are highlighted.

<table>
<tr><td colspan="4">**The whole numbers:**</td></tr>
<tr><td rowspan="3">**Ultimates:**
an ultimate number admits at most one divisor being inferior to it in value</td><td colspan="3">**non-Ultimates:**
a non-ultimate number admits more than one divisor being inferior to it in value</td></tr>
<tr><td rowspan="2">**Raiseds:**
a raised number is a non-ultimate number, power of an ultimate number</td><td colspan="2">**Composites:**
a composite number is a non-ultimate and non-raised number</td></tr>
<tr><td>**pure Composites:**
a pure composite number is a non-ultimate and non-raised number not admitting raised number as divisor</td><td>**mixed Composites:**
a mixed composite number is a non-ultimate and non-raised number admitting at least one raised number as divisor</td></tr>
<tr><td>**level 1**</td><td>**level 2**</td><td>**level 3**</td><td>**level 4**</td></tr>
<tr><td colspan="4">degree of complexity of the final four classes of numbers</td></tr>
</table>

Figure 2: Classification of whole numbers from the definition of ultimate numbers (see also Figures 5 and 7).

4. Novel whole number classification

We will now propose a clear differentiation of all components of the set $\mathbb{N}_0$ into four well-defined number classes. Crucially, this classification unequivocally ranks the exotic numbers zero (0) and one (1), thereby resolving the ambiguities previously discussed.

4.1 The four subsets of whole numbers

By the previous definitions and demonstrations, we propose the classification of the set of whole numbers into four subsets or classes of numbers:

- the ultimate numbers called ***Ultimates*** (***u***),
- the raised numbers called ***Raiseds*** (***r***),
- the pure composite numbers called ***Composites*** (***c***),
- the mixed composite numbers called ***Mixes*** (***m***).

4.1.1 Conventional denominations

We will therefore use the designation "Ultimates" to refer to ultimate numbers (just as "primes" refers to prime numbers). Similarly, the appellations "Raiseds," "Composites," and "Mixes" will designate raised numbers, pure composite numbers, and mixed composite numbers, respectively. Furthermore, we establish the following variable conventions:

- ***u*** will denote an ultimate number
- ***r*** will denote a raised number
- ***c*** will denote a pure composite number
- ***m*** will denote a mixed composite number.

4.2 Organization charts of whole numbers

This new classification of whole numbers necessitates further illustrations of the organization of the set $\mathbb{N}_0$.

4.2.1 Hierarchical organization chart

The set ℕ₀ can thus be described through a hierarchical organization of its components. At the final level of this hierarchy are the four new number classes introduced earlier. Figure 3 illustrates this organization.

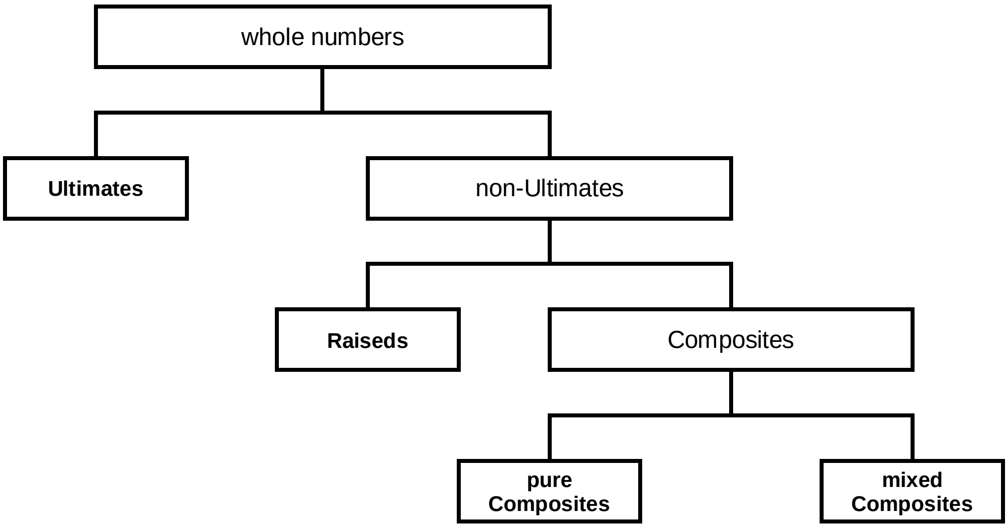


Figure 3: Hierarchical classification of whole numbers since the definition of ultimate numbers.

4.2.2 Inclusive diagram

As illustrated in Figure 4, an inclusive structure is revealed in the organization of the set ℕ₀.

Specifically, the set of whole numbers contains the Ultimates and the non-Ultimates. In turn, the set of non-Ultimates contains the Raiseds and the Composites. Finally, this latter set (the Composites) contains both the pure Composites and the mixed Composites.

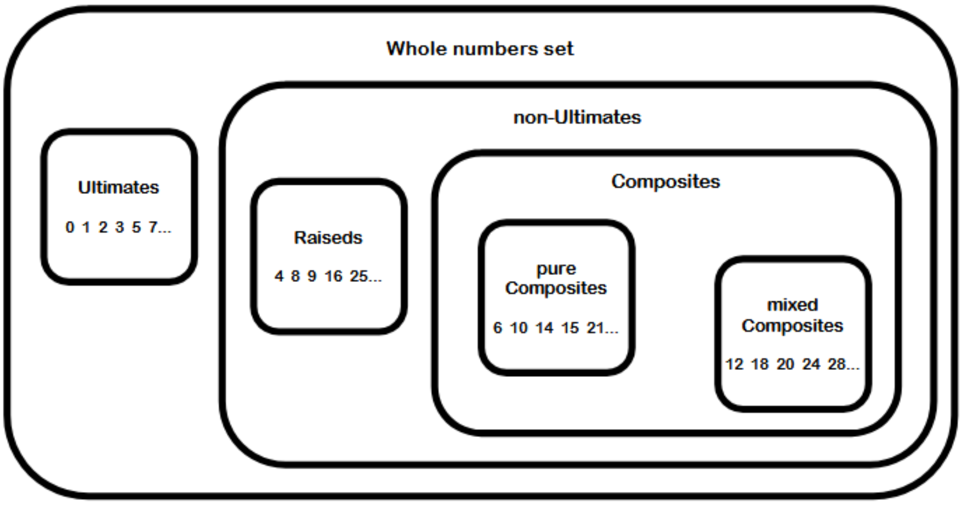


Figure 4: Inclusive (Euler's) diagram of the classification of whole numbers (See also Figure 75 in Chapter15).

Conversely, we can thus conclude that the set of mixed Composites is included within the set of Composites, which is, in turn, included in the set of non-Ultimates, itself included in the set of whole numbers (ℕ₀). The set of pure Composites follows this same chain of inclusions. The set of Raiseds is included in the set of non-Ultimates, which is then included in the set of whole numbers. Finally, the set of Ultimates is included only in the set of whole numbers. Figure 5 summarizes this inclusive organization of the set of whole numbers. In Chapter 15, we will illustrate the depth of inclusion of the first thirty whole numbers where singular arrangements operate.

whole numbers set **($\mathbb{N}_0$)**	$\supseteq$	**Ultimates set**	$\supseteq$	**Raiseds set**	$\supseteq$	**pure Composites set**
		non-Ultimates set		**Composites set**		**mixed Composites set**

Figure 5: Inclusion of the seven sets of numbers constituting the set of whole numbers.

5. Ultimate divisor

The classification of whole numbers into different sets, as deduced from the definition of ultimate numbers, leads to the proposal of the twin concepts of *the ultimate divisor* and *ultimate algebra*.

5.1 Definition of the ultimate divisor

An ultimate divisor of a whole number n is an ultimate number u such that $u < n$ and u is a non-trivial divisor of n.

For example, the number 12 has six divisors (1, 2, 3, 4, 6, and 12), but only two ultimate divisors: 2 and 3. It should also be noted that the numbers zero (0) and one (1), while defined as ultimate numbers themselves, are never ultimate divisors. The division by zero (0) is undefined, and therefore this number cannot be an ultimate divisor. The number one (1) is a trivial divisor because it does not partition a number into some smaller part.

5.2 Concept of ultimate algebra

Ultimate Algebra applies exclusively to the set of whole numbers. Its structure is defined by two key elements: the previously introduced ultimate divisor and the previously introduced ultimate number.

This algebra posits that every whole number n belongs to one of two categories:

- An ultimate number (which has no ultimate divisor).

- A non-ultimate number (which may be Raised, pure Composite, or mixed Composite) that can be decomposed into several ultimate divisors.

In this algebra, a whole number n cannot be expressed in the trivial form $n = n \times 1$. Instead, it must be written in one of the following forms:

- Ultimate: $n = n$

- Raised: $n = u_1 \times u_1 \times \ldots$

- Composite: $n = u_1 \times u_2 \times \ldots$ (where $u_1 \neq u_2 \neq \ldots$)

- Mix: $n = (u_1 \times u_1 \ldots) \times u_2 \times \ldots$

Where u_i are ultimate divisors.

Furthermore, for the number zero, it is not permissible to write for example $0 = 0 \times u_1 \times u_2 \times \ldots$; the only valid representation is $0 = 0$.

5.3 Ultimate divisors and number classes

The table in Figure 6 synthesizes the four interactive definitions of the four classes of whole numbers by incorporating the double concept of ultimate divisor and ultimate algebra.

An ultimate number (***u***) admits at most one divisor being inferior to it in value.
Can be written in the form:

$$u = u$$

→ without
ultimate divisors

A raised number (***r***) is a non-ultimate number, power of an ultimate number.
Can be written in the form:

$$r = u_1 \times u_1$$

→ only with
identical ultimate divisors

$$\begin{tikzcd} u \arrow[r] \arrow[d] & r \arrow[d] \\ c \arrow[r] & m \end{tikzcd}$$

A pure composite number (***c***) is a non-ultimate and not raised number admitting no raised number as divisor.
Can be written in the form:

$$c = u_1 \times u_2$$

→ only with different ultimate divisors

A mixed composite number (***m***) is a non-ultimate and not raised number admitting at least one raised number as divisor
Can be written in the form:

$$m = u_1 \times u_1 \times u_2 = r \times u_2 = c \times u_1$$

→ simultaneously with
identical ultimate divisors and with
different ultimate divisors

Figure 6: Interactions of the four classes of whole numbers (See also Figures 2, 3 and 7).

In contrast to the hierarchical or inclusive organization of the different sets of whole numbers (as illustrated in Figures 2 and 3), the four final natures of numbers also exhibit a linear and semi-circular interaction. As shown in Figure 7, it is therefore possible to contrast the two classes of Ultimates (*u*) and Mixes (*m*) with the two classes of Raiseds (*r*) and Composites (*c*). These two groupings can be respectively designated as the "extreme" and "median" classes.

extreme **nature class**
of Ultimates (*u*)

$$u = u$$

median **nature class**
of Raiseds (*r*)

$$r = u_1 \times u_1$$

$$\begin{tikzcd} u \arrow[r] \arrow[d] & r \arrow[d] \\ c \arrow[r] & m \end{tikzcd}$$

$$c = u_1 \times u_2$$

median **nature class**
of Composites (*c*)

$$m = u_1 \times u_1 \times u_2 = r \times u_2 = c \times u_1$$

extreme **nature class**
of Mixes (*m*)

Figure 7: Nature and interactions of the four classes of whole numbers (See also Figure 6 also).

A critical remark: The class compositions of numbers detailed in the tables of Figures 6 and 7 represent only a minimum (or simplest) form of writing. For example, the composite class (*c*) can also be represented as the product of more than two ultimate numbers, such as $c = u_1 \times u_2 \times u_3$. Similarly, the mixed class (*m*) can be expressed with higher exponents for its components, for instance, $m = u_1 \times u_1 \times u_2 \times u_2$.

5.4 Specific features of the numbers zero and one

Based on the postulates that define this concept of ultimate algebra, we establish and reiterate the following: Although the numbers zero (0) and one (1) are defined as ultimate numbers, they are neither ultimate divisors nor are they composed of ultimate divisors. Consequently, despite being defined by distinct concepts, the set of ultimate divisors and the set of prime numbers are, for practical purposes, identical (or coincident).

→ The concept of the ultimate divisor is further developed in the Appendix, specifically within the study of the matrix of the first hundred numbers.

5.4.1 Prime number definition

Drawing upon the previously introduced definitions of the ultimate number and the ultimate divisor, we propose the following compact definition of prime numbers:

A prime number is an ultimate number that can serve as an ultimate divisor of a whole number.

6. The forty primordial numbers

The new classification of whole numbers into four subsets gives rise to singular arithmetic phenomena related to the initial distribution of these sets. These phenomena manifest as varied and often transcendent ratios, most notably the exact values of 3:2 (and its reciprocal, 2:3). The prime and initial organization of the set of whole numbers in this context highlights forty numbers (four times ten numbers) that we will refer to as "Primordials."

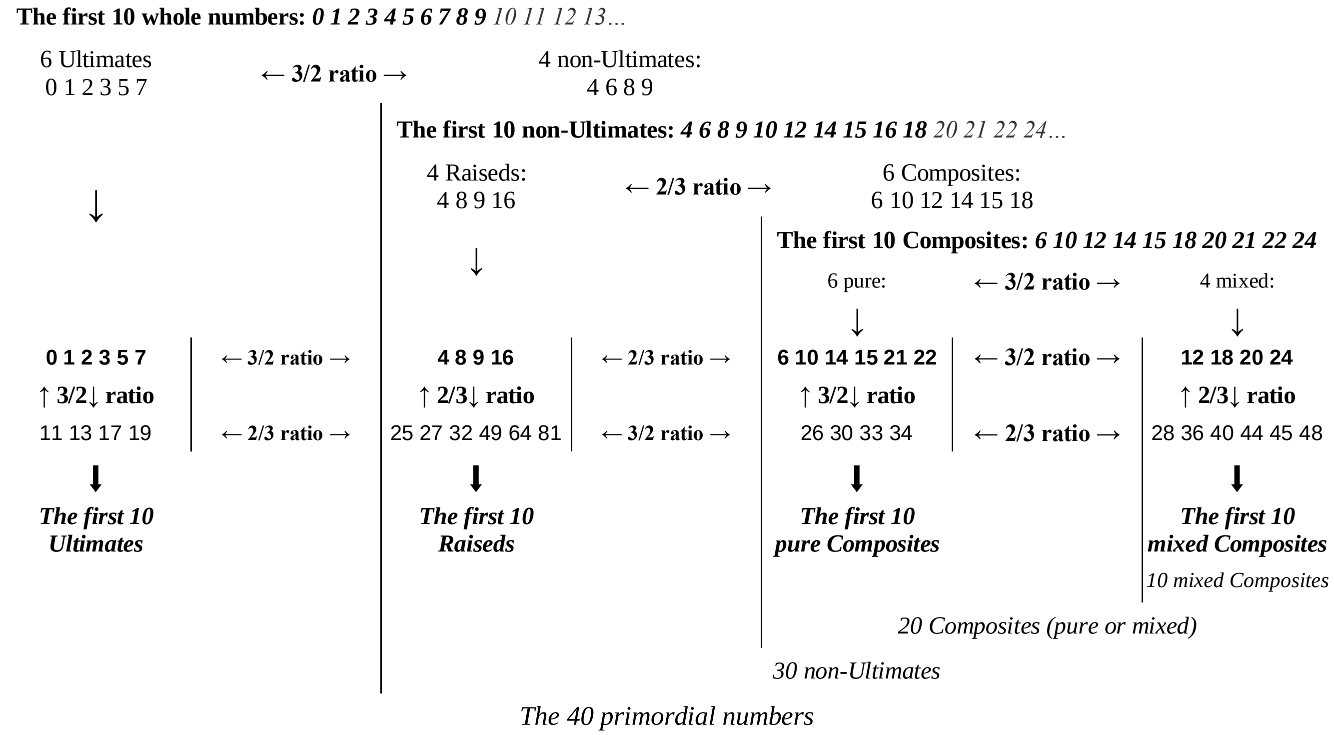


Figure 8: From the first ten numbers of the three source classes of whole numbers, generation inside 3:2 ratios of the first ten numbers of each of the four final number classes: the 40 Primordials (See Figure 2, also Figures 6 and 9).

6.1 Whole number classes and the 3:2 ratio

The progressive differentiation between the source classes and final classes of whole numbers is organized (as shown in Figure 8) into a powerful arithmetic arrangement that consistently generates transcendent 3:2 ratios. This phenomenon is observed across the classification:

The source set of whole numbers includes 6 ultimate numbers versus 4 non-ultimate numbers among its first ten members.

The next source set, that of the non-Ultimates, contains 4 raised numbers versus 6 composite numbers among its first ten members.

Finally, the source set of Composites includes 6 pure Composites versus 4 mixed Composites among its first ten members.

A strong correlation links all these number sets, which oppose one another repeatedly in 3:2 ratios (or the reciprocal 2:3 ratios). For instance, the first 6 Ultimates (0,1,2,3,5,7) are simultaneously opposed:

to the 4 non-Ultimates (4,6,8,9) among the first 10 whole numbers;

to the 4 raised numbers of the first 10 non-Ultimates (4,8,9,16);

and to the following 4 Ultimates (11,13,17,19) among the first 10 ultimate numbers.

6.2 The forty primordial numbers

This entangled classification of whole numbers allows us to define (as detailed in Figures 8 and 9) a set of forty primordial numbers. These forty primordial numbers are defined as the combined set of the first ten numbers from each of the four final classes of whole numbers. Throughout this study, the term "Primordials" will be used to specifically designate this set of forty numbers.

Thus, by convention, these forty numbers qualified as primordial and called “Primordials” are:

- 0-1-2-3-5-7-11-13-17-19 → the first ten Ultimates,
- 4-8-9-16-25-27-32-49-64-81 → the first ten Raiseds,
- 6-10-14-15-21-22-26-30-33-34 → the first ten Composites,
- 12-18-20-24-28-36-40-44-45-48 → the first ten Mixes.

6.3 Primordial numbers and the 3:2 ratio

As illustrated in Figure 8 (and from an alternative perspective in Figure 9), these four sets of ten primordial numbers are all composed of subgroups of six and four entities. This division is consistent with their respective initial formation. Furthermore, due to this initial structure, a 3:2 ratio (or its reciprocal, 2:3) consistently exists between subgroups at adjacent levels of complexity (refer to Figure 2).

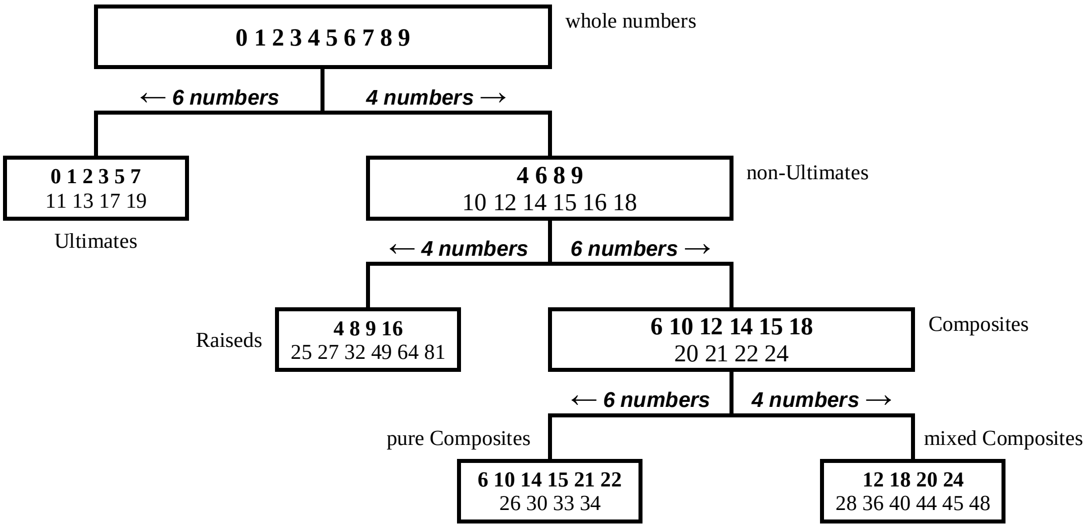


Figure 9: Initial arithmetic arrangements in 3:2 ratios inside hierarchical classification of whole numbers (See also Figures 2 and 8).

6.4 Two sets of primordial numbers

Based on their appearance order in the four final subsets and the origin of their respective source sets (as shown in Figures 8 and 9), the forty primordial numbers can be divided into two distinct groups:

The Primary Primordials (20 members)

The Secondary Primordials (20 members)

As Figure 10 illustrates, these two groups each comprise 20 members: 20 primary Primordials and 20 secondary Primordials. This figure summarizes the initial distribution of the first ten numbers from each of the four whole number categories. In this table, the 3:2 ratio (or its reciprocal, 2:3) is particularly prominent. This consistent appearance order demonstrates a very powerful arithmetic organization governing the initialization of the four whole number subsets we have defined and unequivocally identified.

	first *10 Ultimates*		*first* *10 Raiseds*		*first* *10 Composites*		*first* *10 Mixes*
20 primo Primordials ➡	**6 Ultimates**	← 3/2 →	**4 Raiseds**	← 2/3 →	**6 Composites**	← 3/2 →	**4 Mixes**
	0-1-2-3-5-7		**4-8-9-16**		**6-10-14-15-21-22**		**12-18-20-24**
	↑ 3/2 ↓		↑ 2/3 ↓		↑ 3/2 ↓		↑ 2/3 ↓
20 secondary Primordials ➡	11-13-17-19		25-27-32-49-64-81		26-30-33-34		28-36-40-44-45-48
	4 Ultimates	← 2/3 →	6 Raiseds	← 3/2 →	4 Composites	← 2/3 →	6 Mixes

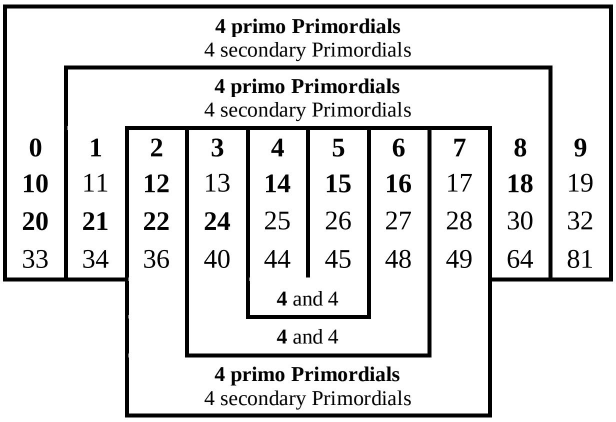
Figure 10: Distinction of primo Primordials and secondary Primordials in the 4 final subsets of whole numbers (See also Figures 8 and 9).

7. Matrix of the Forty Primordials

The magnitude ranking of the forty primordials, organized within a 4 × 10 matrix, shows a non-random distribution when distinguished into primo Primordials and secondary Primordials, as previously defined.

7.1 Symmetrical arrangements

As illustrated in Figure 11, the two types of numbers (primo and secondary Primordials) are always distributed in equal quantities within this matrix of 4 rows by 10 columns. Specifically, this equality is observed in each of the five zones formed by two symmetrically opposite columns. Each of these five zones contains 4 primo Primordials and 4 secondary Primordials.



0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
10	11	**12**	13	**14**	**15**	**16**	17	**18**	19
20	**21**	**22**	**24**	25	26	27	28	30	32
33	34	36	40	44	45	48	49	64	81

Figure 11: Symmetrical distribution of **primo Primordials** and secondary Primordials in the matrix of the Forty Primordials (See also Figure 10).

The specific distribution of **primo Primordials** and secondary Primordials within the Forty Primordials matrix gives rise to numerous singular arithmetic arrangements. As illustrated in Figure 12, these arrangements notably include oppositions characterized by 3:2 or 2:3 ratios, depending on the geometric configuration considered.

12 primo Primordials ↑ 3/2 ↓ 8 secondary Primordials	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**8 primo Primordials** ↑ 2/3 ↓ 12 secondary Primordials
	10	11	**12**	13	**14**	**15**	**16**	17	**18**	19	
	20	**21**	**22**	**24**	25	26	27	28	30	32	
	33	34	36	40	44	45	48	49	64	81	

Figure 12: Distribution of **primo Primordials** and secondary Primordials in the matrix of the Forty Primordials (See also Figures 10 and 11).

Figure 12 provides only an outline of the highly sophisticated entanglement in the organization of the twenty numbers (qualified as primo and secondary Primordials) within the previously defined Forty Primordials matrix.

8 primo Primordials ↑ 2/3 ↓ 12 secondary Primordials	**0**	**1**	**2**	**3**	**4**	***5***	***6***	***7***	***8***	***9***	***0***	***1***	***2***	***3***	***4***	**5**	**6**	**7**	**8**	**9**	**12 primo Primordials** ↑ 3/2 ↓ 8 secondary Primordials
	10	11	**12**	13	**14**	***15***	***16***	*17*	***18***	*19*	***10***	*11*	***12***	*13*	***14***	**15**	**16**	17	**18**	19	
	20	***21***	***22***	***24***	*25*	26	27	28	30	32	**20**	**21**	**22**	**24**	25	*26*	*27*	*28*	*30*	*32*	
	33	*34*	*36*	*40*	*44*	45	48	49	64	81	33	34	36	40	44	*45*	*48*	*49*	*64*	*81*	

Figure 13: Distribution of **primo Primordials** and secondary Primordials in the matrix of the Forty Primordials (See also Figures 10 and 11).

Indeed, as clearly shown in the tables of Figures 13 to 17, within each symmetric sub-matrix (always containing 20 entities), the primo Primordials and secondary Primordials are consistently opposed in a 3:2 or 2:3 ratio. Specifically, this means 12 primo Primordials are opposed to 8 secondary Primordials, or vice versa.

12 primo Primordials ↑ 3/2 ↓ 8 secondary Primordials	**0**	**1**	**2**	**3**	**4**	***5***	***6***	***7***	***8***	***9***	***0***	***1***	***2***	***3***	***4***	**5**	**6**	**7**	**8**	**9**	**8 primo Primordials** ↑ 2/3 ↓ 12 secondary Primordials
	10	*11*	***12***	*13*	***14***	**15**	**16**	17	**18**	19	**10**	11	**12**	13	**14**	***15***	***16***	*17*	***18***	*19*	
	20	**21**	**22**	**24**	25	*26*	*27*	*28*	*30*	*32*	***20***	***21***	***22***	***24***	*25*	26	27	28	30	32	
	33	*34*	*36*	*40*	*44*	45	48	49	64	81	33	34	36	40	44	*45*	*48*	*49*	*64*	*81*	

Figure 14: Distribution of **primo Primordials** and secondary Primordials in the matrix of the Forty Primordials (See also Figures 10, 12 and 13).

So Figure 15, other oppositions of primo and secondary Primordials in 3:2 ratios.

12 primo primordials ↑ 3/2 ↓ 8 secondary primordials	**0**	***1***	**2**	***3***	**4**	***5***	**6**	***7***	**8**	***9***	***0***	**1**	***2***	**3**	***4***	**5**	***6***	**7**	***8***	**9**	**8 primo primordials** ↑ 2/3 ↓ 12 secondary primordials
	10	*11*	**12**	*13*	**14**	***15***	**16**	*17*	**18**	*19*	***10***	11	***12***	13	***14***	**15**	***16***	17	***18***	19	
	20	***21***	**22**	***24***	25	*26*	27	*28*	30	*32*	***20***	**21**	***22***	**24**	*25*	26	*27*	28	*30*	32	
	33	*34*	36	*40*	44	*45*	48	*49*	64	*81*	*33*	34	*36*	40	*44*	45	*48*	49	*64*	81	

Figure 15: Distribution of **primo Primordials** and secondary Primordials in the matrix of the Forty Primordials (See also Figures 10 and 11).

So Figure 16, other oppositions of primo and secondary Primordials in 3:2 ratios.

12 primo primordials ↑ 3/2 ↓ 8 secondary primordials	**0**	***1***	**2**	***3***	**4**	***5***	**6**	***7***	**8**	***9***	***0***	**1**	***2***	**3**	***4***	**5**	***6***	**7**	***8***	**9**	**8 primo primordials** ↑ 2/3 ↓ 12 secondary primordials
	10	*11*	**12**	*13*	**14**	***15***	**16**	*17*	**18**	*19*	***10***	11	***12***	13	***14***	**15**	***16***	17	***18***	19	
	20	**21**	***22***	**24**	*25*	26	*27*	28	*30*	32	**20**	***21***	**22**	***24***	25	*26*	27	*28*	30	*32*	
	33	34	*36*	40	*44*	45	*48*	49	*64*	81	33	*34*	36	*40*	44	*45*	48	*49*	64	*81*	

Figure 16: Distribution of **Primo primordials** and secondary Primordials in the matrix of the Forty Primordials (See also Figures 10 and 11).

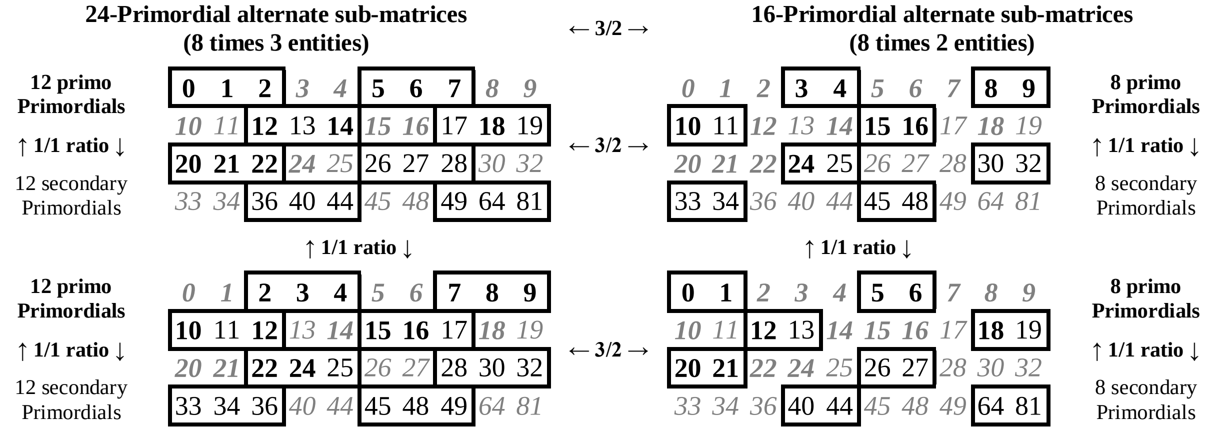
So Figure 17, still other oppositions of primo and secondary Primordials in 3:2 ratios.

12 primo primordials ↑ 3/2 ↓ 8 secondary primordials	**0**	**1**	***2***	***3***	**4**	**5**	***6***	***7***	**8**	**9**	***0***	***1***	**2**	**3**	***4***	***5***	**6**	**7**	***8***	***9***	**8 primo primordials** ↑ 2/3 ↓ 12 secondary primordials
	10	11	***12***	*13*	**14**	**15**	***16***	*17*	**18**	19	***10***	*11*	**12**	13	***14***	***15***	**16**	17	***18***	*19*	
	20	***21***	**22**	**24**	*25*	*26*	27	28	*30*	*32*	**20**	**21**	***22***	***24***	25	26	27	28	30	32	
	33	*34*	36	40	*44*	*45*	48	49	*64*	*81*	33	34	*36*	*40*	44	45	48	49	64	81	

Figure 17: Distribution of **primo Primordials** and secondary Primordials in the matrix of the Forty Primordials (See also Figures 10 and 11).

7.2 Dissymmetrical arrangements

Other unique arithmetic arrangements are revealed within the matrix of the Forty Primordials, again based on the distinction between the twenty primo Primordials and the twenty secondary Primordials. Figure 18 illustrates this: the alternating and dissymmetrical isolation of 3 and 2 numbers in each of the four rows allows for the creation of four sub-matrices. These sub-matrices consistently show oppositions in a 3:2 or 1:1 value ratio, depending on the configurations considered and the nature (primary or secondary) of the forty previously defined primordial numbers.



24-Primordial alternate sub-matrices (8 times 3 entities) ← 3/2 → **16-Primordial alternate sub-matrices (8 times 2 entities)**

12 primo Primordials ↑ **1/1 ratio** ↓ 12 secondary Primordials ← **3/2** → **8 primo Primordials** ↑ **1/1 ratio** ↓ 8 secondary Primordials

0	**1**	**2**	***3***	***4***	**5**	**6**	**7**	***8***	***9***
10	*11*	**12**	13	**14**	***15***	***16***	17	**18**	19
20	**21**	**22**	***24***	*25*	26	27	28	*30*	*32*
33	*34*	36	40	44	*45*	*48*	49	64	81

0	***1***	***2***	**3**	**4**	***5***	***6***	***7***	**8**	**9**
10	11	***12***	*13*	***14***	**15**	**16**	*17*	***18***	*19*
20	***21***	***22***	**24**	25	*26*	*27*	*28*	30	32
33	34	*36*	*40*	*44*	45	48	*49*	*64*	*81*

↑ **1/1 ratio** ↓ ↑ **1/1 ratio** ↓

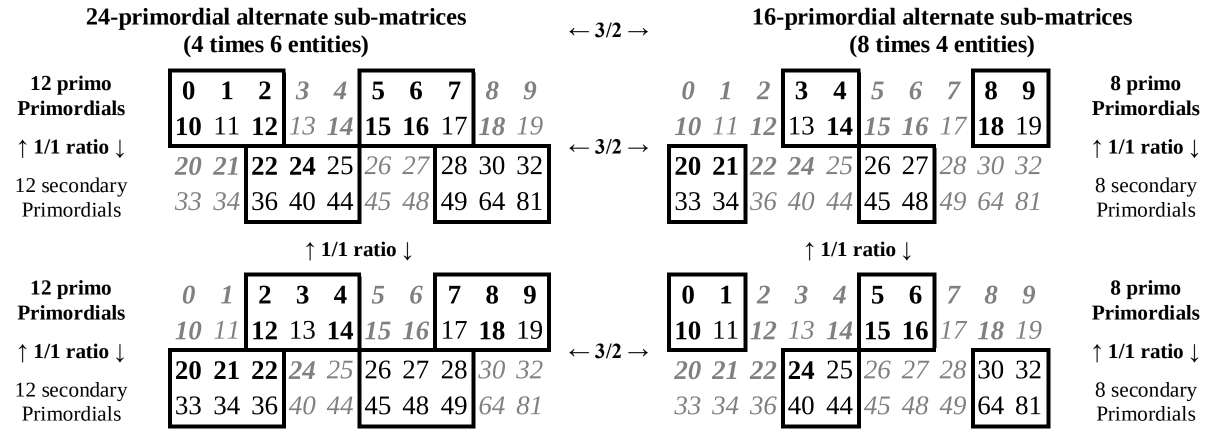
12 primo Primordials ↑ **1/1 ratio** ↓ 12 secondary Primordials ← **3/2** → **8 primo Primordials** ↑ **1/1 ratio** ↓ 8 secondary Primordials

0	***1***	**2**	**3**	**4**	***5***	***6***	**7**	**8**	**9**
10	11	**12**	*13*	***14***	**15**	**16**	17	***18***	*19*
20	***21***	**22**	**24**	25	*26*	*27*	28	30	32
33	34	36	*40*	*44*	45	48	49	*64*	*81*

0	**1**	***2***	***3***	***4***	**5**	**6**	***7***	***8***	***9***
10	*11*	**12**	13	***14***	***15***	***16***	*17*	**18**	19
20	**21**	***22***	***24***	*25*	26	27	*28*	*30*	*32*
33	*34*	*36*	40	44	*45*	*48*	*49*	64	81

Figure 18: Distribution in 3:2 and 1:1 ratios of **primo Primordials** and secondary Primordials in alternative sub-matrices of the Forty Primordials (See also Figure 10).

Figure 19 further demonstrates the same phenomena in the constitution of four additional sub-matrices, where the 3:2 alternation of entities is applied across two rows at a time.



24-primordial alternate sub-matrices (4 times 6 entities) ← 3/2 → **16-primordial alternate sub-matrices (8 times 4 entities)**

12 primo Primordials ↑ **1/1 ratio** ↓ 12 secondary Primordials ← **3/2** → **8 primo Primordials** ↑ **1/1 ratio** ↓ 8 secondary Primordials

0	**1**	**2**	***3***	***4***	**5**	**6**	**7**	***8***	***9***
10	11	**12**	*13*	***14***	**15**	**16**	17	***18***	*19*
20	***21***	**22**	**24**	25	*26*	*27*	28	30	32
33	*34*	36	40	44	*45*	*48*	49	64	81

0	***1***	***2***	**3**	**4**	***5***	***6***	***7***	**8**	**9**
10	*11*	***12***	13	**14**	***15***	***16***	*17*	**18**	19
20	**21**	***22***	***24***	*25*	26	27	*28*	*30*	*32*
33	34	*36*	*40*	*44*	45	48	*49*	*64*	*81*

↑ **1/1 ratio** ↓ ↑ **1/1 ratio** ↓

12 primo Primordials ↑ **1/1 ratio** ↓ 12 secondary Primordials ← **3/2** → **8 primo Primordials** ↑ **1/1 ratio** ↓ 8 secondary Primordials

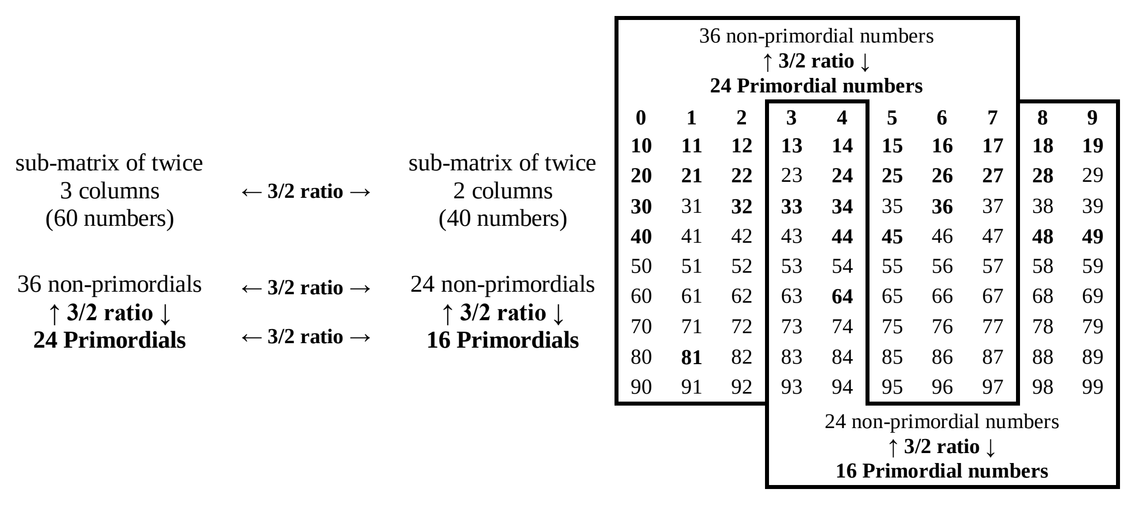
0	***1***	**2**	**3**	**4**	***5***	***6***	**7**	**8**	**9**
10	*11*	**12**	13	**14**	***15***	***16***	17	**18**	19
20	**21**	**22**	***24***	*25*	26	27	28	*30*	*32*
33	34	36	*40*	*44*	45	48	49	*64*	*81*

0	**1**	***2***	***3***	***4***	**5**	**6**	***7***	***8***	***9***
10	11	***12***	*13*	***14***	**15**	**16**	*17*	***18***	*19*
20	***21***	***22***	**24**	25	*26*	*27*	*28*	30	32
33	*34*	*36*	40	44	*45*	*48*	*49*	64	81

Figure 19: Distribution in 3:2 and 1:1 ratios of **primo Primordials** and secondary Primordials in alternative sub-matrices of the Forty Primordials (See also Figure 10).

8. Matrix of the first hundred numbers and primordial numbers

Therefore, within the matrix of the first 100 whole numbers, the 40 Primordials previously defined in Chapter 6.2 are opposed by 60 non-primordial numbers, establishing a 3:2 ratio.



sub-matrix of twice 3 columns (60 numbers) ← **3/2 ratio** → sub-matrix of twice 2 columns (40 numbers)

36 non-primordials ← **3/2 ratio** → 24 non-primordials

↑ **3/2 ratio** ↓ ↑ **3/2 ratio** ↓

24 Primordials ← **3/2 ratio** → **16 Primordials**

36 non-primordial numbers
↑ **3/2 ratio** ↓
24 Primordial numbers

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
20	**21**	**22**	23	**24**	**25**	**26**	**27**	**28**	29
30	31	**32**	**33**	**34**	35	**36**	37	38	39
40	41	42	43	**44**	**45**	46	47	**48**	**49**
50	51	52	53	54	55	56	57	58	59
60	61	62	63	**64**	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	**81**	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

24 non-primordial numbers
↑ **3/2 ratio** ↓
16 Primordial numbers

Figure 20: Distinction and distribution of the **40 primordial** and 60 non-primordial numbers in the matrix of the first hundred numbers.

In the matrix of the first hundred numbers, the position differentiation of the 40 primordials generates singular phenomena of 3:2 ratio depending on the different areas considered to 60 versus 40 entities or to 50 versus 50 entities.

Thus, in this matrix, it turns out in Figure 20, that the distinction of two sub-matrices of twice 3 columns versus twice 2 columns generates sets of primordial numbers and non-primordial numbers which are opposed in 3:2 transcendent ratios to 36 versus 24 entities and 24 versus 16 entities.

In the matrix of the first hundred numbers, the positional differentiation of the 40 primordials generates unique 3:2 ratio phenomena based on the different areas considered, whether they involve 60 vs. 40 or 50 vs. 50 entities.

Figure 20 illustrates that, within this matrix, the creation of two sub-matrices—each comprising two sets of 3 columns opposed to two sets of 2 columns—generates opposing sets of primordial and non-primordial numbers. These sets are defined by a transcendent 3:2 ratio at both the 36 vs. 24 entities and 24 vs. 16 entities levels.

As Figure 21 illustrates, within the equally sized sub-matrices formed by the alternating upper and lower quarters of the complete matrix of the first hundred numbers, the 60 non-primordial numbers are distributed in equal quantities (two sets of 30). These sets of 30 non-primordials are then opposed in a 3:2 ratio by the 40 Primordials, which are similarly divided into two equal sets of 20 entities.

2 times 25-number Matrix (50 numbers)										← **1/1 ratio** →	**2 times 25-number Matrix** (50 numbers)									
0	**1**	**2**	**3**	**4**												**5**	**6**	**7**	**8**	**9**
10	**11**	**12**	**13**	**14**												**15**	**16**	**17**	**18**	**19**
20	**21**	**22**	23	**24**												**25**	**26**	**27**	**28**	29
30	31	**32**	**33**	**34**												35	**36**	37	38	39
40	41	42	43	**44**												**45**	46	47	**48**	**49**
										← **1/1 ratio** →										
					55	56	57	58	59		50	51	52	53	54					
					65	66	67	68	69		60	61	62	63	**64**					
					75	76	77	78	79		70	71	72	73	74					
					85	86	87	88	89		80	**81**	82	83	84					
					95	96	97	98	99		90	91	92	93	94					
30 non-primordials										← **1/1 ratio** →	30 non-primordials									
↑ **3/2 ratio** ↓											↑ **3/2 ratio** ↓									
20 Primordials										← **1/1 ratio** →	**20 Primordials**									

Figure 21: Equal distribution of the 60 non-primordials numbers and the **40 Primordials** in two sub-matrices of the first hundred numbers.

8.1 Linear sub-matrices of sixty and forty numbers

The sub-matrix of 60 entities is formed by alternately selecting the first six numbers, then the last six numbers, of each of the ten rows in the matrix of the first hundred numbers (as introduced in Figure 20). Within this 60-entity sub-matrix, the non-primordial and primordial numbers are opposed, as shown in the left part of Figure 21, into two sets defined by a 3:2 value ratio. Furthermore, these sets are themselves opposed to the two reciprocal sets of the complementary 40-entity sub-matrix by transcendent 3:2 value ratios.

The same phenomena are observed, both internally and between the two sub-matrices of 60 vs. 40 entities, when the alternation of the numbers considered is applied two rows by two rows, as illustrated in the right part of Figure 22.

Sub-matrix to 10 times **6 numbers** (60 numbers)	← **3/2 ratio** →	**Sub-matrix to 10 times** **4 numbers** (40 numbers)	**Sub-matrix to 5 times** **12 numbers** (60 numbers)	← **3/2 ratio** →	**Sub-matrix to 5 times** **8 numbers** (40 numbers)

Sub-matrix to 10 times 6 numbers (60 numbers)

0	**1**	**2**	**3**	**4**	**5**				
				14	**15**	**16**	**17**	**18**	**19**
20	**21**	**22**	23	**24**	**25**				
				34	35	**36**	37	38	39
40	41	42	43	**44**	**45**				
				54	55	56	57	58	59
60	61	62	63	**64**	65				
				74	75	76	77	78	79
80	**81**	82	83	84	85				
				94	95	96	97	98	99

Sub-matrix to 10 times 4 numbers (40 numbers)

						6	**7**	**8**	**9**
10	**11**	**12**	**13**						
						26	**27**	**28**	29
30	31	**32**	**33**						
						46	47	**48**	**49**
50	51	52	53						
						66	67	68	69
70	71	72	73						
						86	87	88	89
90	91	92	93						

Sub-matrix to 5 times 12 numbers (60 numbers)

0	**1**	**2**	**3**	**4**	**5**				
10	**11**	**12**	**13**	**14**	**15**				
				24	**25**	**26**	**27**	**28**	29
				34	35	**36**	37	38	39
40	41	42	43	**44**	**45**				
50	51	52	53	54	55				
				64	65	66	67	68	69
				74	75	76	77	78	79
80	**81**	82	83	84	85				
90	91	92	93	94	95				

Sub-matrix to 5 times 8 numbers (40 numbers)

						6	**7**	**8**	**9**
						16	**17**	**18**	**19**
20	**21**	**22**	23						
30	31	**32**	**33**						
						46	47	**48**	**49**
						56	57	58	59
60	61	62	63						
70	71	72	73						
						86	87	88	89
						96	97	98	99

36 non-primordials ↑ **3/2 ratio** ↓ **24 Primordials**	← **3/2 ratio** → ← **3/2 ratio** →	24 non-primordials ↑ **3/2 ratio** ↓ **16 Primordials**	36 non-primordials ↑ **3/2 ratio** ↓ **24 Primordials**	← **3/2 ratio** → ← **3/2 ratio** →	24 non-primordials ↑ **3/2 ratio** ↓ **16 Primordials**

Figure 22: According to different alternatively linear sub-matrices: distribution of the 60 non-primordials and the **40 Primordials** in opposing sets 3:2 transcendent ratios.

8.2 Concentric and eccentric sub-matrices

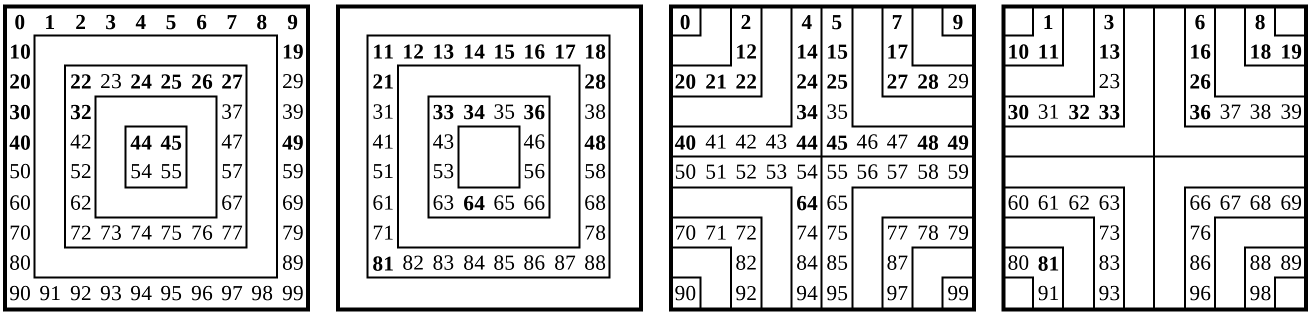
In this matrix of the first hundred numbers, more sophisticated arrangements bring into opposition sets of non-primordial and of primordial numbers in exact 3:2 ratios. Thus, as described in the left part of Figure 23, five concentric zones are opposed, three versus two, in the distribution of their non-primordial and primordial numbers in 3/2 ratios. The same phenomenon is reproduced by considering the five eccentric zones presented in the right part of this Figure 23.

Within this matrix of the first hundred numbers, more sophisticated arrangements place sets of non-primordial and primordial numbers in exact 3:2 ratios of opposition. Specifically, as depicted in the left part of Figure 23, five concentric zones are opposed (three vs. two) based on the distribution of their non-primordial and primordial numbers, resulting in 3:2 ratios. The same phenomenon is reproduced when considering the five eccentric zones presented in the right part of Figure 23.

Concentric configurations: **Eccentric configurations:**

sub-matrix to 3 concentric zones (60 numbers) ← **3/2 ratio** → **sub-matrix to 2 concentric zones** (40 numbers)

sub-matrix to 3 eccentric zones (60 numbers) ← **3/2 ratio** → **sub-matrix to 2 eccentric zones** (40 numbers)



36 non-primordials ↑ **3/2 ratio** ↓ **24 Primordials**	← **3/2 ratio** → ← **3/2 ratio** →	24 non-primordials ↑ **3/2 ratio** ↓ **16 Primordials**	36 non-primordials ↑ **3/2 ratio** ↓ **24 Primordials**	← **3/2 ratio** → ← **3/2 ratio** →	24 non-primordials ↑ **3/2 ratio** ↓ **16 Primordials**

Figure 22: From the matrix of the first hundred numbers, concentric and eccentric configurations of sub-matrices to 60 and 40 entities opposing their non-primordials and their primordials in 3/2 ratios.

Furthermore—using configurations identical to those previously introduced—we can mix the 40- and 60-entity sub-matrices (as presented in Figure 22). After vertically splitting each sub-matrix into two equal parts (30 and 20 entities, respectively), we obtain new 50-entity matrices, as shown in Figure 23. In these mixed configurations, the non-primordials and Primordials are divided into exact 1:1 ratios. Specifically, this involves a division of 30 non-primordials vs. 30 Primordials, and 20 non-primordials vs. 20 Primordials.

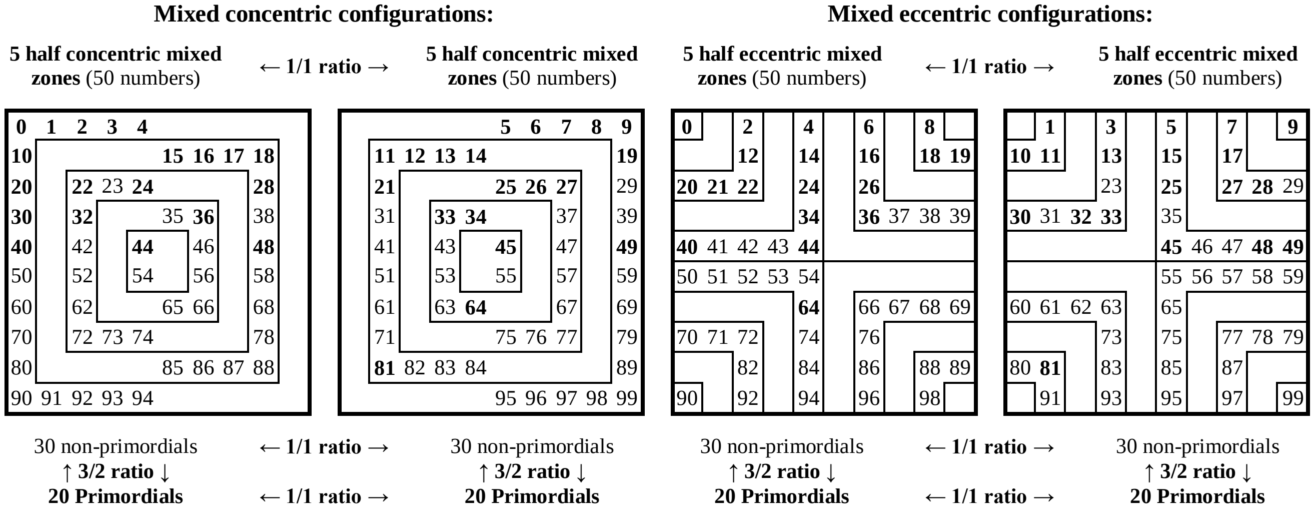


Figure 23: From the matrix of the first hundred numbers, concentric and eccentric mixed configurations of sub- matrices to 50 entities each opposing their non-primordials and their primordials in 3:2 ratios.

It is important to emphasize the total similarity between these arithmetic phenomena and those observed in the analogous sub-matrices introduced in the previous paper, "*"The ultimate numbers and the 3/2 ratio, just two primary sets of whole number"* [1]. This similarity holds despite the fact that the data sources for the analyses (the table of crossed additions of the Twenty Fundamentals) are completely different.

9. Association of opposing classes

Based on their degree of complexity, as introduced in Figure 2 of Chapter 3, the four classes of numbers can be grouped into two sets: extreme classes and median classes. Specifically, ultimate numbers (Complexity Level 1) and mixed composite numbers (Complexity Level 4) form the set of extreme classes. Conversely, raised numbers (Level 2) and composite numbers (Level 3) form the second set of median classes. Therefore, the designation "extremes" is used to refer to numbers of the extreme classes, and the designation "medians" refers to numbers of the median classes.

9.1 100-number matrix and number class.

Figure 24 illustrates that—within the matrix of the first hundred numbers—these two defined sets comprise 55 numbers of extreme classes and 45 numbers of median classes. This means the closed matrix is constituted by 5*x* extremes (→ *x* = 11) and 5*x* medians (→ *x* = 9). . Furthermore, in increasingly diluted sub-matrices of 60 vs. 40 entities, these two families of numbers are always distributed in 3:2 ratios.

27 Ultimates
(level 1 complexity)

u → *r*

10 Raiseds
(level 2 complexity)

27 Ultimates
+
28 Mixes
=
55 numbers
of extreme classes

↓

0	**1**	**2**	**3**	4	**5**	6	**7**	8	9
10	**11**	**12**	**13**	14	15	16	**17**	**18**	**19**
20	21	22	**23**	**24**	25	26	27	**28**	**29**
30	**31**	32	33	34	35	**36**	**37**	38	39
40	**41**	42	**43**	**44**	**45**	46	**47**	**48**	49
50	51	**52**	**53**	**54**	55	**56**	57	58	**59**
60	**61**	62	**63**	64	65	66	**67**	**68**	69
70	**71**	**72**	**73**	74	**75**	**76**	77	78	**79**
80	81	82	**83**	**84**	85	86	87	**88**	**89**
90	91	**92**	93	94	95	**96**	**97**	**98**	**99**

↓

10 Raiseds
+
35 Composites
=
45 numbers
of median classes

35 Composites
(level 3 complexity)

c → *m*

28 Mixes
(level 4 complexity)

Figure 24: Counting of the four classes of numbers in the matrix of the first 100 numbers according to their degree of complexity (See also Figures 2 and 3).

9.1.1 Dilution of sub-matrices

Thus, as shown in the left part of Figure 25, when the matrix is divided into two compact blocks of 60 vs. 40 entities—comprising the first 60 and the subsequent 40 numbers, respectively—the extreme and median numbers are distributed in 3:2 ratios. Specifically, this results in 33 extremes vs. 22 and 27 medians vs. 18 within each set. The same arithmetic phenomena are generated when the matrix of the first hundred numbers is split into 10 blocks of (5×12) vs. (5×8) entities, as illustrated in the right part of Figure 25.

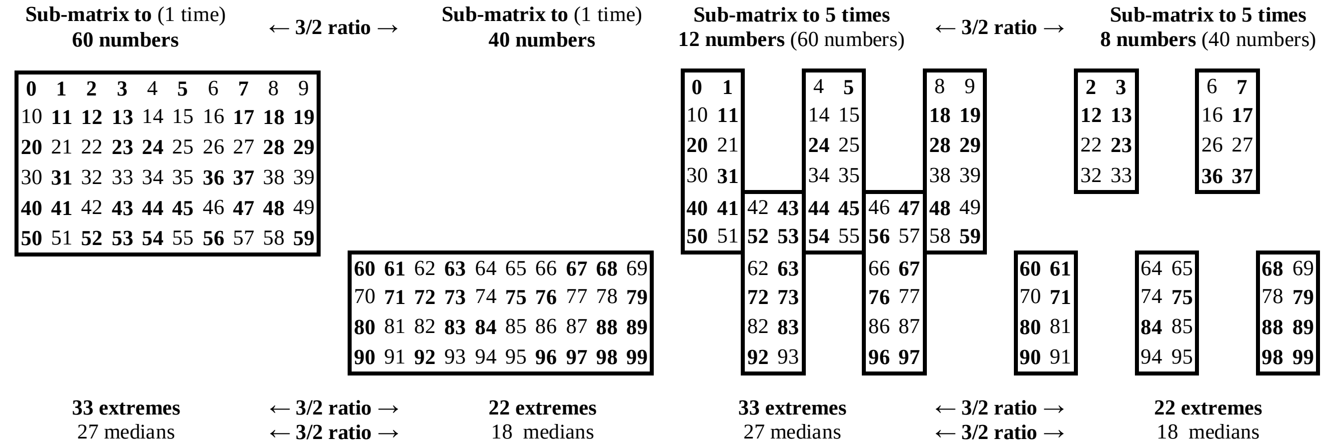


Figure 25: Distribution of numbers belonging to **extreme** and median classes across two double sub-matrices of the first 100 numbers, comparing a less diluted configuration with a more diluted configuration.

Furthermore, the two sets of extreme and median numbers are further divided into 3:2 ratios through a more diluted fractionation of the matrix into 20 blocks of (10×6) and (10×4) entities, as described in the left part of Figure 26. Finally, in a final fractionation of the matrix into 40 blocks of (20×3) vs. (20×2) entities, shown in the right part of Figure 26, the same 3:2 partitions of the two families of numbers are consistently observed: 33 extremes vs. 22 and 27 medians vs. 18.

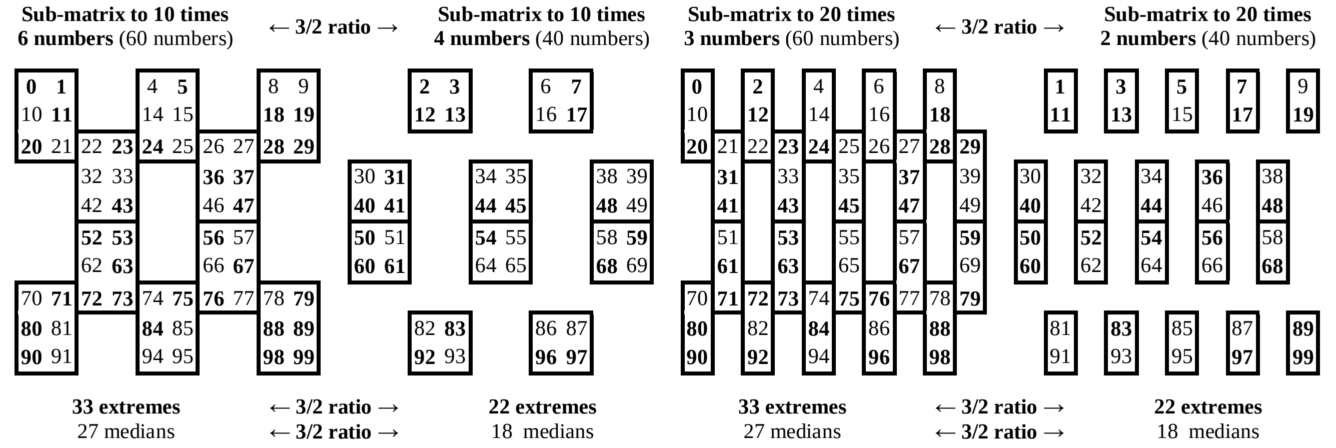


Figure 26: Distribution of the numbers to **extreme** and median classes in two diluted and very diluted double sub-matrices of the first 100 numbers.

9.1.2 Triangular numbers and triangular sub-matrices

The first 100 whole numbers contain 55 extremes and 45 medians. These two quantities are, in fact, triangular numbers: 55 is the sum of the numbers from 1 to 10 (T_{10} = 55) , and 45 is the sum of the numbers from 1 to 9 (T_9 = 45). Coincidentally, a 10×10 number matrix (containing 100 numbers) is precisely formed by assembling two triangular matrices of sizes T_{10} and T_9. Thus, in the matrix of the first hundred whole numbers, the quantities of extreme and median numbers correspond exactly to the dimensions of the two triangular sub-matrices that form this 100-number square matrix.

As demonstrated in Figure 27, within the symmetric triangular sub-matrices of 55 and 45 entities, the number of extremes and medians is consistently a multiple of 5*x*. Furthermore, the values for these two categories of numbers are identical when considering the symmetrically corresponding configurations.

55-number
triangular sub-matrix
→ T_{10}

30 extremes
↑ 6/5 ratio ↓
25 medians

0									
10	**11**								
20	21	22							
30	**31**	32	33						
40	**41**	42	**43**	**44**					
50	51	**52**	**53**	**54**	55				
60	**61**	62	**63**	64	65	66			
70	**71**	**72**	**73**	74	**75**	**76**	77		
80	81	82	**83**	**84**	85	86	87	**88**	
90	91	**92**	93	94	95	**96**	**97**	**98**	**99**

← 1/1 ratio →

← 1/1 ratio →

55-number
triangular sub-matrix
→ T_{10}

									9
								18	**19**
							27	**28**	**29**
						36	**37**	38	39
					45	46	**47**	**48**	49
				54	55	**56**	57	58	**59**
			63	64	65	66	**67**	**68**	69
		72	**73**	74	**75**	**76**	77	78	**79**
	81	82	**83**	**84**	85	86	87	**88**	**89**
90	91	**92**	93	94	95	**96**	**97**	**98**	**99**

30 extremes
↑ 6/5 ratio ↓
25 medians

↑ 6/5 ratio ↓
↑ 5/4 ratio ↓

↑ 11/9 ratio ↓

↑ 11/9 ratio ↓

↑ 6/5 ratio ↓
↑ 5/4 ratio ↓

25 extremes
↑ 5/4 ratio ↓
20 medians

	1	**2**	**3**	4	**5**	6	**7**	8	9
		12	**13**	14	15	16	**17**	**18**	**19**
			23	**24**	25	26	27	**28**	**29**
				34	35	**36**	**37**	38	39
					45	46	**47**	**48**	49
						56	57	58	**59**
							67	**68**	69
								78	**79**
									89

← 1/1 ratio →

← 1/1 ratio →

0	**1**	**2**	**3**	4	**5**	6	**7**	8
10	**11**	**12**	**13**	14	15	16	**17**	
20	21	22	**23**	**24**	25	26		
30	**31**	32	33	34	35			
40	**41**	42	**43**	**44**				
50	51	**52**						
60	**61**	62						
70	**71**							
80								

25 extremes
↑ 5/4 ratio ↓
20 medians

45-number
triangular sub-matrix
→ T_9

45-number
triangular sub-matrix
→ T_9

Figure 27: Distribution of the numbers to **extreme** and median classes in triangular sub-matrices of the first 100-number matrix.

The different observed values are therefore equal to 30, 25 and 20 entities. These values actually correspond to combinations of two other triangular numbers: 10 which is equal to T_4 and 15 which corresponds to T_5. We therefore note that these two two triangular numbers are also equal to 5*x* entities and are opposed in a 2:3 ratio (where *x*=2 and *x*=3, respectively). Specifically, the observed values are constructed as follows: 30 is equal to 2 times T_5 (2 times 15), 20 is equal to 2 times T_4 (2 times 10); and 25 is equal to one time T_5 + one time T_4 (15 + 10). It therefore appears that, within the matrix of the first 100 whole numbers, the two classes of numbers (defined as extreme and median) are subtly distributed in sets that are always multiples of 5*x* and in combinations of triangular numbers that are also multiples of 5*x*.

Figure 28 now demonstrates that the four triangular configurations introduced in Figure 27 correspond to assemblies of four other triangular sub-matrices. The sizes of these newly assembled sub-matrices are themselves arrangements based on the triangular numbers T_4 (→ 10) and T_5 (→ 15).

10 + 15 numbers
triangular sub-matrix
→ T_4 + T_5

0				
10	**11**			
20	21	22		
30	**31**	32	33	
40	**41**	42	**43**	**44**
50	51	**52**	**53**	
60	**61**	62		
70	**71**			
80				

15 extremes
↑ 3/2 ratio ↓
10 medians

← 1/1 ratio →

← 1/1 ratio →

10 + 15 numbers
triangular sub-matrix
→ T_4 + T_5

				9
			18	**19**
		27	**28**	**29**
	36	**37**	38	39
45	46	**47**	**48**	49
	56	57	58	**59**
		67	**68**	69
			78	**79**
				89

15 extremes
↑ 3/2 ratio ↓
10 medians

10 + 10 numbers
triangular sub-matrix
→ T_4 + T_4

1	**2**	**3**	4	**5**	6	**7**	8
	12	**13**	14	15	16	**17**	
		23	**24**	25	26		
			34	35			

10 extremes
↑ 1/1 ratio ↓
10 medians

← 2/3 ratio →

← 2/3 ratio →

15 + 15 numbers
triangular sub-matrix
→ T_5 + T_5

				54	55				
			63	64	65	66			
		72	**73**	74	**75**	**76**	77		
	81	82	**83**	**84**	85	86	87	**88**	
90	91	**92**	93	94	95	**96**	**97**	**98**	**99**

15 extremes
↑ 1/1 ratio ↓
15 medians

Figure 28: Distribution of the numbers to **extreme** and median classes in triangular source sub-matrices of the first 100-number matrix (See also Figures 27 and 29).

In these four source sub-matrices, it turns out that the respective number of extremes and medians is identical to the triangular matrix sub-dimensions which constitute them. Thus, 15 extremes and 10 medians constitute the two sub-matrices of triangular size T_4 (→ 10) + T_5 (→ 15). Also, 10 extremes and 10 medians constitute the sub-matrix of triangular size T_4 (→ 10) + T_5 (→ 10). Finally, 15 extremes and 15 medians constitute the sub-matrix of triangular size T_5 (→ 15) + T_5 (→ 15).

From these four source sub-matrices introduced in Figure 28, other symmetric configurations deserve attention due to their strong singularity. Thus, as illustrated in Figure 29, the assembly of the symmetric matrices allows the creation of two new sub-matrices of 2 times T_4 and 2 times T^5 as size. As well as two others of T_4 + T_5 as another size.

Exactly as in the previous configurations described in Figure 28, in these four new triangular sub-matrices, the respective quantities of the numbers qualified as extremes and medians are always identical to the values T_4 or T_5, so equal to 10 or 15 entities. In second configuration of 15 + 15 numbers, extremes are two time T_4 in number.

---

25 + 25 numbers recombined triangular sub-matrix
→ (T_4 + T_5) + (T_4 + T_5)

0									9
10	**11**							**18**	**19**
20	21	22					27	**28**	**29**
30	**31**	32	33			**36**	**37**	38	39
40	**41**	42	**43**	**44**	**45**	46	**47**	**48**	49
50	51	**52**	**53**			**56**	57	58	**59**
60	**61**	62					**67**	**68**	69
70	**71**							78	**79**
80									**89**

20 + 30 numbers recombined triangular sub-matrix
→ 2T_4 + 2T_5

	1	**2**	**3**	4	**5**	6	**7**	8	
		12	**13**	14	15	16	**17**		
			23	**24**	25	26			
				34	35				
				54	55				
			63	64	65	66			
		72	**73**	74	**75**	**76**	77		
	81	82	**83**	**84**	85	86	87	**88**	
90	91	**92**	93	94	95	**96**	**97**	**98**	**99**

10 + 10 numbers triangular sub-matrix → T_4 + T_4 ← 2/3 ratio → **15 + 15 numbers triangular sub-matrix → T_5 + T_5**

10 + 15 numbers triangular sub-matrix → T_4 + T_5 ← 1/1 ratio → **10 + 15 numbers triangular sub-matrix → T_4 + T_5**

0									9
10	**11**							**18**	**19**
20	21	22					27	**28**	**29**
30	**31**	32	33			**36**	**37**	38	39

40	**41**	42	**43**	**44**	**45**	46	**47**	**48**	49
50	51	**52**	**53**			**56**	57	58	**59**
60	**61**	62					**67**	**68**	69
70	**71**							78	**79**
80									**89**

	1	**2**	**3**	4
		12	**13**	14
			23	**24**
				34
				54
			63	64
		72	**73**	74
	81	82	**83**	**84**
90	91	**92**	93	94

5	6	**7**	8	
15	16	**17**		
25	26			
35				
55				
65	66			
75	**76**	77		
85	86	87	**88**	
95	**96**	**97**	**98**	**99**

10 extremes ↑ 1/1 ratio ↓ 10 medians ← **1/2 ratio** → **10 + 10 extremes** ↑ 2/1 ratio ↓ 10 medians; ← **1/1 ratio** → (medians)

15 extremes ↑ 3/2 ratio ↓ 10 medians ← **3/2 ratio** → **10 extremes** ↑ 2/3 ratio ↓ 15 medians; ← **2/3 ratio** → (medians)

Figure 29: Distribution of the numbers to **extreme** and median classes in triangular source sub-matrices of the first 100-number matrix. (See also Figure 28).

9.1.3 Triangular numbers and alternated sub-matrices

A square matrix of 100 entities is therefore an assembly of two triangular matrices of size T_{10} and T_9. Alternatively, we can consider this matrix as being composed of two alternating sub-matrices, one containing 5 rows and 11 columns, and the other containing 5 rows and 9 columns, as shown Figure 30. These two sub-matrices therefore have sizes T_{10} and T_9 respectively.

55-number sub-matrix → T_{10} **(5 times 11 numbers)**	**← 11/9 ratio →**	**45-number sub-matrix → T_9** **(5 times 9 numbers)**

0	**1**	**2**	**3**	4	**5**	6	**7**	8	9	10
	11	***12***	***13***	*14*	*15*	*16*	***17***	***18***	***19***	
20	21	22	**23**	**24**	25	26	27	**28**	**29**	30
	31	*32*	*33*	*34*	*35*	***36***	***37***	*38*	*39*	
40	**41**	42	**43**	**44**	**45**	46	**47**	**48**	49	**50**
	51	***52***	***53***	***54***	*55*	***56***	*57*	*58*	***59***	
60	**61**	62	**63**	64	65	66	**67**	**68**	69	70
	71	***72***	***73***	*74*	***75***	***76***	*77*	*78*	***79***	
80	81	82	**83**	**84**	85	86	87	**88**	**89**	**90**
	91	***92***	*93*	*94*	*95*	***96***	***97***	***98***	***99***	

0	***1***	***2***	***3***	*4*	***5***	*6*	***7***	*8*	*9*	*10*
	11	**12**	**13**	14	15	16	**17**	**18**	**19**	
20	*21*	*22*	***23***	***24***	*25*	*26*	*27*	***28***	***29***	*30*
	31	32	33	34	35	**36**	**37**	38	39	
40	***41***	*42*	***43***	***44***	***45***	*46*	***47***	***48***	*49*	***50***
	51	**52**	**53**	**54**	55	**56**	57	58	**59**	
60	***61***	*62*	***63***	*64*	*65*	*66*	***67***	***68***	*69*	*70*
	71	**72**	**73**	74	**75**	**76**	77	78	**79**	
80	*81*	*82*	***83***	***84***	*85*	*86*	*87*	***88***	***89***	***90***
	91	**92**	93	94	95	**96**	**97**	**98**	**99**	

30 extremes ↑ 6/5 ratio ↓ 25 medians	**← 3/2 ratio →** **← 1/1 ratio →**	20 medians ↑ 4/5 ratio ↓ **25 extremes**

Figure 30: From the first 100-number matrix, distribution of the numbers to **extreme** and median classes in alternated sub-matrices to 55 (→ $\boldsymbol{T_{10}}$) versus 45 (→ $\boldsymbol{T_9}$) entities (See also Figure 31).

In this new configuration, the extreme and median numbers still correspond to specific and exact ratios of 3/2 and 1/1. However, they also align with the ratios 6/5 and 5/4, yielding a combined ratio of 11/9. This 11/9 ratio is the initial ratio (T_{10} versus T_9) that relates the extreme and median numbers within the original 100-entity matrix.

In an arrangement of sub-matrices consisting of 5 rows of 12 columns versus 5 rows of 8 columns, i.e., in a 3:2 ratio ($4T_5$ versus $4T_4$), the patterns of the extreme and median numbers appear in the same proportions as those found in the geometrically triangular matrices introduced previously.

60-number sub-matrix → $4T_5$	**← 3/2 ratio →**	**40-number sub-matrix → $4T_4$**

0	**1**	**2**	**3**	4	**5**	6	**7**	8	9	10	**11**
		12	***13***	*14*	*15*	*16*	***17***	***18***	***19***		
20	21	22	**23**	**24**	25	26	27	**28**	**29**	30	**31**
		32	*33*	*34*	*35*	***36***	***37***	*38*	*39*		
40	**41**	42	**43**	**44**	**45**	46	**47**	**48**	49	**50**	51
		52	***53***	***54***	*55*	***56***	*57*	*58*	***59***		
60	**61**	62	**63**	64	65	66	**67**	**68**	69	70	**71**
		72	***73***	*74*	***75***	***76***	*77*	*78*	***79***		
80	81	82	**83**	**84**	85	86	87	**88**	**89**	**90**	91
		92	*93*	*94*	*95*	***96***	***97***	***98***	***99***		

0	***1***	***2***	***3***	*4*	***5***	*6*	***7***	*8*	*9*	*10*	***11***
		12	**13**	14	15	16	**17**	**18**	**19**		
20	*21*	*22*	***23***	***24***	*25*	*26*	*27*	***28***	***29***	*30*	***31***
		32	33	34	35	**36**	**37**	38	39		
40	***41***	*42*	***43***	***44***	***45***	*46*	***47***	***48***	*49*	***50***	*51*
		52	**53**	**54**	55	**56**	57	58	**59**		
60	***61***	*62*	***63***	*64*	*65*	*66*	***67***	***68***	*69*	*70*	***71***
		72	**73**	74	**75**	**76**	77	78	**79**		
80	*81*	*82*	***83***	***84***	*85*	*86*	*87*	***88***	***89***	***90***	*91*
		92	93	94	95	**96**	**97**	**98**	**99**		

33 extremes ↑ 11/9 ratio ↓ 27 medians	**← 3/2 ratio →** **← 3/2 ratio →**	**22 extremes** ↑ 11/9 ratio ↓ 18 medians

Figure 31: From the first 100-number matrix, distribution of the numbers to extreme and median classes in alternated sub-matrices to 60 (→$4T_5$) versus 40 (→$4T_4$) entities (See also Figure 31).

Thus, as shown in Figure 31, the extreme and median values exhibit an 11:9 ratio within these two sub-matrices, while these same categories of numbers maintain a 3:2 ratio between the sub-matrices of size $4T_5$ versus $4T_4$.

9.2 Matrix of first twenty-five whole numbers

Within the matrix of the first twenty-five whole numbers, the extremes and medians are distributed in singular arrangements. In relation to both the arithmetic concept of a remarkable identity and that of triangular numbers, the two classes of numbers, designated as extremes and medians, consistently oppose each other in a value ratio of 3:2 across various configurations.

9.2.1 Matrix of first twenty-five whole numbers and remarkable identity

The value 25 is the first that can be subdivided into four constituent values (9 + 6 + 6 + 4), which together generate a triple value ratio based on 3:2. As illustrated in Figure 29, the value 9 (3 × 3) is opposed to the value 6 (3 × 2) in a 3:2 ratio; similarly, the value 6 (2 × 3) is opposed to the value 4 (2 × 2) These four values also oppose one another pair wise in the 15:10 ratio, which is an extension of the 3:2 ratio. This arithmetic demonstration serves as a geometric variant of the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* here hold the values 3 and 2, respectively values that are opposed in the 3:2 ratio.

$(a+b)^2 = a^2 + 2ab + b^2$ → $(3 + 2)^2 = 3^2 + 2(3×2) + 2^2$ → 9 + 6 + 6 + 4 = 25 entities

	0u	***1u***	***2u***	***3u***	*4r*	
9 a^2	***5u***	*6c*	***7u***	*8r*	*9r*	**6** *ab*
	10c	***11u***	***12m***	***13u***	*14c*	
6 *ba*	*15c*	*16r*	***17u***	***18m***	***19u***	**4** b^2
	20m	*21c*	*22c*	***23u***	***24m***	

15 extremes (*u* or *m*) **← 3/2 ratio →** 10 medians (*r* or *c*)

Figure 32: Opposition in 3:2 ratio of the first 15 extremes and the first 10 medians in matrix of 25 entities deduced from the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* have 3 and 2 as respective value.

It so happens that the matrix of the first 25 numbers (which is configured from this remarkable identity) generates these phenomena through the opposition of the extreme class numbers (Ultimates and Mixes) to those of the median classes (Raiseds and Composites) of which it is composed. It appears, as shown in Figure 32, that among the first 25 numbers, there are 15 entities of the extreme classes which oppose the 10 entities of the median classes in a value ratio of 3/2. Furthermore, these two types of number classes are opposed, as shown in Figure 33, in various transcendent value ratios of 3/2 both inside and between the sub-matrices, the configuration of which is deduced from the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* have 3 and 2 as respective value.

9 + 6 numbers
sub-matrix (→ $a^2 + ab$)

	0	**1**	**2**	**3**	4	
a^2	**5**	6	**7**	8	9	***ab***
	10	**11**	**12**	**13**	14	
ba						b^2

9 extremes
↑ 3/2 ratio ↓
6 medians

← 3/2 ratio →

6 + 4 numbers
sub-matrix (→ $ba + b^2$)

a^2						***ab***
ba	15	16	**17**	**18**	**19**	b^2
	20	21	22	**23**	**24**	

6 extremes
↑ 3/2 ratio ↓
4 medians

← 3/2 ratio → (between 9 extremes and 6 extremes; between 6 medians and 4 medians)

9 + 6 numbers
sub-matrix (→ $a^2 + ba$)

	0	**1**	**2**			
a^2	**5**	6	**7**			***ab***
	10	**11**	**12**			
ba	15	16	**17**			b^2
	20	21	22			

9 extremes
↑ 3/2 ratio ↓
6 medians

← 3/2 ratio →

6 + 4 numbers
sub-matrix (→ $ab + b^2$)

				3	4	
a^2				8	9	***ab***
				13	14	
ba				**18**	**19**	b^2
				23	**24**	

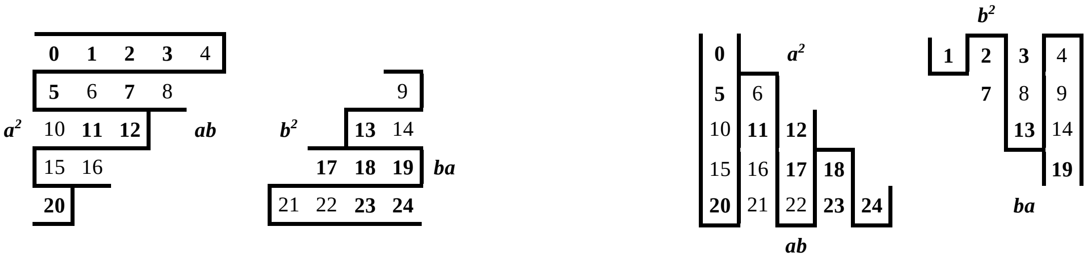
6 extremes
↑ 3/2 ratio ↓
4 medians

← 3/2 ratio →

Figure 33: Distribution of extreme and median class numbers within two double sub-matrices of the first 25 numbers. Configurations inferred from the identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* have the values 3 and 2 (See also Figure 32).

Also, in this matrix of the first 25 numbers, other configurations of the same arithmetic arrangements, but which are geographically redeployed in various other ways still generate the same phenomena opposing the numbers to extreme classes and those to median classes in transcendent 3:2ratios. Figures 34 and 35 illustrate these singular phenomena.

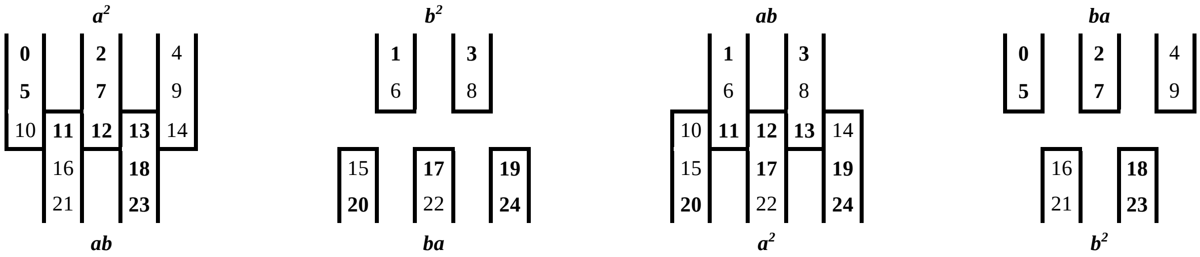
Sub-matrix to 9 + 6 numbers (→ $a^2 + 2ab/2$)	**← 3/2 ratio →**	**Sub-matrix to 6 + 4 numbers (→ $2ba/2 + b^2$)**	**Sub-matrix to 9 + 6 numbers (→ $a^2 + ba$)**	**← 3/2 ratio →**	**Sub-matrix to 6 + 4 numbers (→ $ab + b^2$)**



9 extremes	**← 3/2 ratio →**	**6 extremes**	**9 extremes**	**← 3/2 ratio →**	**6 extremes**
↑ 3/2 ratio ↓		↑ 3/2 ratio ↓	↑ 3/2 ratio ↓		↑ 3/2 ratio ↓
6 medians	**← 3/2 ratio →**	4 medians	6 medians	**← 3/2 ratio →**	4 medians

Figure 34: Distribution of extreme and median class numbers in two double sub-matrices of the first 25 numbers. Configurations inferred from the identity *(a + b...)* .

Sub-matrix to 9 + 6 numbers (→ $a^2 + ab$)	**← 3/2 ratio →**	**Sub-matrix to 4 + 6 numbers (→ $b^2 + ba$)**	**Sub-matrix to 6 + 9 numbers (→ $ab + a^2$)**	**← 3/2 ratio →**	**Sub-matrix to 6 + 4 numbers (→ $ba + b^2$)**



9 extremes	**← 3/2 ratio →**	**6 extremes**	**9 extremes**	**← 3/2 ratio →**	**6 extremes**
↑ 3/2 ratio ↓		↑ 3/2 ratio ↓	↑ 3/2 ratio ↓		↑ 3/2 ratio ↓
6 medians	**← 3/2 ratio →**	4 medians	6 medians	**← 3/2 ratio →**	4 medians

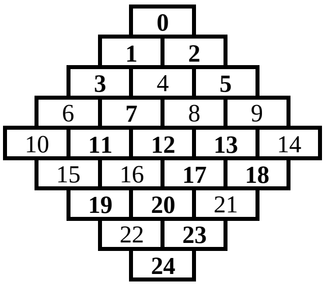
Figure 35: Distribution of extreme and median class numbers in two double sub-matrices of the first 25 numbers. Configurations inferred from the identity *$(a + b^2 = ...)$* .

As seen in Figures 34 and 35, these phenomena are always and again inscribed in different geometric variables of the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* have 3 and 2 to value.

9.2.2 Matrix of first twenty-five whole numbers and triangular numbers

A matrix of 25 entities can also be considered (Figure 36) as consisting of the association of two triangular areas of size T_5 (→ 15) and T_4 (→ 10).

15 + 10 numbers triangular matrix
→ $T_5 + T_4$ entities size



15 extremes (*u* or *m*) **← 3/2 ratio →** 10 medians (*r* or *c*)

Figure 36: the triangular matrix of the first 25 whole numbers

The T_5 and T_4 values are therefore opposite in magnitude in an exact 3:2 ratio. Also, these two values are identical to the respective numbers of extremes and medians which constitute the sequence of the first 25 whole numbers.

As illustrated in Figure 37, this matrix of the first 25 whole numbers can be split into different triangular sub-matrices of size T_5 and T_4. In all these sub-matrices, the 15 extreme numbers and the 10 median numbers consistently oppose each other in a value ratio of 3/2.This opposition holds true both inside the sub-matrices and between these sub-matrices (whether upper or lower), specifically through the opposition of 9 entities vs. 6 and 6 vs. 4. We also note that these four values (9, 6, 6, and 4) echo those used in the remarkable identity presented in the previous chapter.

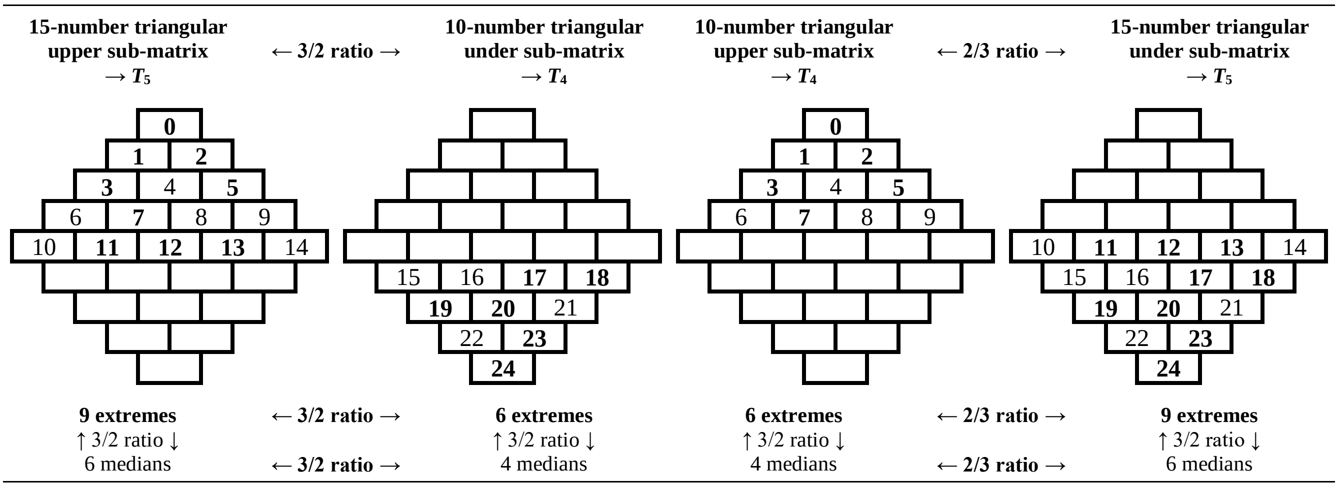


Figure 37: Distribution of **extreme** and median class numbers in triangular sub-matrices of the matrix of the first 25 numbers.

In alternate triangular sub-matrices, extremes and medians are distributed and opposed in exactly the same proportions as illustrated in Figure 38.

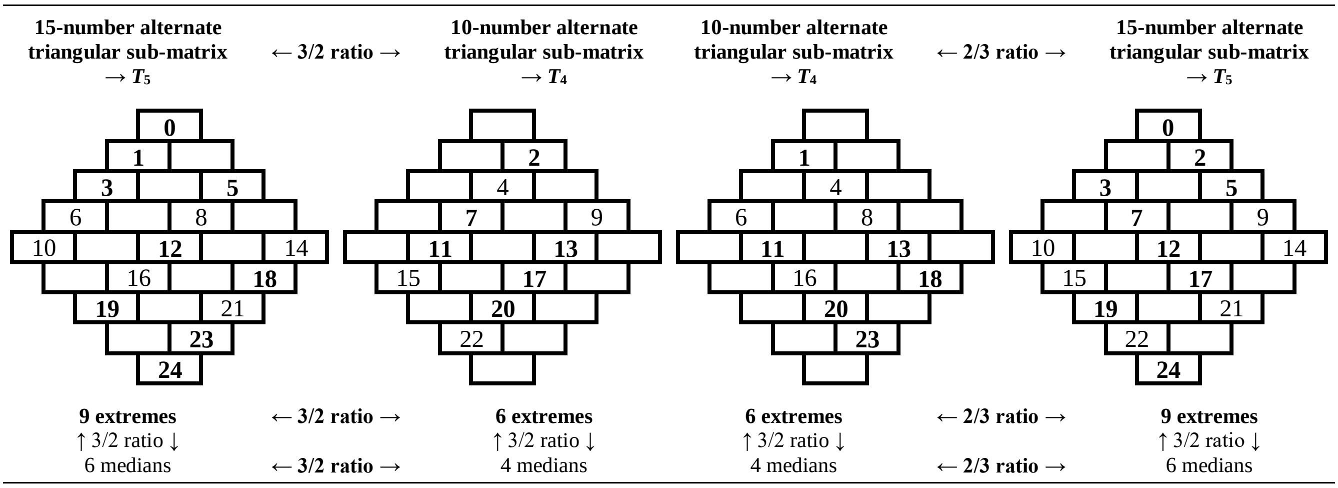


Figure 38: Distribution of **extreme** and median class numbers in alternate triangular sub-matrices of the matrix of the first 25 numbers.

9.3 Matrix of the Forty Primordials and number class

Since the set of forty primordial numbers is composed, as introduced in Chapter 6, of the first ten entities from each of the four classes of whole numbers, it naturally contains 20 extremes and 20 medians. The ranking of the forty primordials in order of magnitude within a 4-row by 10-column matrix, and their distinction into extremes and medians (as previously defined), reveals a non-random distribution of these two groups of numbers.

The distribution of these two classes of numbers is organized into various oppositions of consistently 3/2 magnitude within 20-number symmetric or asymmetric sub-matrices of the original matrix of the forty primordials.

Thus, we count 12 extremes vs. 8 medians in the first two rows of this 40-entity matrix and 8 extremes vs. 12 medians in the last two rows. Similarly, as illustrated in Figure 39, the same proportions of extremes and medians are found in the first 5 columns and the last five.

12 extremes ↑ 3/2 ↓ **8 medians**	**0**	**1**	**2**	**3**	4	**5**	6	**7**	8	9	
	10	**11**	**12**	**13**	14	15	16	**17**	**18**	**19**	
	20	21	22	**24**	25	26	27	**28**	30	32	**8 extremes** ↑ 2/3 ↓ **12 medians**
	33	34	**36**	**40**	**44**	**45**	**48**	49	64	81	

12 extremes ↑ 3/2 ↓ 8 medians	**0**	**1**	**2**	**3**	4	**5**	6	**7**	8	9	**8 extremes** ↑ 2/3 ↓ 12 medians
	10	**11**	**12**	**13**	14	15	16	**17**	**18**	**19**	
	20	21	22	**24**	25	26	27	**28**	30	32	
	33	34	**36**	**40**	**44**	**45**	**48**	49	64	81	

Figure 39: Distribution, in 3:2 opposite ratios, of **extreme** and median class numbers in two symmetric sub-matrices of the matrix of the Forty Primordials.

Still in this matrix of the 40 Primordials, the same phenomena are manifested, Figure 40, in the alternative insulation of five versus five columns and even of 10 semi-columns versus ten others.

← 1/1 ratio →

8 extremes ↑ 2/3 ↓ 12 medians	**0**	**2**	4	6	8	**← 3/2ratio →**	**1**	**3**	**5**	**7**	9	**12 extremes** ↑ 3/2 ↓ 8 medians
	10	**12**	14	16	**18**		**11**	**13**	15	**17**	**19**	
	20	22	25	27	30	**← 2/3 ratio →**	21	**24**	26	**28**	32	
	33	**36**	**44**	**48**	64		34	**40**	**45**	49	81	
			↑ 1/1 ratio ↓						**↑ 1/1 ratio ↓**			
8 extremes ↑ 2/3 ↓ 12 medians	**0**	**2**	4	6	8	**← 3/2ratio →**	**1**	**3**	**5**	**7**	9	**12 extremes** ↑ 3/2 ↓ 8 medians
	10	**12**	14	16	**18**		**11**	**13**	15	**17**	**19**	
	21	**24**	26	**28**	32	**← 2/3 ratio →**	**20**	22	25	27	30	
	34	**40**	**45**	49	81		33	**36**	**44**	**48**	64	

Figure 40: Distribution, in 3/2 opposite ratios, of **extreme** and median class numbers classes in two sub-matrices of the matrix of the Forty Primordials.

10. Classes of numbers and pairs of numbers

According to their classification into four classes as defined in Chapter 3 (*u* = Ultimate, *r* = Raised, *c* = Composite and *m* = Mix, see Figure 2 Chapter 3.1 also), whole numbers can be associated two by two in ten different configurations.

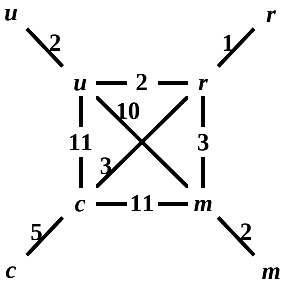
10.1 The ten associations of number classes

Within the matrix of the first hundred whole numbers, classified linearly into ten rows of ten consecutive entities, it's possible to form 50 pairs of consecutive numbers. These pairs can be categorized in ten different ways, depending on the respective congruence class (mod 10) of the two numbers they contain. Figure 41 demonstrates that, in this 10×10 matrix, all ten possible associations are represented. However, there is only one, yet highly significant, association of two perfect powers: the pair 8 and 9 (2^3 and 3^2).

It's worth noting the unique properties of these two numbers. Among the whole numbers, the pair (8, 9) constitutes a remarkable anomaly: this pair represents the only known consecutive perfect powers among the set of whole numbers. They are the last two single-digit cubes and squares. Specifically, their bases (the 2 in 2^3 and the 3 in 3^2) are in the ratio 2:3. Strikingly, their exponents (the 3 in 2^3 and the 2 in 3^2) are in the inverted ratio 3:2. Thus, these two numbers (8 and 9, or more precisely 2^3 and 3^2) constitute the unique solution to Catalan's Theorem, defined by the equation $x^a – y^b = 1$ (for $a,b > 1$).

30 pairs ***r-r*** ***r-m*** ***m-u*** ***u-c*** ***c-c***

0-1	*u-u*	2-3	*u-u*	4-5	*r-u*	**6-7**	***c-u***	**8-9**	***r-r***
10-11	***c-u***	**12-13**	***m-u***	**14-15**	***c-c***	16-17	*r-u*	**18-19**	***m-u***
20-21	*m-c*	**22-23**	***c-u***	**24-25**	***m-r***	26-27	*c-r*	**28-29**	***m-u***
30-31	***c-u***	32-33	*r-c*	**34-35**	***c-c***	**36-37**	***m-u***	**38-39**	***c-c***
40-41	***m-u***	**42-43**	***c-u***	44-45	*m-m*	**46-47**	***c-u***	**48-49**	***m-r***
50-51	*m-c*	**52-53**	***m-u***	54-55	*m-c*	56-57	*m-c*	**58-59**	***c-u***
60-61	***m-u***	62-63	*c-m*	64-65	*r-c*	**66-67**	***c-u***	68-69	*m-c*
70-71	***c-u***	**72-73**	***m-u***	74-75	*c-m*	76-77	*m-c*	**78-79**	***c-u***
80-81	***m-r***	**82-83**	***c-u***	84-85	*m-c*	**86-87**	***c-c***	**88-89**	***m-u***
90-91	*m-c*	92-93	*m-c*	**94-95**	***c-c***	**96-97**	***m-u***	98-99	*m-m*



20 pairs *u-u* *u-r* *r-c* *c-m* *m-m*

Figure 41: Count of the associations of classes of numbers of the pairs of adjacent numbers of the matrix of the first 100 numbers (See also Figure 42).

10.2 Symmetric associations of number classes

By grouping together five particular associations of pairs of numbers and five others, it turns out that, in a ratio of 3/2, 30 couples are made up of these first five associations considered and 20 couples are made up of the other five possible associations. As shown in Figure 39, these two groups of five associations are not arbitrary but are organized in two sub symmetrical hyper configurations which can be called *configuration N* and *configuration Z*. This, with reference to the image released from these hyper configurations of twice five associations of numbers in the schematization of these configurations.

By grouping together five specific associations of number pairs and five other associations, it is found that, in a 3:2 ratio, 30 pairs are formed from the first five associations, vs. 20 pairs formed from the five other possible associations. As shown in Figure 39, these two groups of five associations are not arbitrary. Instead, they are organized into two symmetrical sub-hyperconfigurations, which can be designated as *Configuration N* and *Configuration Z*. These names are chosen with reference to the visual representation (the image) released from the schematization of these twice-five associations of numbers.

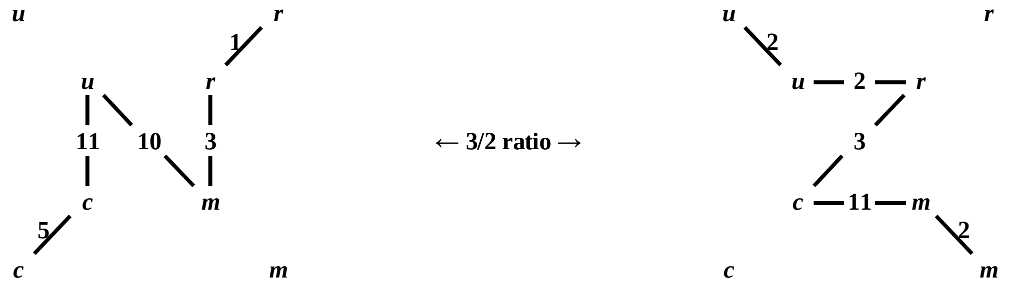
30 pairs (60 numbers)

1	**3**	**10**	**11**	**5**
r-r	***r-m***	***m-u***	***u-c***	***c-c***

← 3/2 ratio →

20 pairs (40 numbers)

2	**2**	**3**	**11**	**2**
u-u	***u-r***	***r-c***	***c-m***	***m-m***



configuration 'N' ***configuration 'Z'***

Figure 42: Classification of the 50 number pairs based on two symmetrical configurations of pair associations. A 3:2 ratio is maintained: 30 pairs belong to the *N configuration* vs. 20 pairs belonging to the *Z configuration*. (See also Figure 41.)

The N-type configuration has two *protuberances* made up of associations of two Raiseds (*r-r*) and two Composites (*c-c*), namely types of numbers of median classes. The Z-type configuration has its two similar and symmetrical protuberances made up of associations of two Ultimates (*u-u*) and two Mixes (*m-m*), namely types of numbers of extreme classes.

Also, illustrated in the left part of Figure 43, among the 30 pairs of configuration *N*, 6 pairs are formed of two numbers of the same classes (5 *c-c* pairs and 1 *r-r* pair) and among the 20 pairs of configuration *Z*, 4 pairs are formed of two numbers of the same classes (2 *u-u* pairs and 2 *m-m* pairs). Again, these sets of pairs are in opposition in a 3:2 ratio. Consequently, illustrated in the right part of Figure 40, the cores of the two configurations (stripped of their protuberances) therefore also oppose in a 3:2 ratio with 24 pairs of configuration *N* versus 16 pairs of configuration *Z*.

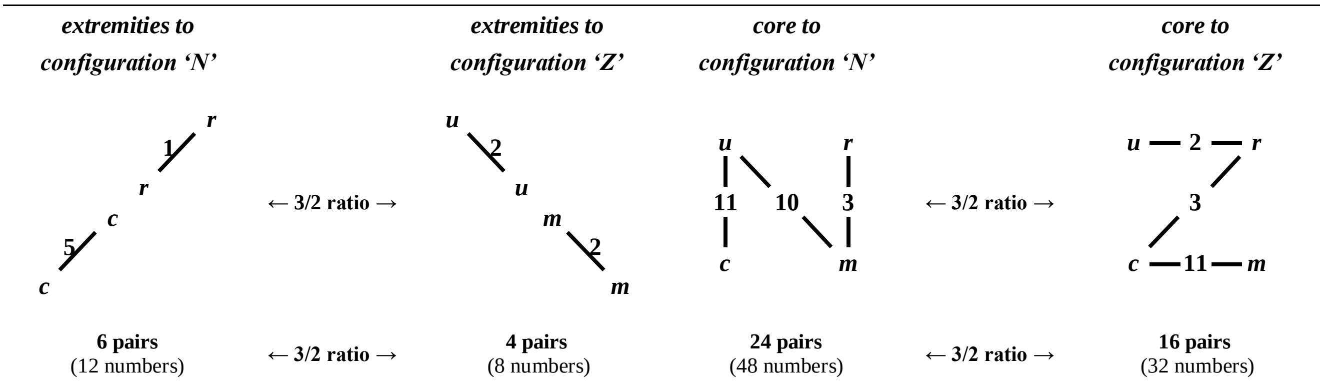
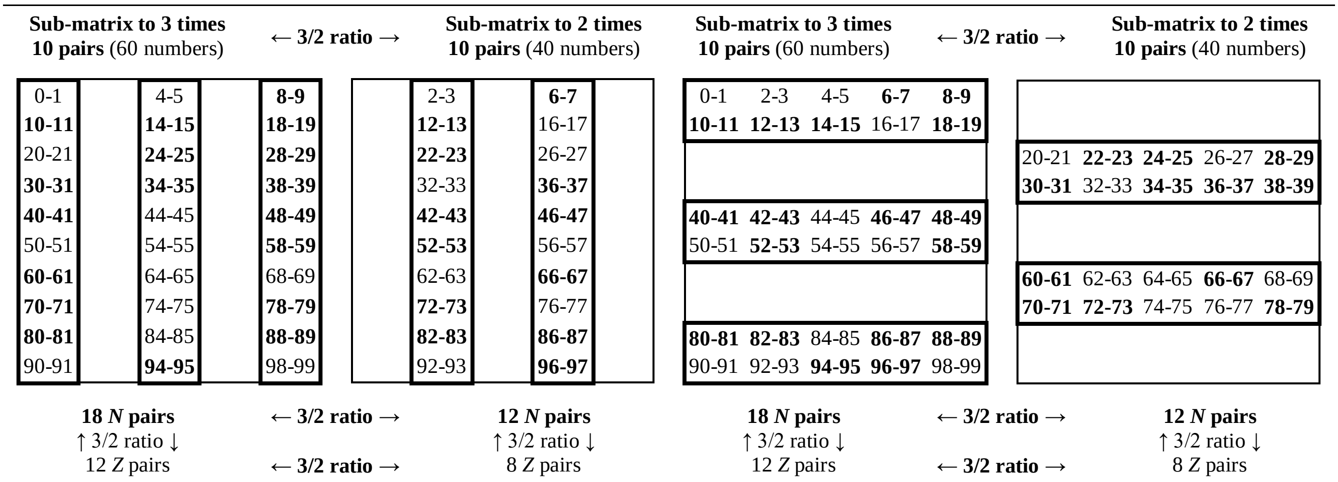


Figure 43: Maintaining of the 3:2 ratio in the protuberances (associations of entities of the same nature) and the cores (associations of entities of different natures) in the *N* and *Z* configurations of number pairs. (See also Figure 42).

10.3 Associations of classes of numbers and 3:2 transcendent ratios



0-1		4-5		**8-9**
10-11		**14-15**		**18-19**
20-21		**24-25**		**28-29**
30-31		**34-35**		**38-39**
40-41		44-45		**48-49**
50-51		54-55		**58-59**
60-61		64-65		68-69
70-71		74-75		**78-79**
80-81		84-85		**88-89**
90-91		**94-95**		98-99

	2-3		**6-7**	
	12-13		16-17	
	22-23		26-27	
	32-33		**36-37**	
	42-43		**46-47**	
	52-53		56-57	
	62-63		**66-67**	
	72-73		76-77	
	82-83		**86-87**	
	92-93		**96-97**	

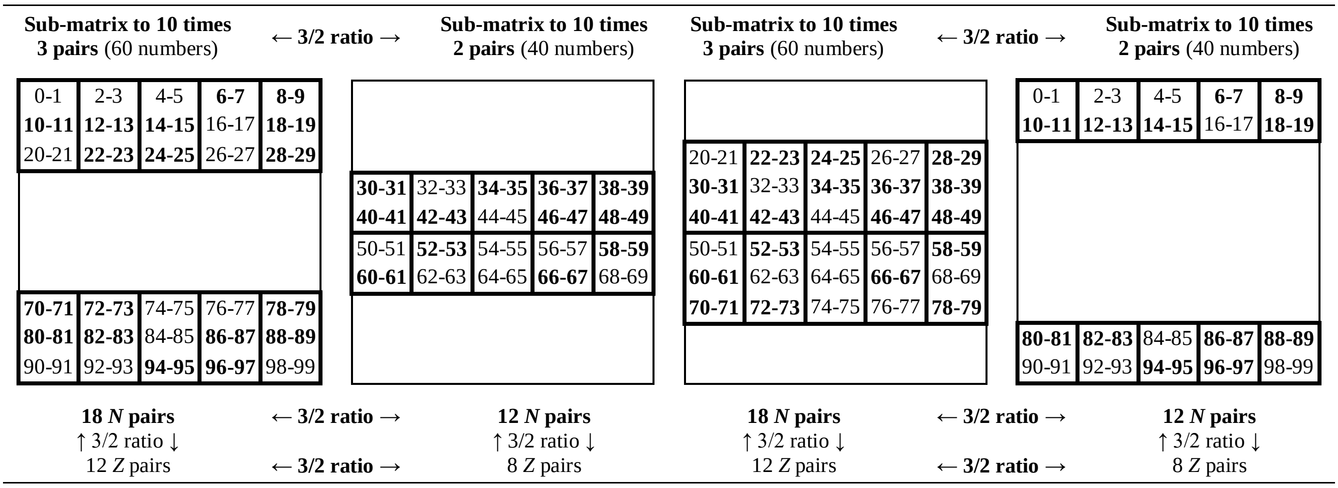
0-1	2-3	4-5	**6-7**	**8-9**
10-11	**12-13**	**14-15**	16-17	**18-19**
40-41	**42-43**	44-45	**46-47**	**48-49**
50-51	**52-53**	54-55	56-57	**58-59**
80-81	**82-83**	84-85	**86-87**	**88-89**
90-91	92-93	**94-95**	**96-97**	98-99

20-21	**22-23**	**24-25**	26-27	**28-29**
30-31	32-33	**34-35**	**36-37**	**38-39**
60-61	62-63	64-65	**66-67**	68-69
70-71	**72-73**	74-75	76-77	**78-79**

Figure 44: Distribution of pairs of ***configuration N*** (***r-r r-m m-u u-c c-c***) and of *configuration Z* (*u-u u-r r-c c-m m-m*) in vertical and horizontal sub-matrices to 30 vs. 20 pairs of adjacent numbers.

Inside two vertical sub-matrices opposing sets of 3 times 10 pairs and sets of 2 times 10 pairs as presented in Figure 44, the pairs of numbers of *configuration N* and those of *configuration Z* are opposed in transcendent 3:2 ratios. In these configurations, 18 *N* pairs are simultaneously opposed to 12 *Z* pairs and 12 other *N* pairs and 8 *Z* pairs are simultaneously opposed to 12 *N* pairs and 12 other *Z* pairs. This could just be by chance, but the fact that exactly the same phenomena occur in identical horizontal configurations (right part of Figure 44) suggests otherwise.

In this matrix of the first hundred numbers, the same arithmetic phenomena appear, Figure 45, both in the opposition of the 30 vertically peripheral pairs to the 20 vertically central pairs as well as in the opposition of the 20 peripheral couples to the 30 central pairs.



0-1	2-3	4-5	**6-7**	**8-9**
10-11	**12-13**	**14-15**	16-17	**18-19**
20-21	**22-23**	**24-25**	26-27	**28-29**
70-71	**72-73**	74-75	76-77	**78-79**
80-81	**82-83**	84-85	**86-87**	**88-89**
90-91	92-93	**94-95**	**96-97**	98-99

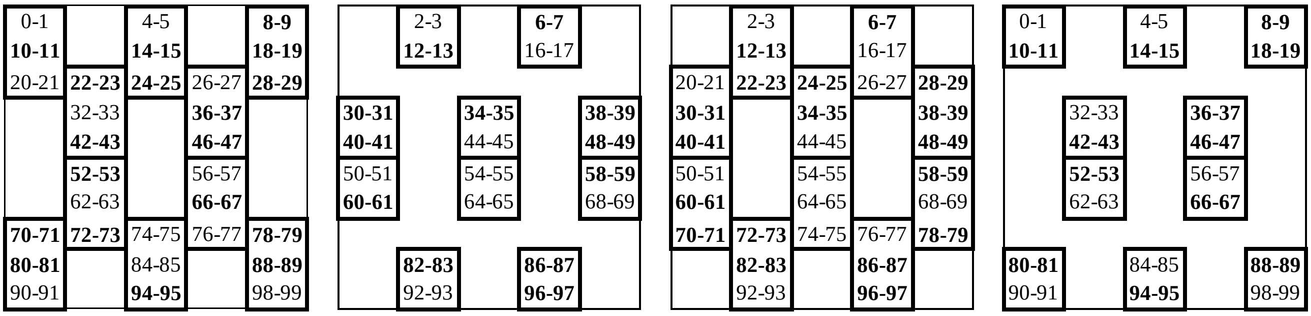
30-31	32-33	**34-35**	**36-37**	**38-39**
40-41	**42-43**	44-45	**46-47**	**48-49**
50-51	**52-53**	54-55	56-57	**58-59**
60-61	62-63	64-65	**66-67**	68-69

20-21	**22-23**	**24-25**	26-27	**28-29**
30-31	32-33	**34-35**	**36-37**	**38-39**
40-41	**42-43**	44-45	**46-47**	**48-49**
50-51	**52-53**	54-55	56-57	**58-59**
60-61	62-63	64-65	**66-67**	68-69
70-71	**72-73**	74-75	76-77	**78-79**

0-1	2-3	4-5	**6-7**	**8-9**
10-11	**12-13**	**14-15**	16-17	**18-19**
80-81	**82-83**	84-85	**86-87**	**88-89**
90-91	92-93	**94-95**	**96-97**	98-99

Figure 45: Distribution of pairs of ***configuration N*** (***r-r r-m m-u u-c c-c***) and of *configuration Z* (*u-u u-r r-c c-m m-m*) in horizontal sub-matrices to 30 vs. 20 pairs of adjacent numbers.

Also, these same arithmetic phenomena still occur in configurations of more diluted alternate sub-matrices such as those illustrated in Figure 46.



Sub-matrix to 10 times 3 pairs (60 numbers) **← 3/2 ratio →** **Sub-matrix to 10 times 2 pairs** (40 numbers) | **Sub-matrix to 10 times 3 pairs** (60 numbers) **← 3/2 ratio →** **Sub-matrix to 10 times 2 pairs** (40 numbers)

0-1		4-5		**8-9**
10-11		**14-15**		**18-19**
20-21	**22-23**	**24-25**	26-27	**28-29**
	32-33		**36-37**	
	42-43		**46-47**	
	52-53		56-57	
	62-63		**66-67**	
70-71	**72-73**	74-75	76-77	**78-79**
80-81		84-85		**88-89**
90-91		**94-95**		98-99

	2-3		**6-7**	
	12-13		16-17	
30-31		**34-35**		**38-39**
40-41		44-45		**48-49**
50-51		54-55		**58-59**
60-61		64-65		68-69
	82-83		**86-87**	
	92-93		**96-97**	

	2-3		**6-7**	
	12-13		16-17	
20-21	**22-23**	**24-25**	26-27	**28-29**
30-31		**34-35**		**38-39**
40-41		44-45		**48-49**
50-51		54-55		**58-59**
60-61		64-65		68-69
70-71	**72-73**	74-75	76-77	**78-79**
	82-83		**86-87**	
	92-93		**96-97**	

0-1		4-5		**8-9**
10-11		**14-15**		**18-19**
	32-33		**36-37**	
	42-43		**46-47**	
	52-53		56-57	
	62-63		**66-67**	
80-81		84-85		**88-89**
90-91		**94-95**		98-99

18 *N* pairs ↑ 3/2 ratio ↓ 12 *Z* pairs **← 3/2 ratio →** **12 *N* pairs** ↑ 3/2 ratio ↓ 8 *Z* pairs | **18 *N* pairs** ↑ 3/2 ratio ↓ 12 *Z* pairs **← 3/2 ratio →** **12 *N* pairs** ↑ 3/2 ratio ↓ 8 *Z* pairs

Figure 46: Distribution of pairs of ***configuration N*** (***r-r r-m m-u u-c c-c***) and of *configuration Z* (*u-u u-r r-c c-m m-m*) in symmetrical sub-matrices to 30 vs. 20 pairs of adjacent numbers.

In these oppositions of pairs of adjacent numbers, The four values are organized into a variant of the remarkable identity:

$$(a + b)^2 = a^2 + 2ab + b^2$$

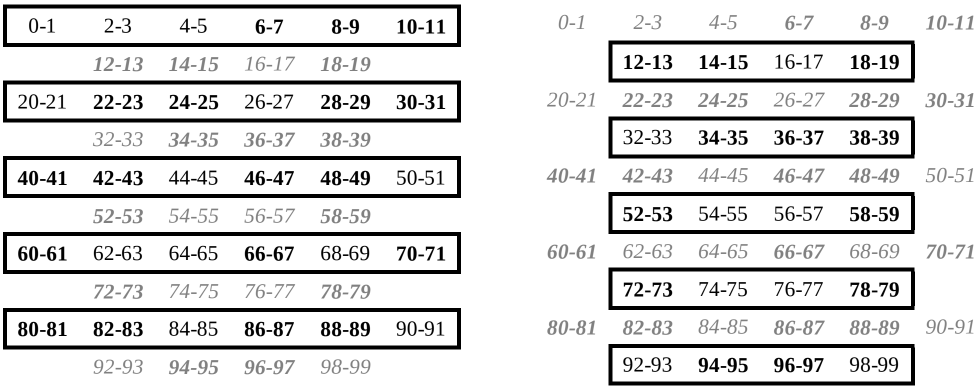
where *a* and *b* have 3 and 2 as original values. For this matrix of 50 pairs of entities, these values are double those of the original remarkable identity, namely:

$$2(a^2) + 2(2ab) + 2(b^2) \rightarrow 18 + 12 + 12 + 8$$

The different classes of whole numbers are thus once again organized in closed matrices within this unique arithmetic system. This organization is also highlighted in the article *"The ultimate numbers and the 3/2 ratio, just two primary sets of whole number"* [1] concerning the introduction of the concept of ultimate numbers.

10.3.1 Alternating sub-matrices

As previously suggested in Chapter 9.3.1 (and illustrated in Figures 30 and 31) for a different matrix, we now examine two sub-matrices with alternating configurations. The first consists of 5 rows and 6 columns of number pairs, and the second consists of 5 rows and 4 columns of number pairs. These sub-matrices are derived from the matrix of the first 50 number pairs that we studied previously.



Sub-matrix to 5 times 6 pairs (60 numbers) **← 3/2 ratio →** **Sub-matrix to 5 times 4 pairs** (40 numbers)

0-1	2-3	4-5	**6-7**	**8-9**	**10-11**
	12-13	***14-15***	*16-17*	***18-19***	
20-21	**22-23**	**24-25**	26-27	**28-29**	**30-31**
	32-33	***34-35***	***36-37***	***38-39***	
40-41	**42-43**	44-45	**46-47**	**48-49**	50-51
	52-53	*54-55*	*56-57*	***58-59***	
60-61	62-63	64-65	**66-67**	68-69	**70-71**
	72-73	*74-75*	*76-77*	***78-79***	
80-81	**82-83**	84-85	**86-87**	**88-89**	90-91
	92-93	***94-95***	***96-97***	*98-99*	

0-1	*2-3*	*4-5*	***6-7***	***8-9***	***10-11***
	12-13	**14-15**	16-17	**18-19**	
20-21	***22-23***	***24-25***	*26-27*	***28-29***	***30-31***
	32-33	**34-35**	**36-37**	**38-39**	
40-41	***42-43***	*44-45*	***46-47***	***48-49***	*50-51*
	52-53	54-55	56-57	**58-59**	
60-61	*62-63*	*64-65*	***66-67***	*68-69*	***70-71***
	72-73	74-75	76-77	**78-79**	
80-81	***82-83***	*84-85*	***86-87***	***88-89***	*90-91*
	92-93	**94-95**	**96-97**	98-99	

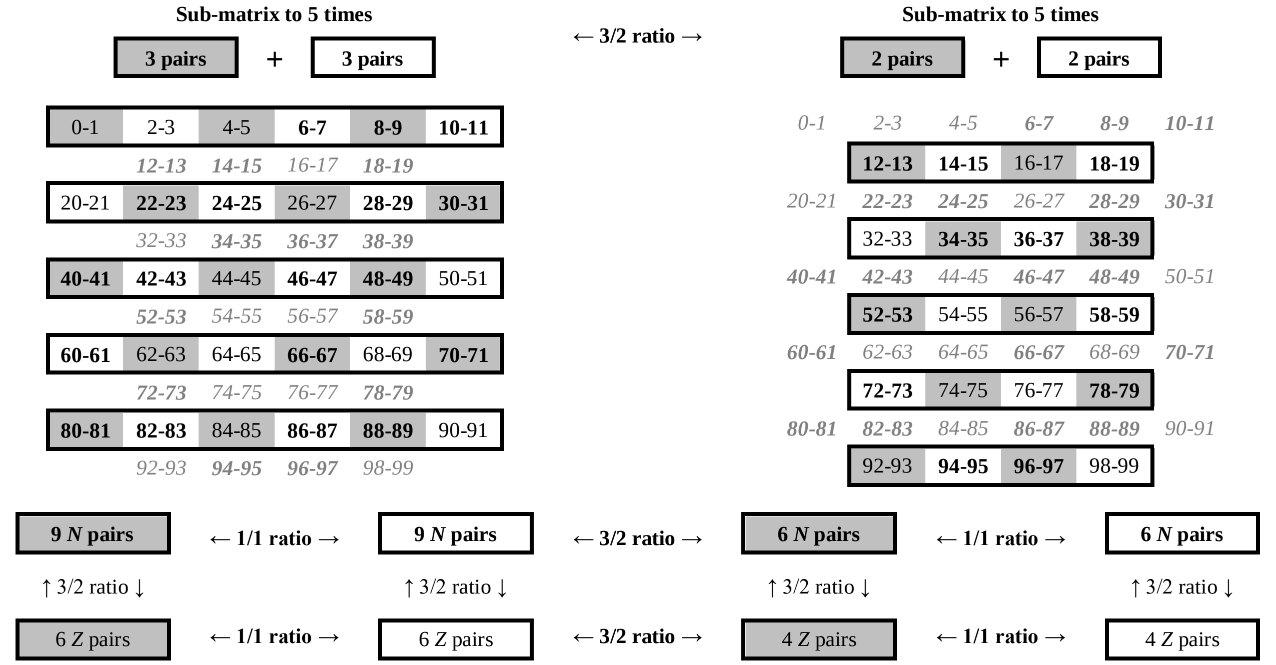
18 *N* pairs ↑ 3/2 ratio ↓ 12 *Z* pairs **← 3/2 ratio →** **12 *N* pairs** ↑ 3/2 ratio ↓ 8 *Z* pairs

← 3/2 ratio →

Figure 47: Distribution of pairs of ***configuration N*** (***r-r r-m m-u u-c c-c***) and of *configuration Z* (*u-u u-r r-c c-m m-m*) in alternating sub-matrices to 30 versus 20 pairs of adjacent numbers.

In this new alternating configuration illustrated Figure 47, and in connection to the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$, once again 18 *N* pairs are simultaneously opposed to 12 *Z* pairs and 12 other *N* pairs and 8 *Z* pairs are simultaneously opposed to 12 *N* pairs and 12 other *Z* pairs.

Also, as shown in Figure 48, a more advanced alternation creating four sub-matrices, where the pairs of numbers are all regularly isolated, reveals the same distribution of the two blocks of configuration *N* and configuration *Z* in the remarkable identity.



Sub-matrix to 5 times | **← 3/2 ratio →** | **Sub-matrix to 5 times**

3 pairs + **3 pairs** | **2 pairs** + **2 pairs**

0-1	2-3	4-5	**6-7**	**8-9**	**10-11**
	12-13	***14-15***	*16-17*	***18-19***	
20-21	**22-23**	**24-25**	26-27	**28-29**	**30-31**
	32-33	***34-35***	***36-37***	***38-39***	
40-41	**42-43**	44-45	**46-47**	**48-49**	50-51
	52-53	*54-55*	*56-57*	***58-59***	
60-61	62-63	64-65	**66-67**	68-69	**70-71**
	72-73	*74-75*	*76-77*	***78-79***	
80-81	**82-83**	84-85	**86-87**	**88-89**	90-91
	92-93	***94-95***	***96-97***	*98-99*	

0-1	*2-3*	*4-5*	***6-7***	***8-9***	***10-11***
	12-13	**14-15**	16-17	**18-19**	
20-21	***22-23***	***24-25***	*26-27*	***28-29***	***30-31***
	32-33	**34-35**	**36-37**	**38-39**	
40-41	***42-43***	*44-45*	***46-47***	***48-49***	*50-51*
	52-53	54-55	56-57	**58-59**	
60-61	*62-63*	*64-65*	***66-67***	*68-69*	***70-71***
	72-73	74-75	76-77	**78-79**	
80-81	***82-83***	*84-85*	***86-87***	***88-89***	*90-91*
	92-93	**94-95**	**96-97**	98-99	

9 *N* pairs **← 1/1 ratio →** **9 *N* pairs** **← 3/2 ratio →** **6 *N* pairs** **← 1/1 ratio →** **6 *N* pairs**

↑ 3/2 ratio ↓ ↑ 3/2 ratio ↓ ↑ 3/2 ratio ↓ ↑ 3/2 ratio ↓

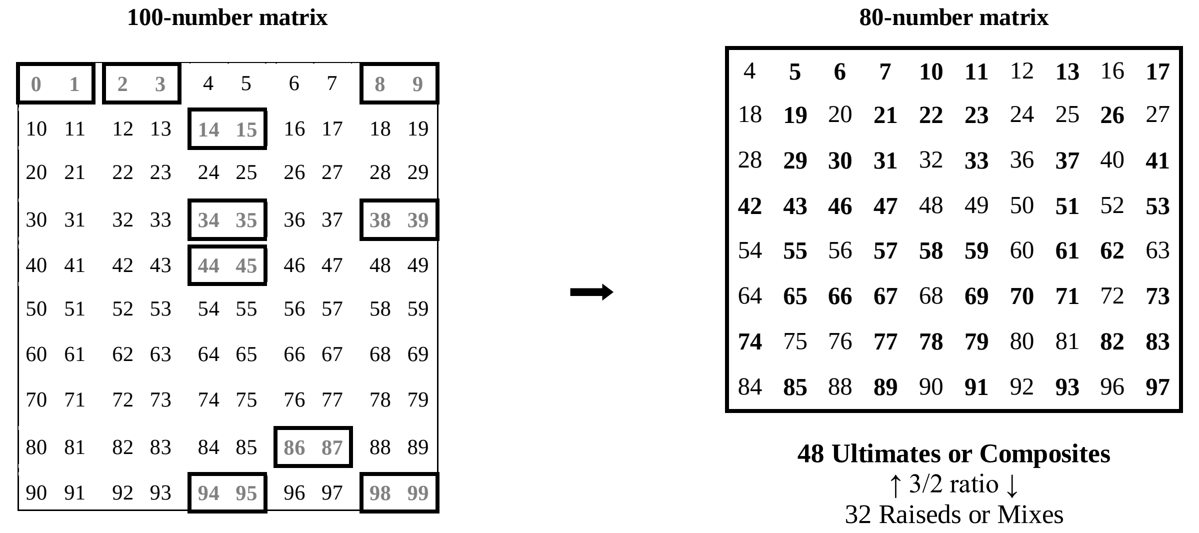
6 *Z* pairs **← 1/1 ratio →** 6 *Z* pairs **← 3/2 ratio →** 4 *Z* pairs **← 1/1 ratio →** 4 *Z* pairs

Figure 48: Distribution of pairs of ***configuration N*** (***r-r r-m m-u u-c c-c***) and of *configuration Z* (*u-u u-r r-c c-m m-m*) in alternated sub-matrices to two times 15 versus 10 pairs of adjacent numbers.

11. From 100-number to 80-number matrix

As we have just demonstrated in the previous chapter (see Figures 41, 42 and 43), among the 50 pairs of numbers in the matrix of the first 100 numbers, there are 10 pairs of the same number class: 2 pairs *u-u*, 1 pair *r-r*, 5 pairs *c-c* and 2 pairs *m-m*.

Based on this matrix of the first 100 numbers, we propose to study an 80-number matrix. This is achieved by subtracting 10 pairs of numbers from the same classes from the original set. We exclusively consider pairs of numbers where the first entity is even (e.g., 0-1, 2-3, etc., but not 1-2, 3-4, etc.).



100-number matrix

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

➡

80-number matrix

4	**5**	**6**	**7**	**10**	**11**	12	**13**	16	**17**
18	**19**	20	**21**	**22**	**23**	24	25	**26**	27
28	**29**	**30**	**31**	32	**33**	36	**37**	40	**41**
42	**43**	**46**	**47**	48	49	50	**51**	52	**53**
54	**55**	56	**57**	**58**	**59**	60	**61**	**62**	63
64	**65**	**66**	**67**	68	**69**	**70**	**71**	72	**73**
74	75	76	**77**	**78**	**79**	80	81	**82**	**83**
84	**85**	88	**89**	90	**91**	92	**93**	96	**97**

48 Ultimates or Composites
↑ 3/2 ratio ↓
32 Raiseds or Mixes

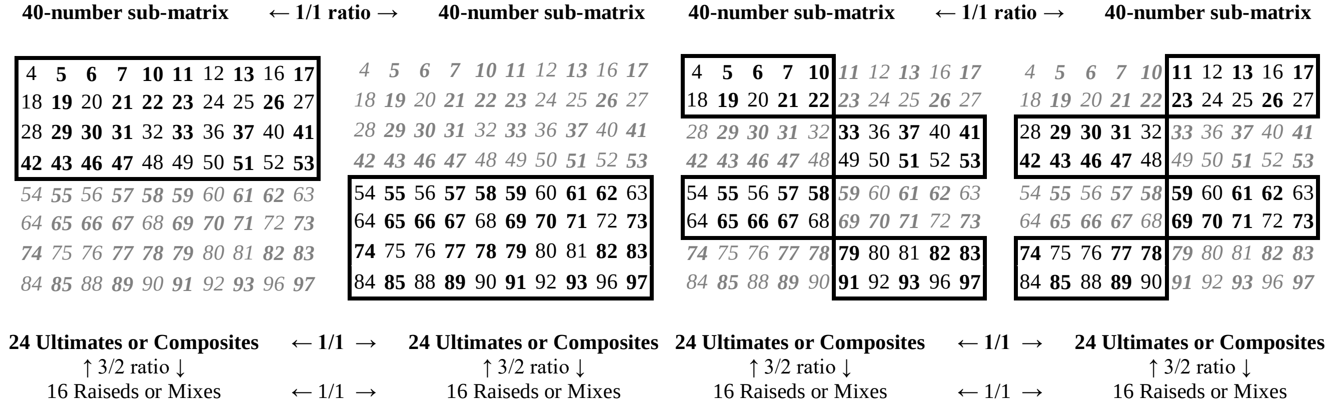
Figure 49: From the 100-number matrix, construction of a 80-number matrix by subtraction of the 10 pairs of numbers from the same classes.

Doing this, as shown in Figure 49, the 10-by-8 matrix appears to consist of 48 Ultimates and Composites versus 32 Raiseds and Mixes, forming an exact 3:2 ratio.

In this new matrix of 8 rows and 10 columns, these two double classes of numbers seem to be systematically opposed in consistently perfect 3:2 ratios, across numerous symmetrical configurations including two sets of 40 numbers and even four sets of 20 numbers.

11.1 Symmetrical 40-number vs. 40-number sub-matrix

As illustrated in Figure 50, the two double classes of predefined numbers oppose each other in an exact 3:2 ratio within the sub-matrices of the first 40 numbers and the next 40 numbers. A similar phenomenon is observed in the right part of Figure 50, where a symmetrically alternating configuration of two sets of five consecutive numbers also reflects this ratio.



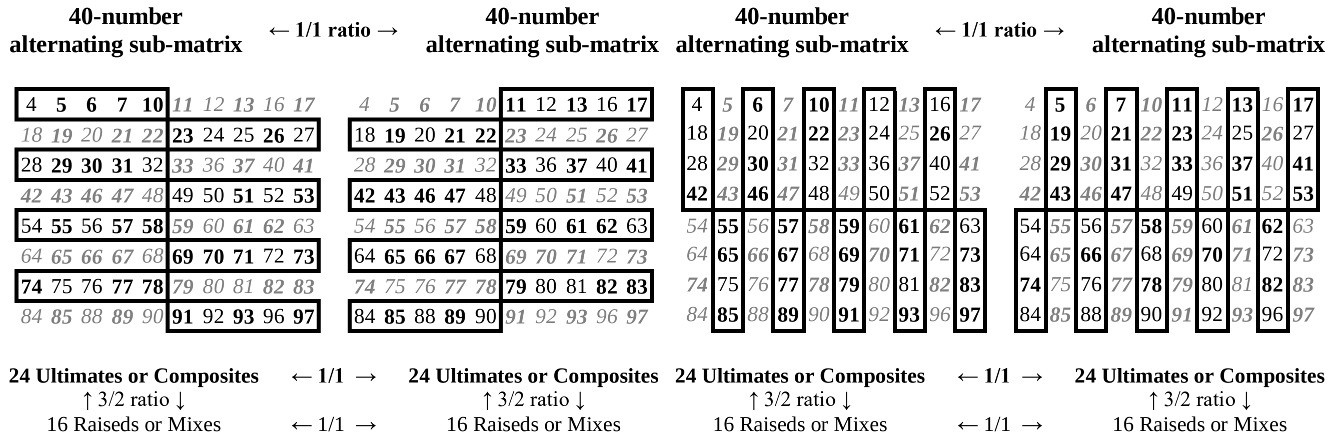
40-number sub-matrix **← 1/1 ratio →** **40-number sub-matrix** **40-number sub-matrix** **← 1/1 ratio →** **40-number sub-matrix**

4	**5**	**6**	**7**	**10**	**11**	12	**13**	16	**17**
18	**19**	20	**21**	**22**	**23**	24	25	**26**	27
28	**29**	**30**	**31**	32	**33**	36	**37**	40	**41**
42	**43**	**46**	**47**	48	49	50	**51**	52	**53**
54	**55**	56	**57**	**58**	**59**	60	**61**	**62**	63
64	**65**	**66**	**67**	68	**69**	**70**	**71**	72	**73**
74	75	76	**77**	**78**	**79**	80	81	**82**	**83**
84	**85**	88	**89**	90	**91**	92	**93**	96	**97**

24 Ultimates or Composites ← 1/1 → **24 Ultimates or Composites** **24 Ultimates or Composites** ← 1/1 → **24 Ultimates or Composites**
↑ 3/2 ratio ↓ ↑ 3/2 ratio ↓ ↑ 3/2 ratio ↓ ↑ 3/2 ratio ↓
16 Raiseds or Mixes ← 1/1 → 16 Raiseds or Mixes 16 Raiseds or Mixes ← 1/1 → 16 Raiseds or Mixes

Figure 50: Distribution of Ultimates or Composites vs. Raiseds or Mixes across two double sub-matrices of the 80-first number matrix without pure pair.

More subtly, the same 3:2 ratio phenomenon always occurs, by alternating all the semi-rows (left part, Figure 51) or all the semi-columns (right part). So, in each configuration, 24 Ultimates or Composites are always opposed to 16 Raiseds or Mixes.



40-number alternating sub-matrix **← 1/1 ratio →** **40-number alternating sub-matrix** **40-number alternating sub-matrix** **← 1/1 ratio →** **40-number alternating sub-matrix**

4	**5**	**6**	**7**	**10**	**11**	12	**13**	16	**17**
18	**19**	20	**21**	**22**	**23**	24	25	**26**	27
28	**29**	**30**	**31**	32	**33**	36	**37**	40	**41**
42	**43**	**46**	**47**	48	49	50	**51**	52	**53**
54	**55**	56	**57**	**58**	**59**	60	**61**	**62**	63
64	**65**	**66**	**67**	68	**69**	**70**	**71**	72	**73**
74	75	76	**77**	**78**	**79**	80	81	**82**	**83**
84	**85**	88	**89**	90	**91**	92	**93**	96	**97**

24 Ultimates or Composites ← 1/1 → **24 Ultimates or Composites** **24 Ultimates or Composites** ← 1/1 → **24 Ultimates or Composites**
↑ 3/2 ratio ↓ ↑ 3/2 ratio ↓ ↑ 3/2 ratio ↓ ↑ 3/2 ratio ↓
16 Raiseds or Mixes ← 1/1 → 16 Raiseds or Mixes 16 Raiseds or Mixes ← 1/1 → 16 Raiseds or Mixes

Figure 51: Distribution of Ultimates or Composites vs. Raiseds or Mixes across two double symmetrical sub-matrices of the 80-first number matrix without pure pair.

11. 2 Symmetrical 20-number vs. 20-number sub-matrix

Even more subtly, we now consider symmetric configurations of only 20 numbers. Thus, in the two following tables, we construct sub-matrices resulting from the mixing of the fine configurations presented in Figure 51 with the larger configuration introduced in the right portion of Figure 50.

As shown in Figure 52, the hyper-alternation of four semi-rows of five numbers continues to oppose the two groups of predefined classes in exact 3:2 ratios.

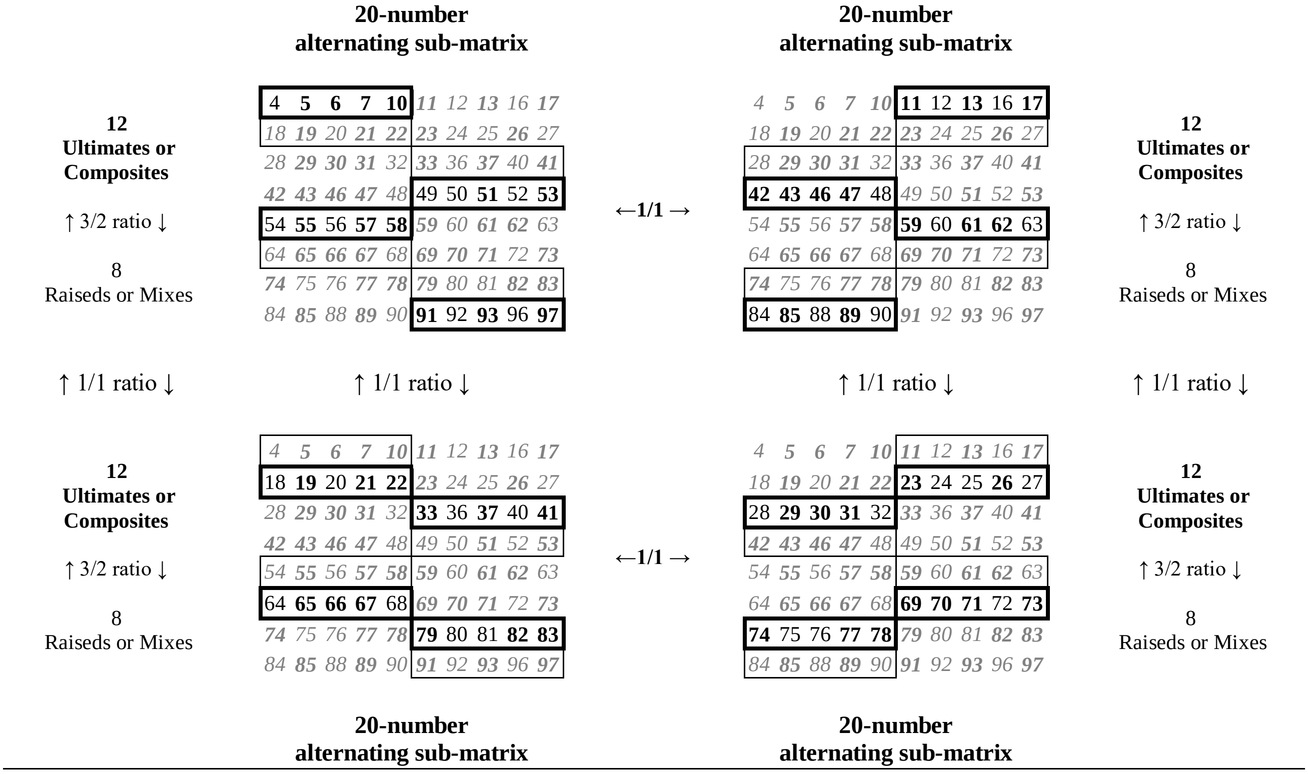


Figure 52: Distribution of Ultimates or Composites vs. Raiseds or Mixes across four symmetrical sub-matrices of the 80-first number matrix without pure pair.

Figure 53, the hyper alternating of ten semi-columns of two numbers continues to oppose the two groups of predefined classes in exact 3:2 ratios.

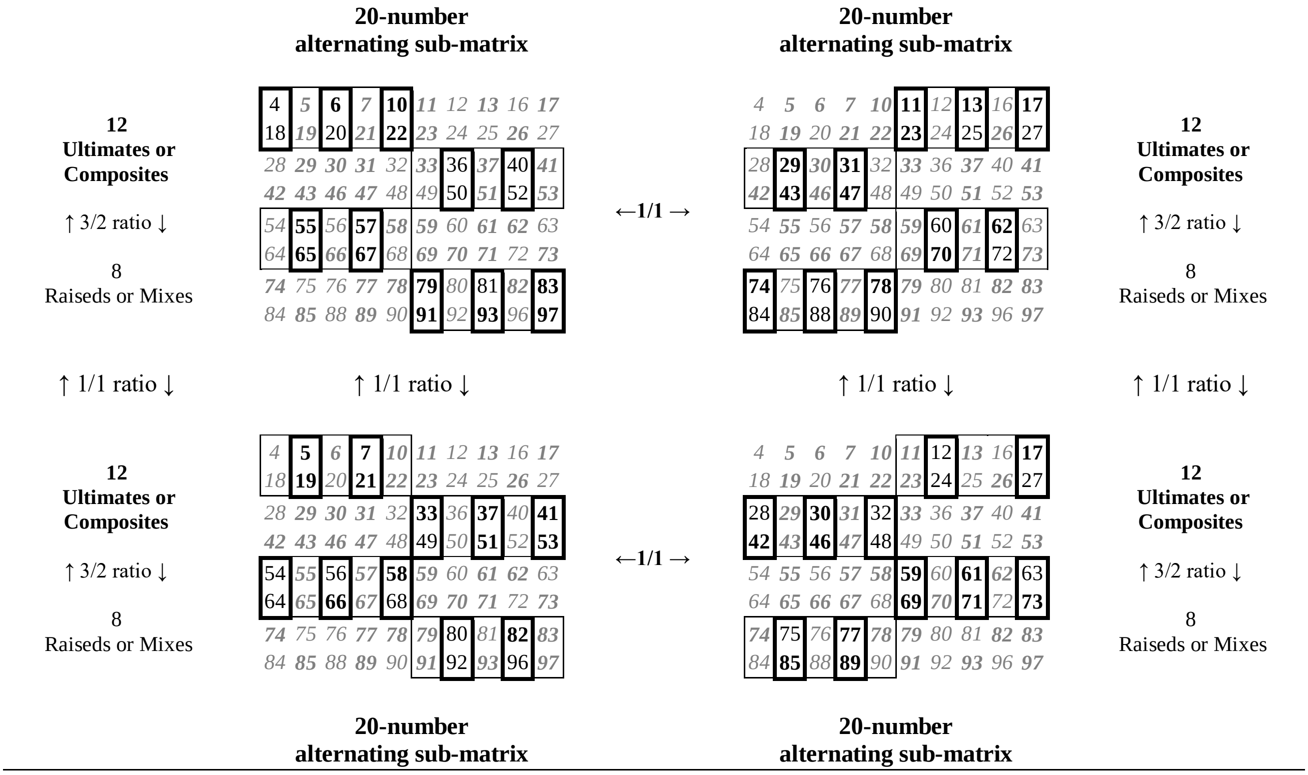
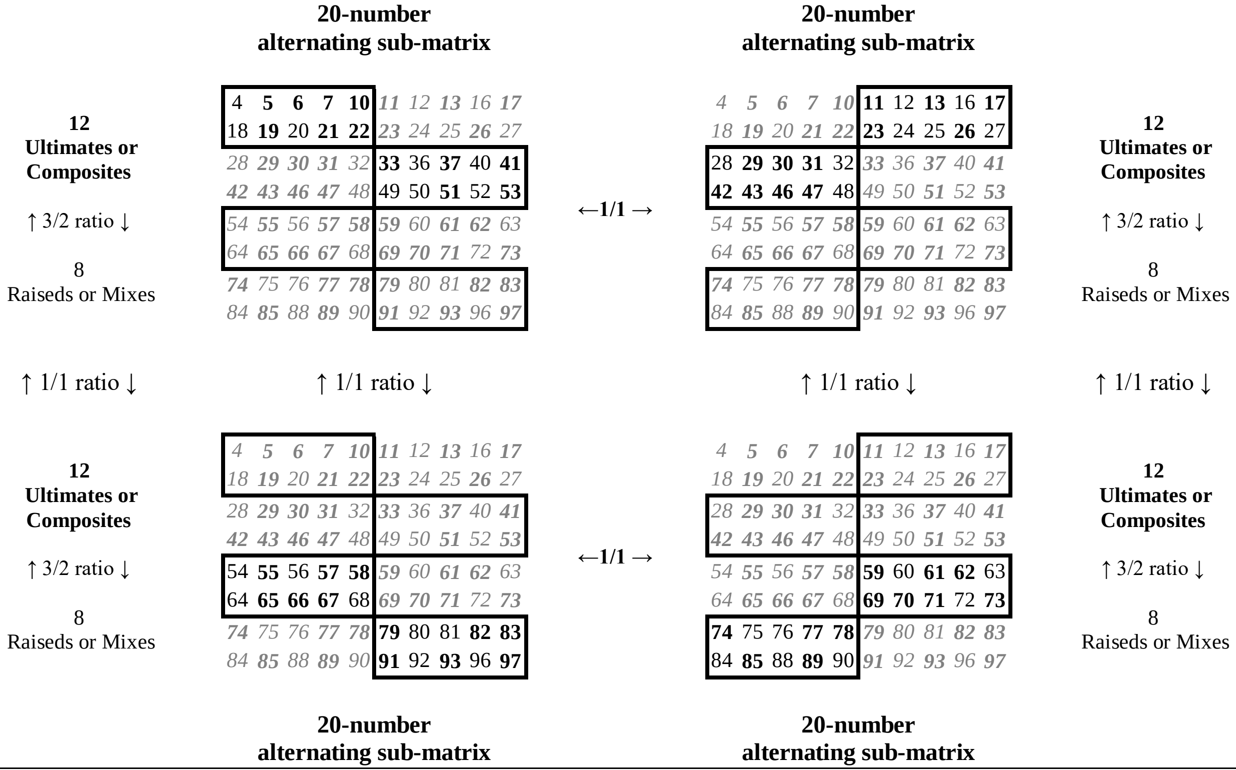


Figure 53: Distribution of Ultimates or Composites vs. Raiseds or Mixes across four symmetrical sub-matrices of the 80-first number matrix without pure pair.

Finally, Figure 54, the mix of the two initial configurations presented in Figure 51 continues to oppose these two category of number in these same 3:2 ratios with always 12 Ultimates or Composites vs. 8 Raiseds or Mixes.



4	**5**	**6**	**7**	**10**	***11***	*12*	***13***	*16*	***17***
18	**19**	20	**21**	**22**	***23***	*24*	*25*	***26***	*27*
28	***29***	***30***	***31***	*32*	**33**	36	**37**	40	**41**
42	***43***	***46***	***47***	*48*	49	50	**51**	52	**53**
54	***55***	*56*	***57***	***58***	***59***	*60*	***61***	***62***	*63*
64	***65***	***66***	***67***	*68*	***69***	***70***	***71***	*72*	***73***
74	*75*	*76*	***77***	***78***	***79***	*80*	*81*	***82***	***83***
84	***85***	*88*	***89***	*90*	***91***	*92*	***93***	*96*	***97***

4	***5***	***6***	***7***	***10***	**11**	12	**13**	16	**17**
18	***19***	*20*	***21***	***22***	**23**	24	25	**26**	27
28	**29**	**30**	**31**	32	***33***	*36*	***37***	*40*	***41***
42	**43**	**46**	**47**	48	*49*	*50*	***51***	*52*	***53***
54	***55***	*56*	***57***	***58***	***59***	*60*	***61***	***62***	*63*
64	***65***	***66***	***67***	*68*	***69***	***70***	***71***	*72*	***73***
74	*75*	*76*	***77***	***78***	***79***	*80*	*81*	***82***	***83***
84	***85***	*88*	***89***	*90*	***91***	*92*	***93***	*96*	***97***

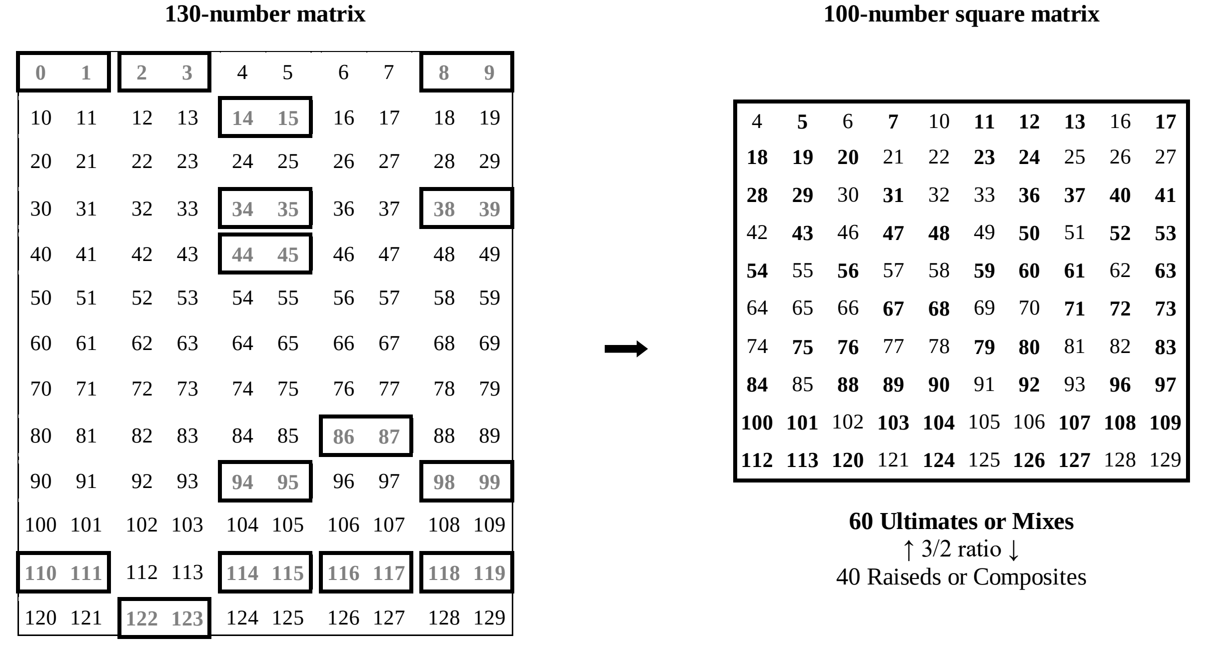
4	***5***	***6***	***7***	***10***	***11***	*12*	***13***	*16*	***17***
18	***19***	*20*	***21***	***22***	***23***	*24*	*25*	***26***	*27*
28	***29***	***30***	***31***	*32*	***33***	*36*	***37***	*40*	***41***
42	***43***	***46***	***47***	*48*	*49*	*50*	***51***	*52*	***53***
54	**55**	56	**57**	**58**	***59***	*60*	***61***	***62***	*63*
64	**65**	**66**	**67**	68	***69***	***70***	***71***	*72*	***73***
74	*75*	*76*	***77***	***78***	**79**	80	81	**82**	**83**
84	***85***	*88*	***89***	*90*	**91**	92	**93**	96	**97**

4	***5***	***6***	***7***	***10***	***11***	*12*	***13***	*16*	***17***
18	***19***	*20*	***21***	***22***	***23***	*24*	*25*	***26***	*27*
28	***29***	***30***	***31***	*32*	***33***	*36*	***37***	*40*	***41***
42	***43***	***46***	***47***	*48*	*49*	*50*	***51***	*52*	***53***
54	***55***	*56*	***57***	***58***	**59**	60	**61**	**62**	63
64	***65***	***66***	***67***	*68*	**69**	**70**	**71**	72	**73**
74	75	76	**77**	**78**	***79***	*80*	*81*	***82***	***83***
84	**85**	88	**89**	90	***91***	*92*	***93***	*96*	***97***

Figure 54: Distribution of Ultimates or Composites vs. Raiseds or Mixes across four symmetrical sub-matrices of the 80-first number matrix without pure pair.

12. Recombined 100-number square matrix

After subtracting the 10 pairs of same-class numbers from the matrix of the first 100 whole numbers, we reconstruct a new 100-number square matrix. We achieve this by adding 20 new numbers to the sequence of whole numbers, while continuously ignoring pairs of numbers belonging to the same class. As before, we exclusively consider number pairs where the first entity is even (e.g., 0-1, 2-3, etc.).It turns out that to finalize this new matrix, we must add the next 30 numbers beyond 99 and subtract 5 pairs of numbers (i.e., ten numbers in total). Consequently, the last number in this recombined matrix is 129 (the 130th whole number). We note here that the number of even numbers to be subtracted is equal to 5*x* (where *x*=1), a recurring value in this study of whole numbers.



0	*1*	*2*	*3*	4	5	6	7	*8*	*9*
10	11	12	13	*14*	*15*	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	*34*	*35*	36	37	*38*	*39*
40	41	42	43	*44*	*45*	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	*86*	*87*	88	89
90	91	92	93	*94*	*95*	96	97	*98*	*99*
100	101	102	103	104	105	106	107	108	109
110	*111*	112	113	*114*	*115*	*116*	*117*	*118*	*119*
120	121	*122*	*123*	124	125	126	127	128	129

➡

4	**5**	6	**7**	10	**11**	**12**	**13**	16	**17**
18	**19**	**20**	21	22	**23**	**24**	25	26	27
28	**29**	30	**31**	32	33	**36**	**37**	**40**	**41**
42	**43**	46	**47**	**48**	49	**50**	51	**52**	**53**
54	55	**56**	57	58	**59**	**60**	**61**	62	**63**
64	65	66	**67**	**68**	69	70	**71**	**72**	**73**
74	**75**	**76**	77	78	**79**	**80**	81	82	**83**
84	85	**88**	**89**	**90**	91	**92**	93	**96**	**97**
100	**101**	102	**103**	**104**	105	106	**107**	**108**	**109**
112	**113**	**120**	121	**124**	125	**126**	**127**	128	129

Figure 55: From the 130-first number matrix, construction of a 100-number square matrix by subtraction of the 15 pairs of same classes numbers.

As shown in Figure 55, the two classes qualified as extreme and the two qualified as median still oppose each other in a perfect 3:2 ratio since this square matrix contains 60 Ultimates or Mixes vs. 40 Raiseds or Composites.

12.1 Recombined 100-number square matrix partitions

In this new 100-number square matrix, the two qualified extreme and median class groups are distributed non-randomly into various sub-matrices of configuration significantly defined according to their size.

60 Ultimates or Mixes (60 extremes)
↑ 3/2 ratio ↓
40 Raiseds or Composites (40 medians)

4	**5**	6	**7**	10	**11**	**12**	**13**	16	**17**
18	**19**	**20**	21	22	**23**	**24**	25	26	27
28	**29**	30	**31**	32	33	**36**	**37**	**40**	**41**
42	**43**	46	**47**	**48**	49	**50**	51	**52**	**53**
54	55	**56**	57	58	**59**	**60**	**61**	62	**63**
64	65	66	**67**	**68**	69	70	**71**	**72**	**73**
74	**75**	**76**	77	78	**79**	**80**	81	82	**83**
84	85	**88**	**89**	**90**	91	**92**	93	**96**	**97**
100	**101**	102	**103**	**104**	105	106	**107**	**108**	**109**
112	**113**	**120**	121	**124**	125	**126**	**127**	128	129

24 Ultimates or Mixes
↑ 3/2 ratio ↓
16 Raiseds or Composites

36 Ultimates or Mixes
↑ 3/2 ratio ↓
24 Raiseds or Composites

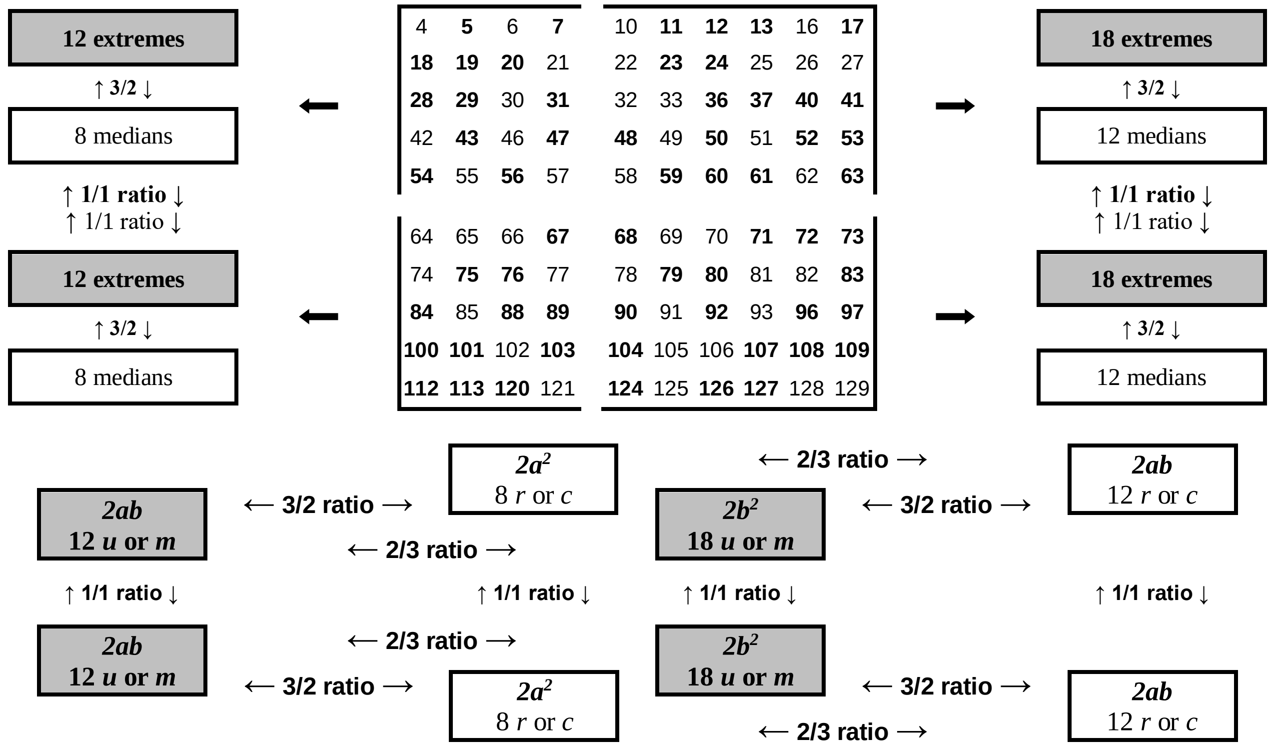
30 Ultimates or Mixes
↑ 3/2 ratio ↓
20 Raiseds or Composites

4	**5**	6	**7**	10	**11**	**12**	**13**	16	**17**
18	**19**	**20**	21	22	**23**	**24**	25	26	27
28	**29**	30	**31**	32	33	**36**	**37**	**40**	**41**
42	**43**	46	**47**	**48**	49	**50**	51	**52**	**53**
54	55	**56**	57	58	**59**	**60**	**61**	62	**63**
64	65	66	**67**	**68**	69	70	**71**	**72**	**73**
74	**75**	**76**	77	78	**79**	**80**	81	82	**83**
84	85	**88**	**89**	**90**	91	**92**	93	**96**	**97**
100	**101**	102	**103**	**104**	105	106	**107**	**108**	**109**
112	**113**	**120**	121	**124**	125	**126**	**127**	128	129

30 Ultimates or Mixes
↑ 3/2 ratio ↓
20 Raiseds or Composites

Figure 56: Based of the 100-number recombined matrix, sub-matrices of size 2:3 or 1:1 with oppositions of extremes and medians in 3:2 ratios.

Thus, as shown in Figure 56, a partition of this square matrix into two vertical sub-matrices of respective size 2:3 generates a distribution of the two predefined class groups in still an exact 3:2 ratio: 24 extremes vs. 16 medians and 36 extremes vs. 24 medians. Also a horizontal partition into two sub-matrices of equal size continues to generate the same phenomena: two times 30 extremes vs. 20 medians.

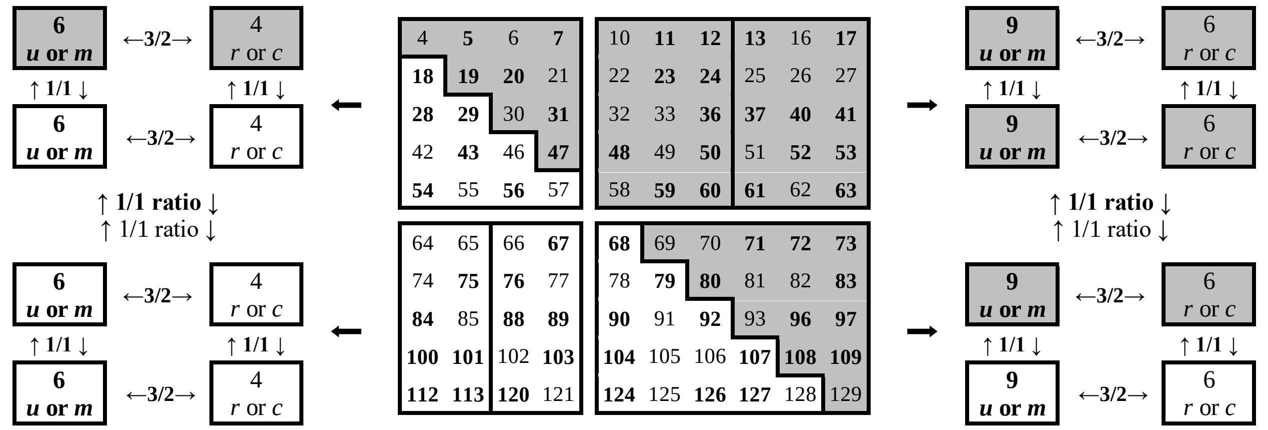


4	**5**	6	**7**	10	**11**	**12**	**13**	16	**17**
18	**19**	**20**	21	22	**23**	**24**	25	26	27
28	**29**	30	**31**	32	33	**36**	**37**	**40**	**41**
42	**43**	46	**47**	**48**	49	**50**	51	**52**	**53**
54	55	**56**	57	58	**59**	**60**	**61**	62	**63**
64	65	66	**67**	**68**	69	70	**71**	**72**	**73**
74	**75**	**76**	77	78	**79**	**80**	81	82	**83**
84	85	**88**	**89**	**90**	91	**92**	93	**96**	**97**
100	**101**	102	**103**	**104**	105	106	**107**	**108**	**109**
112	**113**	**120**	121	**124**	125	**126**	**127**	128	129

Figure 57: From the 100-number matrix, two twin asymmetrical 20-number vs. 30-number sub-matrices . Distribution of extremes and medians numbers in 3:2 ratios and in the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$.

Mixing these two configurations produces four sub-matrices of sizes 2*x* and 3*x* where the numbers of extreme classes and those of median classes always continue to oppose each other in perfect 3:2 ratios. Moreover, as it appears very clearly in Figure 57, the different values of each group of numbers are organized in an entanglement revealing the algebraic mechanism of the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and b have respective value 2 and 3.

A further partition of this matrix reveals a very sophisticated organization of the 100 numbers that compose it. Figure 59 shows us other partitions into sub-matrices of different sizes where the extremes and the medians are always opposed in the ratio 3:2. Thus in the triangular sub-matrices of size T_9 and T_{10} (45 vs. 55 numbers), the distribution of extremes and medians remains in the ratio 3:2. In figure 58 we superimpose this triangular configuration (right in Figure 59) with the previous one introduced in Figure 57. By doing this, we create 8 sub-matrices, 4 of which are triangular and 4 rectangular.

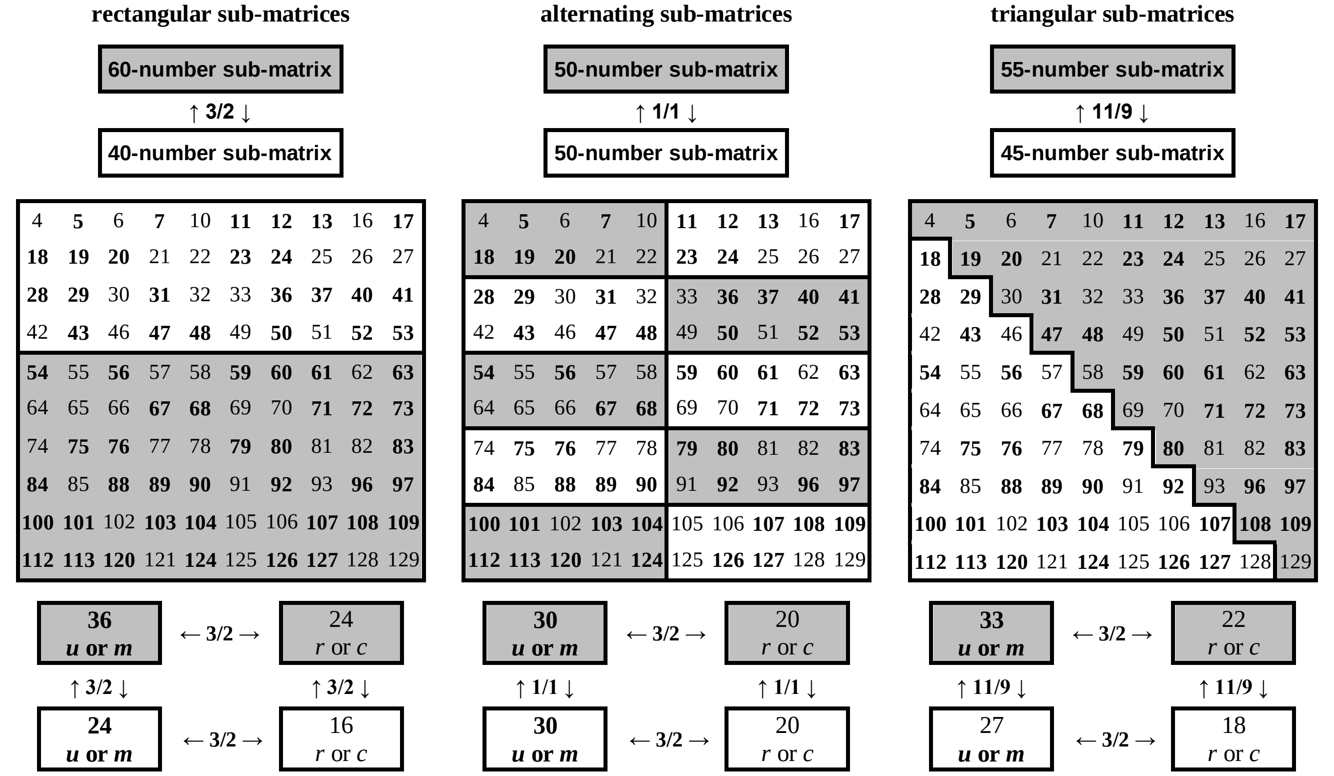


4	**5**	6	**7**	10	**11**	**12**	**13**	16	**17**
18	**19**	**20**	21	22	**23**	**24**	25	26	27
28	**29**	30	**31**	32	33	**36**	**37**	**40**	**41**
42	**43**	46	**47**	**48**	49	**50**	51	**52**	**53**
54	55	**56**	57	58	**59**	**60**	**61**	62	**63**
64	65	66	**67**	**68**	69	70	**71**	**72**	**73**
74	**75**	**76**	77	78	**79**	**80**	81	82	**83**
84	85	**88**	**89**	**90**	91	**92**	93	**96**	**97**
100	**101**	102	**103**	**104**	105	106	**107**	**108**	**109**
112	**113**	**120**	121	**124**	125	**126**	**127**	128	129

Figure 58: From the 100-number matrix, four twin symmetrical 10-number and 15-number sub-matrices . Distribution of extremes and medians numbers in 3:2 ratios and in the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$.

In these 8 sub-matrices, it is remarkable to note that the two groups of numbers considered, namely those that we qualify as extremes and those qualified as medians, are again opposed in an exact ratio of 3:2.

12.2 Other singular partitions of the 100-number square matrix



4	**5**	6	**7**	10	**11**	**12**	**13**	16	**17**
18	**19**	**20**	21	22	**23**	**24**	25	26	27
28	**29**	30	**31**	32	33	**36**	**37**	**40**	**41**
42	**43**	46	**47**	**48**	49	**50**	51	**52**	**53**
54	55	**56**	57	58	**59**	**60**	**61**	62	**63**
64	65	66	**67**	**68**	69	70	**71**	**72**	**73**
74	**75**	**76**	77	78	**79**	**80**	81	82	**83**
84	85	**88**	**89**	**90**	91	**92**	93	**96**	**97**
100	**101**	102	**103**	**104**	105	106	**107**	**108**	**109**
112	**113**	**120**	121	**124**	125	**126**	**127**	128	129

4	**5**	6	**7**	10	**11**	**12**	**13**	16	**17**
18	**19**	**20**	21	22	**23**	**24**	25	26	27
28	**29**	30	**31**	32	33	**36**	**37**	**40**	**41**
42	**43**	46	**47**	**48**	49	**50**	51	**52**	**53**
54	55	**56**	57	58	**59**	**60**	**61**	62	**63**
64	65	66	**67**	**68**	69	70	**71**	**72**	**73**
74	**75**	**76**	77	78	**79**	**80**	81	82	**83**
84	85	**88**	**89**	**90**	91	**92**	93	**96**	**97**
100	**101**	102	**103**	**104**	105	106	**107**	**108**	**109**
112	**113**	**120**	121	**124**	125	**126**	**127**	128	129

4	**5**	6	**7**	10	**11**	**12**	**13**	16	**17**
18	**19**	**20**	21	22	**23**	**24**	25	26	27
28	**29**	30	**31**	32	33	**36**	**37**	**40**	**41**
42	**43**	46	**47**	**48**	49	**50**	51	**52**	**53**
54	55	**56**	57	58	**59**	**60**	**61**	62	**63**
64	65	66	**67**	**68**	69	70	**71**	**72**	**73**
74	**75**	**76**	77	78	**79**	**80**	81	82	**83**
84	85	**88**	**89**	**90**	91	**92**	93	**96**	**97**
100	**101**	102	**103**	**104**	105	106	**107**	**108**	**109**
112	**113**	**120**	121	**124**	125	**126**	**127**	128	129

Figure 59: Three examples of partitioning the predefined 100-number matrix where the extremes and medians are opposed in 3:2 ratios

In numerous other partitions of this 100-number matrix, the entities designated as extremes and those designated as medians also fall into exact 3:2 ratios. Figure 59 presents significant examples of this singular phenomenon. Thus, whether through a rectangular partition (with a 3:2 size ratio), an alternating partition (with a 1:1 ratio), or a more exotic and triangular one with an 11:9 ratio (T_{10} vs. T_9) each generates the same oppositions of numbers, classified as extreme or median, in a 3:2 ratio.

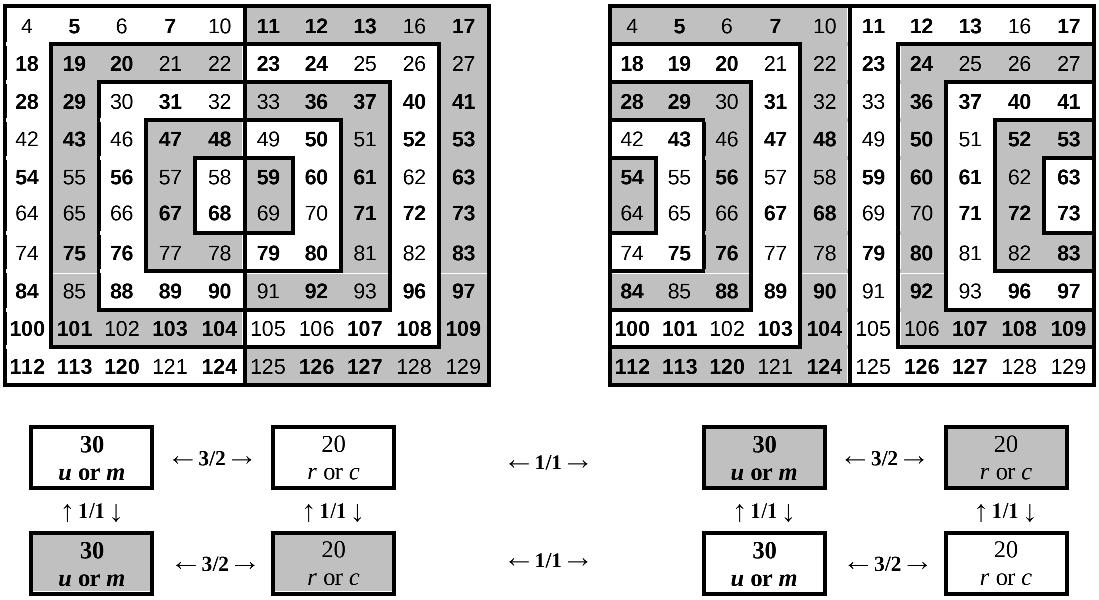
12.3 Crossed intricately together sub-matrices

We have just demonstrated that the 100 elements of the square matrix, previously defined as comprising the first 100 uncoupled whole numbers, are distributed in a highly peculiar manner based on their assignment to either the extreme class (Ultimates or Mixes) or the median class (Raiseds or Composites). We will now definitively demonstrate that these geoalgebraic phenomena cannot be attributed to chance.

Figure 60 illustrates the construction of a fine partition of this square matrix by alternately isolating semicircular sets of numbers along a vertical axis of symmetry. We generate semicircles emanating from the center for one configuration and originating from the periphery for the other. This process thus creates a highly intertwined geometric contrast between two sub-matrices of equal size (50 elements each).

twin symmetrical 50-number sub-matrices in an alternating pattern

twin symmetrical 50-number sub-matrices in an alternating pattern



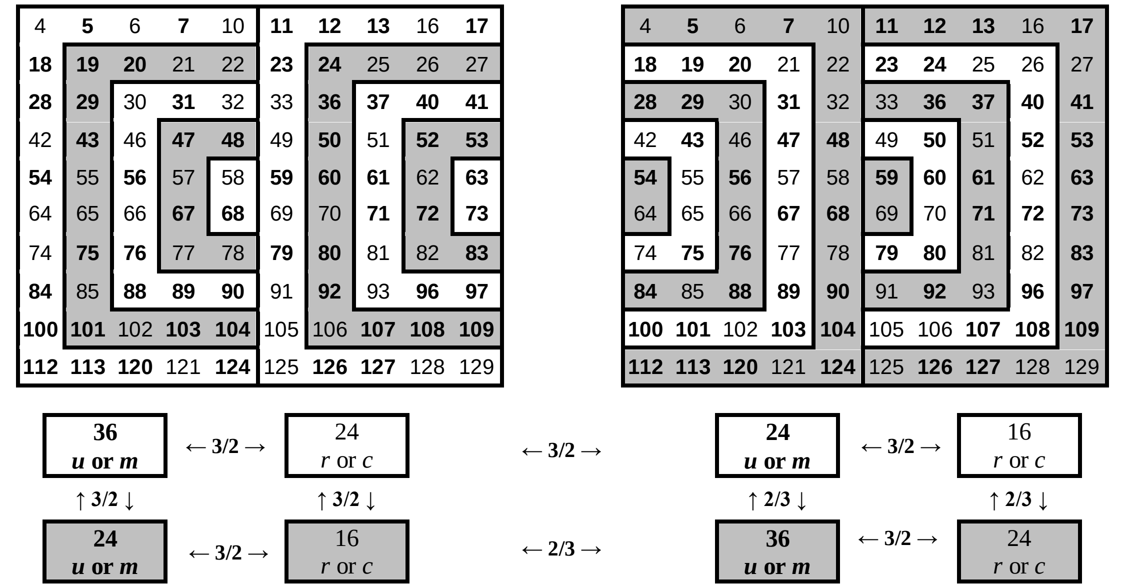
4	**5**	6	**7**	10	**11**	**12**	**13**	16	**17**
18	**19**	**20**	21	22	**23**	**24**	25	26	27
28	**29**	30	**31**	32	33	**36**	**37**	**40**	**41**
42	**43**	46	**47**	**48**	49	**50**	51	**52**	**53**
54	55	**56**	57	58	**59**	**60**	**61**	62	**63**
64	65	66	**67**	**68**	69	70	**71**	**72**	**73**
74	**75**	**76**	77	78	**79**	**80**	81	82	**83**
84	85	**88**	**89**	**90**	91	**92**	93	**96**	**97**
100	**101**	102	**103**	**104**	105	106	**107**	**108**	**109**
112	**113**	**120**	121	**124**	125	**126**	**127**	128	129

4	**5**	6	**7**	10	**11**	**12**	**13**	16	**17**
18	**19**	**20**	21	22	**23**	**24**	25	26	27
28	**29**	30	**31**	32	33	**36**	**37**	**40**	**41**
42	**43**	46	**47**	**48**	49	**50**	51	**52**	**53**
54	55	**56**	57	58	**59**	**60**	**61**	62	**63**
64	65	66	**67**	**68**	69	70	**71**	**72**	**73**
74	**75**	**76**	77	78	**79**	**80**	81	82	**83**
84	85	**88**	**89**	**90**	91	**92**	93	**96**	**97**
100	**101**	102	**103**	**104**	105	106	**107**	**108**	**109**
112	**113**	**120**	121	**124**	125	**126**	**127**	128	129

Figure 60: From the 100-number matrix, twin symmetrical 50-number sub-matrices in an alternating pattern. Distribution of extremes and medians numbers in 3:2 ratios.

In this tangle of symmetrically aesthetic geometric shapes, the extreme and median numbers are again opposed in a perfect value ratio of 3:2: 30 vs. 20 respectively.

Figure 61, we construct two other configurations of this square matrix by vertically crossing the two previous configurations. We thus obtain four asymmetrical sub-matrices whose sizes are in a ratio of 3:2, i.e. 60 numbers vs. 40. In these new configurations, extremes and medians are still opposed in the ratio 3:2 but also in the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where a and b have respective value 2 and 3. with respective values 36 (→$4b^2$), 24 (→$4ab$), 24(→$4ab$) and 16 (→$4a^2$).



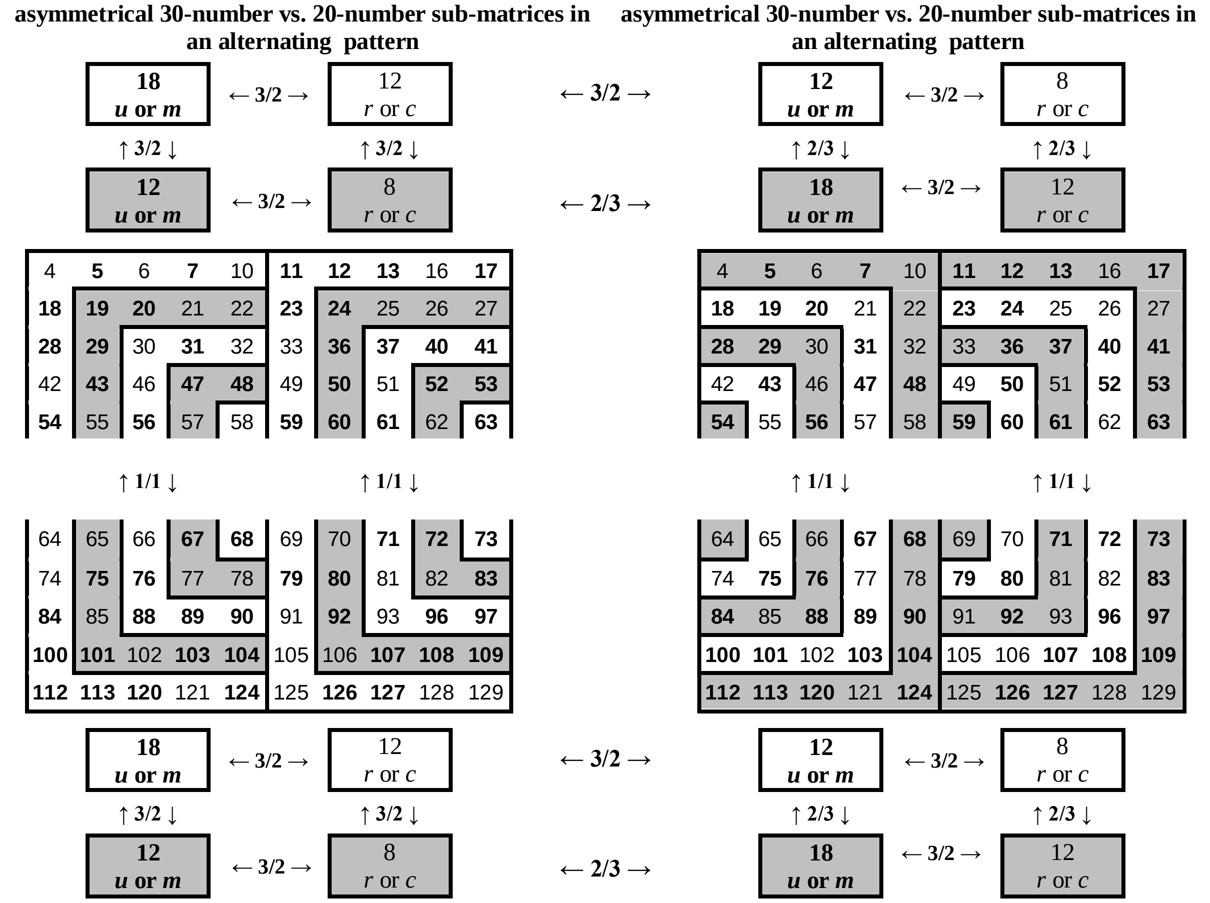
asymmetrical 60-number vs. 40-number sub-matrices in an alternating pattern

asymmetrical 60-number vs. 40-number sub-matrices in an alternating pattern

4	**5**	6	**7**	10	**11**	**12**	**13**	16	**17**
18	**19**	**20**	21	22	**23**	**24**	25	26	27
28	**29**	30	**31**	32	33	**36**	**37**	**40**	**41**
42	**43**	46	**47**	**48**	49	**50**	51	**52**	**53**
54	55	**56**	57	58	**59**	**60**	**61**	62	**63**
64	65	66	**67**	**68**	69	70	**71**	**72**	**73**
74	**75**	**76**	77	78	**79**	**80**	81	82	**83**
84	85	**88**	**89**	**90**	91	**92**	93	**96**	**97**
100	**101**	102	**103**	**104**	105	106	**107**	**108**	**109**
112	**113**	**120**	121	**124**	125	**126**	**127**	128	129

Figure 61: From the 100-number matrix, asymmetrical 60-number vs. 40-number sub-matrices in an alternating pattern. Distribution of extremes and medians numbers in 3:2 ratios and in the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$.

Furthermore, as shown in Figure 62, a horizontal bisection of these two sub-matrices yields four smaller sub-matrices in which the two aforementioned number-class groups maintain a 3:2 opposition ratio, consistently adhering to the mechanism of the same remarkable identity.



asymmetrical 30-number vs. 20-number sub-matrices in an alternating pattern

asymmetrical 30-number vs. 20-number sub-matrices in an alternating pattern

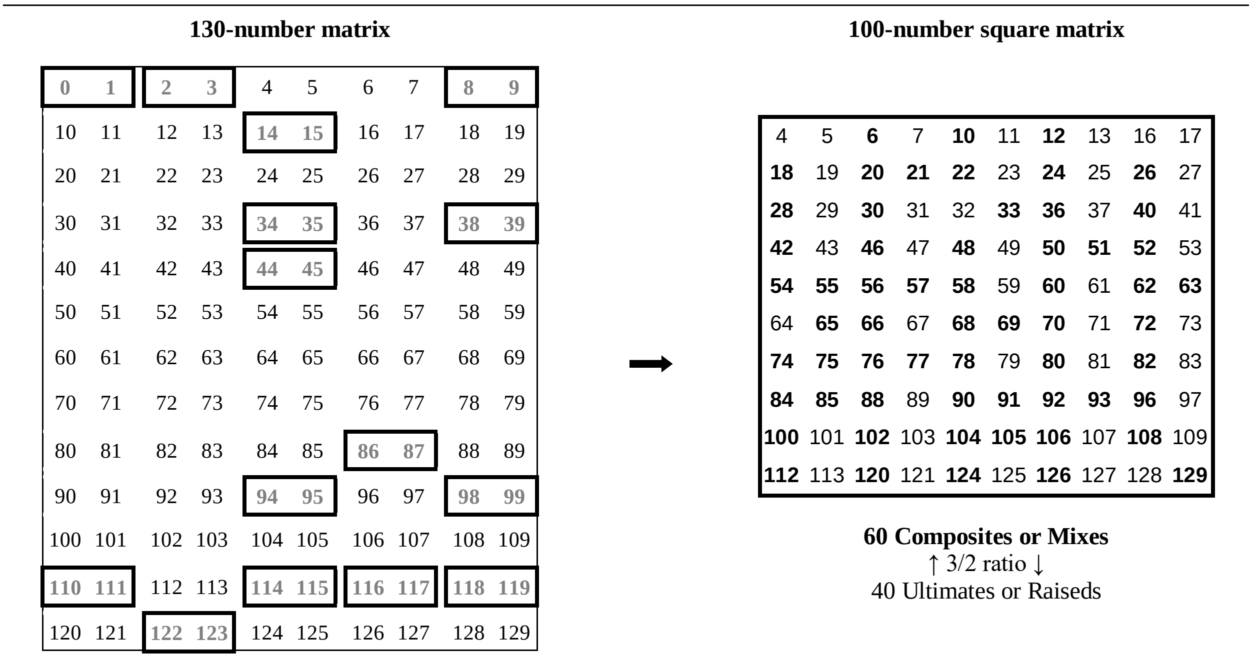
4	**5**	6	**7**	10	**11**	**12**	**13**	16	**17**
18	**19**	**20**	21	22	**23**	**24**	25	26	27
28	**29**	30	**31**	32	33	**36**	**37**	**40**	**41**
42	**43**	46	**47**	**48**	49	**50**	51	**52**	**53**
54	55	**56**	57	58	**59**	**60**	**61**	62	**63**

64	65	66	**67**	**68**	69	70	**71**	**72**	**73**
74	**75**	**76**	77	78	**79**	**80**	81	82	**83**
84	85	**88**	**89**	**90**	91	**92**	93	**96**	**97**
100	**101**	102	**103**	**104**	105	106	**107**	**108**	**109**
112	**113**	**120**	121	**124**	125	**126**	**127**	128	129

Figure 62: From the 100-number matrix, asymmetrical 30-number vs. 20-number sub-matrices in an alternating pattern. Distribution of extremes and medians numbers in 3:2 ratios and in the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$.

13. Alternative analysis of the recombined 100-element matrix

In the preceding chapter, we proposed the construction of a recombined matrix composed of the first 100 uncoupled numbers belonging to distinct classes. We demonstrated that this matrix exhibited an exact 3:2 opposition ratio between the numbers of the extreme classes and those of the median class. We now propose to examine this same matrix but, this time, by contrasting the two classes of Composites and Mixes with the two classes of Ultimates and Raiseds.



130-number matrix

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100	101	102	103	104	105	106	107	108	109
110	111	112	113	114	115	116	117	118	119
120	121	122	123	124	125	126	127	128	129

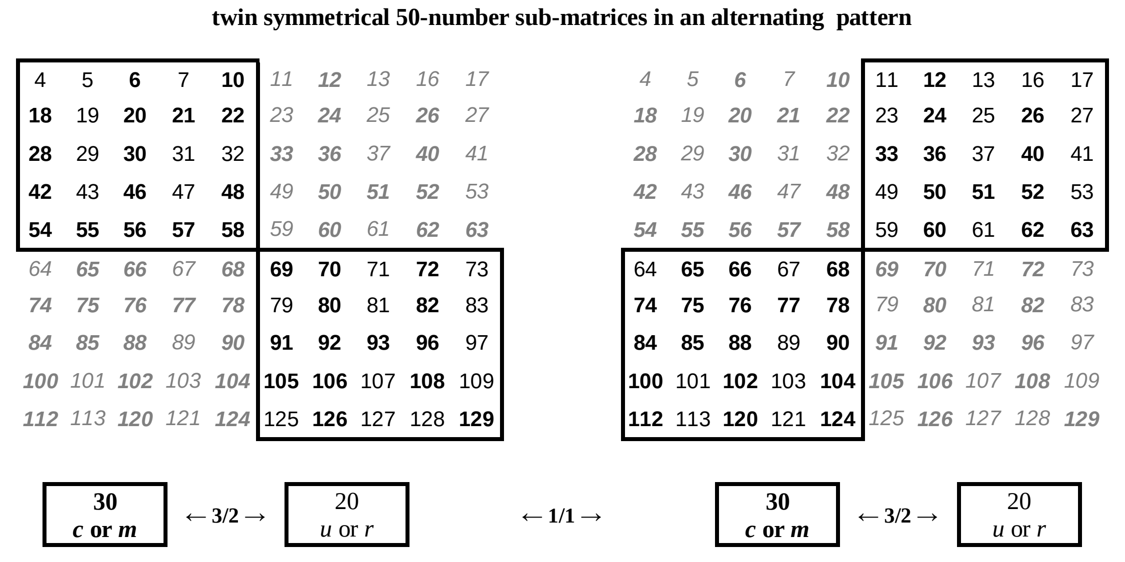
100-number square matrix

4	5	**6**	7	**10**	11	**12**	13	16	17
18	19	**20**	**21**	**22**	23	**24**	25	**26**	27
28	29	**30**	31	32	**33**	**36**	37	**40**	41
42	43	**46**	47	**48**	49	**50**	**51**	**52**	53
54	**55**	**56**	**57**	**58**	59	**60**	61	**62**	**63**
64	**65**	**66**	67	**68**	**69**	**70**	71	**72**	73
74	**75**	**76**	**77**	**78**	79	**80**	81	**82**	83
84	**85**	**88**	89	**90**	**91**	**92**	**93**	**96**	97
100	101	**102**	103	**104**	**105**	**106**	107	**108**	109
112	113	**120**	121	**124**	125	**126**	127	128	**129**

60 Composites or Mixes
↑ 3/2 ratio ↓
40 Ultimates or Raiseds

Figure 63: From the 130-first number matrix, construction of a 100-number square matrix by subtraction of the 15 pairs of same classes numbers: **60 Composites or Mixes** vs. 40 Ultimates or Raiseds.

As illustrated in Figure 63, the two newly defined number-class groups exhibit an exact 3:2 ratio, comprising 60 elements of the Composite or Mix classes versus 40 elements of the Ultimate or Raised classes. Partitioning this matrix into twin, symmetrical 50-element sub-matrices using an alternating pattern, as shown in Figure 64, yields the same 3:2 opposition ratio between these two number groups.



twin symmetrical 50-number sub-matrices in an alternating pattern

4	5	**6**	7	**10**	*11*	***12***	*13*	*16*	*17*
18	19	**20**	**21**	**22**	*23*	***24***	*25*	***26***	*27*
28	29	**30**	31	32	***33***	***36***	*37*	***40***	*41*
42	43	**46**	47	**48**	*49*	***50***	***51***	***52***	*53*
54	**55**	**56**	**57**	**58**	*59*	***60***	*61*	***62***	***63***
64	***65***	***66***	*67*	***68***	**69**	**70**	71	**72**	73
74	***75***	***76***	***77***	***78***	79	**80**	81	**82**	83
84	***85***	***88***	*89*	***90***	**91**	**92**	**93**	**96**	97
100	*101*	***102***	*103*	***104***	**105**	**106**	107	**108**	109
112	*113*	***120***	*121*	***124***	125	**126**	127	128	**129**

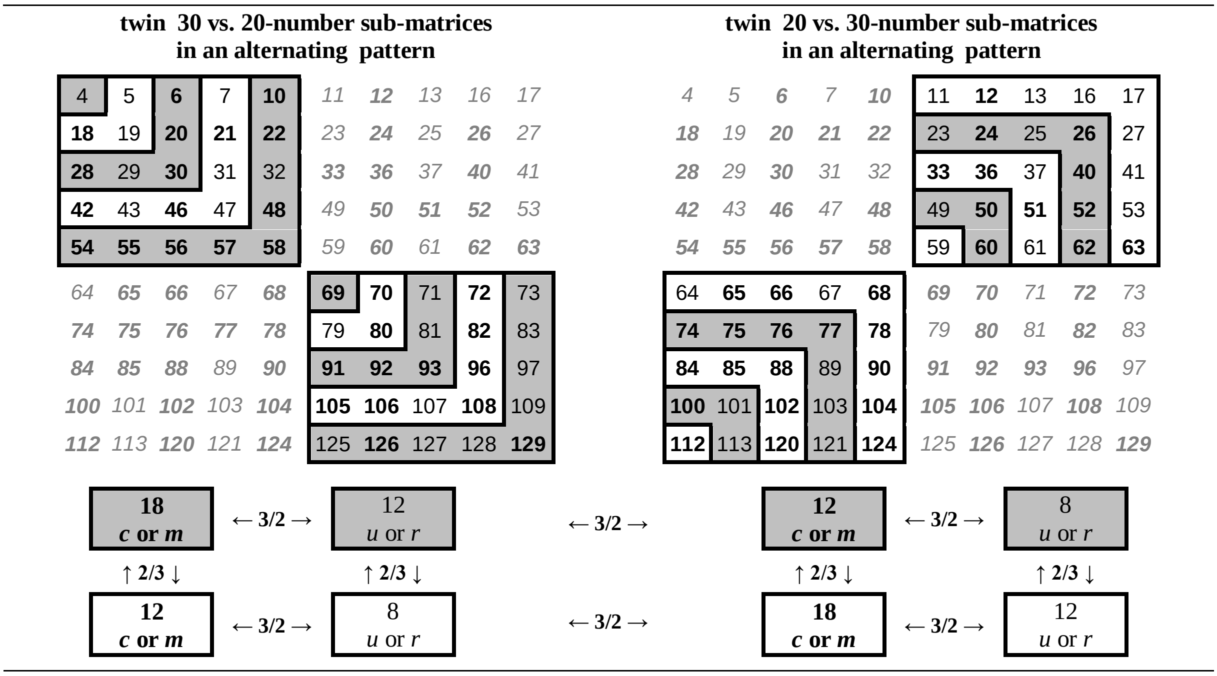
4	*5*	***6***	*7*	***10***	11	**12**	13	16	17
18	*19*	***20***	***21***	***22***	23	**24**	25	**26**	27
28	*29*	***30***	*31*	*32*	**33**	**36**	37	**40**	41
42	*43*	***46***	*47*	***48***	49	**50**	**51**	**52**	53
54	***55***	***56***	***57***	***58***	59	**60**	61	**62**	**63**
64	**65**	**66**	67	**68**	***69***	***70***	*71*	***72***	*73*
74	**75**	**76**	**77**	**78**	*79*	***80***	*81*	***82***	*83*
84	**85**	**88**	89	**90**	***91***	***92***	***93***	***96***	*97*
100	101	**102**	103	**104**	***105***	***106***	*107*	***108***	*109*
112	113	**120**	121	**124**	*125*	***126***	*127*	*128*	***129***

30 ***c* or *m*** **← 3/2 →** 20 *u* or *r* **← 1/1 →** **30** ***c* or *m*** **← 3/2 →** 20 *u* or *r*

Figure 64: From the 100-number matrix, twin symmetrical 50-number sub-matrices in an alternating pattern. Distribution of Composites and Mixes vs. Ultimates and Raiseds in 3:2 ratios.

13.1 Intricately crossed sub-matrices

Figure 65 illustrates the construction of a fine partition of this square matrix by alternately isolating circular sectors originating from the lower-left corner for the first sub-matrix and asymmetrically from the lower-left corner for the symmetrically opposing sub-matrix. This process creates a highly intertwined geometric partition of these two sub-matrices (50 elements each) into four resulting sub-matrices whose sizes are in the 3:2 or 2:3 ratio.



4	5	6	7	10	11	12	13	16	17
18	19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	36	37	40	41
42	43	46	47	48	49	50	51	52	53
54	55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72	73
74	75	76	77	78	79	80	81	82	83
84	85	88	89	90	91	92	93	96	97
100	101	102	103	104	105	106	107	108	109
112	113	120	121	124	125	126	127	128	129

4	5	6	7	10	11	12	13	16	17
18	19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	36	37	40	41
42	43	46	47	48	49	50	51	52	53
54	55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72	73
74	75	76	77	78	79	80	81	82	83
84	85	88	89	90	91	92	93	96	97
100	101	102	103	104	105	106	107	108	109
112	113	120	121	124	125	126	127	128	129

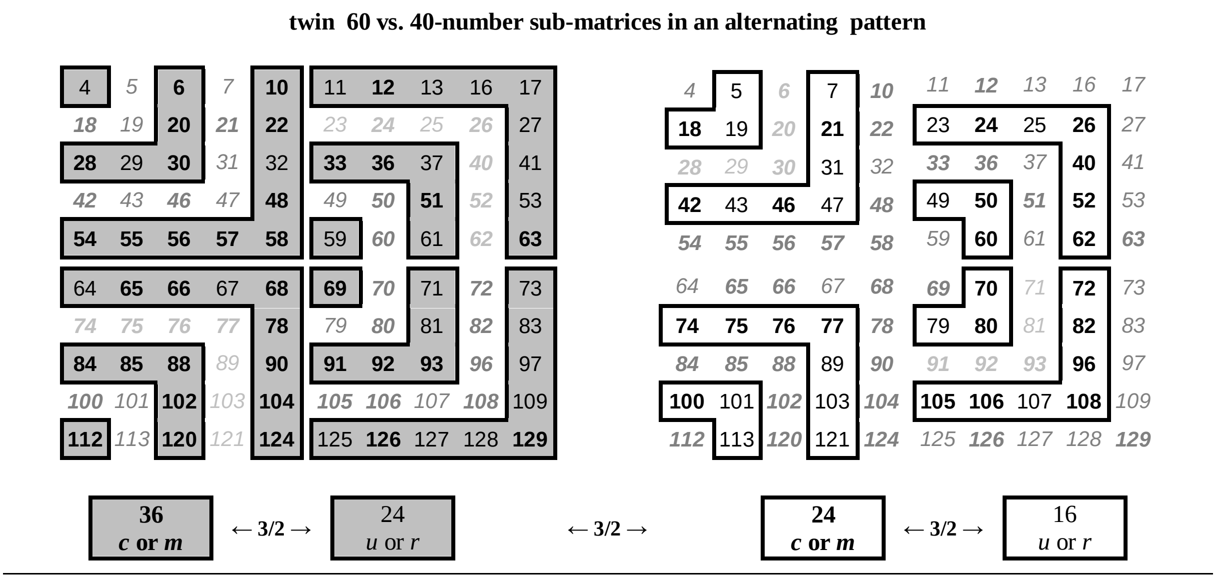
Figure 65: From the 100-number matrix, four twin asymmetrical sub-matrices in an alternating pattern. Distribution of Composites and Mixes vs. Ultimates and Raiseds in 3:2 ratios.

As it appears in Figure 65, within these four interlaced sub-matrices, the two groups of numbers considered oppose each other in 3:2 ratios. Also, globally the different values of these four interlaced sub-matrices are organized, as in many previous introduced configurations, in the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$.

As Figure 65 demonstrates, within these four interlaced sub-matrices, the two number groups maintain their 3:2 opposition ratio. Furthermore, overall, the values within these four interlaced sub-matrices are organized, consistent with many previously introduced configurations, according to the remarkable identity: $(a + b)^2 = a^2 + 2ab + b^2$.

13.2 Recombination of the intricately interlaced sub-matrices

The four sub-matrices just introduced in Figure 65 can be recombined to form two new configurations of 60 and 40 elements, respectively, as shown in Figure 66.



4	5	6	7	10	11	12	13	16	17
18	19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	36	37	40	41
42	43	46	47	48	49	50	51	52	53
54	55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72	73
74	75	76	77	78	79	80	81	82	83
84	85	88	89	90	91	92	93	96	97
100	101	102	103	104	105	106	107	108	109
112	113	120	121	124	125	126	127	128	129

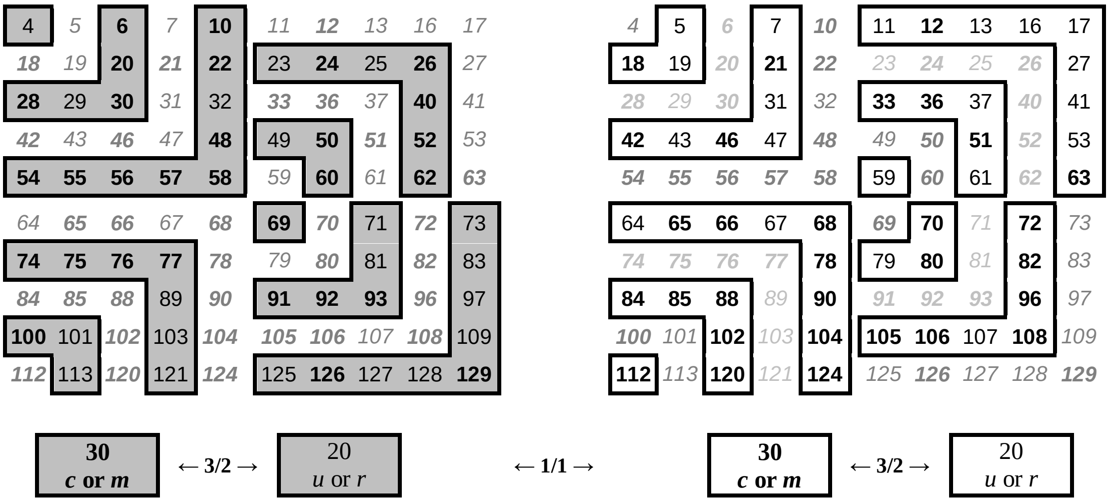
4	5	6	7	10	11	12	13	16	17
18	19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	36	37	40	41
42	43	46	47	48	49	50	51	52	53
54	55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72	73
74	75	76	77	78	79	80	81	82	83
84	85	88	89	90	91	92	93	96	97
100	101	102	103	104	105	106	107	108	109
112	113	120	121	124	125	126	127	128	129

Figure 66: From the 100-number matrix, twin symmetrical 50-number sub-matrices in an alternating pattern. Distribution of Composites and Mixes vs. Ultimates and Raiseds in 3:2 ratios.

In these two newly formed sub-matrices of singular design, the Composites or Mixes and the Ultimates or Raiseds are opposed in a 3:2 ratio and adhere to the remarkable identity previously introduced.

An alternative arrangement of these four sub-matrices facilitates the creation of two new matrices of equal size (50 elements each). As Figure 67 illustrates, these two new 50-element matrices show that the Composites and Mixes are opposed to the Ultimates and Raiseds in a 3:2 ratio.

twin asymmetrical 50-number sub-matrices in an alternating pattern



4	5	6	7	10	11	12	13	16	17
18	19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	36	37	40	41
42	43	46	47	48	49	50	51	52	53
54	55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72	73
74	75	76	77	78	79	80	81	82	83
84	85	88	89	90	91	92	93	96	97
100	101	102	103	104	105	106	107	108	109
112	113	120	121	124	125	126	127	128	129

4	5	6	7	10	11	12	13	16	17
18	19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	36	37	40	41
42	43	46	47	48	49	50	51	52	53
54	55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72	73
74	75	76	77	78	79	80	81	82	83
84	85	88	89	90	91	92	93	96	97
100	101	102	103	104	105	106	107	108	109
112	113	120	121	124	125	126	127	128	129

Figure 67: From the 100-number matrix, twin symmetrical 50-number sub-matrices in an alternating pattern Distribution of Composites and Mixes vs. Ultimates and Raiseds in 3:2 ratios.

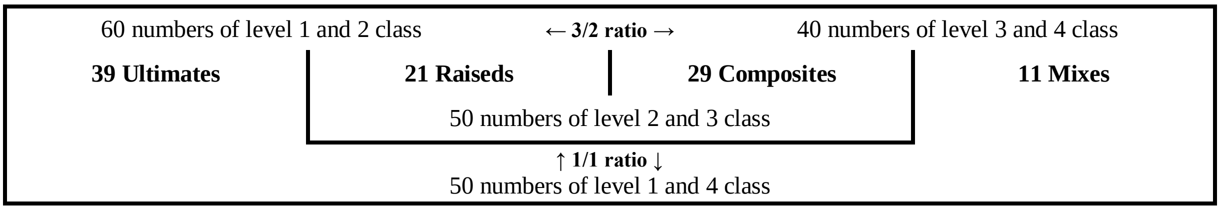
14. Addition matrix of the Twenty Fundamentals and number classes

In the initial article *"The ultimate numbers and the 3/2 ratio, just two primary sets of whole number"* [1], the significant role of the entanglement of the first twenty whole numbers (conventionally known as the "Twenty Fundamentals") is demonstrated. The addition matrix formed by the first ten ultimate numbers and the first ten non-ultimate numbers (which together constitute the first twenty whole numbers) generates one hundred values (Figure 68). These values can be distinguished based on the four number classes defined in Chapter 4: Ultimates (*u*), Raiseds (*r*), Composites (*c*), and Mixes (*m*).

the first 10 non-Ultimates

the first 10 Ultimates

+	4	6	8	9	10	12	14	15	16	18
0	*4r*	*6c*	*8r*	*9r*	*10c*	*12m*	*14c*	*15c*	*16r*	*18m*
1	*5u*	*7u*	*9r*	*10c*	*11u*	*13u*	*15c*	*16r*	*17u*	*19u*
2	*6c*	*8r*	*10c*	*11u*	*12m*	*14c*	*16r*	*17u*	*18m*	*20m*
3	*7u*	*9r*	*11u*	*12m*	*13u*	*15c*	*17u*	*18m*	*19u*	*21c*
5	*9r*	*11u*	*13u*	*14c*	*15c*	*17u*	*19u*	*20m*	*21c*	*23u*
7	*11u*	*13u*	*15c*	*16r*	*17u*	*19u*	*21c*	*22c*	*23u*	*25r*
11	*15c*	*17u*	*19u*	*20m*	*21c*	*23u*	*25r*	*26c*	*27r*	*29u*
13	*17u*	*19u*	*21c*	*22c*	*23u*	*25r*	*27r*	*28m*	*29u*	*31u*
17	*21c*	*23u*	*25r*	*26c*	*27r*	*29u*	*31u*	*32r*	*33c*	*35c*
19	*23u*	*25r*	*27r*	*28m*	*29u*	*31u*	*33c*	*34c*	*35c*	*37u*



60 numbers of level 1 and 2 class ← **3/2 ratio** → 40 numbers of level 3 and 4 class

39 Ultimates | **21 Raiseds** | **29 Composites** | **11 Mixes**

50 numbers of level 2 and 3 class

↑ **1/1 ratio** ↓

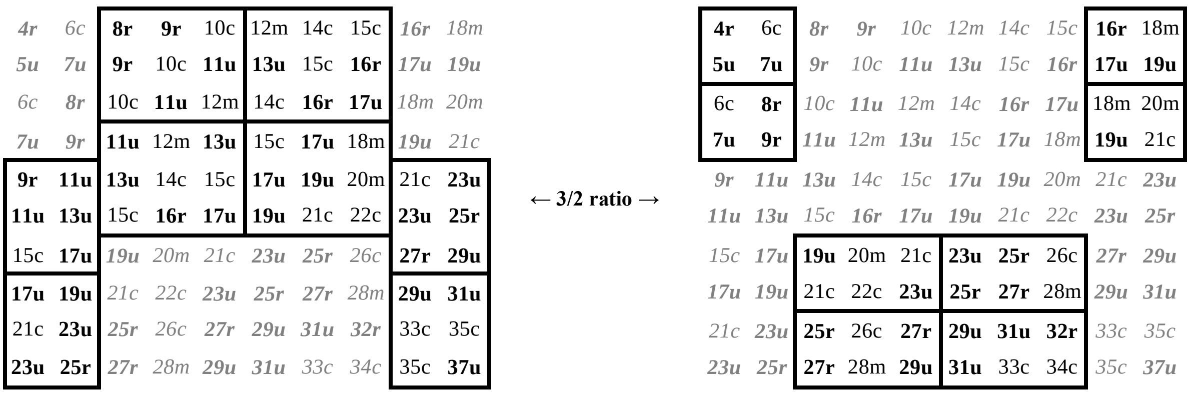
50 numbers of level 1 and 4 class

Figure 68: Distribution of the four number classes generated by the addition matrix of the Twenty Fundamentals, segregated into 10 Ultimates vs. 10 non-Ultimates (See also Figures 2 and 6).

As demonstrated in Figure 68, within this addition matrix of the Twenty Fundamentals, the number classes exhibit two-by-two oppositions with ratios of 3:2 or 1:1, depending on the specific configuration examined. Thus, in this 100-element matrix, the opposition between the extreme classes (complexity levels 1 and 4) and the middle classes (complexity levels 2 and 3) is structured into an exact 1:1 ratio. Furthermore, the opposition between the first two classes (complexity levels 1 and 2) and the last two classes (complexity levels 3 and 4) is structured into an exact 3:2 ratio.

60 Ultimates and Raiseds versus 40 Composites and Mixes (→ **3/2 ratio**)
level 1 and 2 number classes versus level 3 and 4 number classes

Sub-matrix to 36 + 24 numbers (60 numbers) **→ 4($a^2 + ab$)** ← **3/2 ratio** → **Sub-matrix to 24 + 16 numbers** (40 numbers) **→ 4($ba + b^2$)**



4r	*6c*	**8r**	**9r**	10c	12m	14c	15c	***16r***	*18m*
5u	***7u***	**9r**	10c	**11u**	**13u**	15c	**16r**	***17u***	***19u***
6c	***8r***	10c	**11u**	12m	14c	**16r**	**17u**	*18m*	*20m*
7u	***9r***	**11u**	12m	**13u**	15c	**17u**	18m	***19u***	*21c*
9r	**11u**	**13u**	14c	15c	**17u**	**19u**	20m	21c	**23u**
11u	**13u**	15c	**16r**	**17u**	**19u**	21c	22c	**23u**	**25r**
15c	**17u**	***19u***	*20m*	*21c*	***23u***	***25r***	*26c*	**27r**	**29u**
17u	**19u**	*21c*	*22c*	***23u***	***25r***	***27r***	*28m*	**29u**	**31u**
21c	**23u**	***25r***	*26c*	***27r***	***29u***	***31u***	***32r***	33c	35c
23u	**25r**	***27r***	*28m*	***29u***	***31u***	*33c*	*34c*	35c	**37u**

4r	6c	***8r***	***9r***	*10c*	*12m*	*14c*	*15c*	**16r**	18m
5u	**7u**	***9r***	*10c*	***11u***	***13u***	*15c*	***16r***	**17u**	**19u**
6c	**8r**	*10c*	***11u***	*12m*	*14c*	***16r***	***17u***	18m	20m
7u	**9r**	***11u***	*12m*	***13u***	*15c*	***17u***	*18m*	**19u**	21c
9r	***11u***	***13u***	*14c*	*15c*	***17u***	***19u***	*20m*	*21c*	***23u***
11u	***13u***	*15c*	***16r***	***17u***	***19u***	*21c*	*22c*	***23u***	***25r***
15c	***17u***	**19u**	20m	21c	**23u**	**25r**	26c	***27r***	***29u***
17u	***19u***	21c	22c	**23u**	**25r**	**27r**	28m	***29u***	***31u***
21c	***23u***	**25r**	26c	**27r**	**29u**	**31u**	**32r**	*33c*	*35c*
23u	***25r***	**27r**	28m	**29u**	**31u**	33c	34c	*35c*	***37u***

36 Ultimates and Raiseds (level 1 and 2) ← **3/2 ratio** → **24 Ultimates and Raiseds (level 1 and 2)**
↑ 3/2 ratio ↓ **↑ 3/2 ratio ↓**
24 Composites and Mixes (level 3 and 4) ← **3/2 ratio** → 16 Composites and Mixes (level 3 and 4)

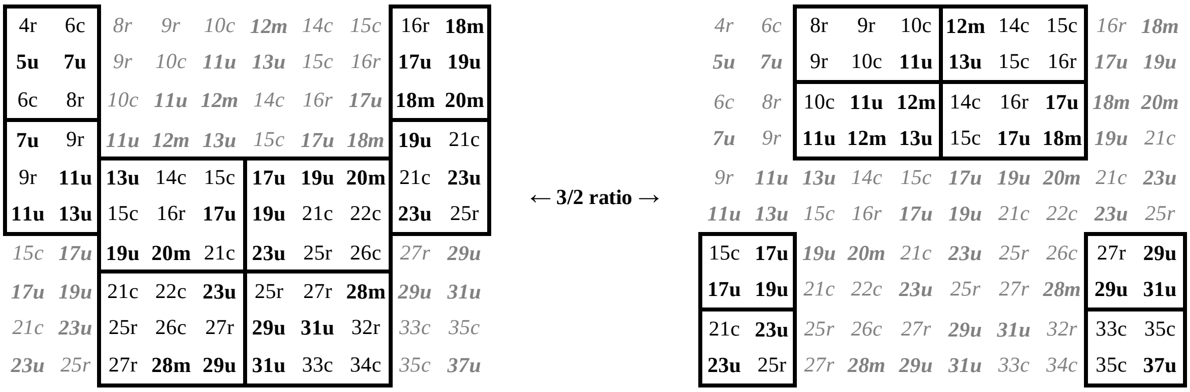
Figure 69: Distribution of the four number classes (level 1 and 2 versus level 3 and 4) generated from the addition matrix of the 20 fundamentals (see Figure 68). Sub-matrices inscribed in the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$.

Thus, 60 Ultimates or Raiseds oppose 40 Composites and Mixes in a 3:2 ratio. Within dissymmetrical sub-matrices of 60 and 40 elements whose magnitudes are components of the remarkable identity $(a + b)^2 = \ldots$, the two groups of number classes (levels 1 and 4 versus levels 2 and 3) continue to exhibit an exact 3:2 opposition ratio, as shown in Figure 69.

Similarly, from this addition matrix of the Twenty Fundamentals, two similar but horizontally inverse sub-matrices of 60 and 40 elements (Figure 70) are generated. Here, the sets of 50 Ultimates and Mixes and 50 Raiseds and Composites (which have an overall 1:1 ratio) continue to oppose each other in the same internal ratios: 30 versus 30 in the 60-element sub-matrix and 20 versus 20 in the 40-element sub-matrix.

50 Ultimates and Mixes versus 50 Raiseds and Composites (→ **1/1 ratio**)
level 1 and 4 number classes versus level 2 and 3 number classes

Sub-matrix to 36 + 24 numbers (60 numbers) **→ 4($a^2 + ab$)** ← **3/2 ratio** → **Sub-matrix to 24 + 16 numbers** (40 numbers) **→ 4($ba + b^2$)**



4r	6c	*8r*	*9r*	*10c*	***12m***	*14c*	*15c*	16r	**18m**
5u	**7u**	*9r*	*10c*	***11u***	***13u***	*15c*	*16r*	**17u**	**19u**
6c	8r	*10c*	***11u***	***12m***	*14c*	*16r*	***17u***	**18m**	**20m**
7u	9r	***11u***	***12m***	***13u***	*15c*	***17u***	***18m***	**19u**	21c
9r	**11u**	**13u**	14c	15c	**17u**	**19u**	**20m**	21c	**23u**
11u	**13u**	15c	16r	**17u**	**19u**	21c	22c	**23u**	25r
15c	***17u***	**19u**	**20m**	21c	**23u**	25r	26c	*27r*	***29u***
17u	***19u***	21c	22c	**23u**	25r	27r	**28m**	***29u***	***31u***
21c	***23u***	25r	26c	27r	**29u**	**31u**	32r	*33c*	*35c*
23u	*25r*	27r	**28m**	**29u**	**31u**	33c	34c	*35c*	***37u***

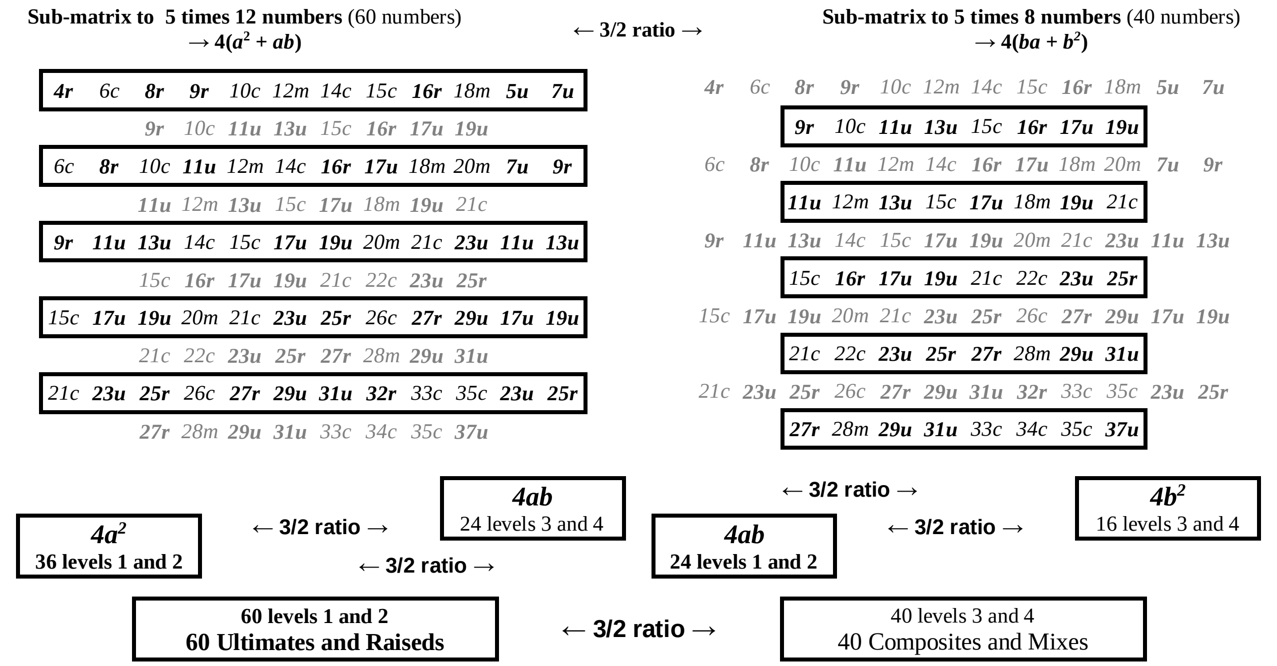
4r	*6c*	8r	9r	10c	**12m**	14c	15c	*16r*	***18m***
5u	***7u***	9r	10c	**11u**	**13u**	15c	16r	***17u***	***19u***
6c	*8r*	10c	**11u**	**12m**	14c	16r	**17u**	***18m***	***20m***
7u	*9r*	**11u**	**12m**	**13u**	15c	**17u**	**18m**	***19u***	*21c*
9r	***11u***	***13u***	*14c*	*15c*	***17u***	***19u***	***20m***	*21c*	***23u***
11u	***13u***	*15c*	*16r*	***17u***	***19u***	*21c*	*22c*	***23u***	*25r*
15c	**17u**	***19u***	***20m***	*21c*	***23u***	*25r*	*26c*	27r	**29u**
17u	**19u**	*21c*	*22c*	***23u***	*25r*	*27r*	***28m***	**29u**	**31u**
21c	**23u**	*25r*	*26c*	*27r*	***29u***	***31u***	*32r*	33c	35c
23u	25r	*27r*	***28m***	***29u***	***31u***	*33c*	*34c*	35c	**37u**

30 Ultimates and Mixes (level 1 and 4) ← **3/2 ratio** → **20 Ultimates and Mixes (level 1 and 4)**
↑ 1/1 ratio ↓ **↑ 1/1 ratio ↓**
30 Raiseds and Composites (level 2 and 3) ← **3/2 ratio** → 20 Raiseds and Composites (level 2 and 3)

Figure 70: Distribution of the four number classes (level 1 and 4 versus level 2 and 3) generated from the addition matrix of the 20 fundamentals (see Figure 68). Sub-matrices inscribed in the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$.

14.1 Alternated sub matrices

Consistent with the methodology introduced for previous matrices, we now focus on two sub-matrices exhibiting alternating configurations: one comprising 5 rows and 12 columns of elements, and the other comprising 5 rows and 8 columns of elements. These sub-matrices are recombinations of the Addition Matrix of the Twenty Fundamentals introduced in Figure 68.



Sub-matrix to 5 times 12 numbers (60 numbers) **→ 4(a^2 + ab)** ← **3/2 ratio** → **Sub-matrix to 5 times 8 numbers** (40 numbers) **→ 4(ba + b^2)**

4r	6c	**8r**	**9r**	10c	12m	14c	15c	**16r**	18m	**5u**	**7u**
		9r	10c	**11u**	**13u**	15c	**16r**	**17u**	**19u**		
6c	**8r**	10c	**11u**	12m	14c	**16r**	**17u**	18m	20m	**7u**	**9r**
		11u	12m	**13u**	15c	**17u**	18m	**19u**	21c		
9r	**11u**	**13u**	14c	15c	**17u**	**19u**	20m	21c	**23u**	**11u**	**13u**
		15c	**16r**	**17u**	**19u**	21c	22c	**23u**	**25r**		
15c	**17u**	**19u**	20m	21c	**23u**	**25r**	26c	**27r**	**29u**	**17u**	**19u**
		21c	22c	**23u**	**25r**	**27r**	28m	**29u**	**31u**		
21c	**23u**	**25r**	26c	**27r**	**29u**	**31u**	**32r**	33c	35c	**23u**	**25r**
		27r	28m	**29u**	**31u**	33c	34c	35c	**37u**		

4r	6c	**8r**	**9r**	10c	12m	14c	15c	**16r**	18m	**5u**	**7u**
		9r	10c	**11u**	**13u**	15c	**16r**	**17u**	**19u**		
6c	**8r**	10c	**11u**	12m	14c	**16r**	**17u**	18m	20m	**7u**	**9r**
		11u	12m	**13u**	15c	**17u**	18m	**19u**	21c		
9r	**11u**	**13u**	14c	15c	**17u**	**19u**	20m	21c	**23u**	**11u**	**13u**
		15c	**16r**	**17u**	**19u**	21c	22c	**23u**	**25r**		
15c	**17u**	**19u**	20m	21c	**23u**	**25r**	26c	**27r**	**29u**	**17u**	**19u**
		21c	22c	**23u**	**25r**	**27r**	28m	**29u**	**31u**		
21c	**23u**	**25r**	26c	**27r**	**29u**	**31u**	**32r**	33c	35c	**23u**	**25r**
		27r	28m	**29u**	**31u**	33c	34c	35c	**37u**		

4a² **36 levels 1 and 2** ← **3/2 ratio** → ***4ab*** 24 levels 3 and 4 ← **3/2 ratio** → ***4ab*** **24 levels 1 and 2** ← **3/2 ratio** → ***4b²*** 16 levels 3 and 4

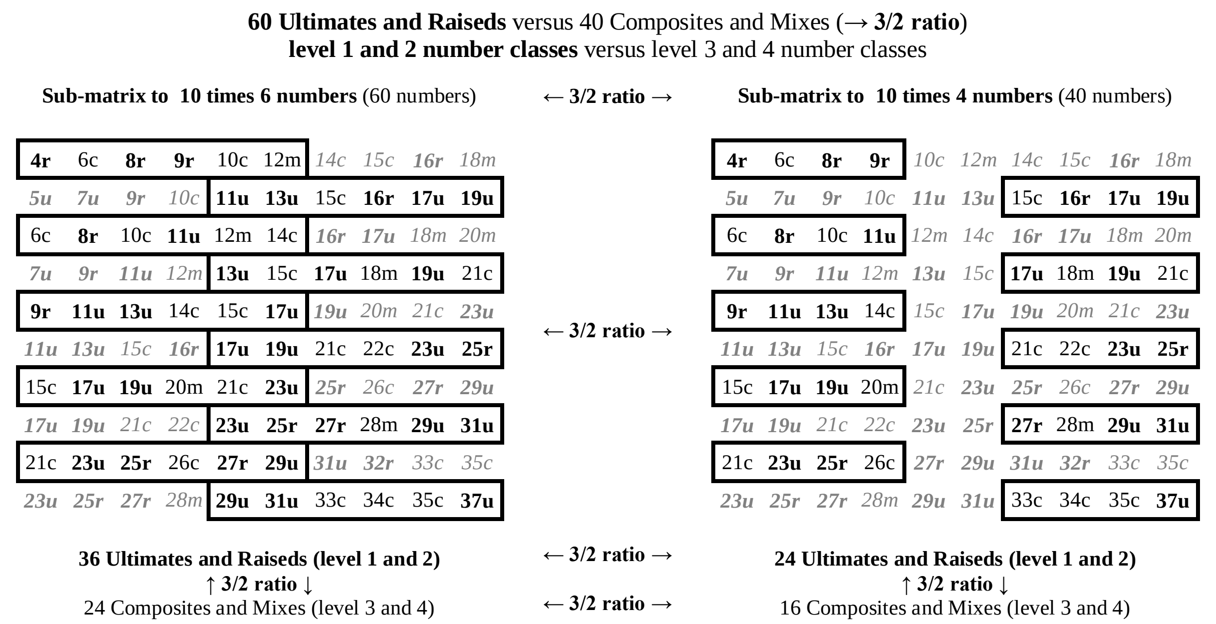
← **3/2 ratio** →

60 levels 1 and 2
60 Ultimates and Raiseds ← **3/2 ratio** → 40 levels 3 and 4
40 Composites and Mixes

Figure 71: Distribution of the four number classes generated from the addition matrix of the 20 fundamentals (see Figure 68). Alternated sub-matrices inscribed in variation of the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$.

Thus, using the Addition Matrix of the Twenty Fundamentals (Figure 68), the alternated sub-matrices of 60 and 40 elements (Figure 71) are analyzed. The sets of 60 elements (Levels 1 and 2) and 40 elements (Levels 3 and 4) continue to exhibit the same 3:2 opposition ratio: 36 versus 24 in the 60-element sub-matrix and 24 versus 16 in the 40-element sub-matrix. This structural organization aligns with the components of the remarkable identity: $(a + b)^2 = a^2 + 2ab + b^2$.

14.2 Other alternating sub matrices



60 Ultimates and Raiseds versus 40 Composites and Mixes (→ **3/2 ratio**)
level 1 and 2 number classes versus level 3 and 4 number classes

Sub-matrix to 10 times 6 numbers (60 numbers) ← **3/2 ratio** → **Sub-matrix to 10 times 4 numbers** (40 numbers)

4r	6c	**8r**	**9r**	10c	12m	14c	15c	**16r**	18m
5u	**7u**	**9r**	10c	**11u**	**13u**	15c	**16r**	**17u**	**19u**
6c	**8r**	10c	**11u**	12m	14c	**16r**	**17u**	18m	20m
7u	**9r**	**11u**	12m	**13u**	15c	**17u**	18m	**19u**	21c
9r	**11u**	**13u**	14c	15c	**17u**	**19u**	20m	21c	**23u**
11u	**13u**	15c	**16r**	**17u**	**19u**	21c	22c	**23u**	**25r**
15c	**17u**	**19u**	20m	21c	**23u**	**25r**	26c	**27r**	**29u**
17u	**19u**	21c	22c	**23u**	**25r**	**27r**	28m	**29u**	**31u**
21c	**23u**	**25r**	26c	**27r**	**29u**	**31u**	**32r**	33c	35c
23u	**25r**	**27r**	28m	**29u**	**31u**	33c	34c	35c	**37u**

← **3/2 ratio** →

4r	6c	**8r**	**9r**	10c	12m	14c	15c	**16r**	18m
5u	**7u**	**9r**	10c	**11u**	**13u**	15c	**16r**	**17u**	**19u**
6c	**8r**	10c	**11u**	12m	14c	**16r**	**17u**	18m	20m
7u	**9r**	**11u**	12m	**13u**	15c	**17u**	18m	**19u**	21c
9r	**11u**	**13u**	14c	15c	**17u**	**19u**	20m	21c	**23u**
11u	**13u**	15c	**16r**	**17u**	**19u**	21c	22c	**23u**	**25r**
15c	**17u**	**19u**	20m	21c	**23u**	**25r**	26c	**27r**	**29u**
17u	**19u**	21c	22c	**23u**	**25r**	**27r**	28m	**29u**	**31u**
21c	**23u**	**25r**	26c	**27r**	**29u**	**31u**	**32r**	33c	35c
23u	**25r**	**27r**	28m	**29u**	**31u**	33c	34c	35c	**37u**

36 Ultimates and Raiseds (level 1 and 2)
↑ 3/2 ratio ↓
24 Composites and Mixes (level 3 and 4)

← **3/2 ratio** →
← **3/2 ratio** →

24 Ultimates and Raiseds (level 1 and 2)
↑ 3/2 ratio ↓
16 Composites and Mixes (level 3 and 4)

Figure 72: Based on the addition matrix of the 20 fundamentals (see Figure 68), distribution of the four classes of numbers (level 1 and 2 versus level 3 and 4) into alternating sub-matrices with sizes of 10×6 numbers and 10×4 numbers.

Still based on the addition matrix of the Twenty Fundamentals, we now conclude our analysis by studying two sub-matrices of alternating configuration: one of size 10 × 6 numbers and 10 ×4 numbers (60 numbers versus 40 numbers).

As shown in Figure 72, the sets within these alternated sub-matrices maintain the overall 3:2 ratio of 60 entities versus 40 entities. Specifically, the 60-entity sub-matrix comprises 36 Ultimates and Raiseds vs. 24 Composites and Mixes, a 3:2 ratio. Similarly, the 40-entity sub-matrix also maintains this 3:2 opposition, consisting of 24 Ultimates and Raiseds vs. 16 Composites and Mixes.

Also in same alternated sub-matrices of 60 versus 40 entities, Figure 73, the sets of 50 Ultimates and Mixes and of 50 Raiseds and Composites which are therefore in an overall 1:1 ratio, continue to oppose each other in this same ratio: 30 versus 30 in the 60-entity sub-matrix and 20 versus 20 in the 40-entity sub-matrix.

Furthermore, within these same alternated sub-matrices (60 versus 40 entities, as shown in Figure 73), the sets of 50 Ultimates and Mixes and 50 Raiseds and Composites maintain their overall 1:1 ratio. This balance is consistently observed in both sub-matrices: 30 versus 30 in the 60-entity sub-matrix, and 20 versus 20 in the 40-entity sub-matrix.

50 Ultimates and Mixes versus 50 Raiseds and Composites (→ **1/1 ratio**)
level 1 and 4 number classes versus level 2 and 3 number classes

Sub-matrix to 10 times 6 numbers (60 numbers) **← 3/2 ratio →** **Sub-matrix to 10 times 4 numbers** (40 numbers)

4r	6c	8r	9r	10c	**12m**	14c	15c	16r	**18m**
5u	**7u**	9r	10c	**11u**	**13u**	15c	16r	**17u**	**19u**
6c	8r	10c	**11u**	**12m**	14c	16r	**17u**	**18m**	**20m**
7u	9r	**11u**	**12m**	**13u**	15c	**17u**	**18m**	**19u**	21c
9r	**11u**	**13u**	14c	15c	**17u**	**19u**	**20m**	21c	**23u**
11u	**13u**	15c	16r	**17u**	**19u**	21c	22c	**23u**	25r
15c	**17u**	**19u**	**20m**	21c	**23u**	25r	26c	27r	**29u**
17u	**19u**	21c	22c	**23u**	25r	27r	**28m**	**29u**	**31u**
21c	**23u**	25r	26c	27r	**29u**	**31u**	32r	33c	35c
23u	25r	27r	**28m**	**29u**	**31u**	33c	34c	35c	**37u**

← 3/2 ratio →

4r	*6c*	*8r*	*9r*	*10c*	***12m***	14c	15c	16r	**18m**
5u	**7u**	9r	10c	***11u***	***13u***	*15c*	*16r*	***17u***	***19u***
6c	*8r*	*10c*	***11u***	***12m***	*14c*	16r	**17u**	**18m**	**20m**
7u	9r	**11u**	**12m**	***13u***	*15c*	***17u***	***18m***	***19u***	*21c*
9r	***11u***	***13u***	*14c*	*15c*	***17u***	**19u**	**20m**	21c	**23u**
11u	**13u**	15c	16r	***17u***	***19u***	*21c*	*22c*	***23u***	*25r*
15c	***17u***	***19u***	***20m***	*21c*	***23u***	25r	26c	27r	**29u**
17u	**19u**	21c	22c	***23u***	*25r*	*27r*	***28m***	***29u***	***31u***
21c	***23u***	*25r*	*26c*	*27r*	***29u***	**31u**	32r	33c	35c
23u	25r	27r	**28m**	***29u***	***31u***	*33c*	*34c*	*35c*	***37u***

30 Ultimates and Mixes (level 1 and 4)
↑ 1/1 ratio ↓
30 Raiseds and Composites (level 2 and 3)

← 3/2 ratio →
← 3/2 ratio →

20 Ultimates and Mixes (level 1 and 4)
↑ 1/1 ratio ↓
20 Raiseds and Composites (level 2 and 3)

Figure 73: Based on the addition matrix of the 20 fundamentals (see Figure 68), distribution of the four classes of numbers (level 1 and 4 versus level 2 and 3) into alternating sub-matrices with sizes of 10×6 numbers and 10×4 numbers.

15. Inclusion depth (set theory)

In Chapters 3 and 4, we gradually introduced an organization of the set ℕ₀ into different subsets based on numbers possessing transcendent and inclusion properties. The various classes of whole numbers proposed in this article are fundamentally rooted in set theory. We will demonstrate this concept here by studying a new initial sequence of the set ℕ₀ the first thirty numbers, which we term *Initials.*

15.1 Classes of the first thirty numbers

30 first numbers

20 fundamental numbers

10 digit numbers

0u 1u 2u 3u 4r 5u 6c 7u 8r 9r	*10c 11u 12m 13u 14c 15c 16r 17u 18m 19u*	*20m 21c 22c 23u 24m 25r 26c 27r 28m 29u*

6 Ultimates / 4 non-Ultimates: → 3/2 ratio
(level class 1 / level classes 2, 3 and 4)

12 Ultimates or Mixes / 8 Raiseds or Composites: → 3/2 ratio
(extreme complexity level classes / middle level classes)

18 Ultimates or Raiseds / 12 Composites or Mixes: → 3/2 ratio
(level 1 and 2 complexity classes / level 3 and 4 classes)

Figure 74: Opposition in various 3:2 ratios of the first thirty numbers according to their belonging to the types of classes of the whole number set (See also Figures 2 and 6).

As illustrated in Figure 74, the first thirty whole numbers, which encompass the ten digits and the Twenty Fundamentals, exhibit various 3:2 opposition ratios based on their classification into different number types.

15.2 Depth of inclusion of number classes

Any whole number belongs to a subset of the set $\mathbb{N}_0$. Also, any whole number occupies a specific depth of inclusion within the set $\mathbb{N}_0$.

As illustrated in Figure 75,

- Ultimates have an inclusion depth of level 1,

- Raiseds have an inclusion depth of level 2,

- pure Composites like mixed Composites, have an inclusion depth of level 3.

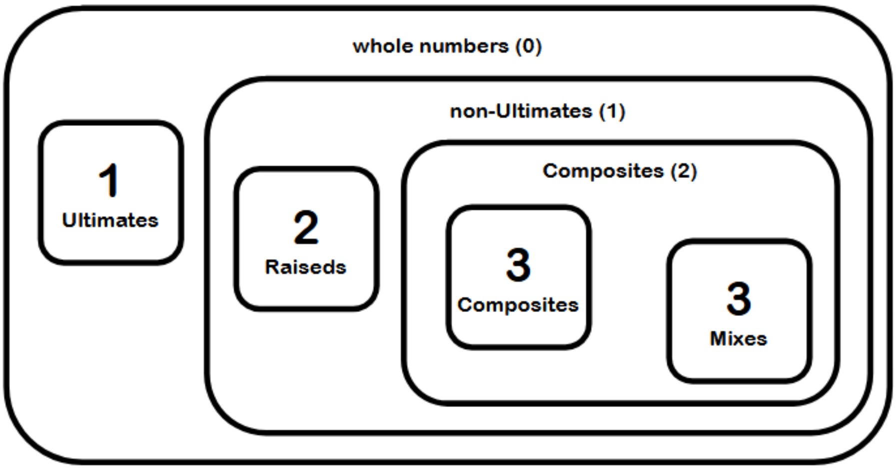


Figure 75: Level of inclusion depth of classes of whole numbers (See also Figures 4 and 5).

15.3 Inclusion depth of the first 30 numbers

As Figure 76 illustrates, the inclusion levels observed among the first thirty numbers are not arbitrary. Specifically, the distribution of inclusion depth follows a precise fractional breakdown: exactly 2/5 of these numbers exhibit a Level 1 inclusion depth, 1/5 exhibit Level 2, and the remaining 2/5 exhibit Level 3.

first 30 numbers inclusion level :

0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*
1	**1**	**1**	**1**	**2**	**1**	**3**	**1**	**2**	**2**	**3**	**1**	**3**	**1**	**3**	**3**	**2**	**1**	**3**	**1**	**3**	**3**	**3**	**1**	**3**	**2**	**3**	**2**	**3**	**1**

inclusion of Level 1 (L_1):	**inclusion of Level 2 (L_2):**	**inclusion of Level 3 (L_3):**
0 1 2 3 5 7 11 13 17 19 23 29	*4 8 9 16 25 27*	*6 10 12 14 15 18 20 21 22 24 26 28*
12 entities	**6 entities**	**12 entities**
2/5	**1/5**	**2/5**

3/5

← 3/2 ratio →

← 2/3 ratio →

3/5

Figure 76: Level of inclusion depth of the first 30 whole numbers (See also Figures 74 and 75).

Figure 76 demonstrates that, following a 3:2 ratio, the 18 entities with an inclusion depth of Level 1 or Level 2 are opposed to the 12 entities with a Level 3 inclusion depth. Simultaneously, when considering the total depth values:

- The 12 entities of Level 1 and the 6 entities of Level 2 yield a cumulative depth of 24 (i.e., (12×1) + (6 ×2) = 24).
- The 12 entities of Level 3 yield a cumulative depth of 36 (i.e., 12×3 = 36).

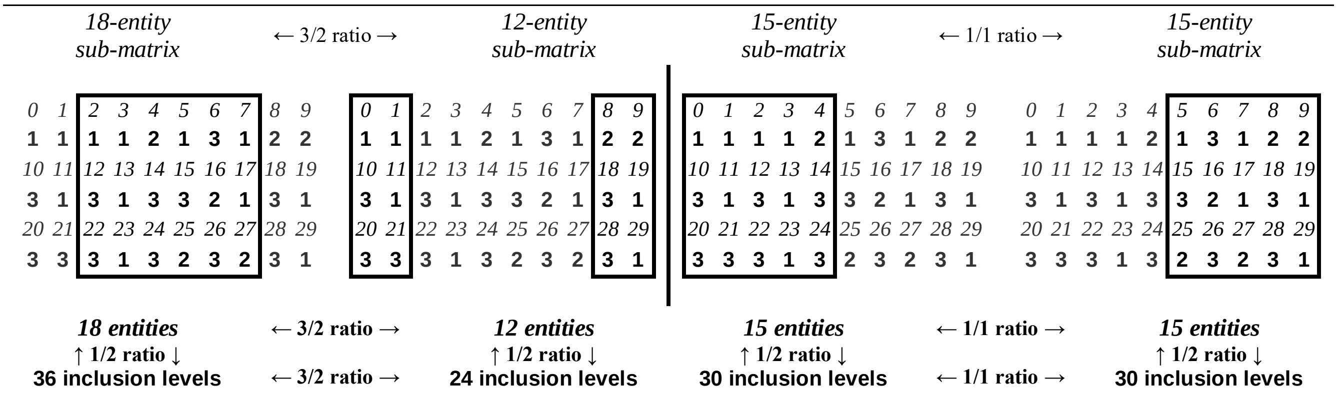
These two cumulative depth values (24 and 36) are thus opposed in the inverse ratio of 2:3.

Following these arithmetic arrangements, as Figure 77 illustrates, the first thirty numbers collectively accumulate 60 levels of inclusion depth within the set $\mathbb{N}_0$, resulting in an exact average depth of Level 2 for this specific set of thirty numbers. Furthermore, similar investigations conducted beyond this range do not reveal any comparable arithmetic phenomena. This finding legitimizes the focused interest taken here in the unique properties observed within the set of the first 30 numbers.

inclusion level:	inclusion of level 1 or 2		inclusion of level 3
entities :	*0 1 2 3 5 7 11 13 17 19 23 29* *4 8 9 16 25 27*		*6 10 14 15 21 22 26* *12 18 20 24 28*
	12 + 6 = 18 entities	**← 3/2 ratio →**	**12 entities**
cumulative levels:	**$12L_1 + 6L_2$ = 12+12=24 levels**	**← 2/3 ratio →**	**$12L_3$ = 36 levels**

Figure 77: Level of inclusion depth and cumulated levels of inclusion of the first 30 whole numbers (See also Figure 74).

Figure 78 further legitimizes the unique inclusion depth properties of the first thirty numbers, both individually and collectively, as presented within a 3 rows by 10 columns number matrix.



18-entity sub-matrix ← 3/2 ratio → *12-entity sub-matrix*

0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1	**1**	**1**	**1**	**2**	**1**	**3**	**1**	**2**	**2**
10	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*
3	**1**	**3**	**1**	**3**	**3**	**2**	**1**	**3**	**1**
20	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*
3	**3**	**3**	**1**	**3**	**2**	**3**	**2**	**3**	**1**

0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1	**1**	**1**	**1**	**2**	**1**	**3**	**1**	**2**	**2**
10	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*
3	**1**	**3**	**1**	**3**	**3**	**2**	**1**	**3**	**1**
20	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*
3	**3**	**3**	**1**	**3**	**2**	**3**	**2**	**3**	**1**

18 entities **← 3/2 ratio →** ***12 entities***

↑ 1/2 ratio ↓ **↑ 1/2 ratio ↓**

36 inclusion levels **← 3/2 ratio →** **24 inclusion levels**

15-entity sub-matrix ← 1/1 ratio → *15-entity sub-matrix*

0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1	**1**	**1**	**1**	**2**	**1**	**3**	**1**	**2**	**2**
10	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*
3	**1**	**3**	**1**	**3**	**3**	**2**	**1**	**3**	**1**
20	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*
3	**3**	**3**	**1**	**3**	**2**	**3**	**2**	**3**	**1**

0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1	**1**	**1**	**1**	**2**	**1**	**3**	**1**	**2**	**2**
10	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*
3	**1**	**3**	**1**	**3**	**3**	**2**	**1**	**3**	**1**
20	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*
3	**3**	**3**	**1**	**3**	**2**	**3**	**2**	**3**	**1**

15 entities **← 1/1 ratio →** ***15 entities***

↑ 1/2 ratio ↓ **↑ 1/2 ratio ↓**

30 inclusion levels **← 1/1 ratio →** **30 inclusion levels**

Figure 78: Symmetrical oppositions of 18 versus 12 entities and 36 levels of inclusion versus 24 in two matrices of the first thirty numbers.

The symmetrical partition of this matrix into two sub-matrices of size $3x$ (18 entities) versus $2x$ (12 entities) reveals a proportional distribution of inclusion levels, yielding 36 levels versus 24 levels.

Furthermore, a symmetrical partition into two other sub-matrices of equal size (15 entities versus 15 entities, or a 1:1 split) also maintains this 1:1 ratio in the distribution of total inclusion levels, with 30 levels versus 30 levels. This outcome stems from the fact that, similar to the original 30-entity matrix, the quantity of numbers with Level 1 inclusion depth is equal to the quantity of numbers with Level 3 inclusion depth within these symmetric sub-matrices. Consequently, the ratio between the total inclusion levels and the quantity of numbers consistently remains 2:1.

16. Discussion

16.1 Summary of fundamental contributions

This study introduced a new conceptual framework for **whole number theory** by proposing an **innovative classification** of the set $\mathbb{N}_0$ into four distinct subsets: the *Ultimates*, the *Raiseds*, the *pure Composites*, and the *mixed Composites*. This partition is founded on the fundamental concepts of **ultimity** and **ultimate divisor**, which incorporate the relative value of the divisor (less than the number in question), thereby resolving a conceptual limitation of classical theory.

The main arithmetic result of this classification lies in the revelation of a unique structural arrangement: an **exact and transcendental ratio of 3:2** (or its inverse 2:3) that organizes the initial distribution of these number classes. This phenomenon is particularly evident in the set of the **Forty Primordial Numbers** (*40 Primordials*), defined as the first ten representatives of each of the four final classes, where 6 vs. 4 or 4 vs. 6 oppositions are systematically observed.

16.2 Theoretical implications and system coherence

The introduction of **Ultimate Algebra** offers an alternative, more coherent, and **conceptually robust** vision of arithmetic. By excluding the trivial decomposition ($n=n\times1$), this algebra prioritizes a representation based on the internal structure of numbers in terms of ultimate divisors.

The most significant impact of this work is the **logical rehabilitation** of the numbers **zero (0) and one (1)**, which are traditionally treated as exceptions in classical prime number theory. The definition of an Ultimate Number, which admits *at most one* smaller divisor, allows 0 and 1 to be unambiguously integrated as the first two Ultimate Numbers, thereby assigning them a structured and essential place in the system. Consequently, the concept of a **Prime Number** can be compactly redefined as an ultimate number that can serve as an ultimate divisor.

16.3 The significance of the 3:2 ratio and fundamental structures

The recurrence of the 3:2 ratio suggests that it is not a random phenomenon, but rather a **reciprocal and/or transcendental magnitude** inherent to the initial constitution of the set $\mathbb{N}_0$. The constant presence of this ratio in sub-matrices based on an opposition between 60 and 40 entities or in triangular configurations T_5 versus T_4 reinforces the idea of a structural constant.

Furthermore, these transcendences frequently integrate fundamental mathematical concepts, notably the **Remarkable Identity** $(a + b)^2 = a^2 + 2ab + b^2$ where the opposed values a=3 and b=2 according to the 3:2 ratio appear naturally to structure the matrices. The distinction between the number classes of an **"extreme"** nature (Ultimates and mixed Composites) and a **"median"** nature (Raiseds and pure Composites) is also organized by this ratio. Finally, the study of the depth of inclusion levels of the first 30 numbers reveals that the 18 entities of level 1 or 2 are opposed to the 12 entities of level 3 according to this same 3:2 ratio, consolidating its role as a **fundamental organizational law**.

16.4 Perspectives and future work

This work opens new avenues for research in number theory:

1. **Exploration of Asymptotic Behaviors:** The detailed analysis focused on the initial segments of $\mathbb{N}_0$. Future investigations should determine whether the prevalence and exactness of the 3:2 ratio is maintained in the organization of numbers as n tends towards infinity, or if new statistical laws emerge.

2. **Properties of Primordial Numbers:** The *40 Primordials* exhibit unique arithmetic properties. It would be relevant to thoroughly how they reveal transcendent and intricate 3:2 various ratios in the different initial sequences of the four classes of numbers introduced and also in the square matrix of the first one hundred whole numbers.

3. **Application of Ultimate Algebra:** The framework of Ultimate Algebra could be applied to **cryptography** or factorization problems, by exploiting the decomposition based on ultimate divisors rather than on traditional prime factors.

4. **Generalization of the 3:2 and $(a+b)^2$ Structures:** Continue the search for matrix configurations (such as those involving triangular matrices T_n that incorporate the 3:2 ratio and the Remarkable Identity, to determine if these arrangements stem from a general law of construction for the sets of whole numbers.

5. **Extension of the Sophie Germain Primes Concept to Ultimate Numbers:** Since ultimates numbers concept allows you to split the set $\mathbb{N}_0$ in just two sub-sets, we propose in another paper [2] the ambitious and innovative idea to applying the Sophie Germain mechanism both to the Ultimates and non-Ultimates.

17. Conclusion

The twin concept of ultimity or non-ultimity of whole numbers, which is based on a new mathematical definition emphasizing the inferiority of the components of the digital entities considered, allows us to propose a novel classification of these whole numbers. Accordingly, every whole number of the set $\mathbb{N}_0$ can belong to only one of the four classes of numbers newly introduced here. These four classes are conventionally named according to their degree of complexity:

The Ultimates (u): The source class, representing the first level of complexity.

The Raiseds (r): The class representing the second level of complexity.

The pure Composites (c), hereafter termed Composites: The class representing the third level of complexity.

The mixed Composites (m), hereafter termed Mixes: The class representing the fourth and final level of complexity.

These four classes of numbers form four subsets of the set $\mathbb{N}_0$ which is also made up, because of this proposed new classification, of the set of non-Ultimates and the global set of Composites.

Thus, the set ℕ₀ is comprised of six distinct sets, all of which derive their characteristics from the original definition of the ultimate numbers. Within ℕ₀, these six sets are assigned a level of inclusion depth ranging from 1 to 3:

> The Ultimates set and the set of non-Ultimates are assigned a Level 1 inclusion depth.
>
> The Raiseds set and the set of Composites are assigned a Level 2 inclusion depth.
>
> The set of pure Composites and the set of mixed Composites are assigned a Level 3 inclusion depth.

Given that ℕ₀ conventionally denotes the set of whole numbers, it is suggested that these six new sets be represented by similar types of designations.

The unique yet consistent arithmetic arrangements observed in the initial organization of these new number sets, most of which fall into 3:2 ratios, strongly validate the legitimacy of this novel classification of whole numbers.

It is therefore essential to emphasize that this classification unambiguously incorporates the exotic numbers 0 (zero) and 1 (one).

This new classification also introduces novel mathematical notions: **fundamental numbers**, **primordial numbers**, and the **extreme** or **median** classes of numbers. These different concepts are extensively highlighted in investigations involving numerous closed matrices, each consistently sized at 5*x* entities. These matrices correspond to the initial sequences of numbers from the set ℕ₀, as it is within the set's initial constitution that unique arithmetic phenomena are revealed concerning the distribution of the different types of numbers considered.

Ultimately, based on multiple approaches and demonstrations, the different identified components within the initial part of the set ℕ₀ are singularly organized in a 3:2 ratio (or its inverse) regarding reciprocal and/or transcendent magnitude. These transcendences frequently operate by integrating the concepts of remarkable identity and triangular numbers.

Whole number glossary

> Warnings: this glossary lists all the main number concepts introduced in this article about the whole number set. These different new names and other words are not yet universally recognized as official terms but they constitute an innovative proposal describing the set ℕ₀.

Listed in alphabetical order:

composite numbers: *a composite number is a non-ultimate and non-raised number.*

Composites: *the abbreviated appellation for* ***pure Composites****.*

fundamental numbers: *20 in number, fundamental numbers are the set of the first ten* ***ultimate numbers*** *and the ten* ***non-ultimate numbers****. Singularly, this is the set of the first twenty numbers also. Also simply called* ***Fundamentals****.*

Fundamentals: *the abbreviated appellation for* ***fundamental numbers****.*

initial numbers: *30 in number, initial numbers are the set of the first thirty numbers. This set includes the 20 fundamentals and the first ten non-fundamental numbers. Also simply called* ***Initials****.*

Initials: *the abbreviated appellation for* ***initial numbers****.*

mixed Composites: *a mixed composite number is a non-ultimate and non-raised number admitting at least one* ***raised number*** *as divisor. Also simply called* ***Mixes****. Abbreviated by m.*

Mixes: *the abbreviated appellation for* ***mixed Composites****.*
non-ultimate numbers: *a non-ultimate number admits more than one divisor being inferior to it in value. Also simply called* ***non-Ultimates****.*

non-Ultimates: *the abbreviated appellation for* ***non-Ultimates numbers****.*

prime numbers: *a prime number is an* ***ultimate number****, which can be a* ***ultimate divisor*** *of a whole number.*

primordial numbers: *a primordial number is one of the first ten entities of the four sequences of numbers defined as* ***Ultimates****,* ***Raiseds****,* ***Composites*** *or* ***Mixes****. So quantity of primordial numbers is 40. Also simply called* ***Primordials****.*

Primordials: *the abbreviated appellation for* ***primordial numbers****.*

pure Composites: *a pure composite number is a* ***non-ultimate*** *and non-raised number not admitting* ***raised number*** *as divisor. Also simply called* ***Composites****. Abbreviated by c.*

raised numbers: *a raised number is a* ***non-ultimate number****, power of an* ***ultimate number****. Also simply called* ***Raiseds****. Abbreviated by r.*

Raiseds: *the abbreviated appellation for* ***raised numbers****.*

ultimate divisor: *An ultimate divisor of a whole number is an* ***ultimate number*** *less than this whole number and non-trivial divisor of this whole number. This definition corresponds to numbers called* ***Prime Numbers****.*

ultimate numbers: *an ultimate number admits at most one divisor being inferior to it in value. Also simply called* ***Ultimates****. Abbreviated by u.*

Ultimates: *the abbreviated appellation for* ***ultimate numbers****.*

References :

1. Jean-Yves Boulay. The ultimate numbers and the 3/2 ratio. Just two primary sets of whole numbers. 2020. https://www.researchgate.net/publication/339943634

2. Jean-Yves Boulay. Sophie Germain primes concept expanded to ultimate numbers. Number Genetics and the 3 to 2 ratio. 2025. https://www.researchgate.net/publication/392508940

Appendix

This appendix serves as a complement to Chapter 5, focusing on the concept of the ultimate divisor.
The demonstrations provided here are intended to further reinforce the validity of this mathematical concept.

A. Ultimate divisors and the matrix of the first hundred numbers

Matrix of the first hundred whole numbers:
27 Ultimates - **73 non-Ultimates**

0 -	1 -	2 -	3 -	**4** 2^2	5 -	**6** 2×3	7 -	**8** 2^3	**9** 3^2
10 2×5	11 -	**12** $2^2\times3$	13 -	**14** 2×7	**15** 3×5	**16** 2^4	17 -	**18** 2×3^2	19 -
20 $2^2\times5$	**21** 3×7	**22** 2×11	23 -	**24** $2^3\times3$	**25** 5^2	**26** 2×13	**27** 3^3	**28** $2^2\times7$	29 -
30 $2\times3\times5$	31 -	**32** 2^5	**33** 3×11	**34** 2×17	**35** 5×7	**36** $2^2\times3^2$	37 -	**38** 2×19	**39** 3×13
40 $2^3\times5$	41 -	**42** $2\times3\times7$	43 -	**44** $2^2\times11$	**45** $3^2\times5$	**46** 2×23	47 -	**48** $2^4\times3$	**49** 7^2
50 2×5^2	**51** 3×17	**52** $2^2\times13$	53 -	**54** 2×3^3	**55** 5×11	**56** $2^3\times7$	**57** 3×19	**58** 2×29	**59** -
60 $2^2\times3\times5$	61 -	**62** 2×31	**63** $3^2\times7$	**64** 2^6	**65** 5×13	**66** $2\times3\times11$	67 -	**68** $2^2\times17$	**69** 3×23
70 $2\times5\times7$	71 -	**72** $2^3\times3^2$	73 -	**74** 2×37	**75** 3×5^2	**76** $2^2\times19$	**77** 7×11	**78** $2\times3\times13$	79 -
80 $2^4\times5$	**81** 3^4	**82** 2×41	83 -	**84** $2^2\times3\times7$	**85** 5×17	**86** 2×43	**87** 3×29	**88** $2^3\times11$	89 -
90 $2\times3^2\times5$	**91** 7×13	**92** $2^2\times23$	**93** 3×31	**94** 2×47	**95** 5×19	**96** $2^5\times3$	97 -	**98** 2×7^2	**99** $3^2\times11$

210 ultimate divisors:

0	0	0	0	**2**	0	**2**	0	**3**	**2**
2	0	**3**	0	**2**	**2**	**4**	0	**3**	0
3	**2**	**2**	0	**4**	**2**	**2**	**3**	**3**	0
3	0	**5**	**2**	**2**	**2**	**4**	0	**2**	**2**
4	0	**3**	0	**3**	**3**	**2**	0	**5**	**2**
3	**2**	**3**	0	**4**	**2**	**4**	**2**	**2**	0
4	0	**2**	**3**	**6**	**2**	**3**	0	**3**	**2**
3	0	**5**	0	**2**	**3**	**3**	**2**	**3**	0
5	**4**	**2**	0	**4**	**2**	**2**	**2**	**4**	0
4	**2**	**3**	**2**	**2**	**2**	**6**	0	**3**	**3**

Figure A1: Distribution of the 15 ultimate divisors (from 2 to 47) in the matrix of the first hundred numbers and distinction (dark areas) of their first appearance. Individual statement of the total quantity of ultimate divisors constituting the 73 non-Ultimates.

In the closed matrix of the first hundred whole numbers, 27 are ultimate numbers and 73 are non-ultimate numbers. The tables in Figure A1 demonstrate that these 73 non-Ultimates are compositions of 15 ($5x \to x = 3$) different ultimate divisors (ultimate numbers from 2 to 47) and locate their first appearance within this matrix. For example, the *ultimate divisor 5* appears to the first time as a ultimate divisor of the *non-ultimate 10.* In the right part of Figure A1, the total of the ultimate divisors individually composing these 73 non-ultimate is counted**.**

As a reminder (see Chapter 5.4), the ultimate numbers are not composed of ultimate divisors and the numbers *zero* (0) and *one* (1) are neither ultimate divisors, nor composed of ultimate divisors.

A.1 Fifteen ultimate divisors and matrix of the first hundred numbers

These fifteen ultimate divisors are grouped, Figure A2, into three sets whose size increases regularly according to whether they make up more or less categories of numbers (classes). Among these fifteen ultimate divisors, 4 are found to be divisors of the three classes of non-ultimate numbers (the Raiseds, the Composites and the Mixes). Then 5 are only divisors of two classes (Composites and Mixes) and finally 6 ultimate divisors are only of one class of numbers, that of Composites. Also, in a ratio of 3:2, 9 ultimate divisors (from 2 to 23) composing more than one class of numbers oppose to 6 divisors (from 29 to 47) composing only one.

The 15 ultimate divisors of the first 100 numbers:

2 3 5 7	***11 13 17 19 23***	***29 31 37 41 43 47***
4 divisors	**5 divisors**	**6 divisors**
of Raiseds, of Composites, of Mixes	of Composites, of Mixes	of Composites

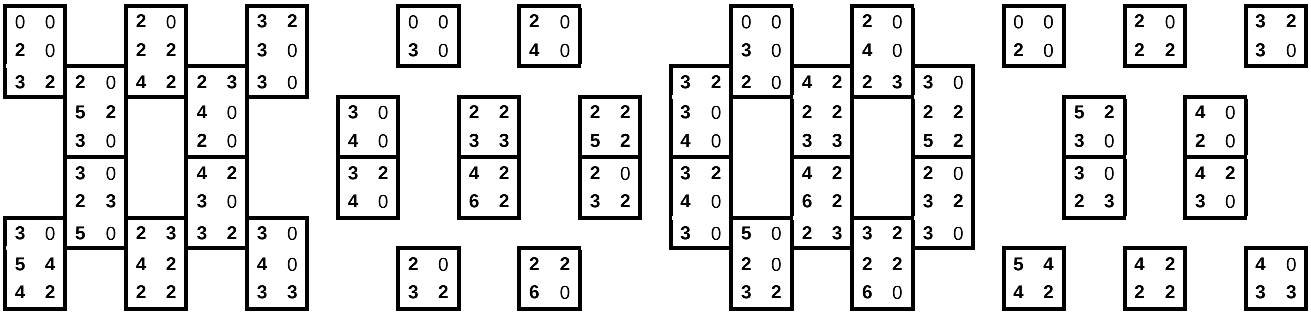
9 divisors of 3 classes or of 2 classes **← 3/2 ratio →** **6 divisors of 1 class only**

Figure A2: Distinction of the 15 ultimate divisors (from 2 to 47) according to the types of classes of whole numbers that they compose (See also Figure A1 and Chapter 3.1).

A.2 Symmetric sub-matrices and 3:2ratio

As shown in the right table of Figure A1, the set of the first hundred numbers (those not composed of the 27 Ultimates) contains a total of 210 ultimate divisors ($5x \rightarrow x = 42$). This total value permits the formation of 3:2 ratios. Indeed, in several symmetrical configurations, when sub-matrices of 60 and 40 numbers are opposed in a 3:2 ratio, their ultimate divisors are also opposed in a 3:2 ratio, with 126 ultimate divisors in the first sub-matrix versus 84 in the second, respectively (3×42 versus 2×42).

Sub-matrix to 10 times 6 numbers (60 numbers) **← 3/2 ratio →** **Sub-matrix to 10 times 4 numbers** (40 numbers) **Sub-matrix to 10 times 6 numbers** (60 numbers) **← 3/2 ratio →** **Sub-matrix to 10 times 4 numbers** (40 numbers)



0 0		2 0		3 2		0 0		2 0			0 0		2 0		0 0		2 0		3 2
2 0		2 2		3 0		3 0		4 0			3 0		4 0		2 0		2 2		3 0
3 2	2 0	4 2	2 3	3 0						3 2	2 0	4 2	2 3	3 0					
	5 2		4 0		3 0		2 2		2 2	3 0		2 2		2 2		5 2		4 0	
	3 0		2 0		4 0		3 3		5 2	4 0		3 3		5 2		3 0		2 0	
	3 0		4 2		3 2		4 2		2 0	3 2		4 2		2 0		3 0		4 2	
	2 3		3 0		4 0		6 2		3 2	4 0		6 2		3 2		2 3		3 0	
3 0	5 0	2 3	3 2	3 0						3 0	5 0	2 3	3 2	3 0					
5 4		4 2		4 0		2 0		2 2			2 0		2 2		5 4		4 2		4 0
4 2		2 2		3 3		3 2		6 0			3 2		6 0		4 2		2 2		3 3

126 ultimate divisors **← 3/2 ratio →** **84 ultimate divisors** **126 ultimate divisors** **← 3/2 ratio →** **84 ultimate divisors**

Figure A3: Respective quantities of the ultimate divisors constituting the first hundred numbers (See also matrix in Figure A1).

Inside symmetrical sub-matrices, composed of 10 twin microzones with 6 versus 4 entities, such as those in Figure A3, the quantities of ultimate divisors constituting the first hundred numbers are also opposed in 3:2 ratios, yielding 126 ultimate divisors in the sub-matrices of 60 entities versus 84 ultimate divisors in the sub-matrices of 40 entities.

A.3 First appearance of the fifteen ultimate divisors

In these same two double geometrically symmetric sub-matrices, it turns out, Figure A4, that the first appearance of the fifteen ultimate divisors (see Figure A1) is also organized in a perfect ratio of 3:2 as value with nine first appearances in sub-matrices of 60 numbers versus six first appearances in sub-matrices of 40 numbers.

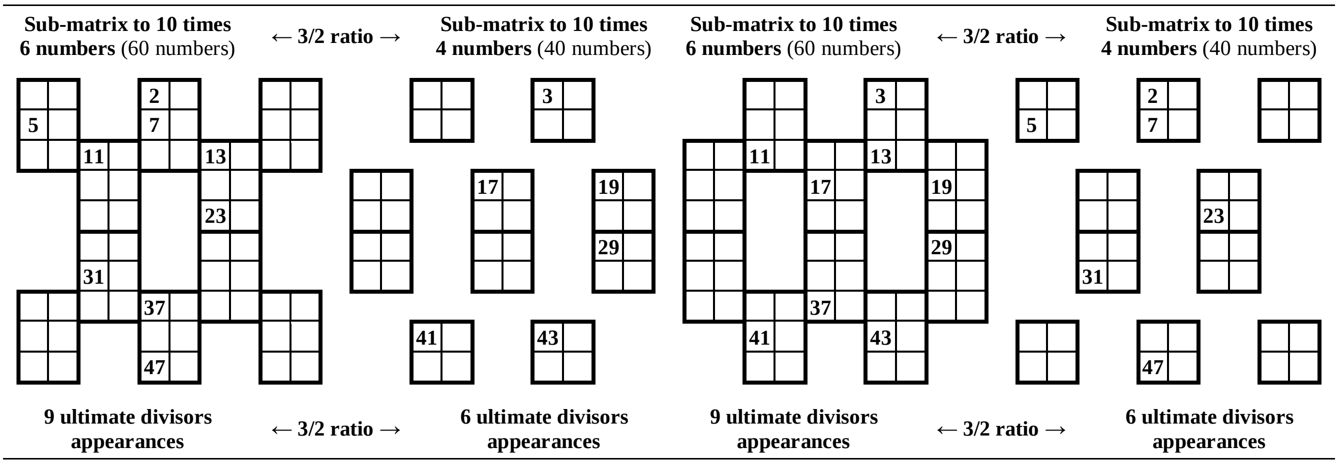


Figure A4: Distribution of the first appearance of the 15 ultimate divisors in the matrix of the first 100 numbers
(See also matrix in Figure A1).