Does graviton really have spin?

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Abstract

In this paper, we calculated the spin of gravitational field theoretically, suspecting the correctness of popular spin 2 postulate.

Introduction

In QFT, spin is came from Lorentz transformation of the field. For scalar field,

$$\phi' = \phi$$

so $s = 0$. For vector field,

$$A'_\nu = A_\mu + \frac{\mathrm{i}}{2}\omega_{\rho\sigma}\mathcal{J}^{\rho\sigma\mu}{}_{\nu}A_\mu$$

so $s = 1$. For spinor field,

$$\psi'_b = \psi_b + \frac{\mathrm{i}}{2}\omega_{\rho\sigma}\mathcal{S}^{\rho\sigma a}{}_{b}\psi_a$$

so $s = \frac{1}{2}$. However, the explanation for graviton is based on linearized GR. Although it had been predicted gravitational waves successfully, approximation couldn't be a legal proof. We need a proof in full theory.

Main Calculation

If we do analogy for metric field above, it would be

$$g'_{\mu\nu} = g_{\mu\nu} + \frac{\mathrm{i}}{2}\omega_{\rho\sigma}\mathcal{J}^{\rho\sigma\alpha}{}_{\nu}g_{\mu\alpha} + \frac{\mathrm{i}}{2}\omega_{\rho\sigma}\mathcal{J}^{\rho\sigma\alpha}{}_{\mu}g_{\alpha\nu}$$

But $\mathcal{J}^{\rho\sigma\mu}{}_{\nu} = \mathrm{i}(g^{\rho\mu}g^{\sigma}_{\nu} - g^{\rho\nu}g^{\sigma}_{\mu})$, hence RHS gives

$$g'_{\mu\nu} = g_{\mu\nu}$$

identical to scalar field! It's actually expected cause $g_{\rho\sigma}\Lambda^{\rho}_{\mu}\Lambda^{\sigma}_{\nu} = g_{\mu\nu}$ for every Lorentz transformation Λ. Hence we suspect if "graviton has spin 2" is wrong.

Aside

If we treat Dirac matrices as a field, then the transformation is

$$\gamma'{}_{\mu b}^{\ \ a} = \gamma_{\mu b}^{\ \ a} + \frac{\mathrm{i}}{2}\omega_{\rho\sigma}\mathcal{J}^{\rho\sigma\alpha}{}_{\mu}\gamma_{\alpha b}^{\ \ a} - \frac{\mathrm{i}}{2}\omega_{\rho\sigma}\mathcal{S}^{\rho\sigma a}{}_{c}\gamma_{\mu b}^{\ \ c} + \frac{\mathrm{i}}{2}\omega_{\rho\sigma}\mathcal{S}^{\rho\sigma c}{}_{b}\gamma_{\mu c}^{\ \ a}$$

By $\mathcal{S}^{\rho\sigma} = \frac{\mathrm{i}}{4}[\gamma^\rho, \gamma^\sigma]$, expanding RHS gives

$$\gamma'{}_{\mu b}^{\ \ a} = \gamma_{\mu b}^{\ \ a}$$

seems Dirac matrices works like “metric”.

Reference

[1] Zhao-Huan Yu “量子场论讲义” (https://yzhxxzxy.github.io/teaching/1807_QFT.pdf)