# The Series Solution to Polynomial Equations

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**Abstract**: Finding the roots of polynomial equations is a fundamental problem in mathematics. This paper discovers that general polynomial equations can be simplified into a canonical or standard form through Tschirnhaus transformations. A power series representation consisting of coefficients in the canonical or standard form is a universal representation of the roots of polynomial equations. If the series converges, a root of the equation is obtained. If the series does not converge, it can be further transformed through one or more Tschirnhaus transformations to obtain a convergent series representation. This method is applicable to higher degree polynomial equations with real and complex coefficients, avoiding the complex determination of whether they are solvable in the radicals , and has universal significance. This advance returns the problem of finding polynomial roots to the realm of pure algebra, using only polynomial transformations and multivariable power series.

**Keywords**: Polynomial equations, Tschirnhaus transformations, canonical form, standard form, power series, convergence

Finding the roots of polynomial equations is one of the fundamental problems in mathematics. Methods for finding the roots of quadratic polynomials existed over 2,000 years ago, but it wasn't until the advent of the Cardano-Tartaglia method in the 16th century that methods for finding the roots of cubic and quartic polynomials were developed. Thereafter, no progress was made on the problem of finding the roots of quintic polynomials. Eventually, Ruffini (1799), Abel (1826), and Galois (1832) proved that the roots of quintic polynomials and higher have no radical solutions [1]. However, Abel and Galois's theory only proved that quintic polynomials and higher have no radical solutions; it did not deny the existence of other general solutions.

Assume a general Nth-degree polynomial equation:

$$f(x) \equiv a_N x^N + a_{N-1} x^{N-1} + \circ \circ \circ + a_1 x^1 + a_0 = 0 \qquad (1)$$

where: $a_N \neq 0$; $a_k, k = 0,1,2,...,N$ are complex numbers.

According to the fundamental theorem of algebra, a polynomial of degree N has N roots in the complex range. A complete polynomial of degree N has (N+1) coefficients. Apparently the problem of finding the roots of a Nth-degree polynomial is to find a function that maps the (N+1) dimensional coefficient space to the N dimensional root space. However, in reality, the (N+1) coefficients are not independent. For example, by properly selecting the parameters of the linear translation transformation, the sub-highest terms can be eliminated, which becomes a root-finding problem with N coefficients. In 1683, Tschirnhaus proposed a nonlinear polynomial transformation that can eliminate the terms $x^{N-1}$ and $x^{N-2}$ [2]. Subsequently, Bring 1786 and Jerrad 1834 proved that for general polynomial equations, the terms $x^{N-1}, x^{N-2}$ and $x^{N-3}$ can be eliminated, resulting in a polynomial equation with fewer coefficients in the new variables [3,4]. How many redundant coefficients of a general polynomial of degree N can be eliminated, or how many independent coefficients or degrees of freedom a polynomial of degree N has is an interesting question. The Problem 13 of the famous Hilbert 23 mathematical problems conjectured that a 9th-degree polynomial can be expressed using functions with no more than 4 parameters and their limited arithmetic operations [3,5].

It has been proven that based on the Tschirnhaus transform, polynomial equations of degree less than or equal to 5 can be transformed into equations with only one coefficient [6,7]; for higher-degree polynomials, Hilbert pointed out that polynomials of degree 6, 7, and 8 can be represented by continuous functions with 2, 3, and 4 parameters, respectively, and this result is optimal [5]. For the Tschirnhaus transform limited to polynomials, when the degree of the polynomial and of transformations increase, there may be more terms that can be eliminated, reducing the complexity of solving the roots of higher-degree equations. However, to the best of the author's knowledge, there is no general conclusion on this. Brauer proposed the concept of resolvent degree in 1975, which is considered an important progress in this field [8]. The study of related issues is related to a series of profound mathematical problems and has attracted people's attention.

In addition to the radical solution of polynomial equations, there are other types of solutions, such as de Fériet's Fourier function [9], hypergeometric function [10], elliptic modular function and inverse Lagrange formula [11], most of which are computationally complex or subject to convergence constraints [12-15]. Recently, Longfellow gave the general solution transformation and inverse canonical beta function of the sixth-degree equation with real coefficients through Tschirnhaus transformation [11]. Wildberger and Rubine discussed the possibility of using super Catalan numbers to solve high-degree polynomial equations [16-17]. For fifth-degree and sixth-degree polynomial equations, a general solution expressed by integer series and fractional series power series was also given [18-20].

For general polynomial equations of degree 5 and above, it is meaningful to seek solutions to general equations expressed by power series. This avoids the debate on whether they can be solved by radicals and provides a convenient choice for practical applications.

This study discovered that a power series consisting of sequences defined by canonical and standard forms provides a universal solution to the roots of polynomials. This method returns the problem of finding polynomial roots to the realm of pure algebra, using only polynomial transformations and multivariable power series. This method is applicable to high-degree polynomial equations with real and complex coefficients, avoiding the complex determination of whether the radical is solvable. It has universal significance and practical value.

## 1. Simplification and classification of polynomial equations

Assume that after the Tschirnhaus transformation, the $(\mathrm{N}-1)$, $(\mathrm{N}-2)$ and $(\mathrm{N}-3)$ terms are eliminated and Eq.(1) becomes:

$$g(y) \equiv b_N y^N + b_{N-4} y^{N-4} + \circ \circ \circ + b_1 y^1 + b_0 = 0 \qquad (2)$$

Further simple transformation, it can be transformed into a form where the highest term coefficient and the constant term are both constant 1:

$$h(z) \equiv z^N + c_{N-4} z^{N-4} + \circ \circ \circ + c_1 z^1 + 1 = 0 \qquad (3)$$

**Definition 1**: Canonical form. The type of polynomial equations that can be represented by only independent coefficients or minimal coefficients after Tschirnhaus transformation, and its number of coefficients is $(\mathrm{N}-2)$.

**Definition 2**: Standard form. The canonical form in which the highest term and the constant term are both constant 1, and its number of non-constant coefficients is $(\mathrm{N}-4)$.

The above Eq.(2) is the canonical form of the polynomial equation, and Eq.(3) is its corresponding standard form.

Taking the sixth-degree equation as an example, the Tschirnhaus transformation can eliminate the three middle terms and transform it into a canonical form equation containing only the sixth-degree term, the constant term, and the remaining two middle terms. The equation contains 4 coefficients. If the two middle terms are quadratic and linear, they are recorded as 6210. It is easy to see that there are three types of equation forms that can be directly derived from the general sixth-degree equation through transformation, that is, according to the different degrees of the remaining two terms, they can form 6210, 6310, and 6320 types. The reciprocal transformation of the variables can also obtain 6430, 6530, and 6540 types. At the same time, there are four types of 6410, 6420, 6510, and 6520 that cannot be directly obtained through transformation. Therefore, there are a total of 10 types of canonical forms for the general sixth-degree equation. Similar reasoning can be used to obtain the number of canonical forms of high-degree polynomial equations.

Furthermore, the canonical form of the sextic equation, which contains four coefficients, can be simplified to a form with only two coefficients. In this case, the two coefficients can appear in any of the four positions. Taking into account the eight cases where the signs in all four positions can be chosen, we can see that there are a total of 480 types of two-coefficient representations for the general sextic equation.

Similarly, we can deduce that there are a total of 120 types of single-coefficient types for equations of degree 5 or less; 3200 types of three-coefficient types for septic equations, and so on.

As the above discussion shows, the number of types of high-degree polynomial equations that can be transformed and simplified increases rapidly, posing a significant challenge to finding solutions that adapt to the general form.

## 2. Series Representation and Properties of Roots of Polynomial Equations in Standard Form

Because the roots of a polynomial are completely determined by its coefficients, according to the above discussion, the simplified polynomial equation will be easier to solve, and its roots can be expressed as the power series of the remaining independent coefficients or the minimum coefficients. Assuming that the independent coefficients in Eq.(1) are $\{p_1, p_2, ..., p_k\}$, then the roots of the equation can be expressed as:

$$x(p_1, p_2, ..., p_k) = \sum_{m_1+m_2+...+m_k \geq 0} \alpha_{m_1 m_2 ... m_k} p_1^{m_1} p_2^{m_2} ... p_k^{m_k} \quad (4)$$

The coefficients can be determined by substituting them into the polynomial equation, they may contain integers or fractions, real numbers or complex numbers, and there may be multiple sequences of solutions for each solution.

For the series representation of Eq.(4), if the series converges, a root of the equation is obtained. Otherwise, further transformation is required, as shown in the following examples and discussions.

The question of the minimum independent coefficients of polynomial equations has always been a hot topic of discussion [4]. They are closely related to the resolvent degree (RD). For polynomials of degree not greater than 9, the number of remaining coefficients after the Tschirnhaus transformation is exactly the same as the conjecture given by Hilbert, that is,

$$\begin{aligned} RD(N) &= 1, N \leq 5; \\ &= N - 4, 5 < N < 9. \end{aligned} \quad (5)$$

where: N is the degree of the equation.

From the discussion in the previous section, the simplified forms of general polynomial equations present a wide variety of types. However, the experiment of solving Eq.(4) shows that the standard form has special significance for solving polynomial equations. The coefficient sequence it obtains is a fractional sequence, the absolute value or modulus of each term is not greater than the constant 1, and the distribution shows the characteristics of alternating positive and negative. Therefore, the standard form of fractional series sequence has good convergence, and the convergence radius is better than other types.

Based on the experiment of solving finite-degree polynomial equations, for polynomial equations of any degree, we have

**Conjecture 1**: For a polynomial equation of any degree, the absolute value or modulus of each coefficient in the fractional series corresponding to its standard form is no greater than constant 1.

Experiments have shown that standard form fractional series have good convergence properties.

**Proposition 1**: If the real absolute value or complex modulus of the coefficients in the standard form equation is no greater than constant 1, then the fractional series converges.

For any given equation, the coefficients of the initial standard form are not guaranteed to have absolute values or modulosity no greater than constant 1, the fractional series may diverge.

In experiments, we found that for the standard form equation corresponding to the series that initially did not converge, we can further perform Tschirnhaus transformations to obtain a new standard form equation. A finite number of iterations of these transformations always yields a standard form equation that satisfies the series convergence conditions. We then return to solving a series of transformed equations, primarily a sequence of quartic Tschirnhaus transformations and scaling transformations. Therefore, the series solution to the high-degree equation presented here means obtaining a root of the high-degree equation using a series of quartic equations. In other words, the problem of finding the root of a high-degree equation is reduced to solving a series of quartic equations.

Geometrically, this series of Tschirnhaus transformations maps any point in (N-4) dimensional space to the unit hypercube at the origin. Theoretically, the ergodicity of the Tschirnhaus transformations is a condition that the mapping must satisfy. We have

**Conjecture 2**: For polynomial equations of any degree, the Tschirnhaus polynomial transformations on which their standard form depends are spatially ergodic, ensuring that after a finite number of transformations, the coefficients of the standard form are all located in the unit hypercube.

To this end, we give

The algorithmic process for solving a general polynomial series is as follows:

1. For a given N-degree equation, apply the quadratic and quartic Tschirnhaus transformations, eliminating the (N-1), (N-2) and (N-3) terms of the equation and obtaining the canonical form.
2. Substitute variables to convert the coefficients of the highest term and the constant term into the canonical form with constant 1.
3. Check whether the values of the equation coefficients are within the convergence interval:
   If the convergence condition is not met, apply the quartic Tschirnhaus transformation to obtain the canonical form and go to step 2.
   If the convergence condition is met, go to the next step.
4. Substitute coefficient values in the standard form into the expression for the series solution and calculate a root of the transformed equation. Then, substituting them back in sequence to obtain a root of the original equation.
5. End.

The next section will provide practical examples to illustrate the solution process.

## 3. Examples of solving roots of polynomial equations

The following experiment uses the above process to solve a root of a polynomial equation of degree 2-7, and compares it with a direct traditional iterative method. The calculations of fractional series in the standard form are all expanded upto no more than 10 power of the argument, and the calculation result is rounded to 7 decimal places. Clearly, increasing the number of summations in the series will improve the accuracy of the solution.

**Example 1**: Solving a quadratic polynomial equation using a fractional sequence

Given

$$x^2 + 2x - 33 = 0$$

Assume the transformation:

$$\mathrm{x} = \sqrt{-33}\ \mathrm{y}$$

The transformed standard form equation is:

$$y^2+py+1=0$$

where： $p=-0.3481553$

The fractional series solution obtained from the standard form of the quadratic equation is:

$$\mathrm{y(p,s)}=\mathrm{s}-\frac{\mathrm{p}}{2}+\frac{\mathrm{p}^2}{8\mathrm{s}}-\frac{\mathrm{p}^4}{128\mathrm{s}^3}+\frac{\mathrm{p}^6}{1024\mathrm{s}^5}+...$$

where： $\mathrm{s}=\pm \mathrm{I}$, $\mathrm{I}$ imaginary unit.

Let $\mathrm{s}=\mathrm{I}$， one root of the y equation is

$$\mathrm{y}=1.1891161$$

Bringing back the original equation, we get a root of the original equation:

$$\mathrm{x}=-6.8309519$$

This is exactly the same as the result obtained by directly solving the original equation.

**Example 2**: Solving a cubic polynomial equation using a fractional sequence

$$x^3+3x-22=0$$

Assume the transformation:

$$\mathrm{x}=\sqrt[3]{-22}\ \mathrm{y}$$

The converted standard form equation is:

$$y^3+py+1=0$$

where： $p=0.1910481+0.3309051I$

The fractional series solutions from the standard form of a cubic equation:

$$\mathrm{y(p,s)}=\mathrm{s}-\frac{\mathrm{p}}{3\ \mathrm{s}}+\frac{\mathrm{p}^3}{81\ \mathrm{s}^5}+\frac{\mathrm{p}^4}{243\ \mathrm{s}^7}-\frac{4\mathrm{p}^6}{6561\ \mathrm{s}^{11}}+...$$

where： $\mathrm{s}=-1,\ \frac{1}{2}\pm\frac{\sqrt{3}}{2}\ \mathrm{I}$

Let $\mathrm{s}=-1$， one root of the transformed standard form equation is:

$$\mathrm{y}=-1.0643258-0.1102260\ \mathrm{I}$$

One root of the original equation is:

$$\mathrm{X} =- 1.2236629 - 2.7371614\ \mathrm{I}$$

Directly iteratively solve a root of the original equation：

$$\mathrm{x} =- 1.2236629 - 2.7371615\ \mathrm{I}$$

**Example 3**: Solving a quartic polynomial equation using a fractional sequence

$$x^4 - 4x + 51 = 0$$

Assume the transformation:

$$\mathrm{x} = \sqrt[4]{51}\ \mathrm{y}$$

The converted standard form equation is:

$$y^4 + py + 1 = 0$$

where：$p =- 0.2095957$

The fractional series solutions obtained from the standard form of the quartic equation:

$$\mathrm{y(p,s)} = \mathrm{s} - \frac{\mathrm{p}}{4\ \mathrm{s}^2} - \frac{\mathrm{p}^2}{32\ \mathrm{s}^5} + \frac{7\mathrm{p}^4}{2048\ \mathrm{s}^{11}} + \frac{\mathrm{p}^5}{512\ \mathrm{s}^{14}} + \ldots$$

where：$\mathrm{s} =\pm \frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2}\ \mathrm{I}$

Let $\mathrm{s} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\ \mathrm{I}$，one root of the transformed equation is:

$$\mathrm{y} = 0.7080729 + 0.6537316\ \mathrm{I}$$

One root of the original equation is：

$$\mathrm{x} = 1.8922151 + 1.7469965\ \mathrm{I}$$

This is exactly the same as directly solving a root of the original equation.

**Example 4**: Solving a quintic polynomial equation using a fractional sequence

$$x^5 - 2x - 5 = 0$$

Assume the transformation：

$$x = \sqrt[5]{-5}\ y$$

The converted standard form equation is:

$$y^5 + py + 1 = 0$$

where： $p =- 0.5518919$

The fractional series solution obtained from the standard form of the quintic equation:

$$y(p, s) = s - \frac{p}{5\ s^3} - \frac{p^2}{25\ s^7} - \frac{p^3}{125\ s^{11}} + \frac{21p^5}{15625\ s^{19}} + \ldots$$

where： $s =- 1$， $\frac{1}{4} + \frac{\sqrt{5}}{4} \pm \frac{\sqrt{2}\ \sqrt{5-\sqrt{5}}}{4}\ I$， $\frac{1}{4} - \frac{\sqrt{5}}{4} \pm \frac{\sqrt{2}\ \sqrt{5+\sqrt{5}}}{4}\ I$

Let $s =- 1$， one root of the transformed equation is：

$$y =- 1.0994929$$

One root of the original equation is：

$$X = 1.5170029$$

Direct iterative method to solve a root of the original equation：

$$x = 1.5170030$$

**Example 5**: Solving the fractional sequence of a sixth-degree polynomial equation 1

$$9x^6 - 2x^2 + 3x - 4 = 0$$

Assume the transformation：

$$x =- 0.8735805I\ y$$

The converted standard form equation is：

$$y^6 + qy^2 + px + 1 = 0$$

where： $p =- 0.6551853\ I, q =- 0.3815714$

The fractional series solution obtained from the standard form is:

$$y(p, q, s) = s - \frac{p}{6s^4} - \frac{q}{6s^3} - \frac{p^2}{24\ s^9} - \frac{pq}{18\ s^8} - \frac{q^2}{72\ s^7} + \ldots$$

where： $s = \pm I, \pm\frac{1}{2}\sqrt{2-2I\sqrt{3}}, \ \pm\frac{1}{2}\sqrt{2+2I\sqrt{3}}$

Let $s = I$， a series solution of the transformed equation is：

$$y = 1.1446979\ I$$

One root of the original equation is:

$$X = -0.9999857$$

The direct iteration method solves a root of the original equation as follows:

$$x = -1.0000000$$

**Example 6**: Solving the fractional sequence of a sixth-degree polynomial equation 2 [14]

$$f(x) \equiv x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0$$

This equation has complex roots represented by trigonometric functions：

$$-\cos\frac{\pi}{7} \pm \sin\frac{\pi}{7}, \ \ \cos\frac{2\pi}{7} \pm \sin\frac{2\pi}{7}, \ \ -\cos\frac{3\pi}{7} \pm \sin\frac{3\pi}{7}$$

Assume the transformation：

$$g_1(x, y) \equiv x^2 - x + y = 0$$

Using the resultant, eliminating the variable x from $f(x)$ and $g_1(x, y)$ , we get the new equation:

$$h_1(y) \equiv y^6 + 7y^2 - 14y + 7 = 0$$

Convert it to standard form:

$$h_2(z) \equiv z^6 + 1.9129312z^2 - 2.7661751z + 1 = 0$$

where： $y = 1.3830876\ z$

The test shows that the series solution obtained by the coefficients of $h_2$ is not converge.

Let’s set the transformation again:

$$g_2(y,u) \equiv y^4 + 2.5y^3 + 8.6896489y^2 + 2.2817974y + 4.6666667 + u = 0$$

Eliminating y by the resultant of $h_1$ and $g_2$, it becomes::

$$h_3(u) \equiv u^6 + 15023.53863u^2 + 2.5720512 * 10^6 u + 3.7647556 * 10^7$$

Make it into standard form:

$$h_4(v) \equiv v^6 + 0.1337461v^2 + 1.2507359v + 1$$

where: $u = 18.3072367v$

The series solution obtained by the coefficients of $h_4$:

$$V = -0.1942500 + 1.0387127I$$

which is close to one of roots of $h_4$:

$$v = -0.1943295 + 1.0383390I$$

Take V as the solution of $h_4$ to go back, and finally get a solution of the original equation:

$$\mathrm{X} = -0.9009689 + 0.4338837\ \mathrm{I}$$

direct solution to the original equation is:

$$\mathrm{x} = -0.9009689 + 0.4338837\ \mathrm{I}$$

The two are exactly the same.

**Example 7**: Solving a seventh-degree polynomial equation using a fractional sequence

$$11x^7 + 3x^3 + 3x^2 + 2x + 2 = 0$$

Assume the transformation:

$$\mathrm{x} = 0.7838517\ \mathrm{y}$$

The transformed standard form equation is:

$$y^7 + ry^3 + qy^2 + \mathrm{p}\ \mathrm{y} + 1 = 0$$

where: $\mathrm{p} = 0.7838517, \mathrm{q} = 0.9216352, \mathrm{r} = 0.7224253$

The fractional series solution obtained from the standard form is

$$y(p,q,r,s) = s - \frac{p}{7s^5} - \frac{q}{7s^4} - \frac{r}{7s^3} - \frac{2p^2}{49\ s^{11}} + \ldots$$

where： $s = -1, \cos\left(\frac{\pi}{7}\right) \pm I \sin\left(\frac{\pi}{7}\right), -\cos\left(\frac{2\pi}{7}\right) \pm I \sin\left(\frac{2\pi}{7}\right),$

$\cos\left(\frac{3\pi}{7}\right) \pm I \sin\left(\frac{3\pi}{7}\right)$

Let $s = -1$, a series solution of the transformed equation is:

$$Y = -0.9076755$$

Get one root of the original equations：

$$X = -0.7114830$$

One root of the original equation by the direct iteration is：

$$x = -0.7114830$$

Within 7 decimal places of precision, they are exactly the same.

## 4. Discussion and Conclusion

The results presented here demonstrate that the roots of any general polynomial equation can be expressed simply as a series consisting of powers of the polynomial coefficients. The solutions to the series corresponding to the standard form used in this experiment are fractional series, which exhibit excellent convergence. However, by selecting different arrangements of signs, integer sequences can also be obtained. For example, when solving equations of degree 5 or lower, if a single coefficient appears in the highest term, a series of Minantu-Catalan numbers or extended Minantu-Catalan numbers is obtained [16,17,18]. These are all integers with known general formulas, and their convergence radius is easy to discuss. However, due to the rapid growth of extended Minantu-Catalan numbers, the convergence radius requirement is extremely demanding. In contrast, the absolute value or modulus of each term in the fractional series in the standard form is no greater than the constant 1, exhibits an alternating positive and negative distribution, and decays rapidly, allowing for a larger convergence radius. But the concise and computationally convenient general formulas needed for further analysis depend on the research of combinatorics.

As the degree of the polynomial equation increases, it is possible to use higher-order Tschirnhaus transformations to eliminate the term $(N-4)$ or more, where further research is needed [4,5,6,7].

In summary, to solve the roots of a polynomial equation, the equation can be converted to standard form using the Tschirnhaus transformation. The corresponding series solution is a series of fractional sequences. Although the exact radius of convergence of this series has not yet been determined, experiments have shown that it converges well when the absolute value or modulus of all coefficients is no greater than constant 1. When this convergence condition is met, a root of the equation is obtained. If the convergence condition is not met, it can be satisfied through appropriate transformations and iterations. This algorithm has universal significance.

Finally, this paper's research brings the problem of finding polynomial roots back to the realm of pure algebra, using only polynomial transformations, fractional sequences, and multivariable power series. An interesting fact is that even without the concepts and methods of radicals, it is still possible to obtain sufficiently precise representations of the roots of any polynomial equation by operating on the powers of the coefficients of the polynomial equation.

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