Gravity at close range

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Abstract

In Einstein's theory of general relativity (GR), gravitation is considered as a consequence of space and time curvature, whereas Newton's law of gravity applies strictly in a Euclidean or flat space. Logically, then, Newtonian gravity must relate solely to the time curvature contribution in GR, and instances where Newton's law does not describe phenomena correctly, such as the perihelion rotation of the planet Mercury and the bending of starlight, must be attributable to an effect of spatial curvature. In this paper, the GR solution for a static point mass is calculated on this basis for correspondence with Newton's law, and found to be crucially different from the usual interpretation in the current scientific literature. It is shown here that gravitational attraction does not diverge to infinity as masses approach each other, but tails off to zero. This means there is no singularity at the origin of coordinates where physical laws would break down, and, furthermore, speeds of free-falling objects do not exceed the speed of light. There is also no event horizon obscuring a black hole at the origin of coordinates, since the spacetime behaves perfectly regularly.

1 Introduction and background theory

Newton's inverse-square law of gravity describes most aspects of gravity very well, including gravitational free fall, and the orbital motion of satellites and spacecraft in the gravitational fields of planets and stars. However, it does not account for the perihelion rotation of the planet Mercury around the Sun or the correct bending of starlight passing close to the Sun. These two phenomena can however be described by Einstein's theory of general relativity GR, which explains gravitation in terms of the curvature of space and time due to stress energy, such as mass [1].

Gravity is usually discussed in the context of the field due to a point mass, and the first solution in GR for this situation was found by Schwarzschild in 1916 [2]. To obtain the solution, a metric line element in a 4D spacetime with spherical spatial symmetry around a point mass may be written in its most general form as

$$d\tilde{s}^2 = c^2 dt'^2 = A(r)\, c^2 dt^2 - B(r)\, dr^2 - C(r)(d\theta^2 + \sin^2\theta\, d\phi^2) \tag{1}$$

(r, θ, ϕ) are spherical polar coordinates and t is time, $d\tilde{s}$ is a spacetime increment, c the speed of light, dt' an increment of proper time, dt an increment of coordinate time, and dr an increment of radial coordinate distance. A, B and C are radially dependent metric coefficients describing the curvature of time, radial distance and tangential distances, respectively. If the spacetime were flat, A and B would be unity and C equal to r^2. In a curved space, the radial coordinate r is the radial distance of a point from the coordinate origin viewed from an infinitely long distance from the mass causing curvature, or if the distance were measured using a hypothetically rigid or non-deformable ruler.

The calculus of variations is applied to the metric to obtain geodesic equations for the four coordinates, from which Christoffel curvature coefficients are obtained to find the Ricci tensor components. These are then set to zero to satisfy Einstein's field equations of GR for the vacuum outside the point mass.

Using the geodesic equation for the radial coordinate the equation of motion of a freely falling particle or test object along a radius can be shown to be given by

$$\ddot{r} = -\frac{A'}{2B} c^2 \dot{t}^2 - \frac{B'}{2B} \dot{r}^2 \tag{2}$$

where $\ddot{r} (= d^2r/dt'^2)$ is the proper acceleration, $\dot{r} (= dr/dt')$ is the proper velocity, $\dot{t} = dt/dt'$, $A' = dA/dr$ and $B' = dB/dr$.

If we now use the metric in the form

$$1 = A\dot{t}^2 - B\dot{r}^2/c^2 \quad [d\theta = d\phi = 0] \tag{3}$$

and substitute this into Equation 2 we obtain

$$\ddot{r} = -\frac{1}{2}\frac{A'}{AB} c^2 - \frac{1}{2}\left(\frac{A'}{A} + \frac{B'}{B}\right)\dot{r}^2 \tag{4}$$

We now need to try to find $A(r), B(r)$ and $C(r)$.

GR itself does not allow all three variables A, B, C to be found explicitly, but only two independent equations are obtained relating them. To circumvent this situation a new radial component is defined, which I shall denote as r^*, where $C = r^{*2}$. The metric of Equation 1 then becomes

$$d\tilde{s}^2 = A(r^*)\, c^2 dt^2 - B(r^*)\, dr^{*2} - r^{*2}(d\theta^2 + \sin^2\theta\, d\phi^2) \tag{5}$$

The *GR* solution using Einstein's field equations is then found to be given by

$$A = \frac{1}{B} = 1 - \frac{\alpha}{r^*} \tag{6}$$

where α is a constant of integration. The quantity r^* is usually called the Schwarzschild radial coordinate. It is very similar to r in most circumstances, but not identical to it when both quantities become very small - which is the situation we are most interested in here.

Every textbook on relativity seems to fail to distinguish between r^* and r, and the superscript is always dropped, (e.g. [3]), resulting in the solution being expressed in the form

$$A = \frac{1}{B} = 1 - \frac{\alpha}{r} \tag{7}$$

Since r can go from zero to infinity, it then appears we have a discontinuity (change of sign or divergence to infinity) for A and B when r passes through α. This discontinuity is referred to as an event horizon and the point mass obscured behind it as a black hole.

Inserting the solution of Equation 7 into Equation 4 gives

$$\ddot{r} = -\frac{1}{2}\, A' c^2 = -\frac{1}{2}\, \alpha c^2 / r^2$$

which when compared with Newton's law of gravitation, in which the acceleration due to gravity of a free-falling particle is $-GM/r^2$, gives

$$\alpha = \frac{2GM}{c^2} \tag{8}$$

G is the universal gravitational constant and M is the mass of the central object causing gravitation.

2 New law of gravitational attraction

I believe the above analysis relating Newton's law and *GR*, conventionally adopted by the scientific community, is fallacious and leads to incorrect predictions, for the following reasons. Newton's inverse-square law of gravitation is only correct for speeds much smaller than c and large distances from the central mass causing gravitation. Under those circumstances it is clear that space is flat: simple observation of astronomical bodies and objects in our surroundings reveals no spatial distortion or gravitational lensing. However, the analysis which is used to make a correspondence between GR and classical physics uses the fact that space and time are curved reciprocally (Equation 7). How can it be correct to compare Newton's law, which is for a specifically flat or Euclidean space, with that? To compare GR with Newton we must use a flat space, viz. $B = 1$. The equation of free-fall motion (Equation 4) then becomes

$$\ddot{r} = -\frac{1}{2}\frac{A'}{A}\left(c^2 + \dot{r}^2\right) \quad [B = 1] \tag{9}$$

Note that this does not yet agree with Newton's law since here the acceleration $\ddot{r}$ seems to depend on the velocity $\dot{r}$, whereas the acceleration or force in Newton's law depends only the position and not the velocity.

Clearly then we need to restrict this to small velocities where $\dot{r} << c$, giving as a weak-field approximation

$$\ddot{r} = -\frac{1}{2}\frac{A'}{A}c^2 \tag{10}$$

which compared with Newton's law gives

$$-\frac{GM}{r^2} = -\frac{1}{2}\frac{A'}{A}c^2$$

or

$$\frac{A'}{A} = \frac{\alpha}{r^2} \tag{11}$$

Integrating this differential equation delivers an expression for A, which is the metric coefficient describing the curvature of time:

$$A = e^{-\alpha/r} \tag{12}$$

Comparing this with the GR solution we then have

$$A = 1 - \frac{\alpha}{r^*} = e^{-\alpha/r}$$

or

$$r^* = \frac{\alpha}{(1 - e^{-\alpha/r})} \tag{13}$$

We see from this (by expanding the exponential function as an infinite series) that the radial coordinate r^* is essentially equal to r for $r >> \alpha$, and regular for all values of r ($\infty > r > 0$). The full solution is then given by

$$A = \frac{1}{B} = e^{-\alpha/r} \quad ; \qquad C = \left(\frac{\alpha}{1 - e^{-\alpha/r}}\right)^2 \tag{14}$$

In this interpretation neither A nor B shows a discontinuity in space or time, and there is therefore no event horizon or black hole.

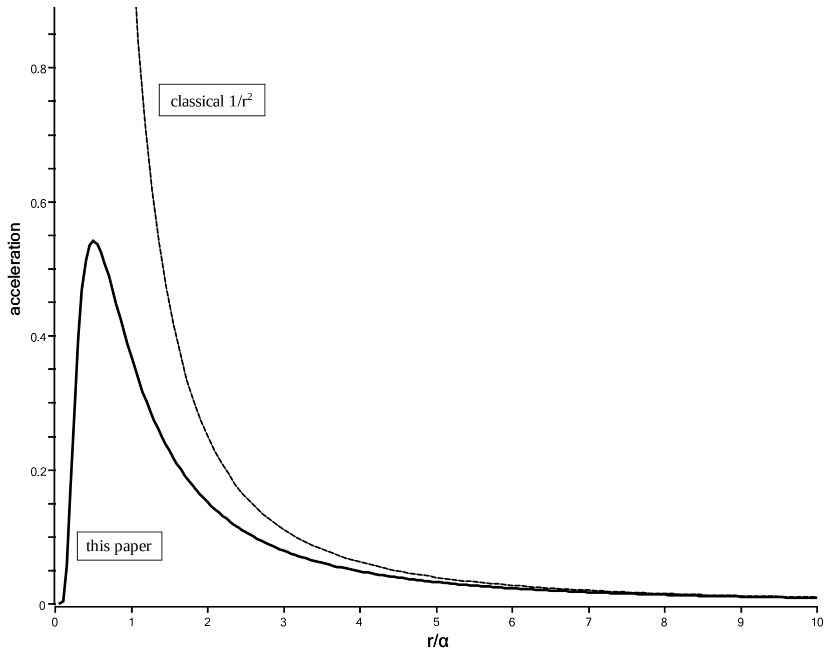

Figure 1: New law of gravity at close range. Acceleration versus separation.

Substituting back into the equation of motion (Equation 4) then gives

$$\ddot{r} = -\frac{1}{2}\,\alpha c^2 \left(\frac{e^{-\alpha/r}}{r^2}\right) \qquad ; \ [\alpha = 2GM/c^2] \tag{15}$$

which is now considered valid for all r. It shows Newtonian behavior for large r but the gravitational acceleration decreases to zero for $r \longrightarrow 0$ (see Figure). This removes the singularity at the origin (where physics would break down) that occurs for the conventional solution, as well as the discontinuity in spacetime called the event horizon.

Integrating the expression for the acceleration $\ddot{r} = -\frac{1}{2}\,c^2 A'$ for the special case of a particle falling from infinity at zero initial speed ($\dot{r} =$

$0; r = \infty$), we obtain the result

$$\dot{r}^2 = c^2(1 - e^{-\alpha/r}) \tag{16}$$

This reduces to the well-known Newtonian expression ($v^2 = 2GM/r$) for large r, but as $r \to 0$ the speed increases just to c, and does not exceed c as predicted by the black-hole solution.

The insight presented in this paper has enormous consequences for the current paradigm in large-scale physics and if accepted by the scientific community would completely alter the understanding of how compact astronomical objects in particular would behave when they approach each other closely.

References

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[3] W.Rindler, *Essential Relativity*, p.136, Springer 1977