Intuition and Game Shows

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abstract

The game show problem originated when Steve Selvin submitted his paper 'A Problem in Probability' to 'The American Statistician' magazine Feb 1975. [1] After receiving some negative reviews from readers, he submitted a 2nd letter to the editor Aug 1975. [1] There was little public awareness of his game show analysis until 1990, when Craig Whitaker posed a question of a winning strategy for a similar game show to Marilyn Savant who wrote articles for Parade magazine.
The debate of probability of success as 2/3 vs 1/2 has continued until today. This paper reveals a common error in both Selvin and Savant interpretations.

key words

statistics, probability, logic, game

probability

A measure of occurrence as the ratio of (an event of interest)/(all possible events of a certain class). The probability is an historical average for a large number of events, but cannot predict the result for a single event. A common example is the weather. So many factors are involved in a dynamic weather system that may change quickly, an accurate prediction will includes a factor based on past weather events. Probability is also a substitute for lack of knowledge, when attempting to predict random events. Possibility is a better factor corresponding more to reality.

Steve Selvin paper

A PROBLEM IN PROBABILITY

It is "Let's Make a Deal"—a famous TV show starring Monte Hall.

Monte Hall: One of the three boxes labeled A, B, and C contains the keys to that new 1975 Lincoln Continental. The other two are empty. If you choose the box containing the keys, you win the car.

Contestant: Gasp!

Monte Hall: Select one of these boxes.

Contestant: I'll take box B.

Monte Hall: Now box A and box C are on the table and here is box B (contestant grips box B tightly). It is possible the car keys are in that box! I'll give you $100 for the box.

Contestant: No, thank you.

Monte Hall: How about $200?

Contestant: No!

Audience: No!!

Monte Hall: Remember that the probability of your box containing the keys to the car is 1/3 and the probability of your box being empty is 2/3. I'll give you $500.

Audience: No!!

Contestant: No, I think I'll keep this box.

Monte Hall: I'll do you a favor and open one of the remaining boxes on the table (he opens box A). It's empty! (Audience: applause). Now either box C or your box B contains the car keys. Since there are two boxes left, the probability of your box containing the keys is now 1/2. I'll give you $1000 cash for your box.

WAIT!!!!

Is Monte right? The contestant knows that at least one of the boxes on the table is empty. He now knows it was box A. Does this knowledge change his probability of having the box containing the keys from 1/3 to 1/2? One of the boxes on the table has to be empty. Has Monte done the contestant a favor by showing him which of the two boxes was empty? Is the probability of winning the car 1/2 or 1/3?

Contestant: I'll trade you my box B for the box C on the table.

Monte Hall: That's weird!!

HINT: The contestant knows what he is doing!

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Solution to "A Problem in Probability"

Certainly Monte Hall knows which box is the winner and, therefore, would not open the box containing the keys to the car. Consider all possible outcomes:

Keys are in box	Contestant chooses box	Monte Hall opens box	Contestant switches	Result
A	A	B or C	A for B or C	loses
A	B	C	B for A	wins
A	C	B	C for A	wins
B	A	C	A for B	wins
B	B	A or C	B for A or C	loses
B	C	A	C for B	wins
C	A	B	A for C	wins
C	B	A	B for C	wins
C	C	A or B	C for A or B	loses

Enumeration shows probability of winning is 6/9 = 2/3. If the contestant does not switch boxes, then his probability of winning the car remains unchanged (1/3) after Monte Hall opens an additional box.

fig.1

Fig.1 is the analysis by Steve Selvin of a hypothetical game show.[1] He casts Monty Hall as the host (forming the popular reference) and uses 3 boxes, 1 with keys to a new car, and 2 that are empty. His list of wins and losses is on the right.

Whitaker-Savant correspondence

Craig Whitaker's question [2] [3]

Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say #1, and the host, who knows what's behind the doors, opens another door, say #3, which has a goat. He says to you, "Do you want to pick door #2?" Is it to your advantage to switch your choice of doors?

Marilyn Savant's 1990 response [3]

Yes; you should switch. The first door has a 1/3 chance of winning, but the second door has a 2/3 chance.

The benefits of switching are readily proven by playing through the six games that exhaust all the possibilities. For the first three games, you choose #1 and "switch" each time, for the second three games, you choose #1 and "stay" each time, and the host always opens a loser...

For the first three games, you choose #1 and "switch" each time, for the second three games, you choose #1 and "stay" each time, and the host always opens a loser. Here are the results.

	DOOR 1	DOOR 2	DOOR 3	RESULT
GAME 1	AUTO	GOAT	GOAT	Switch and you lose.
GAME 2	GOAT	AUTO	GOAT	Switch and you win.
GAME 3	GOAT	GOAT	AUTO	Switch and you win.
GAME 4	AUTO	GOAT	GOAT	Stay and you win.
GAME 5	GOAT	AUTO	GOAT	Stay and you lose.
GAME 6	GOAT	GOAT	AUTO	Stay and you lose.

fig.2

Comparing Selvin's case to Savant's case, keys replace a car, boxes replace doors and nothing replaces the goats. Both offer the player a 2nd guess. The game rules are the same.

The Savant case

	c	g	g		
s	p1	h	r	p2	
1	1	2	3	c	g
2	1	3	2	c	g
3	2	3	1	g	c
4	3	2	1	g	c

fig.3

A session is playing the game with varying player-host actions.
The format is p1(player 1st guess), h (host removes a door), r (remaining closed door). On the left, the 4 possible sessions are listed for the prize distribution of c g g for 3 doors.
Rules. Host must open a door different from the player's guess
and can only open goat doors.
On the right, are the pairs from p1 and r on the left in terms of prizes, which are the choices for p2, the player's 2nd guess. For this part of the session, the host is only required to verify which prize is won.
The player can guess door 1 with the car for 1/2 the sessions since the host can open doors 2 and 3, whereas sessions 3 and 4 allow opening 1 door.
Even though the player 1st guess is correct, there is no prize for that guess.
The prizes are awarded for the player 2nd guess p2.
All p2 guesses are (c or g) with a win car ratio of 1/2.
If Selvin's elements replace Savant's elements, the results are the same as fig.3.
There is no advantage to switch.

The difference
Why do Savant and Selvin get a result different from fig.3?
The common response is to visualize the problem as having 3 sessions, 1 for each possible location. When the prize is in the same door chosen by the player, the host is allowed to open both doors 2 and 3.
The logic of the game rules prohibits opening both in 1 session, revealing the prize location, ending the session and denying the player the 2nd guess. It does allow opening each door in separate sessions, thus the host has 4 choices, 1 for each of 4 sessions, meaning the player also has 4 choices. All 4 sessions are different. Both Selvin and Savant interpret player guessing door 1 when it contains the car, as host opening door 2 half a session and door 3 half a session, i.e. door 2 or door 3 but not both. This produces a bias favoring switching.

	c	g	g	p1	freq		
s	1	2	3		f1	f2	f3
1	p	h	r	c	1	.5	1
2	p	r	h	c	1	.5	1
3	r	p	h	g	1	1	.5
4	r	h	p	g	1	1	.5
			win c =		.50	.33	.67

fig.4

Fig.4 shows manipulating session frequencies determines a strategy. P1 are the prizes for stay, with a win c ratio of .50 for f1. The Selvin-Savant f2 favors a switch strategy with a win c ratio of .67, yet f3 favors a stay strategy with a win c ratio of .67.

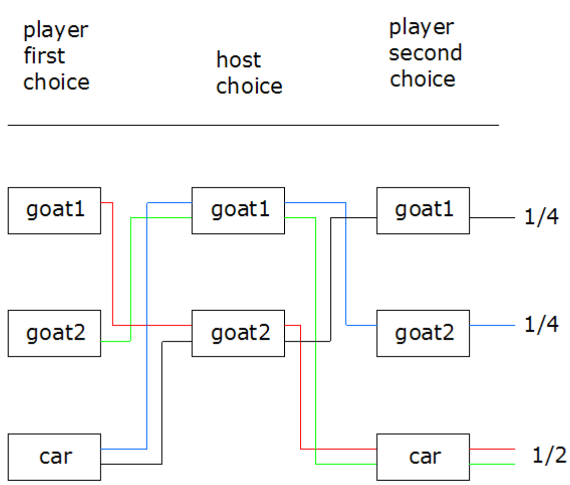

fig.5

All possible sequences of player-host choices are shown for the Whitaker-Savant game with a different colored path in fig.5. Comparison of the stay results in 1st choice and switch results in 2nd choice show no advantage.

conclusion

Both Selvin and Savant focused more on prize locations than possible effects of the host participation. Both made the same errors related to frequency of session play. There is no reason, practical or hypothetical to play a partial session, as shown in fig.4. The actions of the host do not provide the player with any constructive information. The player not knowing the prize location, can only make a random guess, which offers the same opportunity for all players. The switch strategy is only a consequence of their manipulation of game play. There is no basis for a strategy. The answer to Whitaker's question is no.

The player guesses are random and independent, and not conditional.

Winning depends on host choices, which depend on the rules, not location.

The simple intuition of the average person is capable of concluding there are only 2 ways to choose 1 of 2 things.

reference

[1} The American Statistician, August 1975, Vol. 29, No. 3

[2] game show problem, Wikipedia Sep 2024

[3] Marilyn vos Savant,
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