General Relativity from Primordial Dimensional Fluctuations

Ervin Goldfain

Global Institute for Research, Education and Scholarship (GIRES)

E-mail ervingoldfain@gmail.com

Abstract

We develop a theoretical framework in which the equations of General Relativity (GR) emerge from dimensional fluctuations of the early Universe. The derivation points out that primordial fluctuations in the effective dimensionality of spacetime are governed by the *complex Ginzburg–Landau equation* (CGLE), which is a coarse-grained description of complex dynamics near the Planck scale. Elaborating on the behavior of CGLE as generic hydrodynamic flow, our paper offers an intriguing path from fractal dimensionality of primordial cosmology to the onset of gravitational physics in the late Universe.

Key words: complex dynamics, General Relativity, primordial dimensional fluctuations, complex Ginzburg-Landau equation, Madelung transform, stability analysis.

Introduction

A fundamental challenge of theoretical physics is to reconcile the quantum picture of spacetime with the macroscopic description of gravitational fields. While GR treats spacetime as a smooth differentiable manifold endowed with a metric $g_{\mu\nu}$, several approaches—ranging from Causal Dynamical Triangulation to Loop Quantum Gravity—suggest that near the Planck scale, the manifold picture breaks down and spacetime exhibits fluctuating dimensionality. It is important to keep in mind, however, that both Quantum Gravity and the numerous extensions of GR are based on the canonical principles of Lagrangian field theory. There are well-motivated reasons to believe that these principles no longer hold in the far-from-equilibrium conditions characterizing primordial stages of Universe formation [1 - 2]. In contrast, the approach developed here is entirely non-Lagrangian in nature; it exploits the idea that, somewhere below the Planck scale, the effective spacetime dimension D_{eff} undergoes random fluctuations around $D=4$, governed by scaling exponents intrinsic to non-equilibrium statistical systems.

It was argued in [3] that primordial dimensional fluctuations act as a large-scale interacting system whose dynamics is described by the complex Ginzburg–Landau equation (CGLE) [4]. This equation arises naturally when nonlinearities and phase diffusion dominate the dynamics of complex systems close to a bifurcation or critical point—a situation plausibly encountered in the primordial Universe. By analyzing the stationary limit of the CGLE, we recover a Poisson-like equation linking the complex amplitude of CGLE, $W(x^{\mu})$, to an emergent gravitational potential. The phase field $\theta(x^{\mu})$, through its gradient $v_i = \partial_i \theta$, generates a flow velocity field whose quadratic combinations $v_i v_j$ reproduce the components of the emergent metric g_{ij}. This correspondence allows us to identify the energy–momentum tensor $T_{\mu\nu}$ as the coarse-grained expression of $|W|^2$ and its gradients, bridging the fractal dynamics of primordial spacetime with the physics of GR. At the end of this analysis, Einstein field equations arise as stationary solutions of CGLE forming coherent macroscopic structures defined by $\partial_t W = 0$.

The following sections detail the derivation of the emergent gravitational field equations, the identification of G_{eff} and Λ_{eff}, and the interpretation of spacetime stability in terms of the CGLE parameters. To assess the dynamical consistency of this picture, we perform a linear stability analysis of the CGLE. The resulting stability map in the (α, β) plane identifies the Benjamin–Feir line, $1 + \alpha\beta = 0$, as the boundary between coherent (GR-like) and incoherent (chaotic/turbulent) phases.

In a nutshell, our paper promotes the notion that GR corresponds to the self-organized, stable phase of a more fundamental dissipative dynamics driven by primordial dimensional fluctuations.

An important caveat is now in order: the derivation detailed herein is neither airtight nor complete. Many details are left out for follow-up studies, rebuttals or further clarifications.

STEP 1: CGLE from primordial dimensional fluctuations

CGLE is a universal dissipative, driven nonlinear field equation typical of pattern-forming systems [4, 14]. CGLE naturally arises in systems exhibiting

a) Nonequilibrium instabilities,

b) Pattern formation,

c) Coupled diffusion and phase rotation.

Derived from the dynamics of dimensional fluctuations, CGLE in the standard form is given by [3],

$$\boxed{\partial_t W = \mu W + (1+i\alpha)\nabla^2 W - (1+i\beta)|W|^2 W} \tag{1.1}$$

whose coefficients μ, α, β denote, respectively, the linear growth or decay rate (in units of 1/time), and the dispersion / nonlinearity phase parameters.

STEP 2: Madelung transform and elementary differential identities

Introduce the Madelung parametrization [5 - 6],

$$W(\vec{x},t)=\sqrt{\rho(\vec{x},t)}e^{i\theta(\vec{x},t)} \tag{1.2}$$

which enables treating (1.1) as a hydrodynamic flow. Define next two combinations of energy density ρ and phase θ, namely,

$$A\equiv\nabla^2\sqrt{\rho}-\sqrt{\rho}(\nabla\theta)^2 \tag{1.3}$$

$$B\equiv 2\nabla\sqrt{\rho}\cdot\nabla\theta+\sqrt{\rho}\nabla^2\theta \tag{1.4}$$

so that

$$\nabla^2 W=e^{i\theta}(A+iB) \tag{1.5}$$

Inserting (1.3) and (1.4) into the CGLE leads to, after a series of algebraic manipulations,

$$\frac{\partial_t\rho}{2\sqrt{\rho}}+i\sqrt{\rho}\partial_t\theta=R+iI \tag{1.6}$$

where the real (R) and imaginary (I) parts on the right-hand side of (1.6) are, respectively

$$\frac{\partial_t \rho}{2\sqrt{\rho}} = \mu\sqrt{\rho} + A - \alpha B - \rho^{3/2} \tag{1.7}$$

$$\sqrt{\rho}\partial_t \theta = B + \alpha A + \beta\rho^{3/2} \tag{1.8}$$

Using algebraic identities and regrouping terms, the *real* equation (1.7) becomes

$$\partial_t \rho + \nabla\cdot(\rho\vec{v}) = 2\mu\rho + \nabla^2\rho - 2\rho^2 - 2\alpha(2\nabla\sqrt{\rho}\cdot\nabla\theta + \rho\nabla^2\theta) \tag{1.9}$$

in which $\vec{v} \equiv \nabla\theta$. The *imaginary* equation turns into

$$\partial_t \theta + \frac{1}{2}|\vec{v}|^2 + Q(\rho) = \frac{1}{\rho}(B + \alpha A + \beta\rho^{3/2}) \tag{1.10}$$

where the so-called *quantum potential term* is given by

$$Q(\rho) \equiv -\frac{\nabla^2\sqrt{\rho}}{2\sqrt{\rho}} \tag{1.11}$$

STEP 3: Stationary CGLE: algebraic linear system and its exact solution

Stationary solutions of CGLE correspond to *coherent macroscopic structures* and are defined by $\partial_t W = 0$. The complex equation can be shown to take the form,

$$0 = \mu\sqrt{\rho} + (A - \alpha B) - \rho^{3/2} + i\,(B + \alpha A + \beta\rho^{3/2}) \tag{1.12}$$

It follows that the two real constraints for stationarity amount to

$$\mu\sqrt{\rho} + (A - \alpha B) - \rho^{3/2} = 0 \tag{1.13}$$

$$B + \alpha A + \beta\rho^{3/2} = 0 \tag{1.14}$$

This is a linear 2×2 system for the unknown real variables A and B. In matrix form, (1.13) and (1.14) lead to

$$\begin{pmatrix} 1 & -\alpha \\ \alpha & 1 \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \rho^{3/2} - \mu\sqrt{\rho} \\ -\beta\rho^{3/2} \end{pmatrix} \tag{1.15}$$

Invert the matrix and divide A and B by $\sqrt{\rho}$ to obtain two differential equations between amplitude ρ and phase θ,

$$\frac{A}{\sqrt{\rho}} = \frac{\rho(1-\alpha\beta)-\mu}{1+\alpha^2} \tag{1.16}$$

$$\frac{B}{\sqrt{\rho}} = \frac{\rho(\alpha-\beta)-\alpha\mu}{1+\alpha^2} \tag{1.17}$$

By (1.3) and (1.4), the explicit partial differential equations constraints become

$$\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} - (\nabla\theta)^2 = \frac{\rho(1-\alpha\beta)-\alpha\mu}{1+\alpha^2} \tag{1.18}$$

$$\frac{2\nabla\sqrt{\rho}\cdot\nabla\theta}{\sqrt{\rho}} + \nabla^2\theta = \frac{\rho(\alpha-\beta)-\alpha\mu}{1+\alpha^2} \tag{1.19}$$

STEP 4: Emerging effective gravitational constants from CGLE

Following [7 - 9], consider now the weak-gradient/long-wavelength limit of (1.9) and (1.10). There are two natural approximations of this hydrodynamic regime:

1) Quantum potential is small compared to nonlinear terms, that is, $\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} << \rho$.

2) Phase gradients are either small or moderate so that the velocity $\vec{v} = \nabla\theta$ is the dominant kinematic variable of the flow.

Under approximation (1), we drop $\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}$ in (1.18), which then gives

$$(\nabla\theta)^2 \approx -\frac{\rho(1-\alpha\beta)-\mu}{1+\alpha^2} \tag{1.20}$$

Next, define a scalar gravitational potential-like field Φ by choosing,

$$\vec{v} = \nabla\theta = \nabla\Phi \tag{1.21}$$

Note that the identification $\nabla\theta \to \nabla\Phi$ represents a natural weak-field analogy, in which phase gradients are geometric connection / potential gradients. Then,

$$|\nabla\Phi|^2 \approx -\frac{\rho(1-\alpha\beta)-\mu}{1+\alpha^2} \tag{1.22}$$

Using the approximation of slowly varying ρ and differentiating $\nabla^2\Phi$ as source built from ρ, one obtains to linear order (small gradients),

$$\boxed{\nabla^2\Phi \approx \frac{1-\alpha\beta}{1+\alpha^2}\rho - \frac{\mu}{1+\alpha^2}} \tag{1.23}$$

This is a Poisson-like equation, whose interpretation goes as follows:

1. The first term on the right gives a density-sourced potential: it is natural to identify the proportionality constant with an *effective coupling* κ_{eff}, i.e., gravitational constant measured in emergent units. Thus, if ρ_{eff} is taken to stand for the physical energy density arisen from the CGLE via (1.2), we get

$$\nabla^2\Phi = \kappa_{eff}\,\rho_{eff}, \quad \text{with } \kappa_{eff} \propto \frac{1-\alpha\beta}{1+\alpha^2} \tag{1.24}$$

2. The second term acts like a uniform background source, call it "C"– a cosmological constant – like contribution, so it is natural to associate μ with an emergent vacuum energy, up to appropriate overall units.

We conclude that the stationary CGLE produces both a local density source and an effective background source in the Poisson-like equation (1.23) for the emergent potential Φ. The matching conditions are

$$\boxed{\kappa_{eff} \xrightarrow[match]{} 4\pi G_N} \tag{1.25}$$

$$\boxed{C \xrightarrow[match]{} \Lambda} \tag{1.26}$$

STEP 5: Emerging energy-momentum tensor from CGLE

CGLE describes a compressible flow with density ρ, velocity $\vec{v} = \nabla\theta$, pressure-like and gradient-stress terms. The effective, coarse-grained energy-momentum tensor is the standard fluid/stress-energy terms with corrections induced by the quantum potential $Q(\rho)$, that is,

$$\boxed{T_{00}^{eff} = \underbrace{\frac{1}{2}\rho v^2}_{kinetic} + \underbrace{U(\rho)}_{internal} + \underbrace{\rho Q(\rho)}_{quantum}} \quad \text{(energy density)} \tag{1.27}$$

$$\boxed{T_{0i}^{eff} = \rho v_i} \quad \text{(momentum density/energy flux)} \tag{1.28}$$

$$\boxed{T_{ij}^{eff} = \rho v_i v_j + P(\rho)\delta_{ij} + \Pi_{ij}^{grad}} \text{ (stress, pressure, gradient stresses)} \quad (1.29)$$

Here, $U(\rho)$ is the internal energy density produced by the nonlinear term $-(1+i\beta)|W|^2 W$ of the CGLE (1.1). For cubic nonlinearity a natural choice is $U(\rho) = \frac{1}{2} g\rho^2$, with $g = O(1)$, so the pressure term is $P(\rho) = \rho U'(\rho) - U(\rho) = \frac{1}{2}\rho^2$. Also, in (1.29), Π_{ij}^{grad} are gradient (nonlocal) stresses representing higher-derivative corrections which are suppressed in the long-wave limit.

Note that the conservation of the stress tensor recovers the hydrodynamic momentum equation [15]

$$\partial_\mu T_i^{eff,\,\mu} = 0 \Rightarrow \partial_t T_{0i}^{eff} + \partial_j T_{ji}^{eff} = 0 \quad (1.30)$$

The above equation is the continuity/momentum balance obtained from the Madelung transform of the CGLE after coarse graining.

STEP 6: Emergent metric from CGLE

Refer now to (1.9) and (1.10), choose a background $\overline{\rho}(\vec{x},t)$, $\overline{\theta}(\vec{x},t)$ that varies slowly on the scales of interest and account for small perturbations as in

$$\rho=\overline{\rho}+\delta\rho\,,\qquad \theta=\overline{\theta}+\varphi \tag{1.31}$$

Next, assume the following,

a) ***scale separation***: perturbation wavelengths λ satisfy $l_{micro} << \lambda << L$, where l_{micro} is the microscopic coherence scale and L the background variation scale;

b) ***small perturbations***: $|\delta\rho| << \overline{\rho}$, $|\nabla\varphi| << |\nabla\overline{\theta}|$, which states that $\overline{\theta}$ is slowly varying and $\overline{\vec{v}}$ stays finite.

Under a) and b), the linearized continuity equation (1.9) becomes

$$\partial_t\delta\rho+\nabla\cdot(\overline{\rho}\,\delta\vec{v}+\delta\rho\,\overline{\vec{v}})=0 \tag{1.32}$$

with $\delta\vec{v} \propto \nabla\varphi$. In turn, up to the first order in perturbations and dropping terms linked to the quantum potential, the linearized phase equation (1.10) can be written as,

$$\partial_t \varphi + \vec{v} \cdot \nabla \varphi + \frac{c_s^2}{\overline{\rho}} \delta\rho = 0 \tag{1.33}$$

where the *effective sound speed squared* is defined by (in appropriate normalized units)

$$c_s^2(\overline{\rho}) \equiv \frac{1}{\overline{\rho}} \frac{\partial P}{\partial \rho}\bigg|_{\overline{\rho}} \tag{1.34}$$

Differentiating (1.33) w.r.t time and using (1.32) to eliminate $\delta\rho$, yields (after a chain of algebraic manipulations) the second order PDE

$$\partial_t [\overline{\rho}(\partial_t + \vec{v} \cdot \nabla)\varphi] - \nabla \cdot [\overline{\rho} c_s^2 \nabla \varphi + \overline{\rho} \vec{v} (\partial_t + \vec{v} \cdot \nabla)\varphi] = 0 \tag{1.35}$$

This equation can be rearranged to the covariant form of massless scalar wave equation upon writing it as divergence of a current, namely,

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\,g_{eff}^{\mu\nu}\partial_\nu\varphi)=0 \tag{1.36}$$

Comparing (1.35) to (1.36) gives the inverse *effective metric components*

$$\boxed{g_{eff}^{\mu\nu}\propto\frac{1}{\bar{\rho}c_s}\begin{pmatrix}-1 & \bar{v}^j\\ \bar{v}^i & c_s^2\delta^{ij}-\bar{v}^i\bar{v}^j\end{pmatrix}} \tag{1.37}$$

where $\bar{v}^i$ represent the components of the background physical velocity. The metric corresponding to the inverse of (1.37) then reads,

$$\boxed{g_{eff,\,\mu\nu}\propto\bar{\rho}c_s\begin{pmatrix}-(c_s^2-|\bar{v}|^2 & -\bar{v}_j\\ -\bar{v}_i & \delta_{ij}\end{pmatrix}} \tag{1.38}$$

STEP 7: Promoting the emergent gravitational parameters to General Relativity

So far, we have managed to recover the main players of General Relativity from CGLE, namely Newton's constant (1.25), cosmological constant (1.26), energy-momentum tensors (1.27 – 1.29) and metric coefficients (1.37 – 1.38). To justify the full GR tensor equation,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa_{GR} T_{\mu\nu}^{eff} \tag{1.39}$$

we appeal to three arguments [8 - 13]:

(i) ***Conservation*** of the effective energy-momentum tensor derived from coarse-graining the CGLE in the limit of long-wavelengths. If $D^{(g)}$ represents the covariant derivative associated with the emergent metric $g_{\mu\nu}^{eff}$, the conservation requirement amounts to [8 - 9 and the Appendix section]

$$D_{\mu}^{(g)} T_{eff}^{\mu\nu} \approx 0 \tag{1.40}$$

(ii) ***Uniqueness*** of the second-order divergence-free geometric tensors (Lovelock argument): Demand a local, second-order (in derivatives) generic symmetric tensor $\Upsilon_{\mu\nu}[g]$ built only from the metric and having vanishing divergence, $\nabla^{\mu}\Upsilon_{\mu\nu} = 0$. The Lovelock theorem states that requiring that the contracted Bianchi identity to hold, such tensors have the natural structure $\Upsilon_{\mu\nu} \to G_{\mu\nu} + \Lambda g_{\mu\nu}$. Note that this argument does not require writing an action; it uses differential geometry along with the requirement of second-order locality and divergence-free condition.

(iii) ***Matching*** the emergent Poisson equation (1.24) recovers the numerical value of the overall constant (Newton's G_N) and the background contribution (cosmological constant)

Combine these three points: one must have a tensor equation of the form

$$\Upsilon_{\mu\nu}[g] = \kappa_{eff} T^{eff}_{\mu\nu} \tag{1.41}$$

satisfying

$$\nabla^{\mu}\Upsilon_{\mu\nu} = 0 \text{ and } \nabla^{\mu}T^{eff}_{\mu\nu} = 0 \tag{1.42}$$

Then the uniqueness theorem (ii) yields $\Upsilon_{\mu\nu} \to aG_{\mu\nu} + b\,g_{\mu\nu}$. Matching the linearized 00 component per the Poisson equation sets $a=1$, determines $\kappa_{eff} = \kappa_{GR} = 8\pi G_N / c^4$ in standard units and $b = \Lambda$. At the end of this exercise, one is led to Einstein's equations,

$$\boxed{G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu}^{eff}} \tag{1.43}$$

STEP 8: Fixing the Newton and cosmological constants

How does one go about recovering the effective Newton and cosmological constants from this derivation?

To answer this question, refer to first to (1.24). Dimensional consistency requires the left-hand side of (1.24) to be in dimensionless form, since the right-hand side consists of real numbers. To do so, we multiply the effective gravitational constant with an (arbitrary) square of a reference mass M_0 as in

$$\kappa_{eff} M_0^2 = 4\pi G_N M_0^2 \propto \frac{1-\alpha\beta}{1+\alpha^2} \tag{1.44}$$

or, using $G_N \propto M_{Pl}^{-2}$ in natural units,

$$\boxed{4\pi \frac{M_0^2}{M_{Pl}^2} \propto \frac{1-\alpha\beta}{1+\alpha^2}} \tag{1.45}$$

where M_{Pl} stands for the Planck mass.

Next, consider the *closure relationship* relating the effective cosmological constant expressed in energy units ($\Lambda_{eff} = \sqrt{\Lambda}$) to the Planck mass and the electroweak scale $M_{EW} \approx 246\, GeV$ determined by the vacuum expectation value of the Higgs scalar. This relationship stems from a couple of independent arguments,

a) Dimensional regularization of Quantum Field Theory [16],

b) Demanding that neutrino masses are given though the dimension 5 Weinberg operator [17]. In this context, neutrino oscillations can be shown to account for the cosmological constant [19]

We thus obtain,

$$\boxed{\frac{\Lambda_{eff}}{M_{EW}} \approx \frac{M_{EW}}{M_{Pl}} \Rightarrow \Lambda_{eff} \approx \frac{M_{EW}^2}{M_{Pl}} = \frac{\mu}{1+\alpha^2}} \tag{1.46}$$

It is apparent that the pair of relations (1.45) and (1.46) cannot uniquely fix $G_N = G_{eff}$ and Λ_{eff} as we are dealing with three independent parameters α, β, μ. To attempt closing the loop, we appeal to stability considerations applied to CGLE, as briefly discussed in the next section.

STEP 9: Stability analysis of CGLE

Here, we are taking a detour to delve into the stability of CGLE to perturbations. With reference to (1.1), a spatially uniform (homogeneous) steady solution is the *limit cycle* [4]

$$W_0(t) = \sqrt{\mu} e^{-i\omega_0 t}, \quad \omega_0 = \beta\mu \tag{1.47}$$

provided that $\mu > 0$. More generally, one may consider plane waves solutions

$$W_q(\vec{x}, t) = \sqrt{\rho_q} e^{i(\vec{q}\cdot\vec{x} - \omega t)} \tag{1.48}$$

with ρ_q and ω determined from the nonlinear dispersion relation. Perturb (1.47) by a small correction

$$W(\vec{x},t)=[\sqrt{\mu}+\eta(\vec{x},t)]e^{-i\omega_0 t} \tag{1.49}$$

Insert back into CGLE (1.1) and keep linear terms in η. Since η is complex, it is formed by two real fields, $\eta=u+iv$. Now seek normal-mode perturbations proportional to $\exp(\sigma t+i\vec{k}\cdot\vec{x})$. Linearization yields a 2 x 2 eigenvalue problem for $\sigma(\vec{k})$, the real part of which ($\sigma_R(\vec{k})$) determines the growth or decay rate of perturbations.

Solving the 2 x 2 problem gives an explicit dispersion relation in the form

$$\sigma=\sigma(\vec{k};\alpha,\beta,\mu) \tag{1.50}$$

The small wavenumber expansion of the growth/decay rate assumes the form

$$\sigma_R(\vec{k})=-\frac{1+\alpha\beta}{1+\beta^2}k^2+O(k^4) \tag{1.51}$$

The expansion illustrates the so-called *Benjamin-Feir (BF)* instability criterion:

a) If $1+\alpha\beta>0$, then $\sigma_R(\vec{k})<0$ and (1.49) is *stable* to long-wave perturbations,

b) If $1+\alpha\beta<0$, (1.49) becomes *unstable* to long-wave perturbations.

While the BF criterion is a universal condition involving only α and β, the full stability region for plane waves (Eckhaus and BF) produces a relationship among α,β,μ and the wavenumber q. Fig. 1 below displays the stability map in the (α,β) plane upon setting $q=0, \mu=1$.

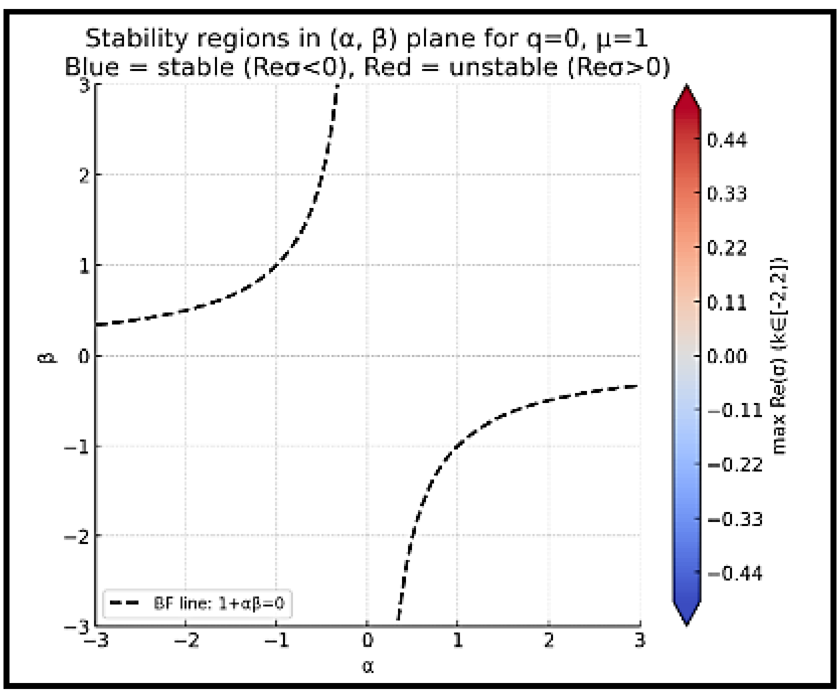

Fig. 1: Stability map associated with the BF criterion

According to this map, the ***blue region*** is linearly stable (perturbations decay $\sigma_R < 0$), the ***red region*** is linearly unstable (perturbations grow as $\sigma_R > 0$) and the dashed black line the BF transition border, defined by $1+\alpha\beta=0$ and marking the onset of instability.

The combined constraints $1-\alpha\beta>0$ (positive Newton constant) and $1+\alpha\beta>0$ (BF criterion) yield

$$\boxed{-1<\alpha\beta<1} \tag{1.52}$$

Although adding (1.52) to (1.45) and (1.46) cannot provide a unique solution for α,β,μ, (1.52) restricts the range of realistic values for α and β.

Note that enforcing BF stability means that the late Universe settles into a homogeneous/slowly varying phase. If, on the other hand, BF stability is violated, the late Universe is expected to evolve into a random web of topological defects and a cascade of chaotic (turbulent) states. While a smooth emergent GR phase matches a positive G_{eff} and the cosmological principle behind standard FRW cosmology, a turbulent and unsettled Universe is consistent with a negative G_{eff} [18].

Concluding remarks

The main findings of our paper can be summarized as follows:

- Dynamics of the early Universe is embodied in a *non-equilibrium field model* described by CGLE,
- Stationary coherence (the Benjamin–Feir stable region $1 + \alpha\beta > 0$) corresponds to a self-consistent, metric-compatible phase that matches GR,
- The structures associated with the stationary coherence define effective tensors $g_{\mu\nu}^{eff}$ and $T_{\mu\nu}^{eff}$,
- Once these tensors are well-defined, a *covariant derivative* can be introduced as the unique operator that preserves the emergent metric (per the Appendix section).
- The Einstein tensor *is not assumed* — it arises because, once a metric field exists, the differential geometry of that metric necessarily encodes curvature,

and Lovelock uniqueness (ii) mandates the Einstein tensor as the only consistent dynamical object at leading derivative order. In different words, curvature is an *emergent descriptor* of how phase gradients evolve in a medium whose coherence is described by the CGLE. Spacetime curvature represents the *memory* of the phase structure in the early Universe. For additional conceptual support of these findings, refer to [8-9].

APPENDIX A

Covariant derivatives in emergent gravity

In a hydrodynamic or Madelung-like reformulation of the CGLE, the real and imaginary parts yield coupled equations for the density ρ and the components of the phase velocity (v_i). In the presence of an effective potential V_{eff}, these equations take the general form

$$\partial_t \rho + \nabla \cdot (\rho \vec{v}) = D_1 \nabla^2 \rho \tag{1.53}$$

$$\partial_t v_i + (\vec{v} \cdot \nabla) v_i = -\nabla_i V_{eff} + D_2 \nabla^2 v_i + \ldots \tag{1.54}$$

in which D_1, D_2 are real coefficients. The nonlinear transport term $(\vec{v}\cdot\nabla)v_i$ contains the *same structure* as the covariant derivative of a vector field in non-Euclidean geometry,

$$v^j \nabla_j v_i = v^j (\partial_j v_i - \Gamma^k_{ij} v_k) \tag{1.55}$$

in which the *effective connection* Γ^k_{ij} is defined through the gradients of the flow as in

$$\Gamma^k_{ij} \propto \frac{1}{2}(\partial_i h^k_j + \partial_j h^k_j - \partial^k h_{ij}) \tag{1.56}$$

where $h_{ij} \equiv v_i v_j / c_s^2$ or a suitable symmetric tensor built from the flow velocity. Thus, in the CGLE picture, the connection coefficients (and hence the covariant derivative) emerge from spatial correlations of the phase flow and act as *analogues* of curvature in non-Euclidean geometry.

This observation parallels [8 – 9], where effective metric and connection arise from the background fluid motion.

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