Gravity is a field in flat space, not geometry of space-time

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Abstract: The result of this article is that the gravitational field is a field in flat space, not the geometry of space-time. This result is reached from eight considerations of General Relativity and a scalar gravitation field. The reasons why gravitation is not space-time geometry include the following. The Einstein equations do not have valid solutions that can describe the gravitation field of the Sun and the Earth and therefore they are wrong. The geodesic metric of General Relativity does not work. General Relativity cannot be quantised. Einstein's equations are not derived from anywhere. But the strongest arguments are from considering when a mass can be reduced into a point mass. This is possible only if the ball of the geometry grows as r^2 implying flat 3-dimensional spatial space.

Keywords: geometrization, scalar gravitation field, General Relativity Theory, flat space.

1. Where does the field equation come from?

The classical formula for a point mass M at the origin in empty space is

$$\Delta\psi = 0 \tag{1}$$

outside the origin. Δ is the Laplace operator.

Let us derive this formula. Let there be a point mass M at the origin and a small test mass m_1 at the distance r from the origin in the Euclidean 3-space. The mass M creates a spherically symmetric time-independent gravitational field $\psi = \psi(r)$. The gradient points towards the origin. Writing the force $\bar{F} = F\bar{e}_r$, we have

$$F(r) = -m_1 \frac{\partial\psi(r)}{\partial r} \tag{2}$$

where the negative sign is because $\bar{e}_r$ points away from the origin. The force spreads to all directions, but the force lines remain. Thus

$$\frac{(A(r+dr)F(r+dr) - A(r)F(r)}{dr} = 0 \tag{3}$$

where $A(r) = 4\pi r^2$ is the area of a 2-sphere of the radius r. Therefore we get

$$\frac{1}{A(r)}\frac{d}{dr}A(r)F(r) = 0 \tag{4}$$

which can be written as

$$\frac{1}{r^2}\frac{\partial}{\partial r}r^2\left(-\frac{m_1\partial\psi}{\partial r}\right) = 0 \tag{5}$$

thus

$$\Delta\psi(r) = 0 \tag{6}$$

outside the origin.

This calculation assumes that the gravitational field is embedded in the Euclidean space $\mathbf{R}^3$. Because of this embedding, the area of a ball of radius r in the space is $4\pi r^2$. Equation (4) shows that the force must grow as the inverse of the area of a ball. If the force grows as r^{-2}, then by (4) the area of a ball grows as r^2. Inversely, if the space is not the Euclidean $\mathbf{R}^3$ but some 3-dimensional manifold, then the area of a ball need not grow as $4\pi r^2$ and if it does not, then the force does not grow as r^{-2}.

The General Relativity Theory (GRT) claims that geometry of the space-time is geometry of the gravitational field. The goal of this article is to show that this is not true and that the traditional view is correct: the spatial part of the gravitational field is embedded in $\mathbf{R}^3$ and time is a human invented coordinate: there is only change in the spatial space, only the present time exists, the future does not yet exist and the past does not exist anymore. We can treat time as a coordinate and it is useful in equations, but time is not quite similar to the spatial coordinates.

In order to defend this traditional view, the article gives the following arguments. The first four are shown in my other papers. The new part of this article is arguments 5, 6 and 7. Each argument is in section number = argument +1.

1) GRT does not manage to show any example of a static field where the force of a point mass in an otherwise empty space does not grow as r^{-2}.

2) There is no convincing derivation of the Einstein equations.

3) The geodesic metric does not work.

4) Einstein's equations cannot be quantised, a scalar gravitation field can.

5) Time cannot be treated in a similar way as spatial coordinates.

6) Movement of masses is modelled in the Newtonian field, not in the D'Alembertian.

7) Reducing mass into a point mass works if the force is r^{-2}

8) Newton's gravitation potential is correct and the space is flat.

The section 10 is a bonus section considering if one can add a term $r^{-\alpha}$ to the force.

Based on 1)-8), gravitation is not geometry, it is a field in the flat space $\mathbf{R}^3 \times \mathbf{R}$ where the time part $\mathbf{R}$ is only partially in the physical world: only the present time and some small environment of it exist in the physical world. T

2. GRT has no examples where the force does not grow as r^{-2}

For the first argument there are the following observations that support the traditional view: The Newtonian gravitation potential grows as r^{-2} and it is a field in the Euclidean $\mathbf{R}^3$. It is not geometry of the space. The Coulomb force also grows as r^{-2}. The electric field is a field in $\mathbf{R}^3$, not geometry. Permittivity μ_0 in Maxwell's equations has the value 4π, which also is related to the area of a ball in $\mathbf{R}^3$.

Article [1] proves that there are no static spherical solutions to the Einstein equations. The Schwarzschild solution, and other similar solutions for the Einstein equations for empty space with a point mass in the origin, are not valid solutions because they do not give a valid metric for the spatial part of the metric in local coordinates [1][3]. This fact comes from the observation that

$$\rho^2 = A(r,\theta,\phi)dr^2 + r^2 A(r,\theta,\phi)d\theta^2 + r^2\sin^2(\phi)A(r,\theta,\phi)d\phi^2 \tag{7}$$

$$= A(r,\theta,\phi)dx^2 + A(r,\theta,\phi)dy^2 + A(r,\theta,\phi)dz^2$$

but if $A(r,\theta,\phi)$, $B(r,\theta,\phi)$ and $C(r,\theta,\phi)$ are not equal and

$$\rho^2 = A(r,\theta,\phi)dr^2 + r^2 B(r,\theta,\phi)d\theta^2 + r^2\sin^2(\phi)C(r,\theta,\phi)d\phi^2 \tag{8}$$

then no Riemannian metric in a small open environment of a point P which has (global) Cartesian coordinates (p_1, p_2, p_2) equals metric (8). We can choose local Cartesian coordinates in the small open environment as $x_1 = x - p_1$, $x_2 = y - p_2$, $x_3 = z - p_3$. Transforming (8) to these local coordinates gives $d\rho^2$ that has terms dx_1dx_2, dx_1dx_3, dx_2dx_4. It cannot have these terms in the metric as Cartesian coordinates are orthogonal. It follows that the only valid metric in spherical coordinates of $\mathbf{R}^3$ has equal functions $A(r,\theta,\phi)$, $B(r,\theta,\phi)$ and $C(r,\theta,\phi)$. Inversely, if in local Cartesian coordinates the metric has the form

$$\rho^2 = A(x_1,x_2,x_3)dx_1^2 + B(x_1,x_2,x_3)dx_2^2 + C(x_,x_2,x_3)dx_3^2 \tag{9}$$

where A, B and C are not the same function in a small environment of the point, then if it is expressed first in x, y, z and then translated to spherical coordinates, there are terms $drd\theta$, $drd\theta$ and $drd\phi$, and it is not of the form (8). A local metric of the form (9) is a valid Riemannian metric, but it cannot be expressed as (8).

A Riemannian metric is not described by global spherical coordinates of $\mathbf{R}^3$. A general smooth 3-manifold does not even embed in $\mathbf{R}^3$. Riemannian metric can only be described with local coordinates in a small open environment and this environment can be infinitely small, so it only has the base point and that point must be the origin of local coordinates. Local coordinates could be spherical coordinates with the origin on the base point, but such spherical coordinates are very difficult to transfer to other points, therefore the local coordinates are best expressed as local Cartesian coordinates. The metric is given by an inner product in these local Cartesian coordinates and a distance between two far away points is calculated by intgrating oves small distances in small environments that have local coordinates. If the 3-manifold is embedded in $\mathbf{R}^3$, a metric defined in spherical coordinates of $\mathbf{R}^3$ is a valid Riemannian metric in local coordinates only if the metric has the same function as A, B and C in (8).

The local metric (9) does not have the same speed of light to every direction, but we can give the local coordinate system velocity and find a rest frame where it has the same speed of light to each direction. In the rest frame

$$A(x_1, x_2, x_3) = B(x_1, x_2, x_3) = C(x_1, x_2, x_3). \tag{10}$$

Requiring that in the rest frame the speed of light is c gives the metric

$$ds^2 = A(x_1, x_2, x_3)c^2 dt_1^2 - A(x_1, x_2, x_3)dx_1^2 \tag{11}$$

$$-A(x_1, x_2, x_3)dx_2^2 - A(x_, x_2, x_3)dx_3^2.$$

It is a scalar metric, i.e., $A(x_1, x_2, x_3) = \psi^2$ for some scalar field ψ.

Article [1] proves that the Einstein equations have no static spherical solutions with a scalar metric. This means that the Einstein equations have no static spherical solutions with any metric, especially not with the Schwarzschild invalid metric. They cannot describe the gravitational field of the Sun and the Earth and therefore the Einstein equations are incorrect. This proves that GRT does not have any examples of a static field where the force of a point mass in an otherwise empty space does not grow as r^{-2} and instead GRT is wrong.

See [1][2][3] and also [4].

3. There is no convicing derivation of the Einstein equations

Article [2] discusses the derivation of the Einstein equations. they are supposedly derived from the Euler-Lagrange equations

$$\mathcal{L} = \frac{c^4}{16\pi G}(R - 2\lambda) - \mathcal{L}_{matter}. \tag{12}$$

Let the matter $\mathcal{L}_{matter}$ be zero outside the origin and we set $\lambda = 0$. Then the Langangian $\mathcal{L}$ is the Ricci scalar curvature R multiplied by a constant. The Euler-Lagrange equations are

$$-\partial_\mu \left(\frac{\partial R}{\partial(\partial_\mu \psi)} \right) + \frac{\partial R}{\partial \psi} = 0. \tag{13}$$

As [2] mentions, a nontrivial scalar metric does not satisfy this equation. The way Einstein may have thought is that if each Ricci tensor entry R_{ab} is zero, then the Euler-Lagrange equations (13) are satisfied. But this requirement gives the Schwarzschild solution which does not give a valid metric in local coordinates for the spatial part, see [1][3]. A better interpretation is that the solution is the trivial scalar metric, i.e., the flat metric. If so understood, then Einstein proves that the space-time is not curved, it is flat. But there is no sense at all in making an Euler-Lagrange equation where the Lagrangean is R. The Ricci scalar R is in fact the Euler-Lagrange equation for scalar metric, see [2]. What is the sense in making Euler-Lagrange equations from an Euler-Lagrange equation? Clearly, there is no convincing derivation of Einstein's equations. Instead, there is a convincing derivation of the field equation of the scalar gravitational theory

$$\Box\psi = 4\pi G\rho. \tag{14}$$

This equation is obtained by replacing the Laplacian of the classical equation with the D'Alembert operator, giving a wave equation for gravitational waves.

See [1][2][3].

4. The geodesic metric does not work

Articles [5][6][7] explain the basic problem with the geodesic metric. The Lagrangean for a test mass is

$$\mathcal{L} = \sqrt{\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}} = 1. \tag{15}$$

The trajectory of the test mass can be calculated for Newtonian gravity. The result is that a freely falling mass in Newtonian gravity does not accelerate, which is a good enough reason to discard this Lagrangean.

There must be a different Lagrangean for light beams because the Lagrangean (15) is minimized over paths that have $ds \neq 0$ while light in GRT always follows paths where $ds = 0$. Articles [5][7] investigate the correct Lagrangean for light beams and conclude that it does not agree with measured bending of light around the Sun during an eclipse. This bending of light very possibly is not caused by geometry, it can be caused by there being some material around the Sun. What ever the reason is for light bending, it is not explained by geodesic metric. The geodesic metric does not work.

See [5][6][7].

5. Einstein's equations cannot be quantised, a scalar gravitation field can

There are the known problems in quantising Einstein's equations. But a scalar gravitational field is simply a scalar field. If it is understood as a field and not as geometry it is a standard textbook case of path integral quantisation.

See [2], [8] and e.g. [9] or any other standard textbook on quantum field theory.

6. Time cannot be treated similarly to space coordinates

Because Section 2 already shows that the Einstein equations are incorrect, let us look at the scalar gravitational theory. It also can be interpreted as a geometric theory and it treats time in a similar way as spatial coordinates. What fails in this way?

Nordström's first gravitation theory's field equation (see [2]) is only a slight generalization to the classical result

$$\Box\psi = -4\pi G\rho(r) \tag{16}$$

It is a geometric theory and shows the geometrization ideas because for scalar fields $\Box$ is closely related to the Ricci scalar curvature:

$$R = -6\psi^{-3}\Box\psi. \tag{17}$$

For time independent fields, Nordström's field equation is exactly the Gauss-type equation in Newtonian gravitation theory. Also Nordström's equation of motion is also exactly the same as $F = ma$ if the field is time independent. Yet, Nordström's theory is not simply Newton's gravity: it gives correctly gravitational and acceleration time dilations.

The Laplacian Δ for a function $\psi(r)$ comes from requiring that through any sphere of radius r goes the same amount of force lines coming from a point mass at the origin.

$$A(r)F(r) = \text{constant} \tag{18}$$

Equation (18) shows directly that our world is 3-dimensional (and not 4-dimensional) because the Newtonian gravitation force grows as r^{-2}, the ball of $\mathbf{R}^3$ grows as r^2.

The D'Alembert operator, and the Ricci scalar, is for a 4-dimensional space. The operator $\Box$ has a solution

$$\psi(r,t) = -\frac{GM}{\sqrt{x^2+y^2+z^2-(ct)^2}}. \tag{19}$$

Does this solution have any physical sense in our world? If $t = 0$, the solution has a physical sense and if time would be similar to other coordinates, then the solution should have a physical sense, but I do not think it has any sense. This means that time cannot be treated in the solution in the same way as spatial coordinates. Time can be understood in the equation only in a certain sense: there is the 3-dimensional Laplacian, which implies that the spatial world is 3-dimensional and has the area of the ball as $4\pi r^2$ implying that the 3-space is $\mathbf{R}^3$, and there is time that changes the equation into a wave equation. This wave equation then is in the space $\mathbf{R}^3 \times \mathbf{R}$, but the time dimension is not all in the physical world: only the present time exists in the physical world, and though the imaginary part of a complex equation does exist, it cannot be directly measured in the physical world. Time can often be treated as a coordinate similarly as spatial coordinates, but this is not always the case and there is no need to try to make it look like that in equations: the physical world is not a four-dimensional space-time.

The time dependency of the D'Alembert operator in (16) only describes gravitational waves. It does not describe the movement of masses in the right side of (16). If the field geometry would be space-time geometry, then mass is space-time, but this is not the case in (16), or in the Einstein equations. Movement and changing of mass is already described in the Newtonian field equation, it is not described by $\Box$. The next section explains how movement of masses is described in the Newtonian field.

7. Modelling of movement of masses in the Newtonian field

Time dependency in a gravitational field includes movement of masses causing a changing gravitational field. Masses can also change, like when a star is burning hydrogen to helium and some of the mass turns into energy. The change of the gravitational field propagates with the speed of light. Most of this is not modelled with the time derivative in $\Box$ where the time dependency only describes gravitational waves. Movement of masses is quite well modelled in the Newtonian field equation. What Newtonian mechanics lacks is the finite propagation speed of gravitation interaction and gravitation and acceleration time dilations in atomic clocks. They are all modelled in Nordström's scalar gravitation theory. Additionally, Newton's mechanics and Nordström's theory both lack apparent mass/weakening of force, see [2].

Let us consider a point mass M_i in the location $\bar{r}_i$ with respect to the origin. We observe the gravitation field at the location $\bar{r}$ with respect to the origin. The gravitation potential at $\bar{r}$ at the time t of the observer at $\bar{r}$ is

$$\psi(\bar{r}, t) = -\frac{GM_i(t - c^{-1}||\bar{r} - \bar{r}_i||)}{||\bar{r} - \bar{r}_i||} \tag{20}$$

Let us define $(x_i, y_i, z_i) = \bar{r}_i,\ x' = x - x_i, y' = y - y_i, z' = z - z_i,\ r' = \sqrt{x'^2 + y'^2 + z'^2}$. By moving the origin to r_i, we have a spherically symmetric field

$$\psi(r', t) = -\frac{GM_i(t - c^{-1}r')}{r'} \tag{21}$$

This spherically symmetric field can be solved from

$$F(r', t) = G\frac{M_i(r', t - c^{-1}r')m}{r'^2} \tag{22}$$

$$\frac{d}{dr'}\psi(r', t) = \frac{F(r', t)}{m} = G\frac{M_i(r', t - c^{-1}r')}{r'^2} \tag{23}$$

We can describe all masses M_i by a density function

$$\rho(r', t) = \sum_i M_i(r', t - c^{-1}r') \tag{24}$$

This is possible: the right side is a function of t and r'. The density function is not necessarily what we think density function should be as it sums masses M_i at different times. But finding this density function simplifies the calculations.

$$\frac{d}{dr'}\psi(r',t = G\frac{Vol*\rho(r',t)}{r'^2} = G\frac{4\pi}{3}r'^3\frac{\rho(r',t)}{r'^2} \tag{25}$$

$$r'^2\frac{d}{dr'}\psi(r',t) = G\frac{4\pi}{3}r'^3\rho(r\ddot{a},t) \tag{26}$$

$$\frac{d}{dr'}r'^2\frac{d}{dr'}\psi(r',t) = G4\pi r'^2\rho(r',t) \tag{27}$$

$$\Delta'\psi(r',t) = \frac{1}{r'^2}\frac{d}{dr'}r'^2\frac{d}{dr'}\psi(r',t) = G4\pi\rho(r',t) \tag{28}$$

Notice that in (r',θ',ϕ') coordinates $\psi(r',t)$ is spherically symmetric and

$$\Delta'\psi(r',t) = \frac{1}{r'^2}\frac{d}{dr'}r'^2\frac{d}{dr'}\psi(r',t) \tag{29}$$

while in (r,θ,ϕ) coordinates $\psi(r') = \psi(\bar{r}-\bar{r}_i,t)$ is not spherically symmetric

$$\Delta\psi(r,\theta,\phi,t) \neq \frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\psi(r,\theta,\phi,t) \tag{30}$$

but

$$\Delta = \partial_x^2 + \partial_y^2 + \partial_z^2 = \partial_{x'}^2 + \partial_{y'}^2 + \partial_{z'}^2 = \Delta' \tag{31}$$

because $dx = dx', dy = dy', dz = dz'$. Therefore

$$\Delta'\psi(r',t) = \Delta'\psi(x',y',z',t) = G4\pi\rho(r',t) \tag{32}$$

$$= \Delta\psi(\bar{r},t) = \Delta\psi(x,y,z,t) = G4\pi\rho(r',t) \tag{33}$$

We get the result

$$\Delta\psi(x,y,z,t) = 4\pi G\rho(x,y,z,t) \tag{34}$$

and notice that this result already includes the finite speed of light, movements of masses and the change of masses. We can solve $\phi(x,y,z,t)$ from this equation, but in order to relate this field to individual masses M_i, we have to use a formula that has time delays from the finite speed of light:

$$\psi(x,y,z,t) = \sum_i -GM_i(x,y,z,t-c^{-1}r)\frac{1}{r}. \tag{35}$$

Movements of masses is modelled in this way and we can conclude that in Nordström's and Einstein's field equations the right side, which corresponds to mass, is not space-time geometry. It is external to the field that is in the left side, described as a field in Nordström's theory and a metric in Einstein's equations. This means that mass is not geometry and it is not the gravitational field. In Newton's gravitational theory masses are singularities of the field. That is a rather good characterization.

8. Mass can be reduced to a point mass of $F \sim r^{-2}$

In this section we check that mass can be reduced into a point if the ball grows as r^2 and the force grows as r^{-2}. This is a property that is much used in Newtonian mechanics: we can think of the mass as being concentrated in the center of weight. Especially, this property makes it much easier to calculate orbits of planets and these orbits are quite well calculated by using this property, the disturbances being explained by effects of other planets. The calculations are so good that previously unknown planets were found because of the disturbances, showing that spherical masses really can be modelled as point masses. The importance of this property is that in the next section we replace the power 2 by a power α and notice that the property that a spherical mass can be considered as a point mass in the center of weight only holds for $\alpha = 2$. This is a strong argument that the area of a ball in the 3-space is $4\pi r^2$ and the 3-space is $\mathbf{R}^3$. Planet orbits are stable over a very long time, if the assumption that a spherical mass can be treated as a point mass were false, the error should have been noticed.

The Poisson (i.e., Gauss) equation is

$$\Delta\psi = -4\pi G\rho \tag{36}$$

where ρ is called mass density. This formula is a residue formula. It does not literally mean that ρ is mass density in the sense that some mass m is evenly distributed with the density $\rho(r)$ to the area $r \in [0, R]$ and there are no point masses. But let us see what we get if $\rho(r)$ actually is radially symmetric mass density, mass being evenly distributed without any point masses creating singularities of the field ψ.

We place a small test mass m_1 to the point $z = h$ in the (x, y, z) coordinates. Let the total mass m is

$$m = \int_0^R 4\pi r^2 \rho(r)dr \tag{37}$$

for some R which we may at the end extend to infinity. If the mass m can be considered as a point mass, then we should get the Newtonian gravitation force acting at the distance h between the two masses m_1 and m. We will see if this is the case.

The angle ϕ between the x and z axes ranges from $-\pi/2$ to $+\pi/2$. All points on a circle of radius r on the (x, y)-plane at the height $z = r\sin\phi$ have the same distance s to m_1. This distance s satisfies

$$s^2 = (r\cos\phi)^2 + (h - r\sin\phi)^2. \tag{38}$$

We take such circles as small masses of the size

$$dm(\phi) = 2\pi r\cos(\phi)\ rd\phi\ \rho(r)dr. \tag{39}$$

The Newtonian gravitation force

$$dF(\phi) = Gm_1 \frac{2\pi r^2\cos(\phi)}{s^2} d\phi\ \rho(r)dr \tag{40}$$

created by the mass $dm(\phi)$ is in the direction of $\bar{s}$ and from the test mass m_1 towards the small mass $dm(\phi)$. The force is not along the z-axis and we need to take a projection on the z-axis

$$dF_z(\phi) = \frac{h - r\sin(\phi)}{s} G m_1 \frac{2\pi r^2 \cos(\phi)}{s^2} d\phi \; \rho(r) dr. \tag{41}$$

This force can be integrated over the angle ϕ. The equation is

$$dF(h) = 2\pi G m_1 r^2 \rho(r) dr I \tag{42}$$

where

$$I = \int_{-\pi/2}^{\pi/2} \frac{(h - r\sin(\phi))\cos(\phi)}{s^3} d\phi. \tag{43}$$

It is integrated by the change of the variable to $x = \sin\phi$

$$I = \int_{-1}^{1} a \frac{\frac{h}{r} - x}{(b-x)^{\frac{3}{2}}} dx \tag{44}$$

where

$$a = h^{-1}(2hr)^{-\frac{1}{2}} \tag{45}$$

$$b = \frac{1}{2}\frac{h}{r}\left(1 + \frac{r^2}{h^2}\right). \tag{46}$$

As

$$\int_{-1}^{1} \frac{a(\frac{h}{r} - x)dx}{(b-x)^{\frac{3}{2}}} = 2a(b-x)^{\frac{1}{2}} \left(\left(b - \frac{h}{r} \right)(b-x)^{-1} + 1 \right) \tag{47}$$

we get

$$I = 2a\left(\frac{h}{r} - b\right)\left((b-1)^{-\frac{1}{2}} - (b+1)^{-\frac{1}{2}}\right) \tag{48}$$

$$- 2a\left((b-1)^{\frac{1}{2}} - (b+1)^{\frac{1}{2}}\right). \tag{49}$$

Inserting

$$\frac{h}{r} - b = \frac{h}{2r}\left(1 - \frac{r^2}{h^2}\right) \tag{50}$$

$$b \pm 1 = \frac{h}{2r}\left(1 \pm \frac{r}{h}\right)^2 \tag{51}$$

we get the final result

$$I = \frac{2}{h^2}. \tag{52}$$

Integrating the force dF_z over r gives the force $F(h)$ on the mass m_1

$$F(h) = \int dF_z = -4\pi G \frac{m_1}{h^2} \int_0^R \rho(r) r^2 dr. \tag{53}$$

Let $\rho(r)$ be a constant ρ. The force is Newtonian

$$F(h) = -4\pi G\frac{m_1}{h^2}\rho\frac{R^3}{3} = -G\frac{m_1 m}{h^2}. \tag{54}$$

The sign is negative because $\bar{F}(h) = F(h)\bar{e}_r$, which is also the reason why the force is the negative of the gradient of the field

$$F(h) = -\frac{\partial\psi(h)}{\partial h}. \tag{55}$$

That is, the mass m can be considered as a point mass at the distance h from the test mass m_1. Let us still continue this analysis with some observations.

The Laplace operator gives zero outside singularities

$$\Delta\psi(h) = \frac{1}{h^2}\frac{\partial}{\partial h}h^2\frac{\partial}{\partial h}\psi(h) \tag{56}$$

$$= \frac{1}{h^2}\frac{\partial}{\partial h}h^2(-F(h)) = 0.$$

We see that outside point masses, which are singularities in Newton's gravitation theory, the Laplace operator vanishes. There is no requirement in the calculation above that R should be smaller than h or even finite. We can write

$$F(h) = \int dF_z = -4\pi G\frac{m_1}{h^2}\int_0^\infty \rho(r)r^2dr. \tag{57}$$

Everything happens at one time t and no time derivatives are taken. We can simply add the time parameter and get

$$F(h,t) = \int dF_z = -4\pi G\frac{m_1}{h^2}\int_0^\infty \rho(r,t)r^2dr. \tag{58}$$

The Laplace equation outside point masses is still

$$\Delta\psi(h,t) = 0. \tag{59}$$

Naturally, if we want to add a delay to the time, we can use a time difference equation inserting instead of $\rho(r,t)$ $\rho(r,t-s/c)$ with a suitable s, but differential-difference equations are difficult to work with.

We can express Δ in Cartesian coordinates

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi(x,y,z,t) = 0 \tag{60}$$

for a radially symmetric ψ. The form of the expression of (60) should logically be the same for any ψ. If so, then outside point masses we should get

$$\Delta\psi(\bar{h},t) = 0. \tag{61}$$

According to literature (61) is correct. We will prove it with point masses in (62)-(69). The solution of (61) is not radially symmetric and especially not $\psi \sim r^{-1}$ in the general case as Δ has other parts than the radial part.

The ρ in (36) is a convention that counts the residues of singularities in point masses in a volume bounded by a closed surface. The value of the residue is calculated as in the Gauss theorem for electromagnetism: a point mass at the origin in empty space gives the residue

$$\Delta\psi(r) = -4\pi GM\delta(\bar{r}) \tag{62}$$

where $\delta(\bar{r})$ is the Dirach delta. For any linear transform $\bar{r}' = \bar{r} - \bar{s}$ where $\bar{s}$ is some constant 3-vector holds

$$\Delta_{x,y,z} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \frac{\partial^2}{\partial z'^2} = \Delta_{x',y',z'} \tag{63}$$

Thus, changing from spherical coordinates to Cartesian, making the linear transform and chanring back we have

$$\Delta_{r,\theta,\phi} = \Delta_{x,y,z} = \Delta_{x',y',z'}\Delta_{r',\theta',\phi'} \tag{64}$$

For any constant vector $\bar{s}_i$ and new variable $\bar{r}' = \bar{r} - \bar{s}_i$ holds

$$\Delta_{r,\theta,\phi}\psi_i(\bar{r} - \bar{s}_i) = \Delta_{r',\theta',\phi'}\psi_i(\bar{r}'). \tag{65}$$

We define the potential as

$$\psi(\bar{r}) = \sum_i \psi_i(\bar{r} - \bar{s}_i). \tag{66}$$

Then

$$\Delta_{r,\theta,\phi}\psi_i(\bar{r}) = \sum_i \Delta_{r'_i,\theta'_i,\phi'_i}\psi_i(\bar{r}'_i) = -4\pi G\sum_i M_i\delta(\bar{r}'_i) \tag{67}$$

which is written as

$$\Delta\psi(\bar{}) = -4\pi G\rho(\bar{r}) \tag{68}$$

where

$$\rho(\bar{r}) = \sum_i M_i\delta(\bar{r} - \bar{s}_i). \tag{69}$$

Equation (68) is (36) and we see from (69) that ρ is not any continuous function.

9. Newton's gravitation potential is correct and the space is flat

This section gives the argument that the 3-space is flat.

We look at a situation when the space is empty outside the single mass body where the mass body is not a point mass. The mass body is a ball of constant mass density ρ centered at the origin and having the radius R and the mass m.

The situation is spherically symmetric, therefore the field must be radially symmetric. Measurements show that Newton's gravitational force is a good approximation. Let us assume that the force is well approximated in the range of r that interests us by the formula

$$F = G\frac{m_1 m}{r^\alpha} \tag{70}$$

where α is a constant and close to two. We are interested in proving that $\alpha = 2$.

We will assume that the geometry of the space is such that the length of a circle of radius r is well approximated in the range of r that interests us by the formula

$$L = \int_0^{2\pi} r^{\frac{\gamma}{2}} d\phi = 2\pi r^{\frac{\gamma}{2}} \tag{71}$$

where γ is a constant and close to two. Then the area of a sphere of radius r is well approximated by

$$A(r) = 4\pi r^\gamma \tag{72}$$

and the volume of a ball of radius r is well approximated by

$$V(r) = \frac{4\pi}{\gamma+1} r^{\gamma+1}. \tag{73}$$

The mass m is distributed over the R-ball centered in the origin and having a constant density ρ, thus

$$m = \rho V(R). \tag{74}$$

The mass m_1 is a much smaller test mass that we will place to the z-coordinate to the place $(0, 0, h)$.

Step (76) below may initially look like a circular assumption when trying to prove that the 3-space is flat, but it is not. In (76) Cartesian coordinates are transformed to spherical coordinates with equations that are valid in $\mathbf{R}^3$, these equations assume that

$$r^2 = x^2 + y^2 + z^2. \tag{75}$$

Then when we get a circle, area or volume, it is given the value from (71), (72) and (73). This way describes correctly a 3-space geometry that is not $\mathbf{R}^3$ in the calculation (77)-(90).

The following spherical coordinates are a convenient choice for this calculation

$$x = r\cos(\phi)\cos(\beta) \qquad y = r\sin(\phi)\cos(\beta) \qquad z = r\sin(\beta). \tag{76}$$

The angle β is between the x-axis and z-axis. At a given value of β there is a circle with the radius $r\cos(\beta)$ with the length $2\pi r^{\frac{\gamma}{2}}$. The volume element having length $r^{\frac{\gamma}{2}} d\beta$ in the β-direction and dr in the radial direction is

$$dV(\beta) = 2\pi r^\gamma dr \cos(\beta) d\beta \tag{77}$$

and the mass is $dm(\beta) = \rho dV(\beta)$. In (77) we have the area of the ball growing as $4\pi r^\gamma$.

The elements of this circular mass are at the distance s from m_1 where

$$s^2 = (r\cos(\beta)^2 + (h - r\sin(\beta))^2 = r^2 + h^2 - 2rh\sin(\beta). \tag{78}$$

This mass creates a gravitational force $dF(\beta, r)$ on m_1. The force from the elements of the mass are not in the direction of $-z$-axis, so we have to add a projection. It is the first multiplier at the right side in the formula below:

$$dF(\beta, r) = \frac{h - r\sin(\beta)}{s} G \frac{m_1 dm}{s^\alpha} \tag{79}$$

The force is towards the origin. In (79) we have the force growing as $r^{-\alpha}$ instead as r^{-2}.

The integral over β is

$$dF(r) = 2\pi G m_1 \rho r^\gamma dr \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(h - r\sin(\beta))\cos(\beta)d\beta}{s^{\frac{\alpha+1}{2}}} \tag{80}$$

$$= 2\pi G m_1 \rho r^\gamma dr I$$

where $x = \sin(\beta)$ simplifies I to

$$I = \int_{-1}^{1} a \frac{(\frac{h}{r} - x)dx}{(b - x)^{\frac{\alpha+1}{2}}} \tag{81}$$

$$a = r(2hr)^{-\frac{\alpha+1}{2}} \tag{82}$$

$$b = \frac{r^2 + h^2}{2hr} \tag{83}$$

A calculation gives the following exact result

$$I = \frac{1}{\alpha - 1} \frac{1}{2r} h^{1-\alpha} I_2 \tag{84}$$

$$I_2 = \left(1 + \frac{r}{h}\right)\left(1 - \frac{r}{h}\right)^{\alpha-2} - \left(1 - \frac{r}{h}\right)\left(1 + \frac{r}{h}\right)^{\alpha-2}$$

$$- \left(1 - \frac{r}{h}\right)^{3-\alpha} + \left(1 + \frac{r}{h}\right)^{3-\alpha} \tag{85}$$

From Taylor series we get the expression:

$$I = \frac{3 - \alpha}{\alpha - 1} 2h^{-\alpha} + p(\alpha) r^2 h^{-\alpha-2} + O(r^4 h^{-4}) \tag{86}$$

where

$$p(\alpha) = \frac{1}{2(\alpha - 1)} \left(20 - \frac{92}{3} + 15\alpha^2 - \frac{7}{3}\alpha^3 \right) \tag{87}$$

If $\alpha = 2$, then $p(\alpha) = 0$ and the $O(r^4 h^{-4})$ term is also zero. Thus, $\alpha = 2$ gives an exact result as the first term of I.

We integrate $-dF(r)$ over r from zero to R and insert m from (75) and (73). The result is

$$F = G\frac{m_1 m}{h^\alpha}\frac{3-\alpha}{\alpha - 1} + G\frac{m_1 m}{h^\alpha}\frac{\gamma+1}{\gamma+3}\frac{p(\alpha)}{2}\frac{R^2}{h^2}\left(1 + O(R^2 h^{-2})\right). \tag{88}$$

Let $h >> R$, then the second term in the right side becomes insignificant. The gravitational force created by a ball of radius R and mass m must approach the gravitational force created by a point mass with the size m. Thus

$$F = G\frac{m_1 m}{h^\alpha}\frac{3-\alpha}{\alpha - 1} \to G\frac{m_1 m}{h^\alpha} \tag{89}$$

when h stays fixed and R approaches zero. This can only happen if

$$3 - \alpha = \alpha - 1 \tag{90}$$

that is, $\alpha = 2$.

We have proven that if a circular mass can be considered as a point mass at the center of weight, then the power α in the force formula must be two, so the force must be Newtonian. It does not matter if we make α a function of h because we are not integrating over h in this calculation. Always (90) must hold in the limit, so $\alpha = 2$.

Let us remark that for $\alpha = 2$ the equation

$$F = G\frac{m_1 m}{h^2} \tag{91}$$

is exact. For any distance $h > R$ the mass of the shape of an R-ball with constant density always gives the same gravitational force as a point mass of the same size.

We did not get any result for the parameter γ from this calculation, but there is an easy way to conclude that in empty space $\gamma = 2$. Force lines from the gravitational force created by a point mass do not disappear in empty space. Thus

$$A(r + dr)F(r + dr) = A(r)F(r) \tag{92}$$

implying that $\gamma = \alpha = 2$, the space geometry is flat.

This is in fact a very strong argument that the 3-space is flat, the force is Newtonian and graviation is a field in $\mathbf{R}^3 \times \mathbf{R}$ and not space-time geometry. Astronomical calculations use the property that a spherical mass can be considered as a point mass and they work well.

10. Adding a $r^{-\alpha}$ term to the force

Let us again look at the equation

$$A(r)F(r) = \text{constant} \tag{93}$$

We see that the fundamental property in the Newtonian field equation is the force, not the potential. The potential is only more convenient as potentials are scalars and are additive while force is a vector and needs vector summing.

Let us investigate the possibility that the gravitational force from a point source is not growing as r^{-2} but has an additional term growing e.g. as r^{-4}. Adding a term that grows as $r^{-\alpha}$ to the potential does not cause a major problem to the field equation (34). As

$$\Delta\left(-\frac{1}{r^{\alpha}}\right) = \alpha(\alpha-1)\frac{1}{r^{\alpha+2}} \tag{94}$$

we only need to add corresponding terms to the right side of (34)

$$\Delta\phi(x,y,z,t) = 4\pi G\rho(x,y,z,t) \tag{95}$$

$$+GM\beta\alpha(\alpha-1)\sum_i \frac{M_i(\bar{r}_i, t-c^{-1}||\bar{r}-\bar{r}_i||)}{||\bar{r}-\bar{r}_i||^{\alpha+2}}$$

These terms do not nicely sum, so the vectors $\bar{r}$, $\bar{r}_i$ remain. The solution of (95) for an empty space with a point mass M at the origin is

$$\psi(r) = -\frac{GM}{r}\left(1-\frac{\beta}{r^{\alpha}}\right) \tag{96}$$

However, (93) makes a compelling argument that $A(r)$ should be different from $4\pi r^2$ if there are additional terms in the force or potential. Let us focus only on a radially symmetric situation.

Let $\phi(r)$ be any field that we want to be the solution for the situation of a point mass M in empty space. Let us define

$$\mathcal{D} = \frac{1}{A(r)}\frac{d}{dr}A(r)\frac{d}{dr} \tag{97}$$

where

$$A(r) = \frac{4\pi GM}{\phi'(r)}. \tag{98}$$

Then

$$\mathcal{D}\phi(r) = 0 \tag{99}$$

and for any $\psi(r)$ holds

$$\mathcal{D}\psi(r) = \Delta\psi(r) - \frac{\Delta\phi(r)}{\Delta\phi(r)}\psi'(r) \tag{100}$$

$$\mathcal{D}\psi(r) = \Delta\psi(r) + \left(\frac{A(r)'}{A(r)} - \frac{2}{r}\right)\psi(r)'. \tag{101}$$

As a conclusion from this section, we can e.g. add a small r^{-3} term to the Newtonian potential. It does not much change the area of a sphere as is seen from (98). If we want that $A(r)$ grows clearly faster than $4\pi r^2$, then (98) shows how the field must grow. The right side of (95), though not a function of r, does approximate a function of r from a long enough distance if the mass distribution is sufficiently spherical.

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