GEOMETRIZED VACUUM PHYSICS. PART 13: CONNECTION WITH QUANTUM MECHANICS

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ABSTRACT

This article is the thirteenth part of the scientific project under the general title "Geometrized Vacuum Physics Based on the Algebra of Signature " [1,2,3,4,5,6,7,8,9,10,11,12]. This article is aimed at substantiating the assertion that there is no difference in the mathematical description of the behavior of objects in the macrocosm and the microcosm. The hierarchical cosmological model proposed in the previous articles of this project assumes that the metric-dynamic models of all "corpuscles", regardless of their size (for example, "elementary particles", naked "planets" and "stars", as well as naked "galaxies") are structured almost identically. The main differences between them are associated primarily with the distinguishability of small details. The larger the "corpuscle", the more subtly its infrastructure is manifested. However, the similarity of "corpuscles" of different sizes is not limited to the coincidence of their shape. Their random movements (i.e., chaotic deviations of the core of the "corpuscles" from their mean positions) also obey the same laws. The article presents the derivation of the stochastic Schrödinger equations and self-diffusion equation, suitable for describing the averaged (including quantized) states of stochastic systems of any scale. It is shown that, for example, the chaotically shifting core of a planet (or star) can have a quantum set of possible averaged states, similar to the excited states of an electron in an atom. It is suggested that when the core of a planet (or star) transitions from one quantum state to another, the interior of this celestial body can absorb or emit gravitational waves. This hypothesis may form the basis of stellar-planetary gravitational spectroscopy.

Keywords: Vacuum physics, stochastic quantum mechanics, derivation of the Schrödinger equations, excited states of the interiors of planets and stars.

BACKGROUND AND INTRODUCTION

1] The Coexistence of General Relativity and Quantum Mechanics

One of the most complex problems in modern physics is the existence of two fundamental, yet completely unrelated, theories: General Relativity (GR) and Quantum Mechanics (QM). All theories describing the deterministic behavior of celestial (i.e., macroscopic) bodies and cosmological models are based on A. Einstein's GR, while all quantum theories, which allow us to predict the most probabilistic states of the smallest particles and the fields surrounding them, are built on the principles of QM.

Both theories, GR and QM, have been convincingly confirmed experimentally within their scope of applicability and underlie virtually all modern advanced technologies.

Within the framework of modern concepts, GR and QM cannot be reconciled. The main intellectual discomfort of scientists lies in the fact that these theories are based on mutually exclusive principles.

The main reason for the incompatibility of general relativity and quantum mechanics lies in the difference in their approaches to understanding the nature of space, time, and matter. General relativity is based on the assumption that spacetime is a smooth, continuously curved four-dimensional extension. In quantum mechanics, however, the smoothness of spacetime must be violated due to the inherent probabilistic causality of events and the uncertainty principle. Therefore, when both theories are applied simultaneously, profound conceptual contradictions arise.

For example, calculating the energy of the physical vacuum within quantum mechanics leads to infinity due to the need to account for strong fluctuations of spacetime on microscopic scales (i.e., the existence of "quantum foam"). On the other hand, attempts to apply quantum mechanics to the quantization of gravity also lead to infinity, which cannot be eliminated by standard methods, as is done in other quantum theories.

2] Quantization in the Modernized General Relativity

Despairing of grasping the fundamental idea for constructing a unified field theory, Einstein once said: "It seems to me that for real progress we must again wring some general guiding principle from Nature." Another of the 20th century's greatest scientists, Wolfgang Pauli, criticized Einstein's program for creating a unified field theory. Pauli famously commented on this topic: "What God has broken apart, let no man put together." Apparently, he was paraphrasing the Bible's dictum: "What God has joined together, let no man put asunder" (Matthew 19:6). Nevertheless, in 1958, Pauli gave a lecture at Columbia University, where he outlined his version of a unified field theory, which was based on Heisenberg's work in this area. Niels Bohr attended this lecture. In a discussion with Pauli, Bohr said, "We in the gallery are convinced that your theory is crazy, but we disagree on whether it is crazy enough to be correct." Later, in response to Pauli's objections, Bohr responded, "Einstein has already tried all the simple ways to create a unified field theory, and they all failed. Therefore, we must consider something radically different from all previous approaches; we must come up with something crazy enough for the theory to be correct." Perhaps the ideas proposed in Geometricized Vacuum Physics based on the Algebra of Signature are radical and unexpected enough to bring us closer to the goal outlined by Einstein.

In the previous papers of the Geometrized Vacuum Physics based on the Algebra of Signature (GVPh&AS) [1,2,3,4,5,6,7, 8,9,10,11,12] we mainly considered a hierarchical cosmological model, which is based on two main modernizations of Einstein's general theory of relativity:

1) All theories and textbooks based on Einstein's General Theory of Relativity (GTR) use metric spaces with one and/or two signatures $(+---)$ and/or $(-+++)$. However, the proposed GVPh&AS takes into account all 16 possible types of 4-dimensional spaces with the following signatures (i.e., topologies, see §4 in [2]):

$$sign\left(ds^{(a,b)2}\right) = \begin{pmatrix} (++++) & (+++-) & (-++-) & (++-+) \\ (---+) & (-+++) & (--++) & (-+-+) \\ (+--+) & (++--) & (+---) & (+-++) \\ (--+-) & (+-+-) & (-+--) & (----) \end{pmatrix}. \quad (1)$$

2) In General Relativity, Einstein's vacuum equation with only one Λ-term is used:

$$R_{ik} + \Lambda g_{ik} = 0, \quad (2)$$

And in the proposed GVPh&AS, Einstein's vacuum equation is also used as conservation laws, but with an infinite number of $\pm\Lambda_i$-terms (see §6 in [5] and the Introduction in [6])

$$R_{ik} + g_{ik}(\Lambda_1 + \Lambda_2 + \Lambda_3 + \cdots + \Lambda_\infty) = 0, \quad (3)$$

where Λ_i can take both positive and negative values

$$\pm\Lambda_i = \pm\frac{3}{r_i^2}, \quad (4)$$

here, r_i are the radii of the corresponding spheres.

Within the framework of the GVPh&AS, Eq. (3) is called Einstein's third vacuum equation.

Analysis of the solutions of Eq. (3) showed (see [6]) that the extent of a closed spherical universe is filled with an infinite number of spherical $\lambda_{m,n}$-vacuum formations (conventionally called "corpuscles" and "anticorpuscles"), which are nested in various ways like Russian dolls (see Figure 1).

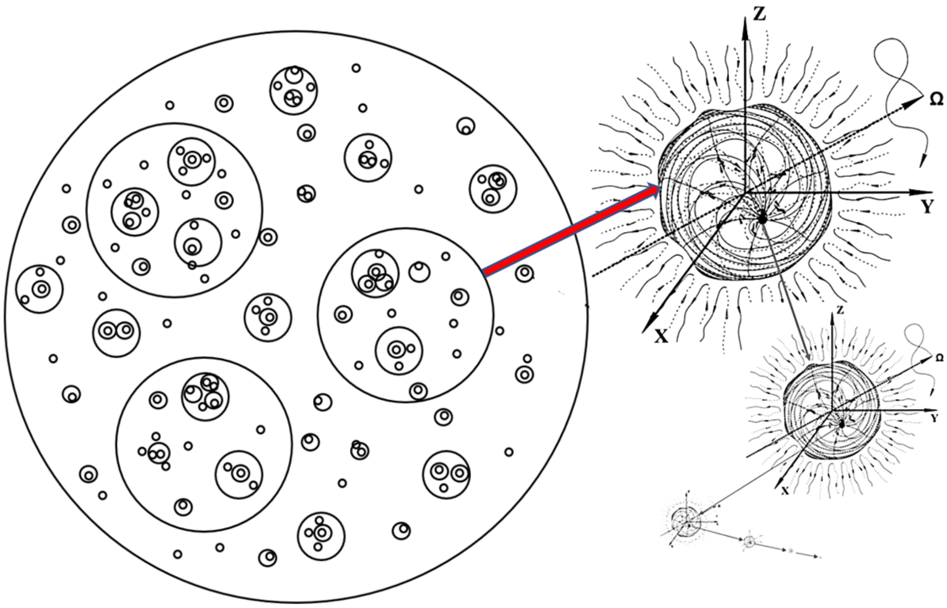


Fig. 1. (Repetition of Fig. 10 in [6]) Illustration of a hierarchical cosmological model consisting of a set of spherical $\lambda_{m,n}$-vacuum formations ("corpuscles" and "anticorpuscles"), nested in different ways like Russian dolls.

Scaling the "corpuscles" and "anticorpuscles" by size allows us to represent equation (3) as

$$R_{ik} + ½\, g_{ik}(\sum_{k=1}^{10}\sum_{m=1}^{\infty} \Lambda_{km} + \sum_{k=1}^{10}\sum_{m=1}^{\infty} - \Lambda_{km}) = 0, \quad (5)$$

where $\pm\Lambda_{km} = \pm\frac{3}{r_{km}^2}$, with hierarchy of radii (44a) in [6]:

$r_1 \sim 10^{39}$ cm is radius commensurate with the radius of the mega-Universe core; (6)
$r_2 \sim 10^{29}$ cm is radius commensurate with the radius of the observable Universe core;
$r_3 \sim 10^{17}$ cm is radius commensurate with the radius of the galactic core;
$r_4 \sim 10^{7}$ cm is radius commensurate with the radius of the core of a planet or star;
$r_5 \sim 10^{-3}$ cm is radius commensurate with the radius of a biological cell;
$r_6 \sim 10^{-13}$ cm is radius commensurate with the radius of an elementary particle core;
$r_7 \sim 10^{-24}$ cm is radius commensurate with the radius of a proto-quark core;
$r_8 \sim 10^{-34}$ cm is radius commensurate with the radius of a plankton core;
$r_9 \sim 10^{-45}$ cm is radius commensurate with the radius of the proto-plankton core;
$r_{10} \sim 10^{-55}$ cm is radius commensurate with the size of the instanton core.

Let's recall that we do not actually know how many spherical objects with radii (6) exist in the largest hierarchical chain. Only radii r_{2m}, r_{3m}, r_{4m}, r_{5m}, r_{6m} correspond to the sizes of the cores of real spherical bodies ("corpuscles"). The existence of "corpuscles" with radii of cores r_{1m}, r_{7m}, r_{8m}, r_{9m}, r_{10m} is only predicted. In other words, we assumed that there is a hierarchical sequence of only 10 types of "corpuscles," differing from each other in size by approximately 10 orders of magnitude, but we are confident in only five of them. However, the hierarchical cosmological model is constructed in such a way that, at the initial stage of research, the number of "corpuscles" of different scales in the hierarchical sequence (6) is not decisive and can be changed as a result of further research.

The initial condition of vacuum balance, formulated in [1,2,3,4], requires that the averaging of the second term in Ex. (5) over the entire volume of the "mega-Universe" (or over all "corpuscles" and "anticorpuscles" of different scales) be equal to zero.

$$\overline{(\sum_{k=1}^{10}\sum_{m=1}^{\infty} \Lambda_{km} + \sum_{k=1}^{10}\sum_{n=1}^{\infty} - \Lambda_{kn})} = 0. \quad (7)$$

Thus, on average, we return to Einstein's original (first) vacuum equation

$$R_{ik} = 0, \tag{8}$$

which predetermines the metric-dynamic model of a closed spherical mega-Universe as a whole.

At the same time, in local regions of the Universe, the result of averaging the second term in Eq. (5) may not be equal to zero.

$$\overline{\left(\sum_{k=1}^{10}\sum_{m=1}^{\infty} \Lambda_{km} + \sum_{k=1}^{10}\sum_{n=1}^{\infty} - \Lambda_{kn}\right)} = \pm\Lambda_l. \tag{9}$$

Then the metric-dynamic properties of such a local region of space can be determined by Einstein's second vacuum equation
.

$$R_{ik} \pm \Lambda_l g_{ik} = 0. \tag{10}$$

In addition to studying averaged cosmological models of the type (8) and (10), from equation (5) one can distinguish an infinite number of individual hierarchical cosmological chains, for example,

$$\begin{cases} R_{ik} + g_{ik}\sum_{m=1}^{10}\Lambda_m = 0, \\ R_{ik} - g_{ik}\sum_{m=1}^{10}\Lambda_m = 0. \end{cases} \tag{11}$$

$$\begin{cases} R_{ik} + g_{ik}\sum_{m=1}^{3}\Lambda_m = 0, \\ R_{ik} - g_{ik}\sum_{m=1}^{3}\Lambda_m = 0. \end{cases} \tag{12}$$

$$\begin{cases} R_{ik} + g_{ik}\sum_{m=1}^{14}\Lambda_m = 0, \\ R_{ik} - g_{ik}\sum_{m=1}^{14}\Lambda_m = 0. \end{cases} \tag{13}$$

etc., depending on how many different-scale spherical $\lambda_{m,n}$-vacuum formations (i.e., "corpuscles" and "anticorpuscles") with the radii of cores from hierarchy (6)) are located inside each other (see Figure 1*a,b*).

Each of the infinite number of hierarchical chains described by systems of equations like (11) – (13) can be solved separately.

However, within the framework of the GVPh&AS, there is a general rule that all hierarchical chains begin with a single common core of the largest "corpuscle" (in particular, with the core of the "mega-Universe" with a radius $r_{1m} \sim 10^{39}$ cm) and end at the core of a single common smallest "corpuscle" (in particular, at the core of an "instanton" with a radius $r_{10m} \sim 10^{-55}$ cm) (see Figure 2).

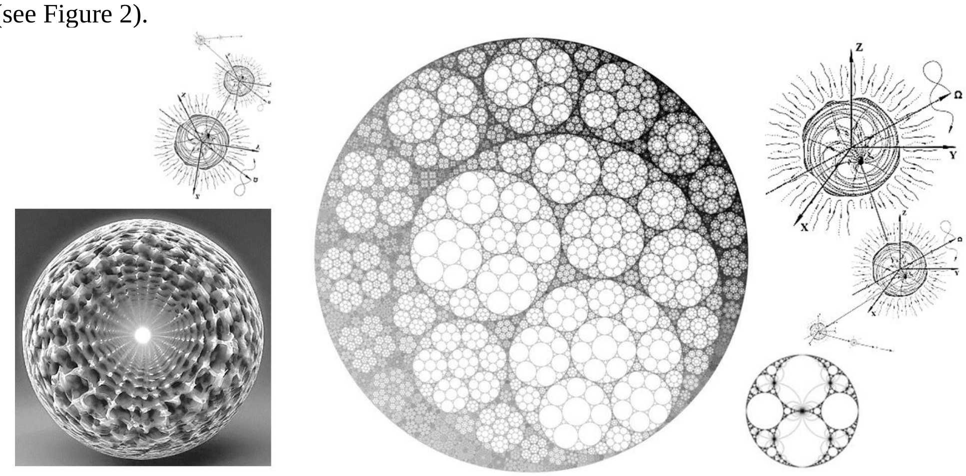


Fig. 2. Illustrations of countless hierarchical chains of "corpuscles" and "anticorpuscles" of different scales, which begin with a single, largest, common "meta-Universe" core and end with a single, smallest, common "instanton" core

For example, within the framework of the GVPh&AS, the metric-dynamic model of a free “electron”, the core of which is located only inside the core of the “mega-Universe” and inside which there is only one “instanton” core, is determined by the equation of a three-level hierarchical chain

$$R_{ik} + g_{ik}(\Lambda_1 + \Lambda_6 + \Lambda_{10}) = R_{ik} + g_{ik}\left(\frac{3}{r_1^2} + \frac{3}{r_6^2} + \frac{3}{r_{10}^2}\right) = 0, \tag{14}$$

more precisely, a system of metric solutions to this equation (of the form (1) – (10) in [7]):

"ELECTRON" (15)

free, valence on average, spherical, stable, "convex" multilayer spherical curvature of the $\lambda_{-12,-15}$-vacuum with signature (+ – – –), consisting of:

The outer shell

in the interval [r_1, r_6] (see Figure 3)

I $$ds_1^{(+---)2} = \left(1 - \frac{r_6}{r} + \frac{r^2}{r_1^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_6}{r} + \frac{r^2}{r_1^2}\right)} - r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{16}$$

H $$ds_2^{(+---)2} = \left(1 + \frac{r_6}{r} - \frac{r^2}{r_1^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_6}{r} - \frac{r^2}{r_1^2}\right)} - r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{17}$$

V $$ds_3^{(+---)2} = \left(1 - \frac{r_6}{r} - \frac{r^2}{r_1^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_6}{r} - \frac{r^2}{r_2^2}\right)} - r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{18}$$

H′ $$ds_4^{(+---)2} = \left(1 + \frac{r_6}{r} + \frac{r^2}{r_1^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_6}{r} + \frac{r^2}{r_1^2}\right)} - r^2(d\theta^2 + \sin^2\theta\, d\phi^2); \tag{19}$$

The core

in the interval [r_6, r_{10}] (see Figure 3)

I $$ds_1^{(+---)2} = \left(1 - \frac{r_{10}}{r} + \frac{r^2}{r_6^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_{10}}{r} + \frac{r^2}{r_6^2}\right)} - r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{20}$$

H $$ds_2^{(+---)2} = \left(1 + \frac{r_{10}}{r} - \frac{r^2}{r_6^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_{10}}{r} - \frac{r^2}{r_6^2}\right)} - r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{21}$$

V $$ds_3^{(+---)2} = \left(1 - \frac{r_{10}}{r} - \frac{r^2}{r_6^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_{10}}{r} - \frac{r^2}{r_6^2}\right)} - r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{22}$$

H′ $$ds_4^{(+---)2} = \left(1 + \frac{r_{10}}{r} + \frac{r^2}{r_6^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_{10}}{r} + \frac{r^2}{r_6^2}\right)} - r^2(d\theta^2 + \sin^2\theta\, d\phi^2); \tag{23}$$

The substratum

in the interval [0, ∞]

i $$ds_5^{(+---)2} = c^2dt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{24}$$

where according to the hierarchy (6):

$r_1 \sim 10^{39}$ cm is radius commensurate with the radius of the mega-Universe core; (25)

$r_6 \sim 10^{-13}$ cm is radius commensurate with the radius of an elementary particle core;

$r_{10} \sim 10^{-55}$ cm is radius commensurate with the size of the instanton core.

An analysis of the set of metrics of type (17) – (24) carried out in [7,8,9] showed that, with certain simplifications, such an “electron” occupies the entire core of the “mega-Universe” with a radius $r_1 \sim 10^{39}$ cm. In this case, three clearly defined zones can be distinguished in such a universal “electron” (see Figure 3): 1) the outer shell of the “electron”, which extends from the core of the “electron” to the edge of the core of the “mega-Universe” with a radius $r_1 \sim 10^{39}$ cm; 2) the core of the “electron” with a radius $r_6 \sim 10^{-13}$ cm; 3) the inner nucleolus of the “electron”, which is the core of the “instanton” with a radius радиусом $r_{10} \sim 10^{-55}$ cm.

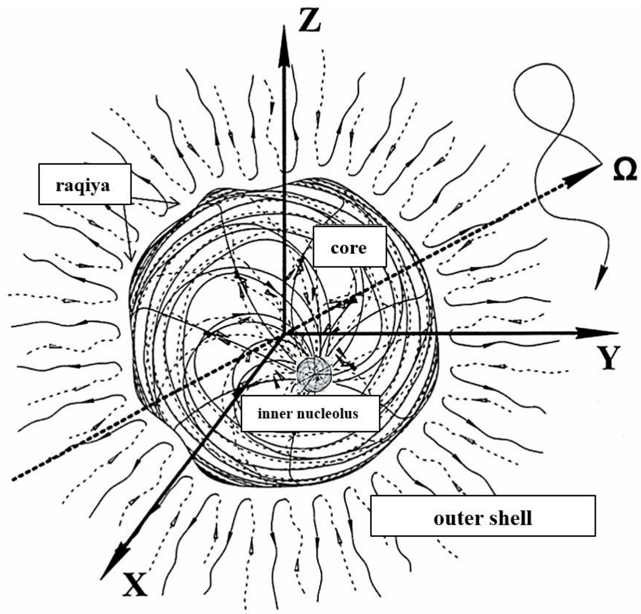


Fig. 3. The universal "electron," which can be divided into three main zones: the outer shell, the core, and the inner nucleolus

In the previous articles [6,7,8,9,10,11,12], we examined in detail a mainly hierarchical cosmological chain consisting of 10 + 10 = 20 cells (i.e., 10 "corpuscles" nested within each other and 10 "anticorpuscles" nested within each other). In other words, we solved the system of equations (11) with 10 + 10 = 20 $\pm\Lambda_i$-terms. As a result, a closed metric-dynamical hierarchical model of one chain was obtained (see Figure 4).

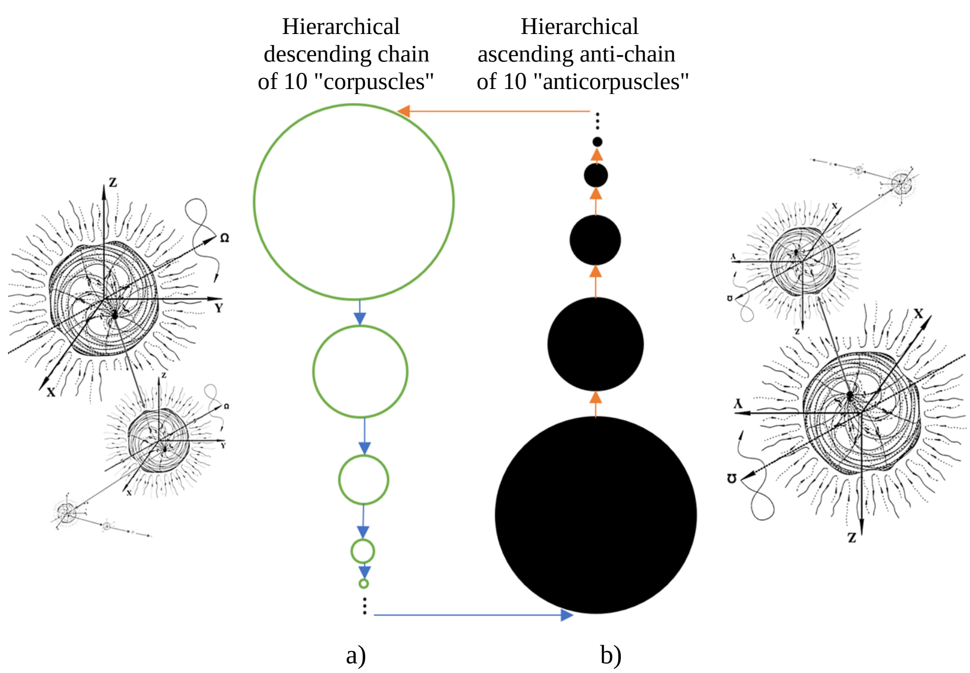


Fig. 4. (Repeat of Figure A1 in [12]). Schematic representation of a closed hierarchical cosmological chain, consisting of 10 nested "corpuscles" (i.e., convexities of the $\lambda_{m,n}$-vacuum) and 10 nested "anticorpuscles" (i.e., concavities of the $\lambda_{m,n}$-vacuum)

In the hierarchical chain shown in Figure 4a, there are: 1 - the core of the "mega-Universe", 2 - inside of which is the core of the "observable Universe", 3 - inside of which is the core of a naked "galaxy", 4 - inside of which is the core of a naked "planet", 5 - inside of which is the core of a "biological cell", 6 - inside of which is the core of an "elementary particle", 7 - inside of which is the core of a "proto-quark", 8 - inside of which is the core of "plankton"; 9 - inside of which is the core of "proto-plankton", 10 inside of which is the core of an "instanton".

Other hierarchical chains may contain, for example, 6 cells (i.e., "corpuscles" of different scales): 1 - the core of the "mega-Universe", 2 - inside which is the core of the "observable Universe", 3 - inside which is the core of a naked "galaxy", 4 - inside which is the core of an "elementary particle", 5 - inside which is the core of a "plankton"; 6 - inside which is the core of an "instanton"; or 8 cells, or 4 cells, etc. (but not less than three).

An infinite number of such hierarchical chains and anti-chains, beginning with a common "mega-Universe" core and ending with a common "instanton" core, form a single hierarchical cosmological model consisting of an infinite number of "corpuscles" of different scales (see Figures 1 and 2). Moreover, the size range of "corpuscles" and "anticorpuscles" is strictly quantized, i.e., each scale type of "corpuscles" and "anticorpuscles" has a strictly defined base size of its smallest core.

The reason for such a large-scale quantization of "corpuscles" and "anticorpuscles" by their sizes is apparently related to the vacuum discontinuity conditions discussed in the section on vacuum kinematics (see §7.3 in [3]) and is due to the finiteness of the speed of light. However, this problem has not been resolved within the framework of the GVPh&AS and requires further consideration.

Within each size class of "corpuscles" and "anticorpuscles," there is an additional possibility of quantization. The point is that only those spherical $\lambda_{m,n}$-vacuum formations (i.e., "corpuscles" and "anticorpuscles") are stable, which are described by the metrics-solutions of the Einstein vacuum equation $R_{ik} \pm \Lambda_k g_{ik} = 0$ with the total (or averaged) signatures (+ – – –) or (– + + +) or (0 0 0 0) (see [5,6,7], as well as [10,12]). Spheroidal $\lambda_{m,n}$-vacuum formations with any other signatures from the matrix (1) are unstable, i.e., they can temporarily locally manifest themselves from the void, but subsequently dissolve or annihilate.

Moreover, the signature of the metric solutions to Einstein's vacuum equation, which define the metric-dynamic model of spheroidal $\lambda_{m,n}$-vacuum formations (i.e., "corpuscles" and "anticorpuscles"), is related to their topology. For example, if a "corpuscle" with an average signature of (+ – – –) is conventionally called a stable spherical convexity of a $\lambda_{m,n}$-vacuum, then an "anticorpuscle" with an averaged, opposite signature of (– + + +) should be called a stable spherical concavity of the same $\lambda_{m,n}$-vacuum.
A striking example of such charged stable spherical $\lambda_{m,n}$-vacuum formations (i.e., “corpuscles” and “anticorpuscles”) are the “electron” with the signature (+ – – –) (see (1) in [7]) and the “positron” with the signature (– + + +) (see (11)) in [7]), as well as various states of the “proton” with the averaged signature (– + + +) (see (84) – (86) in [6]):

$$\begin{array}{l} d_{\text{к}}^{+}(+\ +\ +\ -) \\ u_{\text{з}}^{-}(-\ +\ -\ +) \\ u_{\text{г}}^{-}\underline{(-\ -\ +\ +)} \\ p_1^{-}(-\ +\ +\ +)_{+} \end{array} \qquad (26) \qquad \begin{array}{l} d_{\text{з}}^{+}(+\ +\ -\ +) \\ u_{\text{г}}^{-}(-\ -\ +\ +) \\ u_{\text{к}}^{-}\underline{(-\ +\ +\ -)} \\ p_2^{-}(-\ +\ +\ +)_{+} \end{array} \qquad (27) \qquad \begin{array}{l} d_{\text{г}}^{+}(+\ -\ +\ +) \\ u_{\text{к}}^{-}(-\ +\ +\ -) \\ u_{\text{з}}^{-}\underline{(-\ +\ -\ +)} \\ p_3^{-}(-\ +\ +\ +)_{+} \end{array} \qquad (28)$$

and the “antiproton” with an average signature (– + + +) (see (87) – (89) in [6]):

$$\begin{array}{l} d_{\text{к}}^{-}(-\ -\ -\ +) \\ u_{\text{з}}^{+}(+\ -\ +\ -) \\ u_{\text{г}}^{+}\underline{(+\ +\ -\ -)} \\ p_1^{+}(+\ -\ -\ -)_{+} \end{array} \qquad (29) \qquad \begin{array}{l} d_{\text{з}}^{-}(-\ -\ +\ -) \\ u_{\text{г}}^{+}(+\ +\ -\ -) \\ u_{\text{к}}^{+}\underline{(+\ -\ -\ +)} \\ p_2^{+}(+\ -\ -\ -)_{+} \end{array} \qquad (30) \qquad \begin{array}{l} d_{\text{г}}^{-}(-\ +\ -\ -) \\ u_{\text{к}}^{+}(+\ -\ -\ +) \\ u_{\text{з}}^{+}\underline{(+\ -\ +\ -)} \\ p_3^{+}(+\ -\ -\ -)_{+} \end{array} \qquad ((31)$$

In turn, "corpuscles" with an average zero signature, for example, ½ [(+ – – –) + (– + + +)] = (0 0 0 0) are, on average, flat (i.e., uncharged) $\lambda_{m,n}$-vacuum formations. Some of them can remain stable for a long time, but relatively easily decay and/or transform or merge with other similar "corpuscles" and "anticorpuscles". An example of a neutral "corpuscle" is a "neutron", which can assume the following signature (or topological) configurations (see (103) in [6]):

$$
\begin{array}{llll}
i_{\text{б}}^{-}\ (-\ -\ -\ -) & i_{\text{б}}^{-}\ (-\ -\ -\ -) & i_{\text{б}}^{-}\ (-\ -\ -\ -) & i_{\text{б}}^{-}\ (-\ -\ -\ -) \\
d_{\text{г}}^{+}\ (+\ -\ +\ +) & d_{\text{з}}^{+}\ (+\ +\ -\ +) & d_{\text{г}}^{+}\ (+\ -\ +\ +) & u_{\text{з}}^{-}\ (-\ +\ -\ +) \\
u_{\text{к}}^{-}\ (-\ +\ +\ -) & d_{\text{к}}^{+}\ (+\ +\ +\ -) & u_{\text{з}}^{-}\ (-\ +\ -\ +) & d_{\text{г}}^{+}\ (+\ -\ +\ +) \\
d_{\text{з}}^{+}\ \underline{(+\ +\ -\ +)} & u_{\text{г}}^{-}\ \underline{(-\ -\ +\ +)} & d_{\text{к}}^{+}\ \underline{(+\ +\ +\ -)} & d_{\text{к}}^{+}\ \underline{(+\ +\ +\ -)} \\
n_1^0\ (0\ 0\ 0\ 0)_+ & n_2^0\ (0\ 0\ 0\ 0)_+ & n_3^0\ (0\ 0\ 0\ 0)_+ & n_4^0\ (0\ 0\ 0\ 0)_+ \\
\\
i_{\text{б}}^{+}\ (+\ +\ +\ +) & i_{\text{б}}^{+}\ (+\ +\ +\ +) & i_{\text{б}}^{+}\ (+\ +\ +\ +) & i_{\text{б}}^{+}\ (+\ +\ +\ +) \\
d_{\text{г}}^{-}\ (-\ +\ -\ -) & d_{\text{з}}^{-}\ (-\ -\ +\ -) & d_{\text{г}}^{-}\ (-\ +\ -\ -) & u_{\text{з}}^{+}\ (+\ -\ +\ -) \\
u_{\text{к}}^{+}\ (+\ -\ -\ +) & d_{\text{к}}^{-}\ (-\ -\ -\ +) & u_{\text{з}}^{+}\ (+\ -\ +\ -) & d_{\text{г}}^{-}\ (-\ +\ -\ -) \\
d_{\text{з}}^{-}\ \underline{(-\ -\ +\ -)} & u_{\text{г}}^{+}\ \underline{(+\ +\ -\ -)} & d_{\text{к}}^{-}\ \underline{(-\ -\ -\ +)} & d_{\text{к}}^{-}\ \underline{(-\ -\ -\ +)} \\
n_5^0\ (0\ 0\ 0\ 0)_+ & n_6^0\ (0\ 0\ 0\ 0)_+ & n_7^0\ (0\ 0\ 0\ 0)_+ & n_8^0\ (0\ 0\ 0\ 0)_+
\end{array}
\tag{32}
$$

Recall that each signature in the rankings (26) – (32) corresponds to a colored "quark" or "antiquark" from Table 1 in [6], each of which is described by 10 metrics of the form (72) – (80) in [6] with the corresponding signature.

Unstable spheroidal "corpuscles" and "anticorpuscles" with the remaining signatures from matrix (1) are temporary convex-concave states of the $\lambda_{m,n}$-vacuum: inverted toroids, hyperboloids, paraboloids, etc. The relationship between the metric extension signature and its topology was considered in the works of Felix Klein and is discussed in §4 in [2].

Thus, the GVPh&AS (i.e., the modernized general theory of relativity, hereinafter M-GTR), with the addition of an infinite number of $\pm\Lambda_i$-terms and taking into account all 16 types of signatures from the matrix (1), is quantized. That is, M-GTR is a quantum theory that includes: scaling (or hierarchical) quantization of $\lambda_{m,n}$-vacuum formations (more precisely, discrete sorting of "corpuscles" and "anticorpuscles" by their sizes); and discrete distribution of each type of "corpuscles" and "anticorpuscles" by the criterion of their stable topological (nodal) configuration.

All these types of quantization of $\lambda_{m,n}$-vacuum formations (i.e., "corpuscles" and "anticorpuscles") were considered in previous articles of the GVPh&AS [5,6,7,10,12].

In this article, we will attempt to show that, within the framework of the GVPh&AS, not only scale and topological quantization of stable $\lambda_{m,n}$-vacuum formations can take place, but also quantization in the sense of stochastic quantum mechanics.

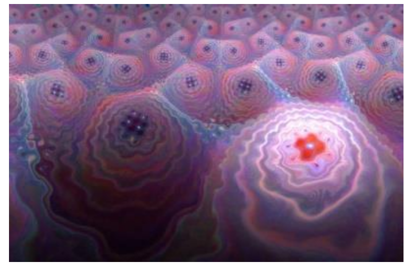
3]. Compromise between "order" and "chaos"

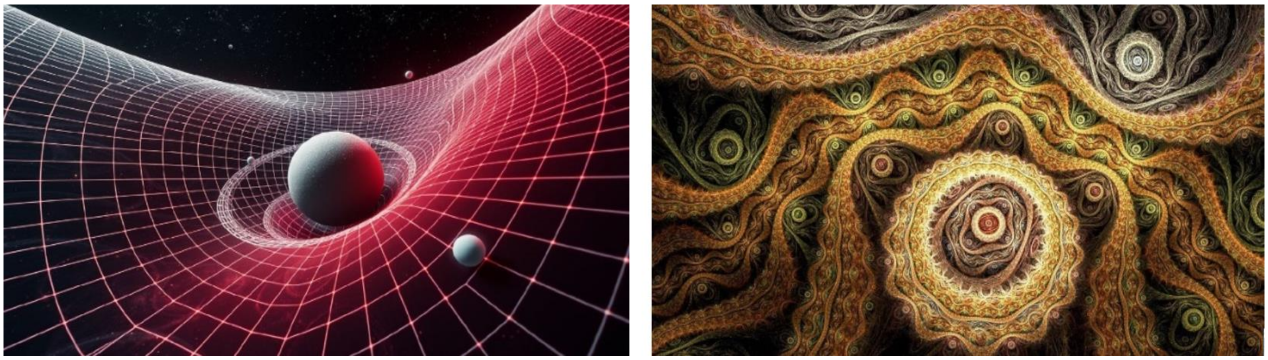
"Freedom is necessity recognized"
ברוך שפינוזה (Baruch Spinoza)

All phenomena in the surrounding world are the result of the desire for a compromise between "order" and "chaos." Order often emerges from chaos at the optimal zoom level, which in many cases is equivalent to averaging. However, with further zoom leveling, order can again dissolve into larger-scale chaos. Periodic (i.e., clearly discrete) alternation of order and chaos is another form of quantization.

In all previous articles of the GVPh&AS [1,2,3,4,5,6,7,8,9,10,11,12], we treated metric-dynamic models of stable $\lambda_{m,n}$-vacuum formations (i.e., "corpuscles" and "anticorpuscles") as spherical objects. In reality, all layers of the $\lambda_{m,n}$-vacuum oscillate chaotically and are bizarrely distorted throughout. Stable forms of "corpuscles" and "anticorpuscles" of any size are revealed from the general chaos by averaging the appropriate scale. For example, in order to identify an “electron” (15) or a “positron” from chaos, it is necessary to average the seething extent of the $\lambda_{m,n}$-vacuum on scales of 10^{-13} cm, and in order to identify the metric-dynamic structure of a bare “planet”, it is necessary to perform averaging over distances of the order of 100 km.

Let's recall that Einstein's third vacuum equation (5) follows from the principle of "extremal curvature of space" and, in essence, is an expression of 10 conservation laws and the conditions for the existence of, on average, stable convex and concave deformations of the $\lambda_{m,n}$-vacuum. However, Eq. (5) allows us to identify only the average framework of a trembling hierarchical chain, since it does not take into account random fluctuations in the extent of $\lambda_{m,n}$-vacuums and, consequently, it does not notice the chaotic movements of the cores of jelly-like "corpuscles" and the complex distortions of their shape (see Figure 5b).





a) b)

Fig. 5. a) Illustration of average distortions of the $\lambda_{m,n}$-vacuum around the averaged core of a "corpuscle"; b) Illustration of chaotic fluctuations in the shape of the "corpuscle" core and the $\lambda_{m,n}$-vacuum surrounding it.

At first glance, the cause of $\lambda_{m,n}$-vacuum fluctuations is simply the superposition of a colossal number of different simultaneous processes disturbing each local elastic-plastic region of each gelatinous layer of the $\lambda_{m,n}$-vacuum. However, not everything is so simple. As will be shown below, chaos has its own patterns, which are determined by the fundamental principle: "The tendency of any stochastic system simultaneously toward a minimum of action (i.e., toward Order) and a maximum of entropy (i.e., toward Chaos)."

Within the framework of the GVPh&AS, the problem associated with averaging the constant distortions of the expanse of a seething vacuum is solved by probing it with light beams (i.e., electromagnetic wave eikonals) with a wavelength $\lambda_{m,n}$ from the corresponding range $\Delta\lambda = 10^m \div 10^n$ cm (see § 1 in [1]). The eikonal diameter of an electromagnetic wave depends on its wavelength $\lambda_{m,n}$, so all vacuum fluctuations smaller than the eikonal diameter are averaged out. In other words, the 3D distortion landscape (i.e., the $\lambda_{m,n}$-vacuum), which is large-scale compared to the wavelength of the probing beam, is initially averaged (see Figure 5a).

Therefore, at this stage of the research, we will be primarily interested not in the random, small-scale metamorphoses of the ubiquitous, seething vacuum, but in the chaotic wanderings of the nuclei of "corpuscles" in a trembling, gelatinous (i.e., elastic-plastic) vacuum.

This article proposes a derivation of the Schrödinger and self-diffusion equations for a chaotically wandering particle based on an approach close to the axiomatics of E. Nelson's stochastic interpretation of quantum mechanics, but employing different principles, in particular the principle of the "extremum of average efficiency" of a stochastic system. Moreover, the new (massless) stochastic quantum mechanics proves applicable not only to describing the state of microscopic objects (e.g., electrons), but also to describing the average state of virtually all trembling and chaotically wandering "corpuscles" of any scale.

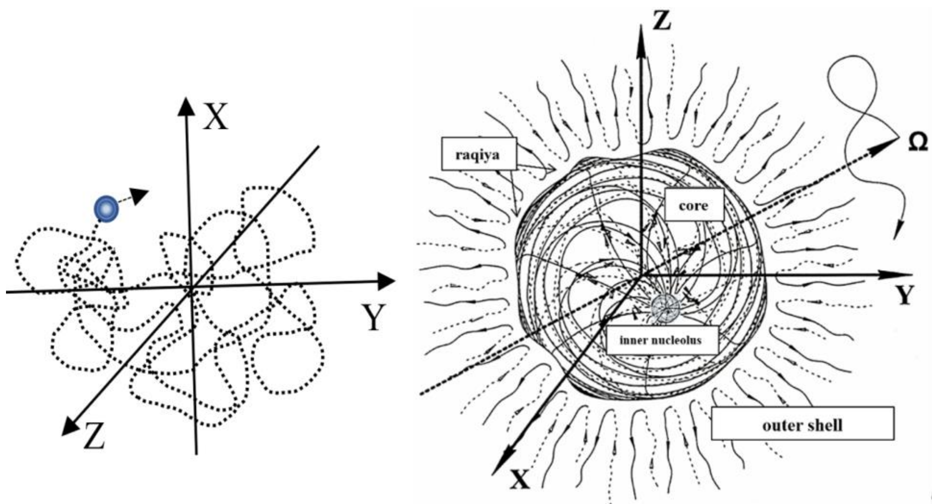
The purpose of this paper is to demonstrate that, within the framework of "Geometrized Vacuum Physics Based on the Algebra of Signature " (GVPh&AS), developed in a series of articles [1,2,3,4,5,6,7,8,9,10,11,12], there are no contradictions between the macro- and microscopic levels of the Universe's organization. The same laws operate at all scales of nature, determined by two fundamental principles: "least action" and "maximum entropy," combined into a single principle of "extremum of average efficiency" of any stochastic system.

MATERIALS AND METHOD

1. Model of a Chaotically Wandering Particle as a Stochastic System

The subject of further consideration is the chaotic motion of any local object. This could be the randomly wandering core of a spherical $\lambda_{m,n}$-vacuum formation ("corpuscle") of any scale, part of a hierarchical cosmological model (see Figure 3), for example, the core of an "electron," the core of a biological cell, the core of a naked "planet," or the core of a naked "galaxy," etc. It could also be a chaotically moving fish in an aquarium, a bird in a cage, a fluttering heart in an animal's chest; a fluttering yolk in a chicken egg; a fluttering moth near a burning lamp; a shifting swarm of mosquitoes; wandering pollen in diluted sugar syrup; a darting air bubble in a boiling liquid; a moving embryo in the mother's womb; a shifting school of fish in the ocean; a flower fluttering in the wind, etc.

The core of the “corpuscle”, which is small compared to the scale of consideration, or the geometric center of each of the above-mentioned local objects, can be conventionally represented as a material point with mass m, which wanders chaotically (i.e. moves along a complex trajectory) in the vicinity of the origin of the reference frame, under the influence of many random influences from the fluctuating environment (in particular, a trembling jelly-like vacuum) (see Figure 6a).



a) b)

Fig. 6. a) Simplified model of a chaotically wandering particle (ChWP) (i.e., a small core of a "corpuscle" or the geometric center of any randomly moving local object); b) Example of a chaotically wandering nucleolus within a larger core. This could be, for example, a wandering nucleus of an "electron" within an atom, or a vibrating organelle within a biological cell, or an iron core within a planet, etc.

We will call such a material point a chaotically wandering particle (ChWP) (this term was first introduced in [13]), and its average behavior in the studied region of space will be called a stochastic system.

Further, we will adhere to the author's article [13], taking into account the features of the hierarchical cosmological model presented in [1,2,3,4,5,6,7,8,9,10,11,12].

As is known, the total mechanical energy E of a particle at each point in space x,y,z and at each moment t is equal to the sum of its kinetic T and potential U energies

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$$E(x,y,z,t) = T(v,x,y,z,t) + U(x,y,z,t). \tag{33}$$

By averaging both sides of Eq. (33), we obtain the averaged total mechanical energy of a chaotically walking particle (ChWP)

$$< E\ (x,y,z,t) >=< T\ (v,x,y,z,t) > + < U\ (x,y,z,t) >, \quad (34)$$

where < > denotes averaging over the entire region of space occupied by the stochastic system (see Figure 6a).

$$< T\ (v,x,y,z,t) > = < T(p_x,p_y,p_z,x,y,z,t) > = < \frac{m[v_x^2(x,y,z,t)+v_y^2(x,y,z,t)+v_z^2(x,y,z,t)]}{2} > \quad (35)$$

is averaged kinetic energy of the CpWP, where $p_i\,(x,y,z,t) = mv_i\,(x,y,z,t)$ is the instantaneous value of the *i*-th component of the CpWP momentum vector, moving with instantaneous velocity $v_i\,(x,y,z,t)$, at time *t* at a point with coordinates *x*,*y*,*z* (see Figure 6a);

$< U(x,y,z,t) >$ is the average potential energy of the ChWP at a point with coordinates *x*,*y*,*z* at time t. This potential energy is due to the fact that the surrounding elastoplastic medium can, on average, elastically stress when the ChWP deviates from the center of the stochastic system. If the surrounding medium, on average, does not elastically stress when the ChWP moves, then its $< U\ (x,y,z,t) > = 0$.

Equation (33) can be represented as

$$< T\ (v,x,y,z,t) > + < U\ (x,y,z,t) > - < \mathrm{E}\ (x,y,z,t) >= 0. \quad (36)$$

We integrate Eq. (35) over time:

$$\int_{t_1}^{t_2} [< T(p_x,p_y,p_z,x,y,z,t) > + < U(x,y,z,t) > - < E(x,y,z,t) >]\, dt = 0. \quad (37)$$

We will call expression (37) the averaged equilibrium of the stochastic system [13].

To simplify the mathematical calculations, we consider a one-dimensional case. This means that we will initially consider only the change in the projection of the ChWP onto the *X*-axis with time *t* (see Figure 7*b*). This approach does not limit the generality of the conclusions, since in the case of three dimensions, only the number of integrations increases.

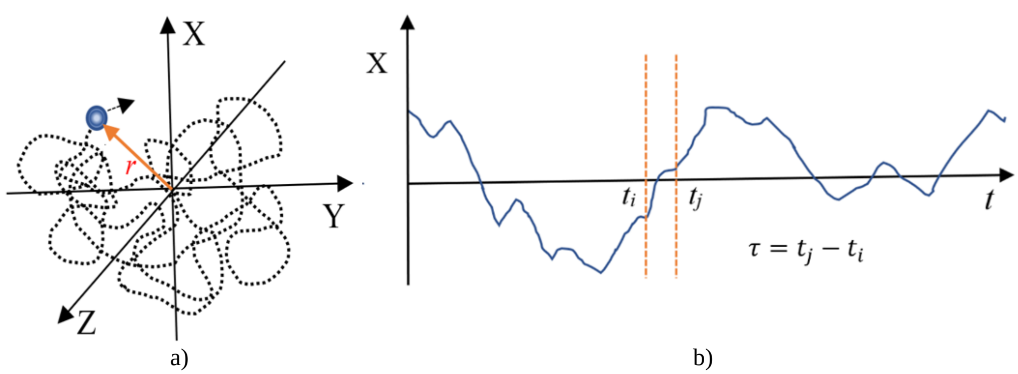


Fig. 7. a) Stochastic system; b) Random change over time *t*
of the projection of a chaotically wandering particle (ChWP) onto the *X*-axis

When considered one-dimensionally, the averaged equilibrium (37) takes on a simplified form.

$$< S_x(t) >= \int_{t_1}^{t_2} [< T(p_x,t) > +< U(x,t) > -< E(x,t) >]\, dt. \quad (38)$$

The average kinetic energy of the particle $< T(p_x,x,t) >$ can be expressed in terms of the probability density function (PDF) $\rho(p_x,t)$ of the *x*-component of the particle momentum p_x

$$< T(p_x,t) >= \frac{1}{2m}\int_{-\infty}^{\infty}\rho(p_x,t)p_x^2dp_x. \quad (39)$$

In Ex. (39), averaging occurs over all possible momenta p_x of the ChWP, regardless of the particle's location within the stochastic system under consideration. In the general case, the PDF $\rho(p_x,t)$ can change with time t.

We represent the average potential energy $< U(x,t) >$ and the average total mechanical energy $< E(x,t) >$ of the ChWP as

$$< U(x,t) >= \int_{-\infty}^{\infty}\rho(x,t)U(x,t)dx, \quad (40)$$

$$< E(x,t) >= \int_{-\infty}^{\infty}\rho(x,t)E(x,t)dx, \quad (41)$$

where $\rho(x,t)$ is the PDF of the ChWP projection onto the X-axis (see Figure 6). In general, $\rho(x,t)$ can change with time t.

Substituting the averaged values (39), (40), and (41) into expression (38), we obtain [13]

$$< S_x(t) >= \int_{t_1}^{t_2}\left\{\frac{1}{2m}\int_{-\infty}^{\infty}\rho(p_x,t)p_x^2dp_x + \int_{-\infty}^{\infty}\rho(x,t)[U(x,t) - E(x,t)]dx\right\}dt. \quad (42)$$

Note that if in the system under consideration the chaotic component of the particle's motion is very small (i.e., when the chaotic behavior of the particle can be neglected), then integral (42) takes the form

$$\overline{S_x}(t_1,t_2) = \int_{t_1}^{t_2}\bar{T}(p_x,x,t) + \bar{U}(x,t)dt\ - \bar{E}(x,t)\Big|_{t_1}^{t_2}, \quad (43)$$

where $\overline{S_x}$, $\bar{T}$, $\bar{U}$, $\bar{E}$ are averaged quantities.

This means returning to the Lagrangian formalism of classical mechanics to determine the average trajectory of a particle.

The next step is to eliminate the concept of "mass."

The entire "Geometrized Vacuum Physics Based on the Algebra of Signature " (GVPh&AS) project [1,2,3,4,5,6,7,8,9,10,11, 12] completely eliminates the concept of mass. The fact is that it is absolutely impossible to introduce the heuristic concept of "mass" with the dimension of kilogram into a completely geometrized theory (where all manifestations of being are the result of deformations, displacements, movements and torsions of various layers of vacuum). At the same time, the concept of mass is deeply imbued with the consciousness of modern scientists, and the dimension of kilogram is part of the dimension of many important physical quantities, such as momentum, energy, temperature, etc. It is absolutely obvious that for further progress in understanding the surrounding reality it is necessary to completely eradicate the concept of mass from science, since at the forefront of knowledge it becomes an insurmountable obstacle to development. Within the framework of GVPh&AS, without involving the concept of mass, the inert properties of stable $\lambda_{m,n}$-vacuum formations ("corpuscles") are easily explained due to the fact that when a "corpuscle" moves in a plane perpendicular to the direction of its motion, spiral-helical (vortex) vacuum currents (see [8]). Also, within the framework of the GVPh&AS, there is no need to invoke the concept of mass to explain the gravitational attraction of large and small "corpuscles" (see [10, 11]). In other words, the massless mathematical apparatus of the GVPh&AS is sufficient to explain and describe most phenomena and effects within the scope of modern physics and chemistry. In order for stochastic quantum mechanics to be fully consistent with the concepts of the GVPh&AS, it is necessary to extract the concept of mass and the dimension of the kilogram from it. In this article (below), we make a primitive attempt to get rid of "mass", but in fact, the theory should be built massless from the very foundation.

Let's eliminate the unnecessary concept of "mass" m of the ChWP. To do this, we must introduce new definitions of massless physical quantities:

$< s_x(t) > = \frac{< S_x(t) >}{m}$ is average x-"efficiency" of the stochastic system; (44)

$\varepsilon(x,t) = \frac{E(x,t)}{m}$ is average *x*-"mechanical energetics" of the ChWP; (45)
$u(x,t) = \frac{U(x,t)}{m}$ is average *x*-"potential energetics" of the ChWP; (46)
$< k(v_x, x, t) > = m < T(\frac{p_x}{m}, t) >= \frac{1}{2}\int_{-\infty}^{\infty} \rho(v_x, t) v_x^2 dv_x$ is averaged *x*-"kinetic energetics" of the ChWP. (47)

In terms of (44) – (47), Ex. (42) takes a massless form [13]

$$< s_x(t) > = \int_{t_1}^{t_2} \left\{ \frac{1}{2}\int_{-\infty}^{\infty} \rho(v_x, t) v_x^2 dv_x + \int_{-\infty}^{\infty} \rho(x,t)[u(x,t) - \varepsilon(x,t)]dx \right\} dt, \quad (48)$$

where $\rho(v_x, t)$ is the probability density function (PDF) of the *x*-component of the ChWP velocity v_x. The PDF $\rho(v_x, t)$ can generally change over time.

Next, to obtain the desired result, it is necessary to write the integral of the averaged *x*-"efficiency" of the stochastic system under consideration (48) in coordinate representation.

2 Coordinate Representation of the PDF $\boldsymbol{\rho(v_x, t)}$

A detailed discussion of the coordinate representation of the PDF $\rho(v_x, t)$ is presented in Appendix 1 of [13]. Here, we present the most important aspects of this mathematical procedure.

2.1 Deriving the velocity distribution law $\rho(v_x, t)$ of a random process x(t)

We consider the change in the projection of the ChWP onto the *X*-axis over time *t* as a random process *x*(*t*) (see Figure 7b), which is characterized by a two-dimensional PDF $\rho(x_i, t_i; x_j, t_j)$.

The two-dimensional PDF of any random process can be represented as [14,15]

$$\rho_2(x_i, t_i; x_j, t_j) = \rho(x_i, t_i)\rho(x_j, t_j / x_i, t_i), \quad (49)$$

where
$\rho(x_i, t_i)$ is the PDF of the random variable *x* at time t_i (see Figure 7*b*);
$\rho(x_j, t_j / x_i, t_i)$ is the conditional PDF.

Let's represent the PDF $\rho(x_j, t_j)$ as a product of two probability amplitudes

$$\rho(x_i, t_i) = \phi(x_i, t_i)\phi(x_i, t_i) = \phi^2(x_i, t_i). \quad (50)$$

As $\tau = t_i - t_j$ tends to zero (see Figure 7*b*), the conditional PDF $\rho(x_j, t_j / x_i, t_i)$ degenerates into a delta function [13]

$$\lim_{\tau \to 0} \rho(x_j, x_j / x_i, t_i) = \delta(x_j - x_i), \quad (51)$$

and the probability amplitudes in two very close time sections t_i and t_j of the random process *x*(*t*) are practically equal

$$\phi(x_i, t_i) \approx \phi(x_i, t_i). \quad (52)$$

Thus, under the condition $\tau = t_i - t_j \to 0$, taking into account Exs. (51) and (52), the two-dimensional PDF (49) can be represented as

$$\lim_{\tau \to 0} \rho(x_i, t_i; x_j, t_j) = \phi(x_i)\delta(x_j - x_i)\phi(x_j). \quad (53)$$

Integrating expression (53) with respect to x_i and x_j, we obtain

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi(x_i)\delta(x_j-x_i)\phi(x_j)dx_idx_j = 1. \quad (54)$$

We use one of the representations of the δ-function (see the theory of generalized functions):

$$\delta(x_j - x_i) = \frac{1}{2\pi}\int_{-\infty}^{\infty} exp\{\, iq(x_j - x_i)\}dq, \quad (55)$$

where q is the generalized frequency.

Substituting the δ-function (55) into Ex. (54), we obtain [13]

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi(x_i)\frac{1}{2\pi}\int_{-\infty}^{\infty} exp\{\, iq(x_j - x_i)\}dq\,\phi(x_j)dx_idx_j = 1. \quad (56)$$

Changing the order of integration in Ex. (56), and taking into account the approximate Eq. (52), we arrive at the following result:

$$\int_{-\infty}^{\infty} w(q)w^*(q)dq = 1, \quad (57)$$

where

$$w(q) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\phi(x)\, exp\{-iqx\}dx, \quad (58)$$

$$w^*(q) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\phi(x)\, exp\{\, iqx\}dx. \quad (59)$$

The integrand $w(q)w^*(q)$ in the normalization condition (57) meets all the requirements of the PDF $\rho(q)$ of the random variable q:

$$\rho(q) = w(q)w^*(q) = |w(q)|^2. \quad (60)$$

Let's now clarify the physical meaning of the generalized frequency q.

The characteristics of the random process under consideration impose the following restrictions on q:
1) q must be a random variable characterizing the random process in the interval $\tau = t_j - t_i$ under consideration, provided that $\tau \to 0$;
2) q must belong to the set of real numbers ($q \in R'$) with the cardinality of the continuum, i.e., q must be able to take any value from the range $]-\infty, \infty[$.

If the random process $x(t)$ under consideration is n times differentiable, then any of the following random variables satisfies the above requirements:

$$x' = \frac{\partial x(t)}{\partial t},\ x'' = \frac{\partial^2 x(t)}{\partial t^2}, \dots ,\ x^{(n)} = \frac{\partial^n x(t)}{\partial t^n}. \quad (61)$$

However, for $\tau = t_i - t_j \to 0$, these quantities characterize the random process $x(t)$ differently.

To demonstrate this, we expand the function $x(t)$ in the neighborhood of the cross-section t_i in a Taylor series.

$$x(t) = x(t_i) + x^{'(t_i)}(t - t_i) + \frac{x^{''(t_i)}}{2}(t - t_i)^2 + \dots + \frac{x^{(n)}(t_i)}{n!}(t - t_i)^n. \quad (62)$$

We write Ex, (62) as follows:

$$\frac{x(t_j)-x(t_i)}{(t_j-t_i)} = x^{'(t_i)} + \frac{x^{''(t_i)}}{2}\left(t_j - t_i\right) + \ldots + \frac{x^{(n)}(t_i)}{n!}\left(t_j - t_i\right)^{n-1}. \quad (63)$$

When $\tau = t_j - t_i \rightarrow 0$, only one term remains on the right-hand side of Ex. (63) [13]

$$\lim_{\tau\rightarrow 0} \frac{x(t_j)-x(t_i)}{\tau} = x^{'} = v_x. \quad (64)$$

Thus, the only random variable that satisfies all the above requirements on the time interval [$t_i = t_k - \tau/2$; $t_j = t_k + \tau/2$], as $\tau \rightarrow 0$, is the first derivative of the original random process $x'(t_k)$ in the moment of time $t_k = (t_j + t_i)/2$. Consequently, it remains to assume that the random variable q in Ex. (57) – (60) is linearly related only to $x^{'} = v_x$, i.e.

$$q = \frac{v_x}{\eta_x}, \quad (65)$$

where $1/\eta_x$ is the dimensional proportionality coefficient.

We present a second argument in favor of the fact that the generalized frequency q corresponds to the velocity v_x. Each exponential, for example, from integral (58) corresponds to a harmonic function with frequency q.

$$exp\{-iqx\} \rightarrow x_k(t) = A\,sin(\,qt), \quad (66)$$

this is one of the harmonic components of the random process $x(t)$.

The derivative of harmonic function (66) is equal to

$$v_{xk} = x'_k = qA\cos(qt). \quad (67)$$

from which it follows that [13]

$$q = \lim_{t\rightarrow 0} \frac{v_x}{A\,cos(qt)} = \frac{v_x}{A}. \quad (68)$$

For $A = \eta$, Exs. (65) and (68) coincide. Therefore, we are confident that each generalized frequency q is directly proportional to the specific velocity v_x of the ChWP.

Substituting Ex. (65) into integrals (58) and (59), we obtain the following desired procedure for obtaining the PDF $\rho(v_x,t)$ of the ChWP velocities given the known one-dimensional PDF $\rho(x,t)$ of the random process $x(t)$ under study [13]:

1] The given PDF $\rho(x,t)$ is represented as a product of two probability amplitudes $\varphi(x)$:

$$\rho(x,t) = \phi(x,t)\phi(x,t). \quad (69)$$

2] Two Fourier transforms are performed

$$w(v_x,t) = \frac{1}{\sqrt{2\pi\eta_x}}\int_{-\infty}^{\infty}\phi(x,t)\,exp\{-iv_x x/\eta_x\}dx, \quad (70)$$

$$w^*(v_x,t) = \frac{1}{\sqrt{2\pi\eta_x}}\int_{-\infty}^{\infty}\phi(x,t)\,exp\{\,iv_x x/\eta_x\}dx. \quad (71)$$

3] Finally, for an arbitrary cross-section t_k of a random process $x(t)$, we obtain the desired PDF of the derivative $x'(t_k)$ (i.e., the velocity $x'(t_k) = v_x$ with which this process can move from any position in cross-section t_i to any position in a nearby cross-section t_j, see Figure 7*b*)

$$\rho(v_x,t) = w(v_x,t)w^*(v_x,t) = |w(v_x,t)|^2. \tag{72}$$

The PDF $\rho(v_x,t)$ can generally change relatively slowly over time, along with the relatively slow change in the PDF $\rho(x,t)$ of the random process $x(t)$ under study (see Figure 7).

To clarify the physical meaning of η_x, we use a comparison with known results. This approach is not flawless from the point of view of mathematical rigor, but it allows us to obtain a practically important result.

In [14,15], a procedure is given for obtaining the PDF $\rho(x')$ of the derivative for a known two-dimensional PDF of a random process $x(t)$

$$\rho_2(x_i,x_j) = \rho_2(x_i,t_i;x_j,t_j). \tag{73}$$

To do this, a substitution of variables is performed in Ex. (73) [14,15]

$$x_i = x_k - \frac{\tau}{2}x_k'; \quad x_j = x_k + \frac{\tau}{2}x_k'; \quad t_i = t_k - \frac{\tau}{2}; \quad t_j = t_k + \frac{\tau}{2}, \tag{74}$$

where $\tau = t_j - t_i$; $t_k = \frac{t_j - t_i}{2}$, with the Jacobian of the transformation $[J] = \tau$.

As a result, from the PDF (73) we obtain

$$\rho_2(x_k,x_k') = \lim_{\tau\to 0} \tau\rho_2\left(x_k - \frac{\tau}{2}x_k', t_k - \frac{\tau}{2}; x_k + \frac{\tau}{2}x_k', t_k + \frac{\tau}{2}\right). \tag{75}$$

Next, integrating the resulting expression over x_k, we find the desired PDF of the derivative of the original random process $x(t)$ in any section t_k [14,15]:

$$\rho(x_k') = \int_{-\infty}^{\infty} \rho\,(x_k,x_k')dx_k \quad \text{or} \quad \rho(x') = \int_{-\infty}^{\infty} \rho\,(x,x')dx. \tag{76}$$

As an example, consider a Gaussian random process $x(t)$. In each section of this process, the random variable x is distributed according to the Gaussian law:

$$\rho(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} exp\{-(x-a_x)^2/2\sigma_x^2\}, \tag{77}$$

where $\sigma_x{}^2$ and a_x are the variance and mathematical expectation of the given process.

Performing the sequence of operations (69) – (72) with the PDF $\rho(x)$ (77), we obtain the PDF $\rho(x') = \rho(v_x)$ of the derivative (or velocity) of this random process:

$$\rho(v_x) = \frac{1}{\sqrt{2\pi[\eta_x/2\sigma_x]^2}} exp\left\{-\frac{v_x^2}{2[\eta_x/2\sigma_x]^2}\right\}. \tag{78}$$

On the other hand, using procedure (73) – (76) for a Gaussian random process with a bivariate normal distribution

$$\rho_2(x_i,x_j) = \frac{1}{2\pi\sigma_{xi}\sigma_{xj}\sqrt{1-\tau_{xcor}^2}}\ exp\left\{-\frac{1}{2(1-\tau_{xcor}^2)}\left[\frac{(x_i-a_{xi})^2}{\sigma_{xi}^2} - \tau_{xcor}\frac{2(x_i-a_{xi})(x_j-a_{xj})}{\sigma_{xi}\sigma_{xj}} - \frac{(x_j-a_{xj})^2}{\sigma_{xj}^2}\right]\right\}, \tag{79}$$

here τ_{xcor} is the autocorrelation coefficient (interval) of the original random process $x(t)$.

As a result, we obtain [14]

$$\rho(x') = \frac{1}{\sqrt{2\pi\sigma_{x'}^2}} exp\{-x'^2/2\sigma_{x'}^2\} = \frac{1}{\sqrt{2\pi\sigma_{v_x}^2}} exp\{-v_x^2/2\sigma_{v_x}^2\}, \quad (80)$$

where $\sigma_{x'} = \sigma_{v_x} = \sigma_x/\tau_{xcor}$.

Comparing the PDFs (78) and (80), we find that for

$$\eta_x = \frac{2\sigma_x^2}{\tau_{xcor}}. \quad (81)$$

They are completely identical [13].

Ex. (81) was obtained for a Gaussian random process, but the standard deviation σ_x and the autocorrelation interval τ_{xcor} are fundamental characteristics of any random process. All other initial and central moments in the case of a non-Gaussian distribution of the random variable x(t) will make an insignificant contribution to Ex. (81). Therefore, it can be stated with a high degree of certainty that Ex. (81) is applicable to a large class of random processes.

In quantum mechanics, the procedure used to transition from the coordinate representation of the wave function $\psi(x)$, which characterizes the state of an elementary particle, to its momentum representation is

$$\phi(p_x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi(x)\, exp\{\, ip_x x/\hbar\} dx = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi(x)\, exp\{\, imx'x/\hbar\} dx, \quad (82)$$

$$\phi^*(p_x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi(x)\, exp\{-ip_x x/\hbar\} dx = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi(x)\, exp\{-imx'x/\hbar\} dx, \quad (83)$$

where $\hbar = 1{,}055 \cdot 10^{-34}$ J/Hz is the reduced Planck constant;

Integrals (82) and (83) take into account that the x-component of the particle's momentum p_x is related to its velocity $v_x = x'$

$$p_x = mv_x = m\frac{dx}{dt} = mx.' \quad (84)$$

The quantum-mechanical procedure for transitioning to the momentum (or more precisely, velocity) representation (82) – (83) is completely identical to the stochastic procedure (70) – (71) for

$$\eta_x = \frac{2\sigma_x^2}{\tau_{xcor}} = \frac{\hbar}{m} \text{ with the dimension } (m^2/s) \text{ [13]}. \quad (85)$$

The difference between these procedures is as follows:

The quantum-mechanical procedure (82) – (83) is derived from Louis de Broglie's heuristic idea of the wave properties of matter and is applicable only when the ratio $\hbar/m$ has a significant value (i.e., only for particles with very small masses). It should be noted that de Broglie's mystical waves of matter are not observed in practice and predetermine the "madness" of the foundations of neopositivistic philosophy, which Einstein was never able to accept.

Whereas the stochastic procedure (70) – (71) is derived from a deep analysis of a differentiable random process [13]. Moreover, the decomposition of such a process into spectral components (i.e., essentially, into a velocity distribution) is associated with the law of the distribution of the heights of irregularities in a given random process. Furthermore, procedure (70) – (71) is applicable to differentiable random processes of any scale for which the scale parameter (85) $\eta_x = 2\sigma_x^2/\tau_{xcor}$ can be determined. An additional advantage of the stochastic approach is that it allows one to completely eliminate the heuristic mass of the particle m, which is, in principle, impossible to measure.

2.2 Coordinate representation of the averaged velocity of a chaotically walking particle (ChWP)

The mathematical calculations performed in this section are well known in quantum mechanics. In particular, they are partially presented in [16] and in Appendix 2 of [16]. However, we repeat these calculations here with some modifications, additions, and refinements as applied to massless stochastic quantum mechanics.

Let's obtain a coordinate representation of the average velocity of a *ChWP* raised to the *n*-th power

$$\overline{v_x^n}(t) = \int_{-\infty}^{+\infty} \rho\,(v_x, t) v_x^n dv_x. \tag{86}$$

Let's use Ex. (72) and write integral (86) in the following form:

$$\overline{v_x^n}(t) = \int_{-\infty}^{+\infty} w\,(v_x, t) v_x^n w^*(v_x, t) dv_x. \tag{87}$$

Substitute integrals (70) and (71) into Ex. (87)

$$\overline{v_x^n} = \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} \phi(x_i) \frac{\mathrm{e}^{-i\frac{v_x x_i}{\eta_x}}}{(2\pi\eta_x)^{\frac{1}{2}}} dx_i v_x^n \int_{-\infty}^{+\infty} \phi(x_j) \frac{\mathrm{e}^{i\frac{v_x x_j}{\eta_x}}}{(2\pi\eta_x)^{\frac{1}{2}}} dx_j \right] dv_x. \tag{88}$$

By direct verification it is easy to verify that [16]

$$v_x^n e^{i\frac{v_x x_j}{\eta_x}} = \left(-i\eta_x \frac{\partial}{\partial x_j}\right)^n e^{i\frac{v_x x_j}{\eta_x}}, \tag{89}$$

$$v_x^n e^{-i\frac{v_x x_i}{\eta_x}} = \left(i\eta_x \frac{\partial}{\partial x_i}\right)^n e^{-i\frac{v_x x_i}{\eta_x}}. \tag{90}$$

We rewrite Ex. (88) taking into account Ex. (89)

$$\overline{v_x^n} = \frac{1}{2\pi\eta_x} \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} \phi\,(x_i) e^{-i\frac{v_x x_i}{\eta_x}} dx_i \int_{-\infty}^{+\infty} \phi(x_j) \left(-i\eta_x \frac{\partial}{\partial x_j}\right)^n e^{i\frac{v_x x_j}{\eta_x}} dx_j \right] dv_x. \tag{91}$$

or taking into account (90)

$$\overline{v_x^n} = \frac{1}{2\pi\eta_x} \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} \phi\,(x_i) \left(i\eta_x \frac{\partial}{\partial x_i}\right)^n e^{-i\frac{v_x x_i}{\eta_x}} dx_i \int_{-\infty}^{+\infty} \phi(x_j) e^{i\frac{v_x x_j}{\eta_x}} dx_j \right] dv_x. \tag{92}$$

We integrate the second integral in the integrand (91) *n* times by parts, assuming that $\phi(x)$ and its derivatives vanish at the integration boundaries *x* = ± ∞. Performing these operations with expression (91), we obtain [16]

$$\overline{v_x^n} = \frac{1}{2\pi\eta_x} \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} \phi\,(x_i) e^{-i\frac{v_x x_i}{\eta_x}} dx_i \int_{-\infty}^{+\infty} e^{i\frac{v_x x_j}{\eta_x}} \left(-i\eta_x \frac{\partial}{\partial x_j}\right)^n \phi(x_j) dx_j \right] dv_x, \tag{93}$$

or

$$\overline{v_x^n} = \frac{1}{2\pi\eta_x} \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} dx_i \int_{-\infty}^{+\infty} \phi(x_i) e^{i\frac{v_x (x_j - x_i)}{\eta_x}} \left(-i\eta_x \frac{\partial}{\partial x_j}\right)^n \phi(x_j) dx_j \right] dv_x. \tag{94}$$

Similarly, we integrate the first integral in the integrand (92) *n* times by parts, assuming that $\phi(x)$ and its derivatives vanish at the integration boundaries *x* = ± ∞. Performing these actions with Ex. (92), we obtain [16]

$$\overline{v_x^n} = \frac{1}{2\pi\eta_x}\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} e^{-i\frac{v_x x_i}{\eta_x}}\left(i\eta_x\frac{\partial}{\partial x_i}\right)^n \phi(x_i)dx_i \int_{-\infty}^{+\infty} e^{i\frac{v_x x_j}{\eta_x}}\phi(x_j)dx_j\right]dv_x, \tag{95}$$

or

$$\overline{v_x^n} = \frac{1}{2\pi\eta_x}\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty}\phi\,(x_i)\left(i\eta_x\frac{\partial}{\partial x_i}\right)^n \phi(x_j)e^{i\frac{v_x(x_j-x_i)}{\eta_x}}dx_i \int_{-\infty}^{+\infty} dx_j\right]dv_x. \tag{96}$$

We change the order of integration in (94) and (96), i.e. First, we integrate over v_x [16]

$$\overline{v_x^n} = \int_{-\infty}^{+\infty} dx_i \int_{-\infty}^{+\infty} dx_j\phi(x_i)\left(-i\hbar\frac{\partial}{\partial x_j}\right)^n \phi(x_j)\frac{1}{2\pi\eta_x}\int_{-\infty}^{+\infty} e^{i\frac{v_x(x_j-x_i)}{\eta_x}}dv_x, \tag{97}$$

$$\overline{v_x^n} = \int_{-\infty}^{+\infty} dx_j \int_{-\infty}^{+\infty} dx_i\phi(x_j)\left(i\hbar\frac{\partial}{\partial x_i}\right)^n \phi(x_i)\frac{1}{2\pi\eta_x}\int_{-\infty}^{+\infty} e^{i\frac{v_x(x_j-x_i)}{\eta_x}}dv_x. \tag{98}$$

These expressions contain a delta function of the type (55) $\delta(x_j - x_i) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{iq(x_j-x_i)}\,dq$:

$$\delta(x_j - x_i) = \frac{1}{2\pi\eta_x}\int_{-\infty}^{+\infty} e^{i\frac{v_x(x_j-x_i)}{\eta_x}}dv_x. \tag{99}$$

Therefore, we represent Exs. (97) and (98) in the following form [16]

$$\overline{v_x^n} = \int_{-\infty}^{+\infty} dx_i \int_{-\infty}^{+\infty}\phi(x_i)\left(-i\eta_x\frac{\partial}{\partial x_j}\right)^n \phi(x_j)\,\delta(x_j - x_i)dx_j, \tag{100}$$

$$\overline{v_x^n} = \int_{-\infty}^{+\infty} dx_i \int_{-\infty}^{+\infty}\phi(x_i)\left(i\eta_x\frac{\partial}{\partial x_j}\right)^n \phi(x_j)\,\delta(x_j - x_i)dx_j. \tag{101}$$

In classical quantum mechanics, only Ex. (100) with $-i\eta_x$, is derived and used, while the possibility of Ex. (101) with $+i\eta_x$, is ignored. In massless stochastic quantum mechanics, both possibilities are taken into account. It is precisely to justify the application of the two cases of $\mp i\eta_x$ that the derivation of these expressions is repeated here.

Using the properties of the δ-function, we finally write for any cross-section t of the random process under study:

$$\overline{v_x^n} = \int_{-\infty}^{+\infty}\phi(x,t)\left(\mp i\eta_x\frac{\partial}{\partial x}\right)^n \phi(x,t)dx. \tag{102}$$

In particular, for n = 2

$$\overline{v_x^2} = \int_{-\infty}^{+\infty}\phi(x,t)\left(\mp i\eta_x\frac{\partial}{\partial x}\right)^2 \phi(x,t)dx, \tag{103}$$

where $\eta_x = \frac{2\sigma_x^2}{\tau_{xacor}} = constant.$ (104)

It should be noted that in the most general case, the scale parameter η_x can depend on time (i.e. $\eta_x(t)$). However, in many cases, if the variance $\sigma_x^2(t)$ of a random process changes over time, then its autocorrelation coefficient $\tau_{xcor}(t)$ also changes in a similar way, so that

$$\frac{2\sigma_x^2(t)}{\tau_{xacor}(t)} \approx \eta_x = constant. \tag{105}$$

For example, it is possible that the variance of a random process changes over time according to the law $\sigma_x^2(t) = \sigma_x^2 \times (t - t_0)$, while its autocorrelation coefficient changes according to the same law $\tau_{xacor}(t) = \tau_{xacor} \times (t - t_0)$, then

$$\eta_x(t) = \frac{2\sigma_x^2(t)}{\tau_{xacor}(t)} \approx \frac{2\sigma_x^2 \times (t-t_0)}{\tau_{xacor} \times (t-t_0)} \approx \eta_x = \frac{2\sigma_x^2}{\tau_{xacor}} = const. \quad (106)$$

We will call the constant ratio of the main averaged characteristics of the random process under study the "rule of constancy of the scale parameter of a stochastic system."

This rule is more general in nature, since the larger the object under consideration, the greater its inertia and, consequently, the more difficult and slower it is to change its speed and direction of motion. Accordingly, a random process involving a large object has a larger variance and autocorrelation coefficient.

Applied to the hierarchical cosmological model, the rule of constancy of the scale parameter η_x manifests itself in the fact that as the radius of the core of a stable spherical $\lambda_{m,n}$-vacuum formation (i.e., a "corpuscle") increases, its inertial properties increase, and, accordingly, so do the variance σ_x^2 and the autocorrelation coefficient τ_{xacor} of the random process in which the core of this "corpuscle" continuously participates.

3 Coordinate representation of the change in the average mechanical energy of the ChWP

Appendix 2 in [13] shows that the slowly changing average mechanical energetics of the ChWP $\varepsilon(x,t)$ can be represented in the coordinate representation

$$\overline{\varepsilon(x,t)} = \int_{-\infty}^{\infty} \rho(x,t)\varepsilon(x,t)dx = \int_{-\infty}^{\infty} \rho(x,t)\varepsilon(x,t_0)dx \mp i\frac{\eta_x^2}{D}\int_{-\infty}^{+\infty} \phi(x,t)\frac{\partial\phi(x,t)}{\partial t}dx, \quad (107)$$

where $\int_{-\infty}^{\infty} \rho(x,t)\varepsilon(x,t_0)dx$ is the initial value of the averaged mechanical energetics of the ChWP at time t_0; D is the self-diffusion coefficient of a ChWP, whose averaged energetics gradually changes (i.e., decreases or increases).

When $D = \eta_x$, the stochastic system under consideration is self-consistent, and Ex. (107) takes on a simplified form.

4 Coordinate representation of the averaged *x*-"efficiency" integral

Let's return to the consideration of the averaged *x*-"efficiency" integral (48)

$$< s_x(t) > = \int_{t_1}^{t_2} \left\{\frac{1}{2}\int_{-\infty}^{\infty} \rho(v_x,t)v_x^2 dv_x + \int_{-\infty}^{\infty} \rho(x,t)u(x,t)dx - \int_{-\infty}^{\infty} \rho(x,t)\varepsilon(x,t)dx\right\} dt. \quad (109)$$

We use Exs. (69), (103) and (108):

$$\rho(x,t) = \phi(x,t)\phi(x,t) = \phi^2(x,t), \quad (110)$$

$$\overline{v_x^2(t)} = \int_{-\infty}^{\infty} \rho(v_x,t)v_x^2 dv_x = \int_{-\infty}^{+\infty} \phi(x,t)\left(\pm i\eta_x\frac{\partial}{\partial x}\right)^2 \phi(x,t)dx = \mp\eta_x^2\int_{-\infty}^{\infty} \psi(x,t)\frac{\partial^2\psi(x,t)}{\partial x^2}dx, \quad (111)$$

$$\overline{\varepsilon_x(t)} = \int_{-\infty}^{\infty} \rho(x,t)\varepsilon(x,t)dx = \int_{-\infty}^{\infty} \phi^2(x,t)\varepsilon(x,t_0)dx \pm i\eta_x\int_{-\infty}^{+\infty} \phi(x,t)\frac{\partial\phi(x,t)}{\partial t}dx \quad (112)$$

to represent integral (48´) (or (109)) in coordinate representation

$$< s_x(t) > = \int_{t_1}^{t_2}\int_{-\infty}^{\infty} \left(\mp\frac{\eta_x^2}{2}\phi(x,t)\frac{\partial^2\phi(x)}{\partial x^2} + \phi^2(x,t)[u(x,t) - \varepsilon(x,t_0)] \mp i\frac{\eta_x^2}{D}\phi(x,t)\frac{\partial\phi(x,t)}{\partial t}\right)dxdt. \quad (113)$$

Thus, we have obtained the one-dimensional integral of the averaged *x*-"efficiency" in its most general form.

5 Coordinate representation of the averaged *r*-“efficiency” integral

In the case of a 3-dimensional consideration of the behavior of the ChWP (see Figure 7a), the averaged “efficiency” integral (113) has the form [13]

$$< s_r(t) >= \int_{t_1}^{t_2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\mp\frac{\eta_r^2}{2}\phi(\vec{r},t)\nabla^2\phi(\vec{r},t)+[u(\vec{r},t)-\varepsilon(\vec{r},t_0)]\phi^2(\vec{r},t)\ \mp i\frac{\eta_r^2}{D}\phi(\vec{r},t)\frac{\partial\phi(\vec{r},t)}{\partial t}\right)dxdydzdt, \qquad (114)$$

where $\vec{r} \coloneqq (x,y,z), \quad \phi(\vec{r},t)=\phi(x,y,z,t)$,

$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is the Laplace operator; (115)

$$\eta_r=\frac{2\sigma_r^2}{\tau_{racor}}=constant \qquad (116)$$

is constant scaling parameter, here for the three-dimensional case:

$$\sigma_r=\sqrt{\sigma_x^2+\sigma_y^2+\sigma_z^2} \qquad (117)$$

is standard deviation of the random 3-dimensional trajectory of the ChWP from the conventional center of the stochastic system under consideration (see Figures 6 and 7);

$$\tau_{racor}=\frac{1}{3}\left(\tau_{xacor}+\tau_{yacor}+\tau_{zacor}\right) \qquad (118)$$

is autocorrelation coefficient (or interval) of the 3-dimensional random process in which the ChWP participates.

In what follows, we will consider the integral of the averaged r-“efficiency” (114) as a functional (i.e., a function of the function $\phi(\vec{r},t)=\phi(x,y,z,t)$), which most fully reflects: on the one hand, the energetics balance of the stochastic system under consideration; on the other hand, the optimization of the relationship between “order” (predetermination) and “chaos” (unpredictability) in the averaged behavior of a chaotically wandering particle ().

The extremal $\phi(\vec{r},t)_{ext}$ of functional (114) (more precisely, the PDF $\rho(\vec{r},t)_{ext}=|\ \phi(\vec{r},t)_{ext}|^2$) determines the state of a stochastic system with maximum entropy (i.e., with the greatest possible chaos), under conditions specified by optimal energetics parameters. In other words, the extremal $\phi(\vec{r},t)_{ext}$ of functional (114) describes the average state of the stochastic system, which it tends to, but never exactly achieves.

6 Derivation of the stationary stochastic equation of the ChWP

Let’s consider the stationary case when the averaged behavior of a chaotically walking particle (ChWP) is independent of time *t*. In this case, the integral of the averaged *r*-"efficiency" takes the form [13]

$$< s_r(t) >= \int_{t_1}^{t_2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\mp\frac{\eta_r^2}{2}\phi(\vec{r})\nabla^2\phi(\vec{r})+[u(\vec{r})-\varepsilon(\vec{r})]\phi^2(\vec{r})\right)dxdydzdt. \qquad (119)$$

The integrand in (119) is time-independent, so we are only interested in the functional

$$w=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\mp\frac{\eta_r^2}{2}\phi(x,y,z)\nabla^2\phi(x,y,z)+[u(x,y,z)-\varepsilon(x,y,z)]\phi^2(x,y,z)\right)dxdydz. \qquad (120)$$

We will look for the extremal $\phi(\vec{r})_{ext}=\phi(x,y,z)_{ext}$ of functional (120).

In the calculus of variations, a functional of general form is considered [17,18,19]

$$I[f] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} L(x_1, x_2, x_3, f, f_1, f_2, f_3, f_{11}, f_{22}, f_{33})dx_1dx_2dx_3, \quad (121)$$

where $f_i := \frac{\partial f}{\partial x_i}$, $f_{ii} := \frac{\partial^2 f}{\partial x_i \partial x_i} = \frac{\partial^2 f}{\partial x_i^2}$ (i =1,2,3). (122)

The extremal $f\,(x_1, x_2, x_3)$ of this functional is determined by the Euler-Poisson-Elsgolts equation [17, pp. 310 – 316, Elsgolts_LE_VI]

$$\frac{\partial L}{\partial f} - \frac{\partial}{\partial x_1}\left\{\frac{\partial L}{\partial f_1}\right\} - \frac{\partial}{\partial x_2}\left\{\frac{\partial L}{\partial f_2}\right\} - \frac{\partial}{\partial x_3}\left\{\frac{\partial L}{\partial f_3}\right\} + \frac{\partial^2}{\partial x_1^2}\left\{\frac{\partial L}{\partial f_{11}}\right\} + \frac{\partial^2}{\partial x_2^2}\left\{\frac{\partial L}{\partial f_{22}}\right\} + \frac{\partial^2}{\partial x_3^2}\left\{\frac{\partial L}{\partial f_{33}}\right\} = 0, \quad (123)$$

where

$$\frac{\partial}{\partial x_i}\left\{\frac{\partial L}{\partial f_i}\right\} = \frac{\partial}{\partial x_i}\left(\frac{\partial L}{\partial f_i}\right) + \frac{\partial}{\partial f}\left(\frac{\partial L}{\partial f_i}\right)\frac{\partial f}{\partial x_i} + \frac{\partial}{\partial f_1}\left(\frac{\partial L}{\partial f_i}\right)\frac{\partial f_1}{\partial x_i} + \frac{\partial}{\partial f_2}\left(\frac{\partial L}{\partial f_i}\right)\frac{\partial f_2}{\partial x_i} + \frac{\partial}{\partial f_3}\left(\frac{\partial L}{\partial f_i}\right)\frac{\partial f_3}{\partial x_i} + \frac{\partial}{\partial f_{11}}\left(\frac{\partial L}{\partial f_i}\right)\frac{\partial f_{11}}{\partial x_i} + \frac{\partial}{\partial f_{22}}\left(\frac{\partial L}{\partial f_i}\right)\frac{\partial f_{22}}{\partial x_1} + $$
$$+ \frac{\partial}{\partial f_{33}}\left(\frac{\partial L}{\partial f_i}\right)\frac{\partial f_{33}}{\partial x_i} \quad (124)$$

is the first total partial derivatives with respect to x_i (i =1,2,3);

$$\frac{\partial^2}{\partial x_i^2}\left\{\frac{\partial L}{\partial f_{ii}}\right\} = \frac{\partial^2}{\partial x_i^2}\left(\frac{\partial L}{\partial f_{ii}}\right) + \frac{\partial}{\partial f}\left(\frac{\partial L}{\partial f_{ii}}\right)\frac{\partial^2 f}{\partial x_i^2} + \frac{\partial}{\partial f_1}\left(\frac{\partial L}{\partial f_{ii}}\right)\frac{\partial^2 f_1}{\partial x_i^2} + \frac{\partial}{\partial f_2}\left(\frac{\partial L}{\partial f_{ii}}\right)\frac{\partial^2 f_2}{\partial x_i^2} + \frac{\partial}{\partial f_3}\left(\frac{\partial L}{\partial f_{ii}}\right)\frac{\partial^2 f_3}{\partial x_i^2} + \frac{\partial}{\partial f_{11}}\left(\frac{\partial L}{\partial f_{ii}}\right)\frac{\partial^2 f_{11}}{\partial x_i^2} + \frac{\partial}{\partial f_{22}}\left(\frac{\partial L}{\partial f_{ii}}\right)\frac{\partial^2 f_{22}}{\partial x_i^2} +$$
$$\frac{\partial}{\partial f_{33}}\left(\frac{\partial L}{\partial f_{ii}}\right)\frac{\partial^2 f_{33}}{\partial x_i^2} \quad (125)$$

is the second total partial derivatives with respect to x_i (i =1,2,3).

In the case of functional (120), we have the Lagrangian:

$$L = \mp\frac{\eta_r^2}{2}\phi(x,y,z)\nabla^2\phi(x,y,z) + [u(x,y,z) - \varepsilon(x,y,z)]\phi^2(x,y,z), \quad (126)$$

at the same time $x_1 = x$, $x_2 = y$, $x_3 = z$; $f(x_1, x_2, x_3) = \phi(x,y,z)$,

$$\phi_x := \frac{\partial\phi}{\partial x}, \quad \phi_y := \frac{\partial\phi}{\partial y}, \quad \phi_z := \frac{\partial\phi}{\partial z}, \quad \phi_{xx} := \frac{\partial^2\phi}{\partial x^2}, \quad \phi_{yy} := \frac{\partial^2\phi}{\partial y^2}, \quad \phi_{zz} := \frac{\partial^2\phi}{\partial z^2}. \quad (127)$$

Substituting Lagrangian (126) into the Euler-Poisson-Elsholtz equation (123), we obtain the equation

$$\pm\frac{3\eta_r^2}{4}\nabla^2\phi(\vec{r}) + [\varepsilon(\vec{r}) - u(\vec{r})]\phi(\vec{r}) = 0 \quad (128)$$

to determine the extremal $\phi(\vec{r}) = \phi(x,y,z)_{ext}$ (see §2.4 in [13]).

Ex. (128) will be called the stationary stochastic equation of the ChWP.

7 Stationary massless stochastic Schrödinger equation

Consider the case where the mechanical energetics of the ChWP $\varepsilon(\vec{r})$ is constant at all points of the stochastic system under study

$$\varepsilon(\vec{r}) = \varepsilon = constant. \quad (129)$$

This is possible if the kinetic energetics of the ChWP $k(x,y,z,t)$ and its potential energetics $u(x,y,z,t)$ are constantly transformed into each other so that their sum at each point $\vec{r} \coloneqq (x, y, z)$ always remains constant.

$$k(\vec{r},t) + u(\vec{r},t) = \varepsilon = constant. \tag{130}$$

That is $k(\vec{r},t)$ and $u(\vec{r},t)$ are random functions of location and time, but their sum is always equal to the same value ε.

In this case, Eq. (128) takes the form

$$\pm \frac{3\eta_r^2}{4} \nabla^2 \phi(\vec{r}) + u(\vec{r})\phi(\vec{r}) = \varepsilon\phi(\vec{r}), \tag{131}$$

or in expanded form

$$- \frac{3\eta_r^2}{4} \nabla^2 \phi(\vec{r}) + u(\vec{r})\phi(\vec{r}) = \varepsilon\psi(\vec{r}), \tag{132}$$

and

$$+ \frac{3\eta_r^2}{4} \nabla^2 \phi(\vec{r}) + u(\vec{r})\phi(\vec{r}) = \varepsilon\psi(\vec{r}). \tag{133}$$

This suggests that for the stochastic system under consideration, we must take into account the superposition of two possible states described by Eqs. (132) – (133).

As is well known, the stationary (i.e., time-independent) Schrödinger equation, which underlies quantum mechanics, has the form

$$- \frac{\hbar^2}{2m} \nabla^2 \psi(\vec{r}) + U(\vec{r})\psi(\vec{r}) = E\psi(\vec{r}). \tag{134}$$

We divide all terms of this equation by the particle mass m:

$$- \frac{\hbar^2}{2m^2} \nabla^2 \psi(\vec{r}) + \frac{U(\vec{r})}{m}\psi(\vec{r}) = \frac{E}{m}\psi(\vec{r}). \tag{135}$$

As a result, taking into account Exs. (45) and (46), the stationary Schrödinger equation takes the form

$$- \frac{1}{2}\left(\frac{\hbar}{m}\right)^2 \nabla^2 \psi(\vec{r}) + u(\vec{r})\psi(\vec{r}) = \varepsilon\psi(\vec{r}). \tag{136}$$

Comparing Eqs. (132) and (136), we find that for

$$\frac{\hbar}{m} = \sqrt{\frac{3}{2}}\eta_r \approx \frac{2{,}45\ \sigma_r^2}{\tau_{rcor}} \quad \text{and} \quad \phi(\vec{r}) = \psi(\vec{r}), \tag{137}$$

these equations are completely identical. Therefore, Exs. (131) will be called the stochastic massless stationary Schrödinger equation.

8 Derivation of the stochastic massless time-dependent equation of the ChWP

We return to the consideration of the integral of the averaged r-"efficiency" of the ChWP (114)

$$< s_r(t) >= \int_{t_1}^{t_2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\mp \frac{\eta_r^2}{2} \phi(\vec{r},t)\nabla^2\phi(\vec{r},t) + [u(\vec{r},t) - \varepsilon(\vec{r},t_0)]\phi^2(\vec{r},t) \ \mp i\frac{\eta_r^2}{D}\phi(\vec{r},t)\frac{\partial\phi(\vec{r},t)}{\partial t} \right) dxdydzdt. \tag{114′}$$

To find the extremals $\phi(\vec{r},t)$ of a given functional, we first recall the condition for extremality of a functional of general form

$$I = \int_{x_{01}}^{x_{02}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} L(x_0, x_1, x_2, x_3, f, f_0, f_1, f_2, f_3, f_{00}, f_{11}, f_{22}, f_{33})dx_0 dx_1 dx_2 dx_3, \quad (138)$$

where $f_i := \frac{\partial f}{\partial x_i}$, $f_{ii} := \frac{\partial^2 f}{\partial x_i \partial x_i} = \frac{\partial^2 f}{\partial x_i^2}$ (i = 0,1,2,3)

is determined by the Euler-Poisson-Elsgolts equation [11, pp. 310 – 316, Elsgolts_LE_VI]

$$\frac{\partial L}{\partial f} - \frac{\partial}{\partial x_0}\left\{\frac{\partial L}{\partial f_0}\right\} - \frac{\partial}{\partial x_1}\left\{\frac{\partial L}{\partial f_1}\right\} - \frac{\partial}{\partial x_2}\left\{\frac{\partial L}{\partial f_2}\right\} - \frac{\partial}{\partial x_3}\left\{\frac{\partial L}{\partial f_3}\right\} + \frac{\partial^2}{\partial x_0^2}\left\{\frac{\partial L}{\partial f_{00}}\right\} + \frac{\partial^2}{\partial x_1^2}\left\{\frac{\partial L}{\partial f_{11}}\right\} + \frac{\partial^2}{\partial x_2^2}\left\{\frac{\partial L}{\partial f_{22}}\right\} + \frac{\partial^2}{\partial x_3^2}\left\{\frac{\partial L}{\partial f_{33}}\right\} = 0, \quad (139)$$

where

$$\frac{\partial}{\partial x_i}\left\{\frac{\partial L}{\partial f_i}\right\} = \frac{\partial}{\partial x_i}\left(\frac{\partial L}{\partial f_i}\right) + \frac{\partial}{\partial f}\left(\frac{\partial L}{\partial f_i}\right)\frac{\partial f}{\partial x_i} + \frac{\partial}{\partial f_0}\left(\frac{\partial L}{\partial f_i}\right)\frac{\partial f_0}{\partial x_i} + \frac{\partial}{\partial f_1}\left(\frac{\partial L}{\partial f_i}\right)\frac{\partial f_1}{\partial x_i} + \frac{\partial}{\partial f_2}\left(\frac{\partial L}{\partial f_i}\right)\frac{\partial f_2}{\partial x_i} + + \frac{\partial}{\partial f_3}\left(\frac{\partial L}{\partial f_i}\right)\frac{\partial f_3}{\partial x_i} + \frac{\partial}{\partial f_{00}}\left(\frac{\partial L}{\partial f_i}\right)\frac{\partial f_{00}}{\partial x_i} + \frac{\partial}{\partial f_{11}}\left(\frac{\partial L}{\partial f_i}\right)\frac{\partial f_{11}}{\partial x_i} + \frac{\partial}{\partial f_{22}}\left(\frac{\partial L}{\partial f_i}\right)\frac{\partial f_{22}}{\partial x_1} + \frac{\partial}{\partial f_{33}}\left(\frac{\partial L}{\partial f_i}\right)\frac{\partial f_{33}}{\partial x_i} \quad (140)$$

is the first total partial derivatives with respect to x_i (i = 0,1,2,3).

$$\frac{\partial^2}{\partial x_i^2}\left\{\frac{\partial L}{\partial f_{ii}}\right\} = \frac{\partial^2}{\partial x_i^2}\left(\frac{\partial L}{\partial f_{ii}}\right) + \frac{\partial}{\partial f}\left(\frac{\partial L}{\partial f_{ii}}\right)\frac{\partial^2 f}{\partial x_i^2} + \frac{\partial}{\partial f_0}\left(\frac{\partial L}{\partial f_{ii}}\right)\frac{\partial^2 f_0}{\partial x_i^2} + \frac{\partial}{\partial f_1}\left(\frac{\partial L}{\partial f_{ii}}\right)\frac{\partial^2 f_1}{\partial x_i^2} + \frac{\partial}{\partial f_2}\left(\frac{\partial L}{\partial f_{ii}}\right)\frac{\partial^2 f_2}{\partial x_i^2} + \frac{\partial}{\partial f_3}\left(\frac{\partial L}{\partial f_{ii}}\right)\frac{\partial^2 f_3}{\partial x_i^2} + \frac{\partial}{\partial f_{00}}\left(\frac{\partial L}{\partial f_{ii}}\right)\frac{\partial^2 f_{00}}{\partial x_i^2} + \frac{\partial}{\partial f_{11}}\left(\frac{\partial L}{\partial f_{ii}}\right)\frac{\partial^2 f_{11}}{\partial x_i^2} + \frac{\partial}{\partial f_{22}}\left(\frac{\partial L}{\partial f_{ii}}\right)\frac{\partial^2 f_{22}}{\partial x_i^2} + \frac{\partial}{\partial f_{33}}\left(\frac{\partial L}{\partial f_{ii}}\right)\frac{\partial^2 f_{33}}{\partial x_i^2} \quad (141)$$

is the second total partial derivatives with respect to x_i (i = 0,1,2,3).

In the case of functional (114) we have the Lagrangian:

$$L = \mp \frac{\eta_r^2}{2}\phi(x,y,z,t)\left(\frac{\partial^2\phi(x,y,z,t)}{\partial x^2} + \frac{\partial^2\phi(x,y,z,t)}{\partial y^2} + \frac{\partial^2\phi(x,y,z,t)}{\partial z^2}\right) + [u(x,y,z,t) - \varepsilon(x,y,z,t_0)]\phi^2(x,y,z,t) \mp i\frac{\eta_r^2}{D}\phi(x,y,z,t)\frac{\partial\phi(x,y,z,t)}{\partial t} \quad (142)$$

at the same time $x_1 = t$, $x_1 = x$, $x_2 = y$, $x_3 = z$; $f(x_0, x_1, x_2, x_3) = \phi(x,y,z,t)$,

$\phi_t := \frac{\partial\psi}{\partial t}$, $\phi_x := \frac{\partial\psi}{\partial x}$, $\phi_y := \frac{\partial\psi}{\partial y}$, $\phi_z := \frac{\partial\psi}{\partial z}$,
$\phi_{tt} := \frac{\partial^2\psi}{\partial t^2}$, $\phi_{xx} := \frac{\partial^2\psi}{\partial x^2}$, $\phi_{yy} := \frac{\partial^2\psi}{\partial y^2}$, $\phi_{zz} := \frac{\partial^2\psi}{\partial z^2}$.

By substituting the Lagrangian (142) into the Euler-Poisson-Elsholtz equation (139), we obtain the desired stochastic time-dependent equation for determining the extremals $\phi(\vec{r},t) = \phi(x,y,z,t)_{ext}$ of the averaged *r*-"efficiency" functional of the ChWP (114) (see §2.6 in [13])

$$\pm i\frac{\eta_r^2}{D}\frac{\partial\phi(\vec{r},t)}{\partial t} = \mp\frac{3\eta_r^2}{2}\nabla^2\phi(\vec{r},t) + 2[u(\vec{r},t) - \varepsilon(\vec{r},t_0)]\phi(\vec{r},t). \quad (143)$$

The stochastic equation (143) in expanded form consists of four equations

I $\quad -i\frac{\eta_r^2}{D}\frac{\partial\phi(\vec{r},t)}{\partial t} = +\frac{3\eta_r^2}{2}\nabla^2\phi(\vec{r},t) + 2[u(\vec{r},t) - \varepsilon(\vec{r},t_0)]\phi(\vec{r},t),$ (144)

H $\quad +i\frac{\eta_r^2}{D}\frac{\partial\phi(\vec{r},t)}{\partial t} = +\frac{3\eta_r^2}{2}\nabla^2\phi(\vec{r},t) + 2[u(\vec{r},t) - \varepsilon(\vec{r},t_0)]\phi(\vec{r},t),$ (145)

V $\quad -i\frac{\eta_r^2}{D}\frac{\partial\phi(\vec{r},t)}{\partial t} = -\frac{3\eta_r^2}{2}\nabla^2\phi(\vec{r},t) + 2[u(\vec{r},t) - \varepsilon(\vec{r},t_0)]\phi(\vec{r},t),$ (146)

H′ $\quad +i\frac{\eta_r^2}{D}\frac{\partial\phi(\vec{r},t)}{\partial t} = -\frac{3\eta_r^2}{2}\nabla^2\phi(\vec{r},t) + 2[u(\vec{r},t) - \varepsilon(\vec{r},t_0)]\phi(\vec{r},t).$ (147)

This suggests that for the stochastic system under consideration, it is necessary to take into account the superposition of all four possible states described by Eqs. (144) – (147).

9 Stochastic massless time-dependent Schrödinger equation

Assume that $D = \eta_r$ and that at the initial time t_0 the averaged mechanical energetics of the ChWP is zero (i.e., $\varepsilon(\vec{r}, t_0) = 0$), then equation (143) takes the form

$$\pm i \frac{\eta_r}{2} \frac{\partial \phi(\vec{r},t)}{\partial t} = \mp \frac{3\eta_r^2}{4} \nabla^2 \phi(\vec{r},t) + u(\vec{r},t)\phi(\vec{r},t). \quad (148)$$

Let us write the time-dependent Schrödinger equation for a similar case:

$$i\hbar \frac{\partial \psi(\vec{r},t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi(\vec{r},t) + U(\vec{r},t)\, \psi(\vec{r},t). \quad (149)$$

We divide both sides of this equation by the particle mass m. As a result, taking into account (46) $u(\vec{r},t) = U(\vec{r},t)/m$, we obtain

$$i \frac{\hbar}{m} \frac{\partial \psi(\bar{r},t)}{\partial t} = -\frac{\hbar^2}{2m^2} \nabla^2 \psi(\bar{r},t) + u(\vec{r},t)\, \psi(\vec{r},t). \quad (150)$$

Obviously, for $\frac{\hbar}{m} = 3\eta_r = \frac{6\sigma_r^2}{\tau_{rcor}}$ and $\phi(\vec{r}) = \psi(\vec{r})$, the stochastic equation (148) and the Schrödinger equation (150) coincide up to the ± signs.

It is interesting to note that Erwin Schrödinger wrote down equation (4") in his article "Quantisierung als Eigenwertproblem, Vierte Mitteilung", Annalen der Physik (1926) [20] in the following form:

$$\Delta\, \psi - \frac{8\pi^2}{h^2} V\psi \pm \frac{4\pi i}{h} \frac{\partial \psi}{\partial t} = 0. \quad (151)$$

We rearrange the terms in this expression and take into account that $\hbar = h/2\pi$ *,*

$$\pm i\hbar \frac{\partial \psi}{\partial t} = -\frac{1}{2} \hbar^2 \nabla^2\, \psi + V\psi. \quad (152)$$

This further demonstrates a more complete analogy between the Schrödinger equation (152), which is fundamental to quantum mechanics, and the stochastic equation (148).

Therefore, equation (148) will be called the stochastic massless time-dependent Schrödinger equation.

10 Advantages of stochastic massless Schrödinger equations

Stochastic Eqs. (131) and (143) have the following advantages over the corresponding Schrödinger equations (134) and (149):

1] Stochastic massless quantum mechanics equations (131) and (143) are derived from a detailed analysis of a random process involving a chaotically walking particle (ChWP), using the principle of "extremum of averaged efficiency" of a stochastic system, which combines two extreme principles: "minimum action" and "maximum entropy." While Schrödinger equations (134) and (149) are practically guessed based on heuristic ideas about de Broglie's mystical matter waves.

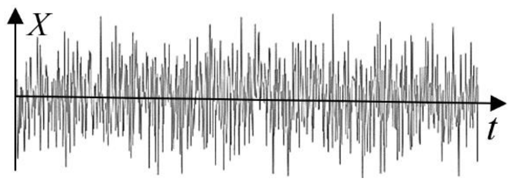
2] Due to the smallness of the reduced Planck constant $\hbar$, Schrödinger equations (134) and (149) are suitable only for describing stochastic systems at the microscopic level, or for extremely precise macroscopic systems such as the LIGO project. Meanwhile, stochastic equations (131) and (143) are suitable for describing quantum systems of any scale, ranging, for example, from chaotic oscillations of proto-quark core within elementary particles to random deviations of the geometric centers of galactic core from the average trajectory of the entire galaxy. To use stochastic equations (131) and (143), it is sufficient to define only three parameters: the standard deviation σ_r, the autocorrelation coefficient τ_{racor}, and the self-diffusion coefficient D, for a random process involving ChWP of any scale and quality.

3] Stochastic equations (131) and (143) lack the heuristic concept of the particle's "mass." Recall that, in reality, the mass of a particle of any scale cannot be directly measured. For example, the ratio of a body's mass to the acceleration due to gravity (m/g) or the ratio of a particle's mass to its charge (m/q) can be varied. The concept of mass is poorly defined and significantly hinders the development of advanced scientific research. The inertial properties of the particle are taken into account using the scaling parameter $\eta_r = 2\sigma_r^2/\tau_{rcor}$. Note again that the ratios $\hbar/m$ and $2\sigma_x^2/\tau_{xcor}$ have the same dimension (m^2/s).

4] Stochastic equations (131) and (143) contain additional signs (±), which expands their applicability.

11 White noise

In this section, we will partially address issues related not to the chaotic behavior of a particle (for example, the core of a "corpuscle"), but to random fluctuations in the extent of the vacuum itself. If we isolate a point in the 3-dimensional extent of a seething vacuum and track the projection of its movement onto the *X*-axis, we will see a practically ideal random process, commonly referred to as white noise.



Recall that one-dimensional white noise is an ideal stationary random process $x(t)$ with an infinite spectrum

$$S(\omega) = \sigma_\omega^2 , \tag{153}$$

and the autocorrelation function

$$\tau_{\omega acor}(\tau) = \sigma_\omega^2 \delta(\tau), \tag{154}$$

where σ_ω^2 is the variance of white noise, $\delta(\tau)$ is the delta function, $\tau = t_2 - t_1$.

Such a random process has the following parameters

$$\varepsilon = \frac{\sigma_\omega^2}{\tau_{\omega acor}^2},\ \eta_r = \frac{2\sigma_\omega^2}{\tau_{\omega acor}} \text{ и } u(x) = 0. \tag{155}$$

In this model, the point under study has no potential energy ($u = 0$). Its chaotic movements around its mean value are due to the influence of a huge number of multidirectional influencing factors.

Substituting parameters (155) into one-dimensional Eq. (131), we obtain

$$\frac{1}{4}\left(\frac{2\sigma_\omega^2}{\tau_{\omega acor}}\right)^2 \frac{d^2\phi(x)}{dx^2} \pm \frac{\sigma_\omega^2}{\tau_{\omega acor}^2}\ \phi(x) = 0. \tag{156}$$

After simple transformations, this equation can be represented in the following form

$$\frac{d^2\phi(x)}{dx^2} \pm \frac{1}{\sigma_\omega^2} \phi(x) = 0, \tag{157}$$

or

$$\frac{d^2\phi(x)}{dx^2} - \frac{1}{\sigma_\omega^2} \phi(x) = 0, \tag{158}$$

$$\frac{d^2\phi(x)}{dx^2} + \frac{1}{\sigma_\omega^2} \phi(x) = 0. \tag{159}$$

In classical quantum mechanics (QM) there is only one Schrödinger equation (158). Let's write its solution in general form

$$\phi(x) = C_1 e^{\frac{x}{\sigma\omega}} + C_2 e^{-\frac{x}{\sigma\omega}}, \text{ where } C_1 \text{ and } C_2 \text{ are integration constants.} \tag{160}$$

Taking into account the normalization condition

$$\int \phi^2(x)dx = \int \rho(x)dx = 1, \tag{161}$$

several variants of the square of the extremal (160) $\phi^2(x) = \rho(x)$ are possible, for example:

1] If in function (160) $C_1 = 0$, then we obtain the Laplace PDF

$$\phi^2(x) = \rho_a(x) = \frac{1}{2\sigma_\omega} e^{-\frac{|x|}{\sigma_\omega}}, \text{ for } x \in [-\infty, \infty]. \tag{162}$$

2] If in function (160) $C_2 = 0$, then

$$\phi^2(x) = \rho_b(x) = \frac{1}{e^x \sigma_\omega \sqrt{2\pi}} exp\left(-\frac{x^2}{2\sigma_\omega^2}\right), \text{ for } x \in [0, \sigma_\omega^2]. \tag{163}$$

3] If in function (160) $C_1 \neq 0$ and $C_2 \neq 0$, then

$$\phi^2(x) = \rho_c(x) = \frac{2}{\pi\sigma_\omega} \frac{2}{e^{\frac{x}{\sigma_\omega}} + e^{-\frac{x}{\sigma_\omega}}}, \text{ for } x \in [-\infty, \infty]. \tag{164}$$

This means that the random variable x in each time t_i of white noise $x(t)$ can be influenced by several random factors, with corresponding PDFs of the type (162), (163), (164).

Within the framework of stochastic massless quantum mechanics (SMQM), this same random process x(t) is equally characterized by the second equation (159), the solution of which is the extremal

$$\phi(x) = C_3 cos\left(\frac{x}{\sigma_\omega}\right) + C_4 sin\left(\frac{x}{\sigma_\omega}\right), \text{ where } C_3 \text{ and } C_4 \text{ are integration constants} \tag{165}$$

In this case, there are several other options, such as:

1] If in function (165) $C_3 = 0$ and $C_4 > 0$, then, taking into account the normalization condition (161), we obtain

$$\phi^2(x) = \rho_d(x) = \frac{2}{\pi\sigma_\omega} sin^2\left(\frac{x}{\sigma_\omega}\right), \text{ for } x \in [0, \pi\sigma_\omega]. \tag{166}$$

2] If in function (165) $C_3 = 0$ and $C_4 < 0$, then

$$\phi^2(x) = \rho_d(x) = -\frac{2}{\pi\sigma_\omega} sin^2\left(\frac{x}{\sigma_\omega}\right), \text{ for } x \in [\pi\sigma_\omega, 2\pi\sigma_\omega]. \tag{167}$$

3] If in function (165) $C_4 = 0$, then

$$\phi^2(x) = \rho_e(x) = \frac{2}{\pi\sigma_\omega} cos^2\left(\frac{x}{\sigma_\omega}\right), \text{ for } x \in \left[-\frac{\pi\sigma_\omega}{2}, \frac{\pi\sigma_\omega}{2}\right]. \tag{168}$$

4] If in function (165) $C_3 \neq 0$ and $C_4 \neq 0$, then

$$\phi^2(x) = \frac{1}{\pi\sigma_\omega} \rho_f(x) = \left(1 + \sin\left(\frac{2x}{\sigma_\omega}\right)\right), \text{ for } x \in [0, \pi\sigma_\omega]. \tag{169}$$

Thus, within the framework of the stochastic massless quantum mechanics (SMQM) developed here, in each cross-section t_i of white noise $x(t)$, the random variable x is influenced by several factors with PDFs (162), (163), (164), (166), (167), (168), (169). Therefore, according to the central limit theorem, we can assume that white noise $x(t)$ is a Gaussian stationary random process with a normal distribution law

$$\rho(x) = \psi(x)\psi(x) = \frac{1}{\sqrt{2\pi\sigma_\omega^2}} exp\left(-\frac{x^2}{2\sigma_\omega^2}\right).$$

At the same time, it turned out that, within the framework of the MSQM, white noise with initial parameters (153) and (154) is the result of the imposition (i.e. superposition) of a discrete (i.e. quantized) series of random processes, and requires additional in-depth study.

12 Stochastic equation of imaginary self-diffusion

If in stochastic equation (143) we set $u(\vec{r},t) = 0$ and $\varepsilon(\vec{r},t_0) = 0$, then it takes the form of a stochastic equation of imaginary self-diffusion (which is a special case of the Fokker-Planck-Kolmogorov equation)

$$\frac{\partial\phi(\vec{r},t)}{\partial t} = \mp i\frac{3}{2}D\nabla^2\phi(\vec{r},t) \quad (170)$$

with a complex self-diffusion coefficient $B = i\frac{3}{2}D$.

Thus, the approach proposed in [13] and in this article, based on searching for extremals $\phi(\vec{r},t)$ of the averaged "efficiency" functional (114) using the methods of the calculus of variations, allows to investigate many stochastic systems (or random processes) with different initial parameters.

13 Quantized states of the chaotically shifting core of a naked "planet" (or "star")

The stochastic massless quantum mechanics (SMQM) proposed in this article is suitable for describing stochastic quantum effects for "corpuscles" of any scale. As an example, we use the stochastic massless stationary Schrödinger equation (131) to study the averaged behavior of a chaotically oscillating "planet" core (see Figure 8).

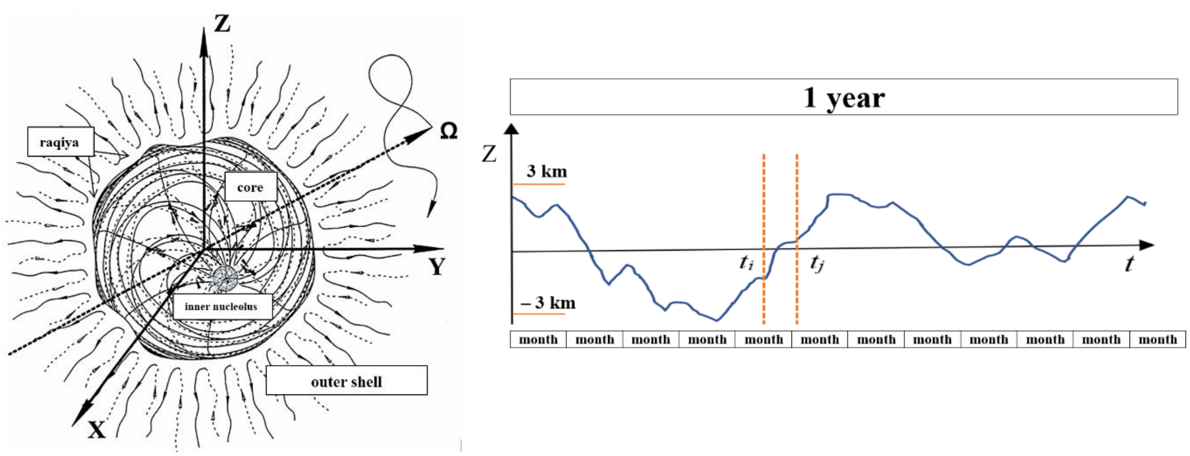


Fig. 8. a) Simplified model of the planet's interior, the geometric center of which slowly and chaotically shifts relative to the geometric center of the entire planet (or star); b) Projection of the chaotic shift of the geometric center of the planet's core onto the Z-axis. Here, it is assumed that one month is 30 revolutions of the planet (or star) around its axis, and 1 year is 360 revolutions of the planet (or star) around its axis

Suppose that, due to complex geophysical processes within the planet's interior, the geometric center of its core continuously and slowly shifts chaotically relative to the geometric center of the entire planet (see Figure 8a).

For example, let the projection of the chaotic shift of the planet's core center (or ChWP) onto any coordinate axis (e.g., the Z axis) be a stationary random process (see Figure 8b) with a standard deviation of $\sigma_{zs} \approx 1\ km = 10^3 m$ and an autocorrelation coefficient of $\tau_{zs\ acor} \approx 1{,}5\ months\ \approx 2{,}6 \cdot 10^6\ sec$. It is assumed that these parameters of the stochastic system under consideration are obtained by averaging over one planetary year (i.e., over 360 revolutions of the planet around its axis), and under the condition that the *XYZ* reference frame rotates with the planet.

According to Exs. (116), (117) and (118), we estimate the value of the scale parameter of the stochastic system under study

$$\eta_{rs} = \frac{2\sigma_{rs}^2}{\tau_{rs\ acor}} \approx \frac{4 \cdot 3 \cdot 10^6}{2{,}6 \cdot 10^6} \approx 4{,}5\ \frac{м^2}{с}.$$

Furthermore, we assume that the randomly varying kinetic energetics of the planet's geometric center of the core (hereinafter, the μ-core, or ChWP) $k(x,y,z,t)$ smoothly transitions into its potential energetics $u(x,y,z,t)$ and vice versa, such that the total mechanical energetics of the ChWP always remains constant:

$$\varepsilon = k(x,y,z,t) + u(x,y,z,t) = const, \tag{171}$$

where $k(x,y,z,t)$ and $u(x,y,z,t)$ are random functions of time t.

The random process under consideration involving the c-core fully corresponds to the stochastic model of ChWP described in §7, and condition (171) coincides with condition (130). Therefore, to describe the averaged states of a chaotically oscillating c-core, the stationary stochastic Schrödinger equation (131) in spherical coordinates can be used (here the notation $\phi = \psi$ is conventionally adopted to exclude the substitution of ϕ for φ)

$$\pm \frac{3\eta_{rs}^2}{4} \nabla^2 \psi(r,\theta,\varphi) + u(r,\theta,\varphi)\psi(r,\theta,\varphi) = \varepsilon\psi(r,\theta,\varphi), \tag{172}$$

where

$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial\phi^2}$ is the Laplace operator in spherical coordinates.

Consider the case where, as the c-core moves away from the center of the stochastic system (which coincides with the geometric center of the entire planet), elastic tension ζ_s, arises in the mantle surrounding the given ChWP, which tends to return the c-core to the center of the system (Fig. 8a). Moreover, this tension $\zeta_s(r)$ increases, on average, as the c-core moves away from the center (i.e., as $r = \sqrt{x^2 + y^2 + z^2}$) increases, see Figure 7a), i.e., an averaged analogue of Hooke's law holds.

$$\zeta_s(r) \approx -k_u r, \tag{173}$$

where $k_u = K_u/m_n$ is the massless elastic tension coefficient of the mantle (K_u is the mantle force constant).

In this case, the average potential energetics of the c-core $u(r)$, which, on average, determines the tendency of a given ChWP to return to the initial center of the system, can be approximately represented as

$$u(r,\theta,\varphi) \approx \left|-\int k_{ux}\, r dr\right| = \left|-\frac{1}{2}k_u r^2\right| = \frac{k_u r^2}{2}. \tag{174}$$

Substituting Ex. (174) into equation (172), we obtain

$$\pm \frac{3\eta_{rs}^2}{4} \nabla^2 \psi(r,\theta,\varphi) + \frac{k_u r^2}{2}\psi(r,\theta,\varphi) = \varepsilon\psi(r,\theta,\varphi), \tag{175}$$

or

$$\nabla^2\psi(r,\theta,\varphi)+\frac{2}{\eta_{nrs}^2}\left[\varepsilon-\frac{k_u r^2}{2}\right]\psi(r,\theta,\varphi)=0, \tag{176}$$

and

$$\nabla^2\psi(r,\theta,\varphi)+\frac{2}{\eta_{nrs}^2}\left[\frac{k_u r^2}{2}-\varepsilon\right]\psi(r,\theta,\varphi)=0, \tag{177}$$

$\eta_{nrs}=\sqrt{3/2}\,\eta_{rs}\approx 7{,}4\ \frac{\text{м}^2}{\text{c}}$ – the scaling parameter for the case under consideration.

Ex. (176) is known in quantum mechanics as the equation of an isotropic three-dimensional harmonic oscillator. The solutions of this equation are functions (i.e., extremals of functional (120) under the conditions specified above) [21]

$$\psi_{kl\text{ɱ}}(r,\theta,\phi)=R_k(r)Y_{l\text{ɱ}}(\theta,\varphi)=\sqrt{\sqrt{\frac{2}{\pi}\left(\frac{\sqrt{k_u}}{2\eta_{n1}}\right)^3}\frac{2^{k+2l+3}k!}{(2k+2l+1)!!}\left(\frac{\sqrt{k_u}}{2\eta_{nrs}}\right)^l}\,r^l\,exp\left\{-\frac{\sqrt{k_u}r^2}{2\eta_{nrs}}\right\}L_l^{(l+1/2)}\left(2\sqrt{\frac{\sqrt{k_u}}{2\eta_{nrs}}}r^2\right)Y_{l\text{ɱ}}(\theta,\varphi),$$

where (178)

$L_l^{(l+1/2)}\left(2\sqrt{\frac{\sqrt{k_u}}{2\eta_{nrs}}}r^2\right)$ is generalized Laguerre polynomials; (179)

$Y_{l\text{ɱ}}(\theta,\varphi)=(-1)^{\text{ɱ}}\left[\frac{(2l+1)(l-\text{ɱ})!}{4\pi(l+\text{ɱ})!}\right]^{\frac{1}{2}}e^{i\text{ɱ}\varphi}P_{l\text{ɱ}}(cos\,\theta)$ is spherical harmonic functions; (180)

$P_{l\text{ɱ}}(cos\,\theta)=\frac{d}{2^l l!}(1-\xi^2)^{\text{ɱ}/2}\frac{d^{l+\text{ɱ}}}{d\xi^{l+\text{ɱ}}}+(\xi^2-1)^l$ is associated Legendre functions; (181)

$\xi=\cos\theta$; (182)

l is the orbital quantum number;

ɱ is the peripheral quantum number.

In classical quantum physics, the number ɱ is called the "magnetic quantum number," but this name is inappropriate for stochastic quantum mechanics. Therefore, in this article, it is proposed to call the number ɱ the "peripheral quantum number." In addition to the quantum numbers k, l, and ɱ, there is also a spin quantum number s, associated with the overall rotation of a stochastic system (for example, with a planet) clockwise or counterclockwise. Thus, there are a total of four quantum numbers.

$$\overset{\text{H}'}{s}\quad\overset{\text{V}}{\text{ɱ}}\quad\overset{\text{H}}{l}\quad\overset{\text{I}}{k}$$

As is known, wave functions (178) correspond to the eigenvalues of the total mechanical energetics of a chaotically wandering *c*-core (or ChWP) [21]

$$\varepsilon_{kl}=\eta_{nrs}\sqrt{k_u}\left(2k+l+\frac{3}{2}\right). \tag{183}$$

The squared modulus of wave functions (178) $|\psi_{klm}(r,\theta,\phi)|^2$ (i.e., the PDF of the possible location of a chaotically wandering *c*-core inside the planet's interior) for $\varphi=0$ and various values of the quantum numbers *k*, *l*, and ɱ are shown in Figure 9. This figure shows that each set of three quantum numbers *k*, *l*, and ɱ corresponds to a unique averaged spatial configuration of the chaotically moving *c*-core (i.e., the ChWP under consideration).

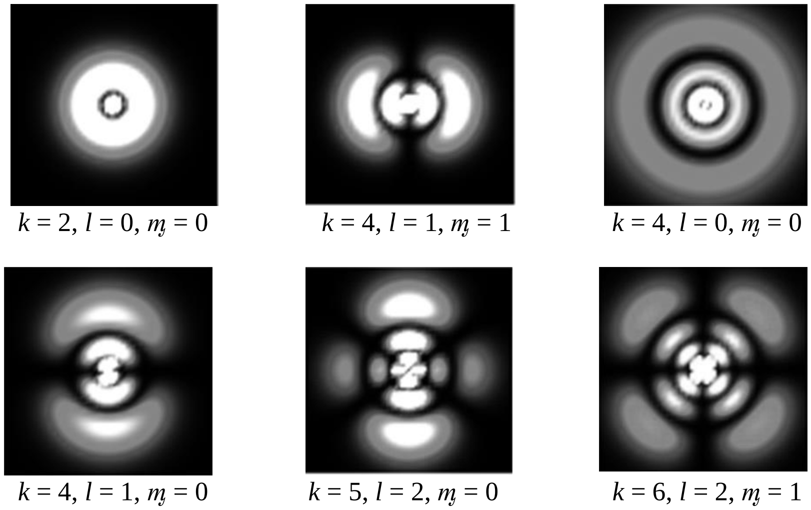


Fig. 9. Probability density functions (PDFs) $|\psi_{klm}(r,\theta,\phi)|^2$ of the possible locations of the *c*-core at $\varphi = 0$ and various values of the quantum numbers *k, l,* and m_l. The brighter the spot, the higher the probability of the *c*-core appearing in this region. Calculations were performed using expression (178) [21] and are presented on the web page: Spherical_Harmonic_Orbitals.png

Eq. (177) with a minus sign before the total mechanical energetics ε has no solutions in the form of averaged states of a chaotically shifting *c*-core (i.e., a ChWP) with a discrete spectrum and square-integrable wave functions. Therefore, no new discrete (quantum) states are added to the stochastic system under study.

Thus, the average behavior of a wandering *c*-core, determined by the simultaneous striving for both minimum action (i.e., maximum possible order and energetics efficiency) and maximum entropy (i.e., maximum possible disorder, determined by the initial conditions), has a discrete set of possible states. In this sense, the chaotic behavior of a planet's *c*-core is no different from other similar chaotically wandering particles (such as, for example, a trembling nucleus in a biological cell, a swimming fish in a round aquarium, an oscillating electron nucleus within an atom, etc.). All these quantum phenomena obey the same laws and differ from each other only in the scale of consideration and the averaging time.

Detecting the averaged quantum states of a wandering *c*-core of a planet (or star) requires years of observation. However, these quantum states of the planet's (or star's) central region should manifest themselves through the passivity or activity of its mantle and many other surface effects.

Furthermore, the transition from the averaged excited state of the wandering *c*-core to its initial averaged state should emit gravitational waves, similar to the way atoms emit electromagnetic waves when an electron transitions from an excited quantum state to its ground state. It is possible that such gravitational waves could be detected by laser interferometers such as LIGO.

If the model concepts presented here are correct, planets and stars are capable of absorbing gravitational perturbations with a wavelength corresponding to the quantum transition of the wandering *c*-core from one averaged state to another. This opens up broad possibilities for gravitational spectroscopy of stars and planets.

CONCLUSION

Nature conceals her mysteries by her Essential Grandeur, not by her cunning."
(from the letter A. Einstein to O. Veblen, April 30, 1930)

This thirteenth part of "Geometrized Vacuum Physics (GVPh) Based on Algebra of signature (AS)" (GVPh&AS) [1,2,3, 4, 5, 6, 7, 8,9,10,11,12] attempts to demonstrate that there is no contradiction between the deterministic description of the macrocosm of stars and planets and the probabilistic description of the microcosm of elementary particles.

All stable spherical $\lambda_{m,n}$-vacuum formations (i.e., “corpuscles”) included in the hierarchical cosmological model [6] (see Figures 1 and 3), regardless of their scale, are subject to the same laws.

The average spherical shape of all "corpuscles" of any scale, from instanton core and proto-quarks to the core of naked stars, galaxies, and the Universe as a whole, is determined by the optimal mutual compensation of the manifestations of $\lambda_{m,n}$-vacuum deformations and intra-vacuum (subcont - antisubcont) currents. In other words, the average spherical shape of all "corpuscles" (see Figure 5a) corresponds to the smallest possible deformations of the $\lambda_{m,n}$-vacuum with the lowest energetics expenditure required to maintain them in a stable state. Therefore, the average metric-dynamic models of all "corpuscles," regardless of their size, are based on metric solutions to Einstein's vacuum equations, which in turn are the result of optimization of the average curvature of 4-dimensional space. Einstein's vacuum equations obtained on the basis of the calculus of variations are, in essence, conservation laws, i.e. conditions under which the averaged 4-deformations of the $\lambda_{m,n}$-vacuum turn out to be stable.

On the other hand, the shape of all "corpuscles," regardless of their scale, is constantly chaotically distorted, i.e., deviates from the average spherical state, like a trembling jelly-like mass (see Figures 1, 3, and 5b). At the same time, the cores of all "corpuscles," regardless of their scale, are constantly chaotically displaced (shifted, oscillated) relative to their average position. The differences between the chaotic behavior of the "corpuscles" are primarily that small "corpuscles" bend and shift rapidly, requiring short periods of time to average out; while large "corpuscles" bend and shift slowly, requiring long periods of observation to reveal their average characteristics.

At first glance, the chaotic behavior of the cores of "corpuscles" of any scale is caused by the influence of a large number of different external factors, such as, for example, disturbances of the $\lambda_{m,n}$-vacuum or force fields of many neighboring "corpuscles", etc. In this case, it is convenient to assume that the cores of the "corpuscle" has a deterministic trajectory of movement, however, it is so complex that it is easier to consider the chaotic behavior of the cores using statistical methods.

In practice, it turned out that the chaotic behavior of elementary particles has its own laws, determined by the Schrödinger equations and Heisenberg's uncertainty relations. This predetermined the triumph of quantum mechanics (and subsequently, all quantum theories). Quantum laws proved so bizarre that common sense had to be abandoned, for example, by abandoning the sizes of elementary particles and their trajectories. As a result, Einstein's deterministic general theory of relativity and neopositivistic quantum mechanics diverged so far from each other that this became one of the fundamental problems of modern physics. Within the framework of quantum theory, Einstein's vacuum should explode, while within the framework of general relativity, the quantized vacuum should collapse into microscopic black holes.

This article attempts to demonstrate that this global problem of our time can be avoided.

First, GVPh&AS, which is essentially a modernized version of Einstein's general theory of relativity, is a quantum theory because it has three types of quantization:

1) “Scale quantization” is the discretization (or size sorting) of "corpuscles" that make up the hierarchical cosmological chain (see Figure 1).

2) “Topological quantization” is the discretization of a set of combinations of 16 signature types (1) that lead to stable spherical $\lambda_{m,n}$-vacuum formations within a single class of "corpuscles" (see, for example, ranking expressions (26) – (32)).

3) “Stochastic quantization”, driven by the tendency of all stochastic systems, including randomly vibrating and chaotically wandering "corpuscle" nuclei of any scale (or class), to seek the most advantageous compromise between order and chaos.

Secondly, relying on the principle of extremality of the average efficiency of all stochastic systems (including chaotically wandering cores of "corpuscles" of any scale), it was possible to derive the stochastic massless Schrödinger equations (131) and (143). The derivation of these stochastic equations was based on an analysis of the random process in which the ChWP (in particular, the wandering core of the "corpuscle") participates, rather than on mystical notions of de Broglie matter waves.

Thus, within the framework of the GVPh&AS, a hierarchical cosmological model has been obtained, consisting of multiple hierarchical chains of "corpuscles" of varying scales, nested like Russian dolls. The core of all these "corpuscles" oscillate chaotically, obeying the uniform laws of massless stochastic quantum mechanics.

This hierarchical cosmological model potentially eliminates many problems that cannot be resolved in principle within the framework of modern natural science. Thus, the GVPh&AS allows us to outline solutions to the following problems:

1) Eliminating the concepts of "mass" and "force" allows us to move closer to realizing the Clifford-Einstein-Wheeler program for the complete geometrization of physics. Metric-dynamic models of virtually all elementary particles included in the Standard Model (bosons, leptons, baryons, and mesons) as vacuum deformations have been obtained;

2) The cause of inertia, the nature of gravity, and electric charge are revealed;

3) Within the framework of the GVPh&AS, there is no baryon asymmetry in the Universe;

4) The Schrödinger equation is derived based on the principle of "extremum of averaged efficiency" of a stochastic system, which includes the principle of "least action" and the principle of "maximum entropy."

5) The cause of quark confinement in hadrons is explained;

6) The fundamental differences between modernized general relativity and quantum mechanics are being erased;

7) The fog regarding dark matter and dark energy is being cleared.

8) GFV&AS can be considered as a theoretical basis for the development of advanced zero-point (i.e., vacuum) technologies, such as: "vacuum energy," "alternative inertialess methods of space travel," "communication channels with superluminal information transfer speeds," "stellar-planetary gravitational spectroscopy," "volume spectral-signature analysis," "unlimited densification of cybernetic power," and many others.

Without a promising theory, new technological breakthroughs are impossible. String theory and loop quantum gravity are clearly at a dead end, leaving the cutting edge of science mired in the darkness of infinite possibilities. GVPh&AS is closely related to these theories, and the Algebra of Signatures (i.e., topologies) opens up ways to constrain these possibilities. In the pitch darkness of the unknown, i.e., beyond the limits accessible to experimental testing, the contours of forms are beginning to emerge that can be subject to deep analysis. It's possible that if GVPh&AS becomes the domain of Artificial Intelligence, we will experience an explosion of new achievements.

At the same time, new, equally complex problems arise in GVPh&AS. For example, it is unknown how many "corpuscles" the longest hierarchical chain contains; If the hierarchical chain is finite, then how can the largest "corpuscle" (the mega-Universe) be located inside the smallest "corpuscle" (the instanton); what is the reason for the large-scale quantization of the sizes of "corpuscles" and "anticorpuscles," etc.

The thirteen proposed papers of the Geometricized Physics of Vacuum Based on the Algebra of Signature (GVPh&AS) [1,2,3,4,5,6,7,8,9,10,11,12], including this paper, are only the trunk of the "tree," from which many branches can sprout. Each paper in this project requires additional in-depth study and further development.

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REFERENCES

[1] Batanov-Gaukhman, M. (2023). Geometrized Vacuum Physics. Part I. Algebra of Stignatures. Avances en Ciencias e Ingeniería, 14 (1), 1-26, https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-14-nro-1-ano-2023-articulo-1/ ; and Preprints, 2023060765. https://doi.org/10.20944/preprints202306.0765.v3, or viXra:2403.0035.

[2] Batanov-Gaukhman, M. (2023).Geometrized Vacuum Physics. Part II. Algebra of Signatures. Avances en Ciencias e Ingeniería, 14 (1), 27-55, https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-14-nro-1-ano-2023-articulo-2/: and Preprints, 2023070716, https://doi.org/10.20944/preprints202307.0716.v1, or viXra:2403.0034.

[3] Batanov-Gaukhman, M. (2023). Geometrized Vacuum Physics. Part III. Curved Vacuum Area. Avances en Ciencias e Ingeniería Vol. 14 nro 2 año 2023 Articulo 5, https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-14-nro-2-ano-2023-articulo-5/; and Preprints 2023, 2023080570. https://doi.org/10.20944/preprints202308.0570.v4, or viXra:2403.0033.

[4] Batanov-Gaukhman, M., (2024). Geometrized Vacuum Physics. Part IV: Dynamics of Vacuum Layers. Avances en Ciencias e Ingeniería Vol. 14 nro 3 año 2023 Articulo 1 https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-14-nro-3-ano-2023-articulo-1/, and Preprints.org. https://doi.org/10.20944/preprints202310.1244.v3, or viXra:2403.0032.

[5] Batanov-Gaukhman, M., (2024). Avances en Ciencias e Ingeniería Vol. 14 nro 3 año 2023 Articulo 2 https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-14-nro-3-ano-2023-articulo-2/, or viXra:2405.0002.

[6] Batanov-Gaukhman, M. (2024) Geometrized Vacuum Physics Part 6: Hierarchical Cosmological Model, Avances en Ciencias e Ingeniería Vol. 14 nro 4 año 2023 https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-14-nro-4-ano-2023-articulo-3/ or viXra:2408.0010.

[7] Batanov-Gaukhman, M. (2025). Geometrized Vacuum Physics Part VII: "Electron" and "Positron", Avances en Ciencias e Ingeniería Vol. 15 nro 1 año 2024 Articulo 3, https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-15-nro-1-ano-2024-articulo-3/, or viXra:2409.0097.

[8] Batanov-Gaukhman, M. (2025) Geometrized Vacuum Physics. Part VIII: Inertial Electromagnetism of Moving «Particles»//Advances en Ciencias e Ingeniería Vol. 15 nro 2 año 2024 Articulo 1, https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-15-nro-2-ano-2024-articulo-1/, or viXra:2411.0086.

[9] Batanov-Gaukhman, M. (2025) Geometrized Vauum Physics. Part IX: «Neutrino»//Avances en Ciencias e Ingeniería Vol. 15 nro 3 año 2024 Articulo 1, https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-15-nro-3-ano-2024-articulo-1/ or viXra:2501.0059

[10] Batanov-Gaukhman, M. (2025) Geometrized Vac-uum Physics. Part X: Naked «Planets» and «Stars»// Avances en Ciencias e Ingeniería Vol. 15 nro 3 año 2024 Articulo 2, https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-15-nro-3-ano-2024-articulo-2/, or viXra:2502.0139.

[11] Batanov-Gaukhman, M. (2025) Geometrized Vacuum Physics. Part XI: Gravity And Levitation. Avances en Ciencias e Ingeniería Vol. 15 nro 4 año 2024 Articulo 1. Publicada el agosto 7, 2025.https://www.executivebs.org/publishing.cl/avances-en-ciencias-e-ingenieria-vol-15-nro-4-ano-2024-articulo-1/, or viXra:2504.0180.

[12] Batanov-Gaukhman, M. (2025) Geometrized Vacuum Physics. Part 12: Naked "Galaxies" - "Particles" of Dark Matter? viXra:2508.0158.

[13] Batanov-Gaukhman M. (2024) Development of the Stochastic Interpretation of Quantum Mechanics by E. Nelson. Derivation of the Schrödinger-Euler-Poisson Equations. *Recent Progress in Materials* **2024**; 6(2): 014; 10.21926/rpm.2402014, or arXiv:2011.09901v10.

[14] Rytov S.M. (1976) Introduction to Statistical Radiophysics Part 1. Nauka, Moscow, p. 494.

[15] Tikhanov V.I. (1982) Statistical Radiophysics. Radio and Communications, Moscow, p. 622.

[16] Blokhintsev D.I. (1963) Fundamentals of Quantum Mechanics. Higher School, Moscow, p. 620.

[17] Elsgolts L.E. (1969) Differential Equations and Calculus of Variations. Nauka, Moscow, p. 424. ((Elsgolts_LE_VI)).

[18] Courant, R. & Hilbert, D. (1953) Methods of Mathematical Physics. Vol. I (First English ed.). New York: Interscience Publishers, Inc. ISBN 978-0471504474.

[19] *Gelfand, I. M. (1963) Calculus of Variations. Dover.* ISBN 0-486-41448-5.

[20] Schrödinger, E. (1926)"Quantisierung als Eigenwertproblem, Vierte Mitteilung", Annalen der Physik, №18.

[21] Fradkin D.M. (1965) Three-dimensional isotropic harmonic oscillator and SU3. American Journal of Physics **33** (3) 207–211, https://en.wikipedia.org/wiki/Quantum_harmonic_oscillator.