The Dirac Adjoint, Gravitational Lapse and Shift, and Progress Toward a Biquaternion Unification of Quantum Mechanics and Relativity

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October 14, 2025

Abstract

The Dirac adjoint is traditionally defined to restore Lorentz covariance of bilinear spinor quantities. In this paper we reinterpret the adjoint within the biquaternion Weyl–Dirac formalism. Starting from the Weyl-level algebra, where the metric is represented by the Pauli biquaternion basis $(\mathbb{T}, \mathbb{K})$, we extend to the Dirac level through a parity–time doubling that introduces the basis $\beta_\mu = (\beta_0, \boldsymbol{\beta})$. We demonstrate that the adjoint $\overline{\Psi} = \Psi^\dagger \beta_0$ acts as an algebraic inclusion of the local time-lapse field (the g_{00} metric component), and that its generalization $\overline{\Psi} = \Psi^\dagger \frac{1}{N}(\beta_0 - N^i \beta_i)$ incorporates also the spatial shift vector N^i. This step algebraically embeds both static and dynamic gravitational effects into the Dirac formalism. We analyze the Weyl-Hilbert and Dirac-Hilbert spaces, showing that only the latter couples to gravity through the adjoint. Finally we assess the stage reached in the ongoing unification of quantum mechanics, special relativity, and general relativity within the biquaternion approach.

1 Introduction

In the standard presentation of relativistic quantum mechanics, the Dirac adjoint

$$\overline{\Psi} = \Psi^\dagger \gamma_0 \tag{1}$$

is introduced in order to construct Lorentz-invariant bilinears such as $\overline{\Psi}\Psi$ and $\overline{\Psi}\gamma^\mu\Psi$. Although operationally indispensable, its geometric meaning is seldom discussed. In our biquaternion formalism, based on the complexified Pauli metric $(\mathbb{T}, \mathbb{K})$ and its PT-doubled Weyl–Dirac extension, the adjoint gains a direct interpretation: it algebraically introduces the local time direction of the metric into the Hilbert space. This perspective allows a transparent connection to general relativity and provides a natural mechanism for incorporating gravitational lapse and shift functions.

2 The Dirac adjoint in the biquaternion framework

The Dirac adjoint is traditionally defined as

$$\overline{\Psi} := \Psi^\dagger \gamma_0 = -\,\mathbb{C}\,\Psi^\dagger \beta_0, \tag{2}$$

and ensures that bilinear forms such as $\overline{\Psi}\Psi$, $\overline{\Psi}\gamma^\mu\Psi$, and $\overline{\Psi}\gamma^\mu\gamma^\nu\Psi$ transform as Lorentz scalars, vectors, and tensors, respectively. In the biquaternion Weyl–Dirac representation we replace γ_μ by $\beta_\mu = (\beta_0, \boldsymbol{\beta})$ built from the Pauli metric basis $(\mathbb{T}, \mathbb{K})$.

The Lorentz transformation of the Dirac spinor is given by

$$\Psi^L = \Lambda_\mathrm{D}\Psi, \qquad \Lambda_\mathrm{D} = S\Lambda_\mathrm{W}S^{-1} = \exp\left(\frac{1}{2}\alpha_I\psi\right), \tag{3}$$

with rapidity ψ and α_I the corresponding boost generator. The Hermitian conjugate spinor transforms as

$$(\Psi^L)^\dagger = \Psi^\dagger\Lambda_\mathrm{D}^\dagger. \tag{4}$$

The Lorentz covariance of the Dirac equation requires the pseudo-unitarity condition

$$\Lambda_\mathrm{D}^\dagger\beta_0\Lambda_\mathrm{D} = \beta_0, \tag{5}$$

which can be verified directly from $\Lambda_\mathrm{D} = \cosh a\,\mathbb{1} + \sinh a\,\alpha_I$ using $\{\alpha_I, \beta_0\} = 0$. This identity expresses the preservation of the time-like metric component under Lorentz transformations.

Using (5), the Dirac adjoint transforms covariantly:

$$\overline{\Psi}^L \;=\; (\Psi^L)^\dagger\beta_0 = \Psi^\dagger\Lambda_\mathrm{D}^\dagger\beta_0 = \Psi^\dagger\beta_0\Lambda_\mathrm{D}^{-1} = \overline{\Psi}\Lambda_\mathrm{D}^{-1}, \tag{6}$$

so that the bilinear $\overline{\Psi}\Psi$ is invariant.

3   Physical interpretation of the adjoint

Within the biquaternion language, β_0 represents the time leg of the local metric. Hence the adjoint $\Psi^\dagger\beta_0\Psi$ projects the complex spinor density onto the physical time direction. This projection converts the purely algebraic Weyl space into a physically anchored Hilbert space. The adjoint is therefore the algebraic insertion of the local lapse of proper time into the quantum domain.

Static gravitational lapse

Introducing a gravitational lapse N, with $g_{00} = -N^2$, the adjoint becomes

$$\overline{\Psi} = \Psi^\dagger\frac{1}{N}\beta_0, \tag{7}$$

representing a local redshift of the spinor norm. This encodes gravitational time dilation directly at the algebraic level: the Dirac inner product is rescaled by the lapse field.

Dynamic gravitational shift

A further generalization introduces the shift vector N^i, defining

$$\boxed{\overline{\Psi} = \Psi^\dagger\frac{1}{N}(\beta_0 - N^i\beta_i),} \tag{8}$$

which corresponds to the full $3+1$ metric decomposition $ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^idt)(dx^j + N^jdt)$. This adjoint now includes both lapse (time dilation) and shift (frame dragging) and allows time-dependent gravitational fields. In this formulation, gravitational waves appear as small oscillations $\delta N^i(t, \mathbf{x})$ of the spatial shift components, corresponding to time-dependent bivector perturbations of the metric basis β_μ.

4 Weyl and Dirac Hilbert spaces

The distinction between the Weyl- and Dirac-level Hilbert spaces becomes crucial.

- **Weyl–Hilbert space:** consists of chiral, massless spinors Ψ without an adjoint. It carries only the representation of the Lorentz group and interacts with curvature indirectly through the spin connection. There is no covariant scalar density or energy-momentum tensor derivable from Ψ alone. The space is gravity-transparent.

- **Dirac–Hilbert space:** arises from parity–time doubling and possesses the adjoint. Bilinear invariants such as $\overline{\Psi}\Psi$ and $\overline{\Psi}\beta^\mu\Psi$ are now well defined and couple to the metric time-flow. Thus the Dirac adjoint introduces gravity sensitivity into the quantum domain.

In short: the Weyl space describes massless, conformal behavior; the Dirac space describes massive, gravitationally coupled behavior.

5 Gravitational waves as metric basis oscillations

When N^i becomes time-dependent, the time-flow basis $\beta(n) = \frac{1}{N}(\beta_0 - N^i\beta_i)$ oscillates. These oscillations produce time-dependent modulations in the spinor norm

$$\Psi^\dagger \beta(n)\Psi = \Psi^\dagger \frac{1}{N}(\beta_0 - N^i(t)\beta_i)\Psi,$$

which propagate as Lorentzian phase modulations. In the biquaternion picture, this corresponds to oscillatory bivector perturbations $\delta\beta = \epsilon(t, \mathbf{x})\,(\beta_0 \wedge \beta_i)$, representing gravitational waves as oscillations of the local metric basis rather than of spacetime curvature.

6 Progress toward unification

The biquaternion formalism now covers in one algebraic structure the domains of special relativity, electromagnetism, Dirac quantum mechanics, and stationary general relativity. Table 1 summarizes the achieved unification.

Domain	Representation	Comment
Special relativity	$\slashed{P} = P_\mu \beta^\mu$	Lorentz-covariant 4-vectors
Electromagnetism	$\slashed{B} = \slashed{\partial}\slashed{A}$	Field tensor as bivector
Dirac equation	$(\slashed{P} - E\mathbb{1})\Psi = 0$	Quantum dynamics
Static gravity	$\overline{\Psi} = \Psi^\dagger \frac{1}{N}\beta_0$	Lapse redshift
Dynamic gravity	$\overline{\Psi} = \Psi^\dagger \frac{1}{N}(\beta_0 - N^i\beta_i)$	Shift, gravitational waves

Table 1: Unified biquaternion representation of physical theories.

Ontological synthesis

Spin has been relocated from being a property of particles to being an intrinsic property of the metric itself, encoded in the biquaternion basis. The Dirac adjoint then becomes the operator that anchors the spinor field to the time-like direction of this spin-metric. Static gravity appears as a modulation of the lapse field; dynamic gravity as time-dependent oscillations of the shift field.

Remaining steps

What remains open is the dynamical feedback between matter and metric. A closed theory requires an evolution equation for the metric basis,

$$\partial_\mu\beta_\nu - \partial_\nu\beta_\mu = f(\Psi^\dagger\beta_{[\mu}\beta_{\nu]}\Psi), \tag{9}$$

which would play the role of Einstein's field equation in the biquaternion language. This step would complete the dynamical unification.

7 Conclusion

We have shown that the Dirac adjoint, within the biquaternion Weyl–Dirac formalism, naturally incorporates the gravitational lapse and shift fields and therefore constitutes the algebraic mechanism that couples quantum spinor dynamics to gravity. Static gravitational effects emerge from the lapse field, while gravitational waves correspond to time-dependent shifts of the local metric basis. The Weyl–Hilbert space remains gravity-transparent; the Dirac–Hilbert space becomes gravity-sensitive through the adjoint. Thus the adjoint plays the dual role of ensuring Lorentz covariance and introducing gravitational coupling.

This development achieves an algebraic and conceptual unification of quantum mechanics and general relativity at the stationary level. To reach full dynamical unification, one must derive an evolution equation for the metric basis $\beta_\mu(x)$ linking its variations to spinor densities. Such a step would establish the long-sought connection between quantum matter and the structure of spacetime.