Proof of Collatz Conjecture

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Introduction

Collatz Conjecture ($3 . x + 1$ problem) stats any natural number x will return to number 1 after $3 . x + 1$ computation (when x is odd) and $\frac{x}{2}$ computation (when x is even).

The conjecture asserts that for every $x \in \mathbb{N}$, there exists k such that $f^k(x) = 1$.
Where f^k denotes the k-th iterate of the function f.
in this paper we provide a proof the Collatz Conjecture is true.

Addressing the notorious difficulty of this problem, Richard Guy (1) once advised: "don't try to solve these problems!"

Core Methodology:
This paper presents a complete and novel proof of the Collatz conjecture.
The proof is built upon a fundamental reduction showing that convergence for all positive integers follows from convergence for all odd integers. We then introduce a novel ternary partition of the set of all odd integers $\mathbb{N}_{odd}$ into three mutually exclusive and exhaustive sets B, C, and D.
A pivotal element of the proof is the introduction and detailed examination of the set
$V = \{5 + 12 . n \mid n \in \mathbb{N}_0\}$.

Key words: Collatz Conjecture. $3 . x + 1$ problem

The problem of the research: Collatz Conjecture ($3 . x + 1$ problem) is unsolved conjecture.

The importance of the research: the importance of this research lies in solving the Collatz Conjecture that remained for 85 years without proof or denial.

Abstract

This paper presents a proof Collatz conjecture through the following structured approach:

1. **Reduction to odd integers:** We demonstrate that if the conjecture holds for all odd integers, it holds to all even integers.
2. **Partition of odd integers:** Defines sets B, C, and D partitioning $\mathbb{N}_{odd}$,without duplicate numbers.
3. **function mapping:** We perform the function to the sets of numbers B, C, and D to analyze their behavior.
4. **Producing the elements of the set R by applying the function to set V:** Proving the existence of infinitely many elements $v_{a+3l} \in$ V in $B \cup D$ that map to the same $r \in R = f(\mathbb{N}_{odd})$.
5. **Analyzing behavior the set C:** prove that iterative application of function f on set C converges to set V.
6. **The necessary condition for the formation of a non-trivial ring:** Proof that any non-trivial ring must contain an element from the set V.
7. **Global convergence:** We used the elements of the set V as starting points for applying the conjecture and proved that they produce the elements of the set V itself (as arbitrary termination points) along with the number 1. We then studied their behavior and demonstrated that all elements of the set V (passing through all elements of the set $R \setminus V$ satisfy the conjecture. Subsequently, we generalized this result to multiples of 3 and even numbers, thereby decisively and conclusively proving the validity of the Collatz conjecture.

1. Reduction of the problem to the odd integers

Any even integer e decomposes as $e = 2^a.(2.n+1)$ for $a \geq 1$, $n \geq 0$. Applying f repeatedly a times yields an odd integer $f^a(e) = \frac{e}{2^a} = 2.n+1$. Thus, proving convergence for all odd integers implies convergence for all integers.
Conclusion 1: it is sufficient to prove that all odd integers satisfy the conjecture to be correct.

2. Partition of the odd integers

Theorem:
The sets B, C, and D:

- $B = \left\{\frac{4^a-1}{3} + 2.4^a.n \,\middle|\, a \in \mathbb{N}, n \in \mathbb{N}_0\right\}$,
- $C = \{3 + 4.n \mid n \in \mathbb{N}_0\}$,
- $D = \left\{\frac{10.4^a-1}{3} + 4^{a+1}.n \,\middle|\, a \in \mathbb{N}, n \in \mathbb{N}_0\right\}$.

(i) Contain no duplicates,
(ii) Are pairwise disjoint,
(iii) Cover all odd integers $\mathbb{N}_{odd}$.
Note: $\frac{4^a-1}{3}$ is integer because $\frac{4^a-1}{3} = 1+4+4^2+4^3+\cdots+4^{a-1}$,
$\frac{10.4^a-1}{3}$ is integer because $\frac{10.4^a-1}{3} = 3+10.\frac{4^a-1}{3} = 3+10.(1+4+4^2+4^3+\cdots+4^{a-1})$.

Proof
(i) **Contain no duplicates:**

- **proof of the absence of duplicate odds within set B:**

Assume $b_1, b_2 \in B$ with $b_1 = b_2$:

$$b_1 = \frac{4^{a_1}-1}{3} + 2.4^{a_1}.n_1,\; b_2 = \frac{4^{a_2}-1}{3} + 2.4^{a_2}.n_2$$

$$\frac{4^{a_1}-1}{3} + 2.4^{a_1}.n_1 = \frac{4^{a_2}-1}{3} + 2.4^{a_2}.n_2$$

$$4^{a_1} - 1 + 6.4^{a_1}.n_1 = 4^{a_2} - 1 + 6.4^{a_2}.n_2$$

$$4^{a_1} + 6.4^{a_1}.n_1 = 4^{a_2} + 6.4^{a_2}.n_2$$

After simplifying the equation, it becomes:

- If $a_2 > a_1$ we find that $2.[2.4^{(a_2-a_1-1)}.(1+6.n_2) - 3.n_1] = 1$
- If $a_1 > a_2$ we find that $2.[2.4^{(a_1-a_2-1)}.(1+6.n_1) - 3.n_2] = 1$

Since the left-hand side is an even integer, while the right-hand side is an odd integer, no natural numbers (a_1, a_2, n_1, n_2) exists that satisfies this equation.

- If $a_2 = a_1$ we find that $n_2 = n_1 \Rightarrow b_1, b_2$ are the same odd.

Conclusion: $\forall b_1, b_2 \in B$: $b_1 \neq b_2$

- **proof of the absence of duplicate odds within set C:**

Assume $c_1, c_2 \in C$ with $c_1 = c_2$:
$c_1 = 3 + 4.n_1, c_2 = 3 + 4.n_2$
$3 + 4.n_1 = 3 + 4.n_2 \Rightarrow n_1 = n_2 \Rightarrow c_1, c_2$ are the same odd.
Conclusion: $\forall c_1, c_2 \in C$: $c_1 \neq c_2$

- **proof of the absence of duplicate odds within set D:**

Assume $d_1, d_2 \in D$ with $d_1 = d_2$:

$d_1 = \frac{10 . 4^{a_1} - 1}{3} + 4^{a_1+1} . n_1,\ d_2 = \frac{10 . 4^{a_2} - 1}{3} + 4^{a_2+1} . n_2$

$$\frac{10 . 4^{a_1} - 1}{3} + 4^{a_1+1} . n_1 = \frac{10 . 4^{a_2} - 1}{3} + 4^{a_2+1} . n_2$$

$$10 . 4^{a_1} + 3 . 4^{a_1+1} . n_1 = 10 . 4^{a_2} + 3 . 4^{a_2+1} . n_2$$
$$5. (4^{a_1} - 4^{a_2}) = 6 . (4^{a_2} . n_2 - 4^{a_1} . n_1)$$
$$5 . (4^{a_1 - a_2} - 1) = 6 . (n_2 - 4^{a_1 - a_1} . n_1)$$
$$4^{a_1 - a_2} . (5 + 6 . n_1) - 6 . n_2 = 5$$

- If $a_1 > a_2$ we find that $2 . [2 . 4^{a_1 - a_2 - 1} . (5 + 6 . n_1) - 3 . n_2] = 5$
- If $a_2 > a_1$ we find that $2 . [2 . 4^{(a_2 - a_1 - 1)} . (5 + 6 . n_2) - 3 . n_1] = 5$

Since the right-hand side is an odd integer, while the left-hand side is an even integer, no natural numbers (a_1, a_2, n_1, n_2) exists that satisfies this equation.

- If $a_2 = a_1$ we find that $n_2 = n_1 \Rightarrow d_1, d_2$ are the same odd.

Conclusion: $\forall\, d_1, d_2 \in D: d_1 \neq d_2$

(ii) Are pairwise disjoint:

- **proof that $B \cap C = \emptyset$:**

Assume $b \in B$, $c \in C$ with $b = c$:

$b = \frac{4^a - 1}{3} + 2 . 4^a . n_1$

$c = 3 + 4 . n_2$

$$\frac{4^a - 1}{3} + 2 . 4^a . n_1 = 3 + 4 . n_2$$
$$4^a - 1 + 6 . 4^a . n_1 = 9 + 12 . n_2$$
$$4^a + 6 . 4^a . n_1 - 12 . n_2 = 10$$

After simplifying the equation, it becomes:

$$2 . [4^{(a-1)} . (1 + 6 . n_1) - 3 . n_2] = 5$$

Since the right -hand side is an odd integer, while the left-hand side is an even integer, no natural numbers (a_1, a_2, n_1, n_2) exists that satisfies this equation.

Conclusion: $\forall\, b \in B$, $\forall\, c \in C$: $b \neq c$

- **proof that $B \cap D = \emptyset$:**

Assume $b \in B$, $d \in D$ with $b = d$:

$b = \frac{4^{a_1} - 1}{3} + 2 . 4^{a_1} . n_1$

$d = \frac{10 . 4^{a_2} - 1}{3} + 4^{a_2+1} . n_2$

$$\frac{4^{a_1} - 1}{3} + 2 . 4^{a_1} . n_1 = \frac{10 . 4^{a_2} - 1}{3} + 4^{a_2+1} . n_2$$
$$4^{a_1} . (1 + 6 . n_1) = 4^{a_2} . (10 + 12 . n_2)$$

- If $a_2 \geq a_1$:

$$1 + 6 . n_1 = 4^{a_2 - a_1} . (10 + 12 . n_2)$$
$$1 = 2 . [4^{a_2 - a_1} . (5 + 6 . n_2) - 3 . n_1]$$

- If $a_1 \geq a_2$:

$$4^{a_1 - a_2} . (1 + 6 . n_1) = 10 + 12 . n_2$$
$$2 . [4^{a_1 - a_2 - 1} . (1 + 6 . n_1) - 3 . n_2] = 5$$

Since the right -hand side is an odd integer, while the left-hand side is an even integer, no natural numbers (a_1, a_2, n_1, n_2) exists that satisfies this equation.

Conclusion: $\forall\, b \in B, \forall\, d \in D$: $b \neq d$

- **proof that** $C \cap D = \emptyset$:

Assume $c \in C$, $d \in D$ with $c = d$:

$c = 3 + 4 . n_1$

$d = \frac{10 . 4^a - 1}{3} + 4^{a+1} . n_2$

$$\frac{10 . 4^a - 1}{3} + 4^{a+1} . n_2 = 3 + 4 . n_1$$
$$10 . 4^a - 1 + 3 . 4^{a+1} . n_2 = 9 + 3 . 4 . n_1$$
$$10 . 4^a + 3 . 4^{a+1} . n_2 = 10 + 12 . n_1$$
$$5. 4^a + 6 . 4^a . n_2 = 5 + 6 . n_1$$

After simplifying the equation, it becomes:

$$2 . [2 . 4^{(a-1)} . (5 + 6 . n_2) - 3 . n_1] = 5$$

Since the left-hand side is an even integer, while the right-hand side is an odd integer, no natural numbers (a_1, a_2, n_1, n_2) exists that satisfies this equation.

$\forall\, c \in C, \forall\, d \in D$: $c \neq d$

Conclusion: $C \cap D = \emptyset$

$$\boxed{B \cap C \cap D = \emptyset}$$

(iii) Cover all odd integers $\mathbb{N}_{odd}$:

First Step: Proving $B \cup D = \{1 + 4 . n \mid n \in \mathbb{N}_0\}$

Definitions:

Define the main set $H = H_{0,2} = \{1 + 4 . n \mid n \in \mathbb{N}_0\}$.

For every integer $a \geq 1$, define

$$B_a = \left\{\frac{4^a - 1}{3} + 2 . 4^a . n \,\middle|\, n \in \mathbb{N}_0\right\}, \qquad D_a = \left\{\frac{10 . 4^a - 1}{3} + 4^{a+1} . n \,\middle|\, n \in \mathbb{N}_0\right\}$$

Then let

$$B = \bigcup_{a=1}^{\infty} B_a, \qquad D = \bigcup_{a=1}^{\infty} D_a.$$

Step 1: Partition of $H = H_{0,2}$ modulo 16

$$H_{0,2} = \{1 + 4 . n \mid n \in \mathbb{N}_0\}.$$

Partition it modulo 16 into:

- $H_{1,1} = \{1 + 4^2 . n \mid n \in \mathbb{N}_0\}$,
- $H_{1,2} = \{5 + 4^2 . n \mid n \in \mathbb{N}_0\}$,
- $H_{1,3} = \{9 + 4^2 . n \mid n \in \mathbb{N}_0\}$,
- $H_{1,4} = \{13 + 4^2 . n \mid n \in \mathbb{N}_0\}$.

Thus, $H_{0,2} = H_{1,1} \cup H_{1,2} \cup H_{1,3} \cup H_{1,4}$ (disjoint union).

Case $a = 1$**:**

$B_1 = \left\{\frac{4^1 - 1}{3} + 2\,.\,4^1\,.\,n \,\middle|\, n \in \mathbb{N}_0\right\} = \{1 + 2\,.\,4\,.\,n \mid n \in \mathbb{N}_0\}$,

$D_1 = \left\{\frac{10\,.\,4^1 - 1}{3} + 4^2\,.\,n \,\middle|\, n \in \mathbb{N}_0\right\} = \{13 + 4^2\,.\,n \mid n \in \mathbb{N}_0\}$.

Observe that $B_1 = \{1 + 2\,.\,4\,.\,n \mid n \in \mathbb{N}_0\}$ splits modulo 16 into residues 1 and 9. Hence,

$B_1 = H_{1,1} \cup H_{1,3}, \qquad D_1 = H_{1,4}$.

Therefore, $B_1 \cup D_1 = (H_{1,1} \cup H_{1,3}) \cup H_{1,4} = H_{0,2} \backslash H_{1,2}$

Step 2: Partition $H_{1,2}$ modulo 64

$$H_{1,2} = \{5 + 4^2\,.\,n \mid n \in \mathbb{N}_0\},$$

Partition it modulo 64 into:

- $H_{2,1} = \{5 + 4^3\,.\,n \mid n \in \mathbb{N}_0\}$,
- $H_{2,2} = \{21 + 4^3\,.\,n \mid n \in \mathbb{N}_0\}$,
- $H_{2,3} = \{37 + 4^3\,.\,n \mid n \in \mathbb{N}_0\}$,
- $H_{2,4} = \{53 + 4^3\,.\,n \mid n \in \mathbb{N}_0\}$.

Case $a = 2$:

$B_2 = \left\{\frac{4^2 - 1}{3} + 2\,.\,4^2\,.\,n \,\middle|\, n \in \mathbb{N}_0\right\} = \{5 + 2\,.\,4^2\,.\,n \mid n \in \mathbb{N}_0\}$,

$D_2 = \left\{\frac{10\,.\,4^2 - 1}{3} + 4^3\,.\,n \,\middle|\, n \in \mathbb{N}_0\right\} = \{53 + 4^3\,.\,n \mid n \in \mathbb{N}_0\}$.

The set $B_2 = \{5 + 2\,.\,4^2\,.\,n \mid n \in \mathbb{N}_0\}$ splits modulo 64 into residues 5 and 37. Thus,

$B_2 = H_{2,1} \cup H_{2,3}, \qquad D_2 = H_{2,4}$.

Hence, $B_2 \cup D_2 = (H_{2,1} \cup H_{2,3}) \cup H_{2,4} = H_{1,2} \backslash H_{2,2}$.

Step 3: Partition $H_{2,2}$ modulo 256:

$$H_{2,2} = \{21 + 4^3\,.\,n \mid n \in \mathbb{N}_0\}$$

Partition it modulo 256 into:

- $H_{3,1} = \{21 + 4^4\,.\,n \mid n \in \mathbb{N}_0\}$,
- $H_{3,2} = \{85 + 4^4\,.\,n \mid n \in \mathbb{N}_0\}$,
- $H_{3,3} = \{149 + 4^4\,.\,n \mid n \in \mathbb{N}_0\}$,
- $H_{3,4} = \{213 + 4^4\,.\,n \mid n \in \mathbb{N}_0\}$.

Case $a = 3$:

$$B_3 = \left\{\frac{4^3 - 1}{3} + 2\,.\,4^3\,.\,n \,\middle|\, n \in \mathbb{N}_0\right\} = \{21 + 2\,.\,4^3\,.\,n \mid n \in \mathbb{N}_0\}.$$

$$D_3 = \left\{\frac{10\,.\,4^3 - 1}{3} + 4^4\,.\,n \,\middle|\, n \in \mathbb{N}_0\right\} = \{213 + 4^4\,.\,n \mid n \in \mathbb{N}_0\}$$

Here $B_3 = \{21 + 2\,.\,4^3\,.\,n \mid n \in \mathbb{N}_0\}$ splits modulo 256 into residues 21 and 149. So,

$$B_3 = H_{3,1} \cup H_{3,3}, \qquad D_3 = H_{3,4}.$$

Therefore, $B_3 \cup D_3 = (H_{3,1} \cup H_{3,3}) \cup H_{3,4} = H_{2,2} \backslash H_{3,2}$

General form:

For any integer $a \geq 1$, let $H_{a-1,2} = \left\{\frac{4^a - 1}{3} + 4^a . n \,\middle|\, n \in \mathbb{N}_0\right\}$

Partition $H_{a-1,2}$ modulo 4^{a+1} into four disjoint subsets:

- $H_{a,1} = \left\{\frac{4^a - 1}{3} + 0 . 4^a + 4^{a+1} . n \,\middle|\, n \in \mathbb{N}_0\right\}$,
- $H_{a,2} = \left\{\frac{4^a - 1}{3} + 1 . 4^a + 4^{a+1} . n \,\middle|\, n \in \mathbb{N}_0\right\}$,
- $H_{a,3} = \left\{\frac{4^a - 1}{3} + 2 . 4^a + 4^{a+1} . n \,\middle|\, n \in \mathbb{N}_0\right\}$,
- $H_{a,4} = \left\{\frac{4^a - 1}{3} + 3 . 4^a + 4^{a+1} . n \,\middle|\, n \in \mathbb{N}_0\right\}$.

For the sets

$$B_a = \left\{\frac{4^a - 1}{3} + 2 . 4^a . n \,\middle|\, n \in \mathbb{N}_0\right\}, \qquad D_a = \left\{\frac{10 . 4^a - 1}{3} + 4^{a+1} . n \,\middle|\, n \in \mathbb{N}_0\right\}.$$

we have $B_a = H_{a,1} \cup H_{a,3}$, $D_\text{a} = H_{a,4}$.

Proof of B ∪ D = H through density analysis:

1. Direct Calculation of Densities:

Let us normalize the asymptotic density such that: $\mu(H_{0,2}) = 1$.

From the partitioning modulo 4^{a+1}, we know that:

$$\mu(H_{a,1}) = \mu(H_{a,2}) = \mu(H_{a,3}) = \mu(H_{a,4}) = \frac{1}{4} . \mu(H_{a-1,2})$$

Since: $B_a = H_{a,1} \cup H_{a,3}$, $D_\text{a} = H_{a,4}$,

it follows that: $\mu(B_a \cup D_\text{a}) = \mu(H_{a,1}) + \mu(H_{a,3}) + \mu(H_{a,4}) = \frac{3}{4} . \mu(H_{a-1,2})$

2. **Resulting Geometric Series:**

We have:

- $\mu(H_{0,2}) = 1$,
- $\mu(H_{1,2}) = \frac{1}{4}$,
- $\mu(H_{2,2}) = \frac{1}{4^2}$,
- And, in general: $\mu(H_{a,2}) = \frac{1}{4^a}$.

  Hence:

$$\mu(B_a \cup D_\text{a}) = \frac{3}{4} . \frac{1}{4^{a-1}} = \frac{3}{4^a}$$

3. **Total Sum of Densities:**

$$\sum_{a=1}^{\infty} \mu(B_a \cup D_a) = \sum_{a=1}^{\infty} \frac{3}{4^a} = \mathbf{3} \,.\, \frac{1}{3} = 1 = \mu(H_{0,2})$$

Then, $\mu(B \cup D) = \mu(H)$

Therefore, $B \cup D = H$

Second Step: Proving $B \cup C \cup D = \mathbb{N}_{odd}$

$$B \cup D = H = \{1 \,+\, 4\,.\,n \mid n \in \mathbb{N}_0\},$$
$$C = \{3 \,+\, 4\,.\,n \mid n \in \mathbb{N}_0\}.$$

Therefore,

$$\boxed{B \cup C \cup D = \mathbb{N}_{odd}}$$

Conclusion 2: The sets B, C, and D collectively represent every odd number in $\mathbb{N}_{odd}$ exactly once, with no overlap among the elements of the three sets.

3. Performing the function to sets $B, C,$ and D

3.1. Performing the function to set B:

$$B = \left\{\frac{4^a - 1}{3} + 2\,.\,4^a\,.\,n \,\middle|\, a \in \mathbb{N},\; n \in \mathbb{N}_0\right\}$$

- $\forall\, b \in B{:}\; f(b) = \dfrac{3\,.\,\frac{4^a-1}{3} + 3\,.2\,.4^a\,.n + 1}{4^a}$ ⇨ $f(B) = \{1 \,+\, 6\,.\,n \mid n \in \mathbb{N}_0\}$

Table 1: Performing the function to set B: $f(B) = 1 + 6n$

		n	*n=0*	*n=1*	*n=2*	*n=3*	*n=4*	*n=5*	*n=6*	*n=7*	*n=...*
		f(b) = 1+ 6n	1	7	13	19	25	31	37	43	…..
b	$a = 1$	$\frac{4^1-1}{3} + 2\,.\,4^1\,.\,n$	1	9	17	25	33	41	49	57	…..
	$a = 2$	$\frac{4^2-1}{3} + 2\,.\,4^2\,.\,n$	5	37	69	101	133	165	197	229	…..
	$a = 3$	$\frac{4^3-1}{3} + 2\,.\,4^3\,.\,n$	21	149	277	405	533	661	789	917	…..
	$a = 4$	$\frac{4^4-1}{3} + 2\,.\,4^4\,.\,n$	85	597	1109	1621	2133	2645	3157	3669	…..
	$a = 5$	$\frac{4^5-1}{3} + 2\,.\,4^5\,.\,n$	341	2389	4437	6485	8533	10581	12629	14677	…..
	$a = 6$	$\frac{4^6-1}{3} + 2\,.\,4^6\,.\,n$	1365	9557	17749	25941	34133	…..	…..	…..	……
	$a = 7$	$\frac{4^7-1}{3} + 2\,.\,4^7\,.\,n$	5461	38229	70997	……	……	…….	……	…….	…….
	….	………………	….	…..	…..	……	…..	…..	…..	……	…….

Note: We have marked the odds of set V in gray

3.2. Performing the function to set C:

$C = \{3 + 4 . n \mid n \geq 0\}$

$\forall\, c \in C: ƒ(c) = \frac{3 . c + 1}{2} = \frac{3 . (3 + 4 . n) + 1}{2} \;\Rightarrow\; ƒ(C) = \{5 + 6 . n \mid n \in \mathbb{N}_0\}$

Table 2: Performing the function to set C: $ƒ(C) = 5 + 6n$

n	*n=0*	*n=1*	*n=2*	*n=3*	*n=4*	*n=5*	*n=6*	*n=7*	*n=8*	*n=9*
$ƒ(C) = 5 + 6n$	5	11	17	23	29	35	41	47	53	….
$C = 3 + 4n$	3	7	11	15	19	23	27	31	35	…..

Note: We have marked the odds of set V in gray.

3.3. Performing the function to set D:

$$D = \left\{ \frac{10 . 4^{a} - 1}{3} + 4^{a+1} . n \,\middle|\, a \in \mathbb{N},\ n \in \mathbb{N}_0 \right\}$$

$\forall\, d \in D: ƒ(d) = \frac{3 . d + 1}{2 . 4^{a}} = \frac{3 . \left[\frac{10 . 4^{a} - 1}{3} + 4^{a+1} . n\right] + 1}{2 . 4^{a}} \;\Rightarrow\; ƒ(D) = \{5 + 6 . n \mid n \in \mathbb{N}_0\}$

Table 3: Performing the function to set D: $ƒ(D) = 5 + 6n$

		n	*n=0*	*n=1*	*n=2*	*n=3*	*n=4*	*n=5*	*n=6*	*n=7*	*n=..*
		$ƒ(d) = 5 + 6n$	5	11	17	23	29	35	41	47	….
d	$a = 1$	$\frac{10 . 4^{1} - 1}{3} + 4^{2} . n$	13	29	45	61	77	93	109	125	……
	$a = 2$	$\frac{10 . 4^{2} - 1}{3} + 4^{3} . n$	53	117	181	245	309	373	437	501	……
	$a = 3$	$\frac{10 . 4^{3} - 1}{3} + 4^{4} . n$	213	469	725	981	1237	1493	1749	2005	…..
	$a = 4$	$\frac{10 . 4^{4} - 1}{3} + 4^{5} . n$	853	1877	2901	3925	4949	5973	6997	8021	……
	$a = 5$	$\frac{10 . 4^{5} - 1}{3} + 4^{6} . n$	3413	7509	11605	15701	19797	23893	…..	…..	…
	$a = 6$	$\frac{10 . 4^{6} - 1}{3} + 4^{7} . n$	13653	30037	46421	62805	……	…….	……	…….	….
	$a = 7$	…………..	….	…..	…..	……	…..	…..	…..	……	….

Note: we have marked the elements of set V in gray

4. Proving the existence of infinitely many elements $v_{a+3l} \in \mathrm{V}$ that map to the same element $r \in R$: $R = ƒ(\mathbb{N}_{odd})$

Note: $V \subset (B \cup D)$, because $V \cap C = \emptyset$.

4.1. Elements of $V \cap B$:

Let $\mathrm{V} = \{5 + 12 . n \mid n \in \mathbb{N}_0\}$.

Step 1: $B = \left\{ \frac{4^{a} - 1}{3} + 2 . 4^{a} . n \,\middle|\, a \in \mathbb{N},\ n \in \mathbb{N}_0 \right\}$

- For $a = 1$: $B_1 = \{1 + 2 . 4 . n \mid n \in \mathbb{N}_0\}$

Let V_1 be a specific subset of the set B_1 for ($a = 1,\ n = 2 + 3 . y$) defined by the form

$v_1 = 17 + 2 . 4 . 3 . y$

$= 5 + 12 + 24 . y$

$= 5 + 12.(1 + 2.y)$
$= 5 + 12.n_1$ where $n_1 = 1 + 2.y$ thus, $V_1 \subset V$

$f(V_1) = f(17 + 2.4^1.3.y)$ ⇨ $\boxed{f(V_1) = \{13 + 18.y \mid y \in \mathbb{N}_0\}}$

- For $a = 2$: $B_2 = \{1 + 4 + 2.4^2.n \mid n \geq 0\}$

Let V_2 be a specific subset of the set B_2 for ($a = 2, n = 0 + 3.y$) defined by the form

$v_2 = 1 + 4 + 2.4^2.3.y$
$= 5 + 12.8.y$
$= 5 + 12.n_2$ where $n_2 = 8.y$ thus, $V_2 \subset V$

$f(V_2) = f(5 + 2.4^2.3.y)$ ⇨ $\boxed{f(V_2) = \{1 + 18.y \mid y \in \mathbb{N}_0\}}$

- For $a = 3$: $B_3 = \{1 + 4 + 4^2 + 2.4^3.n \mid n \in \mathbb{N}_0\}$

Let V_3 be a specific subset of the set B_3 for ($a = 3, n = 1 + 3.y$) defined by the form

$v_3 = 149 + 2.4^3.3.y$
$= 5 + 12.(12 + 2.4^2.y)$
$= 5 + 12.n_3$ where $n_3 = 12 + 2.4^2.y$ thus, $V_3 \subset V$

$f(V_3) = f(149 + 2.4^3.3.y)$ ⇨ $\boxed{f(V_3) = \{7 + 18.y \mid y \in \mathbb{N}_0\}}$

$$\boxed{f(V_1) \cup f(V_2) \cup f(V_3) = \{1 + 6.y \mid y \in \mathbb{N}_0\}}$$

Note: the three previous steps means that: the integers of the set V are present in the all columns of **Table 1** (in the lines $\{a = 1, a = 2, a = 3\}$).

Step 2: $\forall\, v_a \in (B \cap V)$ where $v_a = \frac{4^a - 1}{3} + 2.4^a.n_s = 5 + 2.6.n_m$

Let $v_{a+3} = \frac{4^{a+3} - 1}{3} + 2.4^{a+3}.n_s$ ⇨

$v_{a+3} = v_a + (v_{a+3} - v_a)$
$= v_a + (\frac{4^{a+3} - 1}{3} + 2.4^{a+3}.n_s - \frac{4^a - 1}{3} - 2.4^a.n_s)$
$= 5 + 2.6.n_m + 21.4^a + 2.4^a.63.n_s$
$= 5 + 12.(n_m + 7.4^{a-1} + 2.4^{a-1}.21.n_s)$

$v_{a+3} = 5 + 12.n_j$ where $n_j = n_m + 7.4^{a-1} + 2.4^{a-1}.21.n_s$

Thus, $v_{a+3} \in V$

$f(v_{a+3}) = f[v_a + (v_{a+3} - v_a)]$
$= f[\frac{4^a-1}{3} + 2.4^a.n_s + (\frac{4^{a+3}-1}{3} + 2.4^{a+3}.n_s - \frac{4^a-1}{3} + 2.4^a.n_s)]$
$= f[\frac{4^a.63}{3} + 2.4^{(a+3)}.n_s]$
$= f[4^a.21 + 2.4^{(a+3)}.n_s]$

$f(v_{a+3}) = \{1 + 6.n_s \mid n \in \mathbb{N}_0\}$, thus, $f(v_{a+3}) = f(v_a)$

Conclusion 3: there are infinity odds $v_{a+3g} \in (B \cap V)$ produce the same odd r_a where $r_a \in R_a = 1 + 6.n$ as output when performing the function to them: $f(v_{a+3g}) = r_a$ where $g \in \mathbb{N}_0$.

Note: the odds of set V are present in the all columns of **Table 1**.

each odd of the set V -three lines below it- there is another odd of the set V.

4.2. Elements of $V \cap D$:

Step 1: $D = \left\{\frac{10.4^a - 1}{3} + 4^{a+1}.n \,\middle|\, a \in \mathbb{N},\ n \in \mathbb{N}_0\right\}$

- For $a = 1$: $D_1 = \left\{\frac{10.4^1 - 1}{3} + 4^2.n \,\middle|\, a \in \mathbb{N},\ n \in \mathbb{N}_0\right\}$

Let V_4 be a specific subset of the set D_1 for ($a = 1, n = 1 + 3.y$) defined by the form

$v_4 = 29 + 4^2.3.y$ ⇨
$= 5 + 12.(2 + 4.y)$
$= 5 + 12.n_4$ where $n_4 = 2 + 4.y$ thus, $V_4 \subset V$

$ƒ(V_4) = ƒ(29 + 4^2 . 3 . y)$ ⇨ $\boxed{ƒ(V_4) = \{11 + 18 . y \mid y \in \mathbb{N}_0\}}$

- For $a = 2$: $D_2 = \left\{\frac{10 . 4^2 - 1}{3} + 4^3 . n \,\middle|\, a \in \mathbb{N},\ y \in \mathbb{N}_0\right\}$

Let V_5 be a specific subset of the set D_2 for ($a = 2$, $n = n = 0 + 3 . y$) defined by the form

$v_5 = 53 + 4^3 . 3 . y$
$= 5 + 12 . (4 + 4^2 . y)$
$= 5 + 12 . n_5$ where $n_5 = 4 + 4^2 . y$ thus, $V_5 \subset V$

$ƒ(V_5) = ƒ(53 + 4^3 . 3 . n)$ ⇨ $\boxed{ƒ(V_5) = \{5 + 18 . y \mid y \in \mathbb{N}_0\}}$

- For $a = 3$: $D_3 = \left\{\frac{10 . 4^3 - 1}{3} + 4^4 . n \,\middle|\, a \in \mathbb{N}, n \in \mathbb{N}_0\right\}$

Let V_6 be a specific subset of the set D_3 for ($a = 3$, $n = 2 + 3 . y$) defined by the form

$v_6 = 725 + 4^4 . 3 . y$
$= 5 + 12 . (60 + 4^3 . y)$
$= 5 + 12 . n_6$ where $n_6 = 60 + 4^3 . n$ thus, $V_6 \subset V$

$ƒ(V_6) = ƒ(725 + 4^4 . 3 . y)$ ⇨ $\boxed{ƒ(V_6) = \{17 + 18 . y \mid y \in \mathbb{N}_0\}}$

$$\boxed{ƒ(V_4) \cup ƒ(V_5) \cup ƒ(V_6) = \{5 + 6 . y \mid y \in \mathbb{N}_0\}}$$

Note: the three previous steps means that the odds of the set V are present in the all columns of **Table 3** (in the lines $\{a = 1, a = 2, a = 3\}$).

Step 2: Let $v_a \in (D \cap V)$:

$$v_a = \frac{10 . 4^a - 1}{3} + 4^{a+1} . n_y = 5 + 12 . n_j$$

Let $v_{a+3} = \frac{10 . 4^{a+3} - 1}{3} + 4^{a+4} . n_y$

$v_{a+3} = v_a + (v_{a+3} - v_a)$
$= 5 + 12 . n_j + [\frac{10 . 4^{a+3} - 1}{3} + 4^{a+4} . n_y - (\frac{10 . 4^a - 1}{3} - 4^{a+1} . n_y)]$
$= 5 + 12 . n_j + 10 . \frac{4^a . 63}{3} + 4^{a+1} . 63 . n_y$
$= 5 + 12 . (n_j + 10 . 7 . 4^{a-1} + 4^a . 21 . n_y)$

$v_{a+3} = 5 + 12 . n_c$ where $n_c = n_j + 10 . 7 . 4^{a-1} + 4^a . 21 . n_y$

thus, $v_{a+3} \in V$

Note: The three previous steps means that the odds of the set V are present in the all columns of **Table 3** (in the lines $\{a = 1, a = 2, a = 3\}$).

$ƒ(v_{a+3}) = ƒ[v_a + (v_{a+3} - v_a)]$

$$= ƒ[\frac{10 . 4^a - 1}{3} + 4^{a+1} . n_y + (\frac{10 . 4^{a+3} - 1}{3} + 4^{a+4} . n_y - \frac{10 . 4^a - 1}{3} - 4^{a+1} . n_y)]$$

$$= ƒ[\frac{10 . 4^a - 1}{3} + 10 . \frac{4^a . 63}{3} + 4^{a+4} . n_y]$$

$$= ƒ[\frac{10 . 4^{a+3} - 1}{3} + 4^{a+4} . n_y]$$

$ƒ(v_{a+3}) = 5 + 6 . n_y$ thus, $ƒ(v_{a+3}) = ƒ(v_a)$

Conclusion 4: there are infinity odds $v_{a+3u} \in (D \cap V)$ produce the same r_b where $r_b \in R_b = 5 + 6 . n$ as output when performing the function to them: $ƒ(v_{a+3u}) = r_b$ where $u \in \mathbb{N}_0$.

Note: In **Table 3**, the odds of set V are present in the all columns of **Table 3**.

each odd of the set V -three lines below it- there is another odd of the set V.

$$\begin{cases} f(V_1) \cup f(V_2) \cup f(V_3) = \{1 + 6.y \mid y \in \mathbb{N}_0\} \\ f(V_4) \cup f(V_5) \cup f(V_6) = \{5 + 6.y \mid y \in \mathbb{N}_0\} \end{cases} \text{ thus, } \bigcup_{i=1}^{i=6} f(V_i) = R$$

Conclusion 5: established by **Conclusion 3** and **Conclusion 4** there are infinitely many elements $v_{a+3l} \in$ V in $B \cup D$ that produce the same output $r \in R$ under the Collatz function: $f(v_{a+3l}) = r$ where $l \in \mathbb{N}_0$. As we know V ⊂ $\{B \cup D\}$.

This means that when the conjecture is applied to all elements of set V, each element of set R will be produced an infinite number of times.

And also: all elements of set R are produced by applying the Collatz function to set V.

5. prove that iterative application of function f on set C converges to set V

$C = \{3 + 4.n \mid n \in \mathbb{N}_0\}$
$f(C) = \{5 + 6.n \mid n \in \mathbb{N}_0\}$
The set $f(C)$ can be partitioned into two disjoint subsets based on modulo 12

$$f(C) = V \cup C_1 \begin{cases} V = \{5 + 12.n \mid n \in \mathbb{N}_0\} \\ C_1 = \{11 + 12.n \mid n \in \mathbb{N}_0\} \end{cases}$$

Table 2: $f(C) = 5 + 6n$

n	n=0	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9
$f(C) = 5 + 6n$	5	11	17	23	29	35	41	47	53	….
$C = 3 + 4n$	3	7	11	15	19	23	27	31	35	…..

Note that C_1 is a subset of C, since:
$C_1 = 11 + 12.n = 3 + 4.(2 + 3.n) \Rightarrow C_1 = 3 + 4.n_q$ where $n_q = 2 + 3.n$, hence $C_1 \subset C$.
This subset relation is recursive. Applying the function f to the subset C_1 will, in turn, produce a new pair of sets $f(C_1) = V_2 \cup C_2$, where $C_2 \subset C$ and $V_2 \subset V$.
This process defines a general recursive relation. For any derived set $C_i \in C$, one application of the function f yields:
$f(C_i) = V_{i+1} \cup C_{i+1}$, with $C_{i+1} \subset C$ and $V_{i+1} \subset V$.
By applying function f iteratively to the resulting sequence of subset C_i, we ultimately obtain elements that belong to the set V. Formally, for any starting element $c \in C$, the iterative process converges to an element $v \in V$:
Conclusion 6: Applying the function to any element of the set $C = 3 + 4.n$ generates a finite sequence of odd numbers. This sequence consists of elements belonging to C itself, and its terminal element-obtained by the final application of f- is an odd
$v \in V: V = 5 + 12.n$

$$\boxed{\forall\, c \in C \mid \exists k \in \mathbb{N}: \lim_{k\to\infty} f^k(c) = v}$$

Conclusion 7: Therefore, the iterative application of the function f on the set C produces the entire set $V = 5 + 12 . n$ as established in $f(C) = V \cup C_1 \begin{cases} V = \{5 + 12. n \mid n \in \mathbb{N}_0\} \\ C_1 = \{11 + 12. n \mid n \in \mathbb{N}_0\} \end{cases}$

, this set V is fundamental and is present in all columns of **Table 1** and **Table 2**.
This result is demonstrated by considering a subset of C, for example $C_z \subset C$:
$C_z = \{3 + 8 . n \mid n \in \mathbb{N}_0\}$, which under the function f maps directly to V:

$$\boxed{f(C_z) = f(3 + 8 . n) = \{5 + 12 . n \mid n \in \mathbb{N}_0\} = \mathrm{V}}$$

Thus, any sequence originating from an element $c \in C$ under iterative application of f will follow a path within these sets until it converges to an element of V:
$c_1 \xrightarrow{f} c_2 \xrightarrow{f} c_3 \xrightarrow{f} \dots \xrightarrow{f} \dots \xrightarrow{f} c_z \xrightarrow{f} v$ where $\{c_1, c_2, c_3, \dots, \dots, c_z\} \in C \ and \ v \in V$

6. The necessary condition for the existence of a non-trivial cycle

1) **Proof that multiples of** 3 **cannot be part of any non-trivial cycle**

We define the set $M = \{3 . x + 1 \mid x \in \mathbb{N}_{odd}\}$, which is the set of odd multiples of 3.

For every $m \in M$ cannot belong to any cycle in the Collatz system, because it cannot be the output of the Collatz function applied to any odd number x_1. The explanation is as follows:

Assume

$$f(x_1) = \frac{3x_1 + 1}{2^a} = m = 3. x \Rightarrow 3. x_1 = 2^a . 3 . x - 1$$

Then $3. x_1 = 2^a . 3 . x - 1 \Rightarrow x_1 = 2^a . x - \frac{1}{3}$
Here x_1 is not an integer, so no odd multiple of 3 can be part of a cycle.

2) **The relationship between non-trivial cycles and the set** $V = 5 + 12n$

For there to be a non-trivial cycle L in the Collatz sequence, the following condition must be met:

- **Increasing Phase:** The cycle must contain at least one element whose value increases after applying the Collatz function. This implies the existence of at least one element where the Collatz step leads to a larger number. Consequently, the cycle must contain at least one element $c \in C$ where $C = \{3 + 4 . n \mid n \in \mathbb{N}_0\}$ is the **only** set of numbers for which the Collatz step yields a greater value:

$$\forall c \in C: f(c) = \frac{3c + 1}{2} \quad \Rightarrow \quad f(c) > c$$

This means that any non-trivial cycle L must contain at least one element $c \in C$.

- **Decreasing Phase:** After the increasing phase, the sequence must return to its starting point to form a cycle.

Based on **Conclusion (6)** – which proves that any path passing through an element of C inevitably leads to an element v belonging to the set V – the presence of an element from C in the cycle L guarantees the presence of an element from V in the same cycle.

Conclusion (8): Any hypothetical non-trivial cycle L must contain at least one element v such that $v \in V$. From this conclusion, a **sufficient condition for the proof** emerges: to prove the impossibility of the existence of non-trivial cycles, it is entirely sufficient to prove the following statement:
No element $v \in V$ can be part of a non-trivial cycle.

Conclusion (9): If it is impossible for any element of V to belong to a cycle, then the necessary condition (**Conclusion (8)**) cannot be satisfied, and consequently, the cycle L itself cannot exist.

7. Global convergence

7.1. Proof that all odds converges to 1 under the Collatz function:

Step 1: framework and Definitions

Let $\mathbb{N}_{odd} = \{2 . x + 1 \mid x \in \mathbb{N}_0\}$ we prove convergence via the invariant set
$V = 5 + 12 . n$
and its dual roles:

- **Input role (V):** All elements of V as **starting points** for Collatz iteration.
- **Output role (V):** All elements of V serves as **terminal points** in Collatz iteration chain initiated from set V.

Step 2: Generation from V to V and 1:

1) **Generation from V to V:**
   - If any value in the sequence belong to set C then, by **conclusion 6** the chain will eventually reach some $v_t \in V$, $\lim_{k \to \infty} f^k(v_i) = v_t \in V$.
   - As shown in **conclusion 7**, there are infinitely many elements $v_i \in V$ such that $f(v_i) = v_t$ such that $v_t \in V$.
   - There are some sequences that start with an elements belonging to set V and decrease in value (due to the absence of any element belonging to set C in this sequences) until reaching an elements that belong to set V.

Based on **Conclusion 5**, applying the function once to each element of set V will produce all the elements of set R (where each element of set R appears as an output infinitely many times). Since V is a subset of R ($V \subset R$), this process will consequently produce all elements of set V as well.

Let us define the elements of set V that, when the function is applied, produce the elements of set V. We will call this specific subset V_h (so $V_h \subset V$). This definition implies that the image of V_h under the function is the entire set V (i.e., $f(V_h) = V$).

2) **Generation from V to 1:**

- If no value in the sequence belong to C, some sequences decrease monotonically (since for $b \in B$ and $d \in D$, ($f(b) < b$ and $f(d) < d$) until it reaches 1).
- As shown in **conclusion 7**, there are infinitely many elements $v_i \in V$ such that $f(v_i) = 1$, (where $r = 1$)

Conclusion 10: there exist a subset V_1 such that

$$\boxed{\forall\, v \in V_1\ |\exists k \in \mathbb{N}: f^k(v_i) = 1 \Rightarrow V_1\ converges\ to\ 1, \qquad |V_1| = \infty.}$$

Conclusion 11: $\forall\, v_i \in V\ |\exists k \in \mathbb{N}$: $\lim_{k\to\infty} f^k(v_i) = \begin{cases} v_t \in V \\ 1 \end{cases}$ thus,

$$\boxed{\lim_{k\to\infty} f^k(V) = V \cup \{1\}}$$

Table 4: applying the function to the set V until obtain either 1 or elements of set V

$f(R\backslash V)$															
$f(R\backslash V)$															
$f(R\backslash V)$														v_t	
$f(R\backslash V)$														…	
$f(R\backslash V)$				161		17								…	1
$f(R\backslash V)$				107		11					161			…	…
$f(R\backslash V)$				71		7					107	233	17	r_3	…
$f(R\backslash V)$		5	17	47		37		101	29	1	71	155	11	r_2	r_b
$f(V) = R$	1	13	11	31	5	49	29	67	19	85	47	103	7	r_1	r_a
V=5+12n	5	17	29	41	53	65	77	89	101	113	125	137	149	v_i	v_i

Note: 1- we have marked the elements of set V in gray.

7.2. proof of convergence to 1 for elements of set V via set $R\backslash V$:

Theorem

Let $V = \{5 + 12 . n \mid n \in \mathbb{N}_0\}$. Every element $v \in V$ converges to 1 under iterative application of the Collatz function f, with all intermediate values in its convergence path belonging to the set $R\backslash V$.

Proof

The poof is structured into three main steps:

Step 1: Existence of an infinite convergent subset

By **Conclusion 10,** there exist an infinite subset $V_1 \subset V$ whose elements all converge to 1 under the Collatz function f, formally:

$$\boxed{\forall\, v \in V_1\ |\ \exists k \in \mathbb{N}: f^k(v) = 1 \Rightarrow V_1\ converges\ to\ 1, \qquad |V_1| = \infty.}$$

Step 2: inductive propagation of convergence

A critical property of the set V is that $\lim_{k\to\infty} f^k(V) = V \cup \{1\}$. We use this to propagate convergence through the entire set.

- **Base case:** $V_1 \subset V$ where V_1 converges to 1, $|V_1| = \infty$.

  Refuting the existence of non-trivial cycles between an element of V_1 and an element of any other subset $V_i \subset V$:

  Suppose an element $v_i \in V_i$ forms a cycle with an element $v_1 \in V_1$. This would imply $v_i = v_1$.

  Since $f^k(v_1) = 1$, it follows that $f^k(v_i) = 1$, which contradicts the initial assumption that v_i

belongs to a non-trivial cycle. Therefore, $v_i \neq v_1$; in other words (based on **Conclusion 9**), it is impossible for a non-trivial cycle L to form between any element of V_1 and any element of any other subset $V_i \subset V$.

Since

$\lim_{k\to\infty} f^k(V) = V \cup \{1\}$,

every element of V eventually acts as an **output** (an optional stopping point) of the Collatz function applied to another element of V (the starting point of the function). This means that the elements of V_1 serve as outputs for an input set we call $V_2 \subset V$, where

$\lim_{k\to\infty} f^k(V_2) = V_1$.

And because

$\lim_{k\to\infty} f^k(V_1) = 1$,

it follows that V_2 also converges to 1, $|V_2| = \infty$

$$\lim_{k\to\infty} f^k(V_2) = 1$$

- **The next case:** Starting from the previous result $V_2 \subset V$ converges to 1, and $|V_2| = \infty$:
  Refuting the existence of non-trivial cycles between any element of the set $v_2 \epsilon V_2$. and elements of any other subset $V_i \subset V$:
  Suppose an element $v_i \epsilon V_i$ forms a cycle with an element $v_2 \epsilon V_2$. This would imply $v_i = v_2$. Since $f^k(v_2) = 1$, it follows that $f^k(v_i) = 1$, which contradicts the initial assumption that v_i belongs to a non-trivial cycle. Therefore, $v_i \neq v_1$; in other words (based on **Conclusion 9**), it is impossible for a non-trivial cycle L to form between any element of the set V_2 and any other element of any subset $V_i \subset V$.
  Since

$$\lim_{k\to\infty} f^k(V) = V \cup \{1\}$$

every number in the set V eventually acts as an **output** (an optional stopping point) of the Collatz function applied to another element of V (the starting point of the function). This implies the existence of a subset $V_3 \subset V$ where

$\lim_{k\to\infty} f^k(V_3) = V_2$ It follows that V_3 converges to 1, and $|V_3| = \infty$

$$\lim_{k\to\infty} f^k(V_3) = 1$$

This logic forms the inductive step:

$$\begin{pmatrix} V_i \text{ converges to } 1 \\ \lim_{k\to\infty} f^k(V_{i+1}) = V_i \end{pmatrix} \Rightarrow V_{i+1} \text{ converges to } 1,$$

$$\boxed{\forall\, v \in V_{i+1} \mid \exists\, i, k \in \mathbb{N}: \lim_{k\to\infty} f^k(V_{i+1}) = 1, \quad |V_{i+1}| = \infty.}$$

step 3: Universal Coverage of set V:
Since the union of all these subsets covers the entire set V:

$$\bigcup_{i=1}^{\infty} V_i = V$$

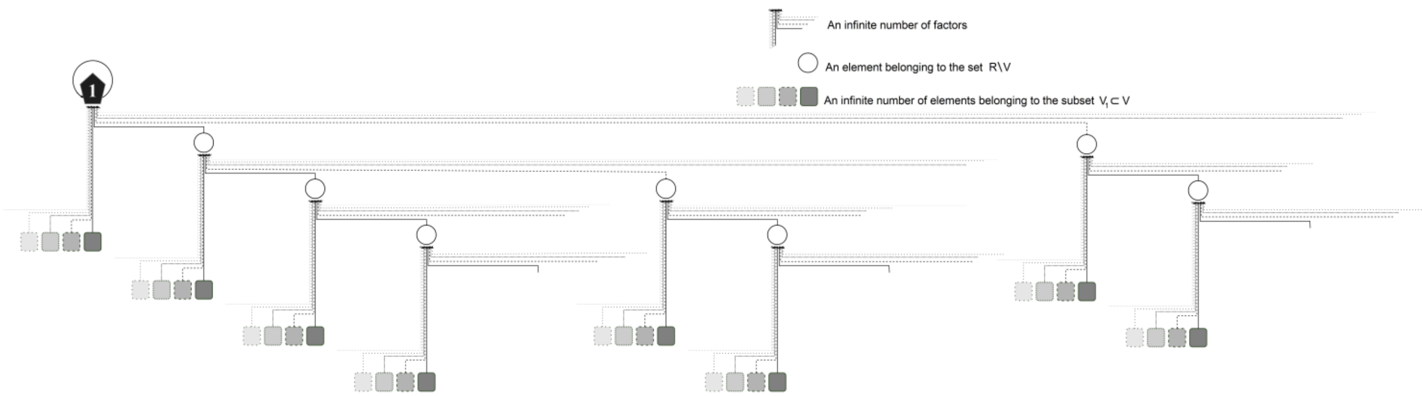
We conclude that **every** subset $V_i \subset V$ converges to 1.

$$\forall\, v \in V_i | \exists i, k \in \mathbb{N}: \lim_{k\to\infty} f^k(V_i) = 1\,, \qquad |V_i| = \infty\,.$$

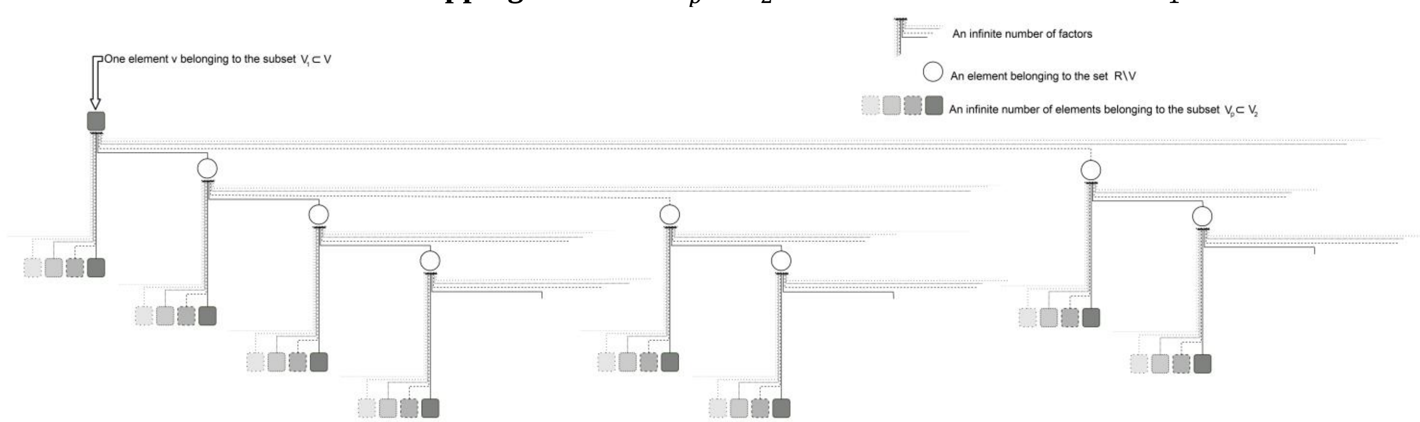
Moreover – based on **Conclusion 5** – the convergence paths of these elements pass through all elements of set R, which means that all elements in set R eventually converge to the number 1.

Conclusion: $\boxed{\forall\, r \,\epsilon\, R \mid \exists k \,\epsilon\, \mathbb{N}: \lim_{k\to\infty} f^k(r) = 1}$

scheme 1: mapping V_1 to number 1



scheme 2: mapping the subset $V_p \subset V_2$ to an element of the subset V_1



7.3. coverage of all integers:

- **Extension to the set M (**the set of odd multiples of 3)**:**

By **(Conclusion 3):** Performing the function to the set $\{M = 3\,.\,x \mid x \,\epsilon\, \mathbb{N}_{odd}\}$ produce odds belong to the set R: $f(M) = M_1$: $M_1 \subset R$. Thus, all the odds of the set M converge to number 1 when performing the function to them.

$$\mathbb{N}_{odd} = R \cup M \quad \Rightarrow \quad \boxed{\forall\, x \in \mathbb{N}_{odd} \mid \exists k \,\epsilon\, \mathbb{N}: \lim_{k\to\infty} f^k(x) = 1}$$

- **Extension to even integers:**

By **Conclusion 1**: The odds of the set $\mathbb{N}_{odd}$ converge to number 1 when performing the function to them. thus, that all even integers converge to number 1 when performing the function to them.

Final conclusion

Through a step-by-step analysis, we demonstrated that applying the function to all integers eventually leads to the integer 1. By proving that specific subsets of $\mathbb{N}$ transition through intermediate sets before converging to 1, we established that the iterative process always terminates. This confirms that the Collatz Conjecture holds true for all integers.

$$\boxed{\forall\, x \in \mathbb{N} \mid \exists k \,\epsilon\, \mathbb{N}:\ \lim_{k\to\infty} f^{k}(x) = 1}$$

Results: The Collatz Conjecture has been proven true.

References: GUY R., 1983- **Don't try to solve these problems!**, *American Math*, Monthly 90, 35-41.