Titus Planck Units: Derivations & Numerical Evaluations

Quinton R. D. Tharp

October 26, 2025

Abstract

This paper presents a detailed and foundational analysis of Planck-scale quantities within the Quantum Phase Lattice Model (Titus framework). Unlike conventional treatments, which often introduce Planck units as derived dimensional combinations, the Titus approach begins from a deterministic lattice interpretation of energy and time, expressed through the Planck loop condition $E_P t_P = h$. This definition implies $E_P = h/t_P$, a form larger than the conventional ("$\hbar$-based") Planck energy by a factor of 2π. By anchoring all substitutions to this loop identity, the derivations remain internally consistent, deterministic, and directly linked to observable quantities. For clarity, we denote conventional reduced forms with a subscript std (e.g., $E_{P,\mathrm{std}} = \hbar/t_P$); Titus "loop" forms use h and satisfy $X = 2\pi\, X_{\mathrm{std}}$ for energy-like X.

We provide symbolic derivations, numerical evaluations, and explicit unit checks for the full set of Planck units, using CODATA 2022 values. Each equation is cross-referenced with NIST standards to ensure clarity and reproducibility. The electromagnetic quartet (Planck charge, voltage, current, and power), together with Planck impedance, are shown to emerge transparently from the Titus substitutions. These quantities demonstrate an algebraic structure, making transparent that the Planck impedance equals half the vacuum impedance ($Z_P = Z_0/2$).

The analysis highlights how the Titus framework unifies Planck-scale physics with atomic-scale structure: the Bohr identities $h = 2\pi m_e v_B a_0$ and $\hbar = m_e v_B a_0$ arise naturally and consistently from the lattice formalism. By presenting all Planck units within this structure, the work establishes a rigorous foundation for later sections of the Quantum Phase Lattice Model, where phase-coherence mechanics and cosmological implications can be developed upon this Planck-scale base.

1 Introduction

Planck units are often presented as dimensional combinations of $\hbar$, c, and G that set a natural scale for quantum gravity. In this paper we take a different starting point, motivated by the Quantum Phase Lattice ("Titus") framework: the *loop* action identity

$$E_P t_P = h \tag{1.1}$$

which we adopt as fundamental.[1] Equation (1.1) implies $E_P = h/t_P$ and leads to a systematic 2π enhancement relative to the reduced-action (Std) convention $E_{P,\text{std}} = \hbar/t_P$, namely $E_P = 2\pi\, E_{P,\text{std}}$. We show that this bookkeeping choice propagates consistently and transparently across the Planck set, electromagnetism, and Bohr-anchored atomic identities.

Perspective and inputs. We treat t_P (and ℓ_P) as primitive lattice spacings linked by

$$c = \frac{\ell_P}{t_P} \tag{1.2}$$

so that space and time are set by a common cycle. The analysis uses only SI-defined exact constants (h, e, k_B, c) and CODATA 2022 recommended values for the remaining quantities; after the 2019 SI redefinition, h and e are exact, while μ_0, ϵ_0, and Z_0 are inferred and inherit the small uncertainty mainly from α. All numerical evaluations are carried out with CODATA 2022 and are cross-checked for consistency.

Unifying Bohr, EM, and Planck structure. A key bridge is the Bohr action identity

$$\hbar = m_e v_B a_0 \tag{1.3}$$

from which the fine-structure constant emerges as a pure velocity ratio,

$$\alpha = \frac{v_B}{c} \tag{1.4}$$

and the Planck charge admits equivalent Titus/EM forms,

$$q_P^2 = \frac{2E_P t_P}{Z_0} = \frac{4\pi}{Z_0}\, m_e v_B a_0 = 4\pi\, \epsilon_0\, \hbar\, c \tag{1.5}$$

These identities culminate in a compact electromagnetic *quartet*

$$\{V_P,\ I_P,\ P_P,\ Z_P\}$$

with closure relations $P_P = V_P I_P = V_P^2/Z_P = I_P^2 Z_P$ and the notably simple impedance result

$$Z_P = \frac{h}{q_P^2} = \frac{Z_0}{2} \tag{1.6}$$

The 2π map. Using the loop action h rather than $\hbar$ produces a uniform correspondence

$$X \;=\; 2\pi\, X_{\text{std}} \quad \text{for} \quad X \in \{E_P,\ m_P,\ p_P,\ F_P,\ u_P,\ \rho_P,\ T_P,\ P_P\}$$

while leaving dimensionless quantities (e.g. α) and vacuum-impedance relations unchanged. We emphasize that this is a matter of algebraic bookkeeping, not a change to SI definitions; the two conventions are related by explicit, exact factors of 2π.

[1] Throughout, we tag the conventional *$\hbar$-based* Planck definitions with Std to avoid confusion with the particle-physics $\hbar$-based convention.

Contributions

- A closed, minimal dictionary linking lattice (E_P, t_P, ℓ_P), EM (Z_0, μ_0, ϵ_0), and Bohr (m_e, a_0, v_B) quantities with exact cancellations.
- Derivations of the electromagnetic quartet and the Planck impedance $Z_P = Z_0/2$ directly from lattice and Bohr inputs.
- Parallel Titus (loop) and Std (reduced) forms for all major Planck units, exposing a transparent 2π mapping.
- Length–time representations that eliminate G and $\hbar$ where possible, clarifying which identities are definitional versus inferred.
- Comprehensive numerical validations (CODATA 2022) and compact unit checks for reproducibility.

Roadmap. Table 1 fixes notation and inputs. We then establish the action constants and gravitational-consistency forms, followed by the core Planck set (time, length, energy; frequency/angle; mass/momentum; forces and densities; temperature). The charge and EM section derives q_P, $\alpha = v_B/c$, and the vacuum relations. The electromagnetic quartet consolidates V_P, I_P, P_P, and Z_P. We confirm the Bohr radius and close the elementary-charge loop. A brief note summarizes the induced 2π scalings, followed by compact unit checks and an appendix of raw numerics.

Unless stated otherwise, numerical values are from CODATA 2022 (NIST) [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 1]. Exact SI definitions after the 2019 redefinition are noted.

Table 1: CODATA 2022 constants used.

Quantity	Symbol	Value (CODATA 2022)	Source
Speed of light (exact)	c	$2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}$	[13]
Planck constant (exact)	h	$6.626\,070 \times 10^{-34}\,\mathrm{J\,s}$	[14]
Reduced Planck constant	$\hbar$	$h/2\pi$ (exact from h)	[14]
Elementary charge (exact)	e	$1.602\,177 \times 10^{-19}\,\mathrm{C}$	[13]
Boltzmann constant (exact)	k_B	$1.380\,649 \times 10^{-23}\,\mathrm{J\,K^{-1}}$	[13]
Fine-structure constant	α	$7.297\,353 \times 10^{-3}$	[15, 1]
Planck time	t_P	$5.391\,247 \times 10^{-44}\,\mathrm{s}$	[16]
Planck length	ℓ_P	$1.616\,255 \times 10^{-35}\,\mathrm{m}$	[17]
Electron mass	m_e	$9.109\,384 \times 10^{-31}\,\mathrm{kg}$	[18]
Bohr radius	a_0	$5.291\,772 \times 10^{-11}\,\mathrm{m}$	[19]
Vacuum impedance	Z_0	$3.767\,303 \times 10^{2}\,\Omega$	[20]
Vacuum permeability	μ_0	$1.256\,637 \times 10^{-6}\,\mathrm{N\,A^{-2}}$	[21, 1]
Vacuum permittivity	ϵ_0	$8.854\,188 \times 10^{-12}\,\mathrm{F\,m^{-1}}$	[22, 1]
Newtonian constant of gravitation	G	$6.674\,300 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$	[24, 1]

Numerical evaluations (CODATA 2022). Using the constants in Table 1:

$$\begin{aligned} E_P t_P &= h \\ &\Rightarrow E_P = \frac{h}{t_P} \\ &= \frac{6.626\,070 \times 10^{-34}\,\mathrm{J\,s}}{5.391\,247 \times 10^{-44}\,\mathrm{s}} \\ &= 1.229\,042 \times 10^{10}\,\mathrm{J} \end{aligned} \tag{1.7}$$

$$\begin{aligned} v_B &= \alpha\, c \\ &= (7.297\,353 \times 10^{-3})(2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}) \\ &= 2.187\,691 \times 10^{6}\,\mathrm{m\,s^{-1}} \end{aligned} \tag{1.8}$$

$$\begin{aligned} h &= 2\pi\, m_e v_B a_0 \\ &= 2\pi\,(9.109\,384 \times 10^{-31}\,\mathrm{kg})(5.291\,772 \times 10^{-11}\,\mathrm{m})(2.187\,691 \times 10^{6}\,\mathrm{m\,s^{-1}}) \\ &= 6.626\,070 \times 10^{-34}\,\mathrm{J\,s} \end{aligned} \tag{1.9}$$

$$\begin{aligned}\hbar &= m_e v_B a_0 \\ &= (9.109\,384 \times 10^{-31}\,\mathrm{kg})(5.291\,772 \times 10^{-11}\,\mathrm{m})(2.187\,691 \times 10^{6}\,\mathrm{m\,s^{-1}}) \\ &= 1.054\,572 \times 10^{-34}\,\mathrm{J\,s}\end{aligned} \tag{1.10}$$

$$\begin{aligned}h &= 2\pi\,\hbar \\ &= 2\pi \times 1.054\,572 \times 10^{-34}\,\mathrm{J\,s} \\ &= 6.626\,070 \times 10^{-34}\,\mathrm{J\,s} \quad \text{(consistent with CODATA value)}\end{aligned} \tag{1.11}$$

$$\begin{aligned}c &= \frac{\ell_P}{t_P} \qquad \text{(identity in this framework)} \\ &\approx \frac{1.616\,255 \times 10^{-35}\,\mathrm{m}}{5.391\,247 \times 10^{-44}\,\mathrm{s}} \\ &\approx 2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}} \quad \text{(matches exact SI value of } c\text{)}\end{aligned} \tag{1.12}$$

$$\begin{aligned}Z_0 &= \mu_0\,c \qquad \text{(definition)} \\ &\approx (1.256\,637 \times 10^{-6}\,\mathrm{N\,A^{-2}})\,(2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}) \\ &\approx 3.767\,303 \times 10^{2}\,\Omega\end{aligned} \tag{1.13}$$

$$\begin{aligned}Z_0 &= \frac{1}{\epsilon_0\,c} \qquad \text{(definition)} \\ &\approx \frac{1}{(8.854\,188 \times 10^{-12}\,\mathrm{F\,m^{-1}})\,(2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}})} \\ &\approx 3.767\,303 \times 10^{2}\,\Omega\end{aligned} \tag{1.14}$$

Bohr velocity and fine-structure constant. From Eq. (1.8), the ratio of Bohr velocity to c gives

$$\begin{aligned}\frac{v_B}{c} &= \frac{2.187\,691 \times 10^{6}\,\mathrm{m\,s^{-1}}}{2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}} \\ &= 7.297\,353 \times 10^{-3} \\ &= \alpha\end{aligned} \tag{1.15}$$

which matches the CODATA 2022 fine-structure constant value.

1.1 Action Constants

Using Eq. (1.1) together with the Bohr identities $\hbar = m_e v_B a_0$ and $h = 2\pi\hbar$,

$$h = E_P t_P = \frac{E_P}{f_P} = 2\pi\, m_e v_B a_0 \tag{1.16}$$

$$\hbar = \frac{E_P}{\omega_P} = \frac{h}{2\pi} = m_e v_B a_0 \tag{1.17}$$

Titus lattice forms for h. Directly from the lattice parameters ($E_P = \frac{h}{t_P}$, $c = \frac{\ell_P}{t_P}$):

$$h = E_P\, t_P \tag{1.18}$$

$$= E_P\, \frac{\ell_P}{c} \tag{1.19}$$

An electromagnetic expression $h = \frac{Z_0}{2}\, q_P^2$ is shown later, after establishing $Z_P = h/q_P^2 = Z_0/2$.

Titus lattice forms for $\hbar$. Using the angular pairing ($E_P = \hbar\,\omega_P$) and the frequency identities:

$$\hbar = \frac{h}{2\pi} \tag{1.20}$$

$$= \frac{E_P\, t_P}{2\pi} = \frac{E_P}{\omega_P} \tag{1.21}$$

$$= \frac{Z_0}{4\pi}\, q_P^2 \tag{1.22}$$

The last line is equivalent to $q_P^2 = 4\pi\,\epsilon_0\,\hbar\, c$ together with $Z_0 = 1/(\epsilon_0 c)$ and matches (3.1).

Bohr forms (kept explicitly)

$$E_P^{\text{Titus}}/f_P = h = 2\pi\, m_e v_B a_0 \tag{1.23}$$

$$E_P^{\text{Titus}}/\omega_P = \hbar = m_e v_B a_0 \tag{1.24}$$

Numeric validations (CODATA 2022) All forms below evaluate to the same values for h and $\hbar$.

Planck loop:

$$\begin{aligned} h = E_P t_P &= \left(1.229\,042 \times 10^{10}\,\mathrm{J}\right)\left(5.391\,247 \times 10^{-44}\,\mathrm{s}\right) \\ &= 6.626\,070 \times 10^{-34}\,\mathrm{J\,s} \quad \text{(consistent with CODATA value [14])} \end{aligned} \tag{1.25}$$

Length–time lattice:

$$\begin{aligned} h = E_P\, \frac{\ell_P}{c} &= \frac{\left(1.229\,042 \times 10^{10}\,\mathrm{J}\right)\left(1.616\,255 \times 10^{-35}\,\mathrm{m}\right)}{2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}} \\ &= 6.626\,070 \times 10^{-34}\,\mathrm{J\,s} \end{aligned} \tag{1.26}$$

Impedance/charge lattice:

$$h = \frac{Z_0}{2}\, q_P^2 = \frac{3.767\,303 \times 10^{2}\,\Omega}{2} \left(3.517\,673 \times 10^{-36}\,\mathrm{C}^2\right)$$
$$= 6.626\,070 \times 10^{-34}\,\mathrm{J\,s} \tag{1.27}$$

using q_P^2 from (3.1) (numeric value quoted in the charge section).

Angular (frequency) pairing:

$$\hbar = \frac{E_P}{\omega_P} = \frac{1.229\,042 \times 10^{10}\,\mathrm{J}}{1.165\,442 \times 10^{44}\,\mathrm{s}^{-1}}$$
$$= 1.054\,572 \times 10^{-34}\,\mathrm{J\,s} \quad \text{(exact from } h/2\pi \text{ [14])} \tag{1.28}$$

Summary. Across lattice (loop and length–time), electromagnetic (impedance/charge), angular (frequency), and Bohr constructions, we obtain the same action constants:

$$\boxed{h = E_P t_P = \frac{E_P \ell_P}{c} = \frac{Z_0}{2}\, q_P^2 = 2\pi m_e v_B a_0} \tag{1.29}$$

$$\boxed{\hbar = \frac{h}{2\pi} = \frac{E_P}{\omega_P} = \frac{Z_0}{4\pi}\, q_P^2 = m_e v_B a_0} \tag{1.30}$$

1.2 Gravitational Constant Consistency

In the Titus framework, the gravitational constant G admits several equivalent forms, depending on whether one emphasizes time or length as the scaling basis. A convenient starting point is the Planck–Einstein identity

$$G = \frac{2\pi\, \ell_P\, c^4}{E_P} \tag{1.31}$$

obtained from the lattice relations $E_P = \frac{h}{t_P}$ and $c = \frac{\ell_P}{t_P}$.

Temporal form (with $\hbar$). Using $t_P^2 = \dfrac{\hbar G}{c^5}$ gives

$$G_{\text{temporal}} = \frac{c^5 t_P^2}{\hbar} \tag{1.32}$$

Temporal form (Bohr-anchored). With the Bohr identity $\hbar = m_e v_B a_0$ one has

$$G_{\text{temporal,Bohr}} = \frac{c^5 t_P^2}{m_e v_B a_0} \tag{1.33}$$

Length–time (ℓ/t) form (Bohr-anchored). Eliminating c with $c = \ell_P/t_P$ and using $\hbar = m_e v_B a_0$,

$$G_{\ell/t} = \frac{\ell_P^5}{t_P^3\, m_e v_B a_0} \tag{1.34}$$

Equivalence. From $c = \ell_P/t_P$,

$$\begin{aligned} \frac{c^5 t_P^2}{m_e a_0 v_B} &= \frac{\left(\frac{\ell_P}{t_P}\right)^5 t_P^2}{m_e a_0 v_B} \\ &= \frac{\ell_P^5}{t_P^3\, m_e a_0 v_B} \\ &= \frac{\ell_P^5}{t_P^3\, \hbar} \end{aligned} \tag{1.35}$$

so (1.33) and (1.34) are identical; all representations are mutually consistent.

Numeric (from Titus form). Using ℓ_P (CODATA 2022) [17], c (exact) [13], and E_P from Eq. (1.7),

$$G = \frac{2\pi \left(1.616\,255 \times 10^{-35}\,\mathrm{m}\right) \left(2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}\right)^4}{1.229\,042 \times 10^{10}\,\mathrm{J}} = 6.674\,300 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} \tag{1.36}$$

Numeric (temporal, with $\hbar$). Using t_P and $\hbar$ (CODATA 2022) [16, 14, 13],

$$G_{\mathrm{temporal}} = \frac{c^5 t_P^2}{\hbar} \tag{1.37}$$

$$= \frac{\left(2.997\,925 \times 10^{8}\right)^5 \left(5.391\,247 \times 10^{-44}\right)^2}{1.054\,572 \times 10^{-34}}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} \tag{1.38}$$

$$= 6.674\,300 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} \tag{1.39}$$

Numeric (temporal, Bohr-anchored). With $v_B = \alpha c$ see Eq. (1.8); CODATA 2022 α, c) [15, 13],

$$G_{\mathrm{temporal,Bohr}} = \frac{c^5 t_P^2}{m_e v_B a_0} \tag{1.40}$$

$$= \frac{\left(2.997\,925 \times 10^{8}\right)^5 \left(5.391\,247 \times 10^{-44}\right)^2}{\left(9.109\,384 \times 10^{-31}\right)\left(5.291\,772 \times 10^{-11}\right)\left(2.187\,691 \times 10^{6}\right)}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} \tag{1.41}$$

$$= 6.674\,300 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} \tag{1.42}$$

Numeric (ℓ/t form, Bohr-anchored). Using v_B from Eq. (1.8),

$$G_{\ell/t} = \frac{\ell_P^5}{t_P^3\, m_e v_B a_0} \tag{1.43}$$

$$= \frac{\left(1.616\,255\times 10^{-35}\right)^5}{\left(5.391\,247\times 10^{-44}\right)^3} \frac{1}{\left(9.109\,384\times 10^{-31}\right)\left(5.291\,772\times 10^{-11}\right)} \tag{1.44}$$

$$\times \frac{1}{\left(2.187\,691\times 10^{6}\right)}\ \mathrm{m^3\,kg^{-1}\,s^{-2}} \tag{1.45}$$

$$= 6.674\,300\times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}} \tag{1.46}$$

Bohr identity closure. Substituting any of (1.31), (1.32), or (1.34) into

$$t_P^2 = \frac{\hbar G}{c^5} \qquad \ell_P^2 = \frac{\hbar G}{c^3} \tag{1.47}$$

yields $\hbar = m_e v_B a_0$, confirming that the Titus and Bohr-anchored forms are algebraically closed and consistent with CODATA 2022.

2 Core Planck Set

2.1 Planck Time

The Planck time t_P represents the fundamental unit of duration in the Planck system of natural units. It is defined as the time required for light to travel one Planck length in vacuum. In conventional form, it is derived by combining $\hbar$, G, and c:

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \tag{2.1}$$

In the Titus framework, t_P is treated as a primitive lattice unit of time, directly linked to the Planck loop relation ($E_P t_P = h$, Eq. (1.1)). Thus, t_P is the base temporal spacing of the Einstein lattice, from which other quantities are deterministically constructed.

Titus Form Using $c = \omega_P/k_P$,

$$t_P = \frac{\ell_P}{c} = \frac{\ell_P k_P}{\omega_P} \tag{2.2}$$

Square-root check (Std form). Starting from the standard Planck combination,

$$t_P = \sqrt{\left(\frac{E_P\,\ell_P}{2\pi c}\right)\left(\frac{2\pi\ell_P c^4}{E_P}\right)\frac{1}{c^5}} \tag{2.3}$$

$$= \sqrt{\frac{\ell_P^2 c^3}{c^5}} \quad \text{(cancel } E_P \text{ and } 2\pi;\ c\cdot c^4 = c^5\text{)} \tag{2.4}$$

$$= \sqrt{\frac{\ell_P^2}{c^2}} \tag{2.5}$$

$$= \frac{\ell_P}{c} \tag{2.6}$$

$$= t_P \quad \text{since } \ell_P = c\,t_P \text{ by (2.16).} \tag{2.7}$$

Square-root check (Bohr form). Using the Bohr identity $\hbar = m_e a_0 v_B$ and the equivalent form,

$$\begin{aligned} G &= \frac{\ell_P^5}{(\ell_P/c)^3\,(m_e a_0 v_B)} \\ &= \frac{\ell_P^2 c^3}{m_e a_0 v_B} \end{aligned} \tag{2.8}$$

we have

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \tag{2.9}$$

$$= \sqrt{\frac{(m_e v_B a_0)\left(\dfrac{\ell_P^5}{(\ell_P/c)^3\,(m_e v_B a_0)}\right)}{c^5}} \tag{2.10}$$

$$= \sqrt{\frac{\ell_P^5}{(\ell_P/c)^3\,c^5}} \tag{2.11}$$

$$= \sqrt{\frac{\ell_P^5}{(\ell_P^3/c^3)\,c^5}} = \sqrt{\frac{\ell_P^5}{\ell_P^3\,c^2}} \tag{2.12}$$

$$= \sqrt{\frac{\ell_P^2}{c^2}} = \frac{\ell_P}{c} = t_P \tag{2.13}$$

Numeric:

$$\begin{aligned} t_P &= \sqrt{\frac{(1.054\,572\times10^{-34}\,\mathrm{J\,s})(6.674\,300\times10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}})}{(2.997\,925\times10^{8}\,\mathrm{m\,s^{-1}})^5}} \\ &= 5.391\,247\times10^{-44}\,\mathrm{s} \end{aligned} \tag{2.14}$$

This matches the CODATA 2022 recommended value [16], and both the Std and Bohr routes collapse consistently to the same result $t_P = \ell_P/c$.

2.2 Planck Length

The Planck length ℓ_P represents the fundamental unit of spatial distance in the Planck system of natural units. It is defined as the distance light travels in one Planck time. In conventional form, it is derived by combining $\hbar$, G, and c:

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \tag{2.15}$$

In the Titus framework, ℓ_P is treated as a primitive lattice unit of length, directly linked to the temporal spacing t_P by

$$\ell_P = c\, t_P = \frac{t_P \omega_P}{k_P} \tag{2.16}$$

so that space and time emerge coherently from the same lattice cycle.

Square-root check (Std form). Starting from

$$\ell_P = \sqrt{\left(\frac{E_P\,(t_P c)}{2\pi c}\right)\left(\frac{2\pi\,(t_P c)\,c^4}{E_P}\right)\frac{1}{c^3}} \tag{2.17}$$

$$= \sqrt{\left(\frac{t_P c}{c}\right)\,(t_P c)\,c^4 \cdot \frac{1}{c^3}} \tag{2.18}$$

$$= \sqrt{t_P \cdot (t_P c) \cdot c} \tag{2.19}$$

$$= \sqrt{t_P^2 c^2} \tag{2.20}$$

$$= t_P c \tag{2.21}$$

Square-root check (Bohr form). Using the Bohr identity $\hbar = m_e v_B a_0$,

$$\ell_P = \sqrt{\left(m_e v_B a_0\right)\left(\frac{(t_P c)^5}{t_P^3\, m_e v_B a_0}\right)\frac{1}{c^3}} \tag{2.22}$$

Inside the root,

$$\sqrt{\left(m_e v_B a_0\right)\left(\frac{(t_P c)^5}{t_P^3\, m_e v_B a_0}\right)\frac{1}{c^3}} = \sqrt{\left(m_e v_B a_0\right)\left(\frac{t_P^2 c^5}{m_e v_B a_0}\right)\frac{1}{c^3}} \tag{2.23}$$

so multiplying by $m_e v_B a_0$ cancels it cleanly, leaving

$$\ell_P = \sqrt{t_P^2 c^5 \cdot \frac{1}{c^3}} = \sqrt{t_P^2 c^2} = t_P c \tag{2.24}$$

Numeric:

$$\ell_P = (2.997\,925 \times 10^8\,\mathrm{m\,s^{-1}})(5.391\,247 \times 10^{-44}\,\mathrm{s}) = 1.616\,255 \times 10^{-35}\,\mathrm{m} \tag{2.25}$$

which matches the CODATA 2022 recommended value [17]; constants c and t_P from [13, 16].

2.3 Planck Energy

The Planck energy E_P represents the characteristic loop energy of a single Planck cycle of duration t_P. In the Titus framework it is

$$E_P = \frac{h}{t_P} \tag{2.26}$$

with the Planck loop relation $E_P t_P = h$ defining h as the energy–frequency exchange constant of the lattice [14, 16].

Square-root form ($\hbar$-based (Std)). The conventional expression is

$$E_{P,\mathrm{std}} = \sqrt{\frac{\hbar c^5}{G}} \tag{2.27}$$

Using $t_P^2 = \hbar G/c^5$ (CODATA) [16], one may eliminate G:

$$E_{P,\mathrm{std}} = \sqrt{\frac{\hbar c^5}{\frac{t_P^2 c^5}{\hbar}}} = \sqrt{\frac{\hbar^2}{t_P^2}} = \frac{\hbar}{t_P} \tag{2.28}$$

With the Bohr identity

$$\hbar = m_e v_B a_0 \qquad v_B = \alpha\, c \tag{2.29}$$

one obtains

$$E_{P,\mathrm{std}} = \frac{m_e v_B a_0}{t_P} \tag{2.30}$$

where m_e, a_0, α, and c are CODATA 2022 [18, 19, 15, 13].

Titus form and the 2π enhancement. In the Titus framework, the loop relation uses h rather than $\hbar$:

$$E_P = \frac{h}{t_P} = \frac{2\pi\hbar}{t_P} \tag{2.31}$$

Comparing (2.28) and (2.31),

$$\boxed{E_P = 2\pi\, E_{P,\mathrm{std}}} \tag{2.32}$$

Thus, the Titus definition is a full 2π increase over the Std value, reflecting the use of the complete loop action h rather than the reduced action $\hbar$.

Bohr form (Titus). Using Bohr's action quantization,

$$\hbar = m_e v_B a_0 \qquad h = 2\pi\,\hbar \tag{2.33}$$

write energy as an explicit phase–rate product,

$$E = \left(\frac{\vartheta}{t}\right)\hbar \tag{2.34}$$

$$E = \omega\,\hbar \qquad \omega = \frac{\vartheta}{t} \tag{2.35}$$

Setting $\vartheta = 2\pi$ and $t = t_P$ reproduces the Titus loop relation,

$$E_P = \left(\frac{2\pi}{t_P}\right)\hbar = \omega_P\,\hbar \quad (\omega_P = 2\pi/t_P) = \frac{h}{t_P} = \frac{2\pi\,m_e v_B a_0}{t_P} \tag{2.36}$$

Numeric (CODATA 2022, Titus value):

$$E_P = \frac{h}{t_P} = \frac{6.626\,070 \times 10^{-34}\,\mathrm{J\,s}}{5.391\,247 \times 10^{-44}\,\mathrm{s}} = 1.229\,042 \times 10^{10}\,\mathrm{J} \tag{2.37}$$

using exact h and t_P from CODATA [14, 16].

Equivalently, using the Bohr form with the constants from Table 1 and $v_B = \alpha c$ [15, 13]:

$$\begin{aligned} E_P &= \frac{2\pi\,9.109\,384 \times 10^{-31}\,\mathrm{kg}\,5.291\,772 \times 10^{-11}\,\mathrm{m}\,2.187\,691 \times 10^{6}\,\mathrm{m/s}}{5.391\,247 \times 10^{-44}\,\mathrm{s}} \\ &= 1.229\,042 \times 10^{10}\,\mathrm{J} \end{aligned} \tag{2.38}$$

which agrees with (1.7) within the precision of the tabulated α digits.

Numeric (CODATA 2022, Std value):

$$\begin{aligned} E_{P,\mathrm{std}} &= \frac{m_e v_B a_0}{t_P} \\ &= \frac{9.109\,384 \times 10^{-31}\,\mathrm{kg}\,5.291\,772 \times 10^{-11}\,\mathrm{m}\,2.187\,691 \times 10^{6}\,\mathrm{m/s}}{5.391\,247 \times 10^{-44}\,\mathrm{s}} \\ &= 1.956\,081 \times 10^{9}\,\mathrm{J} \end{aligned} \tag{2.39}$$

Summary

$$\boxed{E_P = \frac{h}{t_P} = \frac{2\pi\,m_e v_B a_0}{t_P} = 2\pi\,E_{P,\mathrm{std}} = 2\pi\left(\frac{m_e v_B a_0}{t_P}\right)} \tag{2.40}$$

2.4 Planck Frequency

Definition

$$f_P = \frac{1}{t_P} \tag{2.41}$$

By the Titus loop identity $E_P t_P = h$ Eq. (2.26),

$$E_P = h\,f_P \tag{2.42}$$

equivalently $E_P = \hbar\,\omega_P$ once $\omega_P = 2\pi f_P$ (see Eq. (2.49)).

$\hbar$-based (Std) square-root route. From $t_P^2 = \hbar G/c^5$ we have

$$f_P = \frac{1}{t_P} = \sqrt{\frac{c^5}{\hbar G}} \tag{2.43}$$

Using the Titus forms $\hbar = \dfrac{E_P t_P}{2\pi} = \dfrac{E_P \ell_P}{2\pi c}$ (with $\ell_P = c\,t_P$; Eq. (2.16) and $G = \dfrac{2\pi\,\ell_P c^4}{E_P}$ Eq. (1.31),

$$f_P = \sqrt{\frac{c^5}{\left(\frac{E_P \ell_P}{2\pi c}\right)\left(\frac{2\pi\,\ell_P c^4}{E_P}\right)}} \tag{2.44}$$

$$= \sqrt{\frac{c^5}{\ell_P^2 c^3}} = \frac{c}{\ell_P} \tag{2.45}$$

Since $c = \ell_P/t_P$ Eq. (2.16), this gives $f_P = c/\ell_P = 1/t_P$, consistent with (2.41).

Std square-root (Bohr-anchored) cancellation. Using $\hbar = m_e v_B a_0$ and the temporal form $G = \dfrac{c^5 t_P^2}{\hbar}$,

$$f_P = \sqrt{\frac{c^5}{\hbar G}} = \sqrt{\frac{c^5}{(m_e v_B a_0)\left(\frac{c^5 t_P^2}{m_e v_B a_0}\right)}} = \frac{1}{t_P} \tag{2.46}$$

Numeric (CODATA 2022)

$$f_P = \frac{1}{5.391\,247 \times 10^{-44}\,\mathrm{s}} = 1.854\,858 \times 10^{43}\,\mathrm{Hz} \tag{2.47}$$

Cross-check:

$$h\,f_P = \left(6.626\,070 \times 10^{-34}\,\mathrm{J\,s}\right)\left(1.854\,858 \times 10^{43}\,\mathrm{s^{-1}}\right) = 1.229\,042 \times 10^{10}\,\mathrm{J} \tag{2.48}$$

matching E_P from Eq. (2.26).

2.5 Planck Angular Frequency

Definition

$$\omega_P = 2\pi f_P = \frac{2\pi}{t_P} \tag{2.49}$$

Then by $E_P t_P = h$ Eq. (2.26),

$$E_P = \hbar\,\omega_P \tag{2.50}$$

which is equivalent to Eq. (2.42) since $\omega_P = 2\pi f_P$.

Std square-root form. Using $f_P = \sqrt{c^5/(\hbar G)}$ Eq. (2.43),

$$\omega_P = 2\pi f_P = \frac{2\pi}{t_P} = 2\pi\sqrt{\frac{c^5}{\hbar G}} \tag{2.51}$$

Numeric (CODATA 2022)

$$\omega_P = \frac{2\pi}{5.391\,247 \times 10^{-44}\,\mathrm{s}} = 1.165\,442 \times 10^{44}\,\mathrm{s}^{-1} \tag{2.52}$$

$$\hbar\,\omega_P = \left(\frac{6.626\,070 \times 10^{-34}\,\mathrm{J\,s}}{2\pi}\right)\left(1.165\,442 \times 10^{44}\,\mathrm{s}^{-1}\right) = 1.229\,042 \times 10^{10}\,\mathrm{J} \tag{2.53}$$

in agreement with E_P from Eq. (2.26).

2.6 Planck Wavenumber

Definition (spatial dual)

$$k_P = \frac{2\pi}{\ell_P} \tag{2.54}$$

This represents the spatial reciprocal of the Planck length and forms the direct dual to the angular frequency ω_P in the Titus lattice.

Wave–frequency closure. Using $\omega_P = 2\pi/t_P$ Eq. (2.49) and $c = \ell_P/t_P$ Eq. (2.16),

$$\frac{\omega_P}{k_P} = \frac{\frac{2\pi}{t_P}}{\frac{2\pi}{\ell_P}} = \frac{\ell_P}{t_P} = c \qquad \Rightarrow \qquad \frac{\ell_P}{t_P} = \frac{\omega_P}{k_P} \tag{2.55}$$

This is the deterministic Titus identity linking the temporal and spatial Planck intervals through the invariant phase velocity c.

Numeric (CODATA 2022)

$$k_P = \frac{2\pi}{1.616\,255 \times 10^{-35}\,\mathrm{m}} = 3.887\,496 \times 10^{35}\,\mathrm{m}^{-1} \tag{2.56}$$

$$\frac{\omega_P}{k_P} = \frac{1.165\,442 \times 10^{44}\,\mathrm{s}^{-1}}{3.887\,496 \times 10^{35}\,\mathrm{m}^{-1}} = 2.997\,925 \times 10^{8}\,\mathrm{m\,s}^{-1} = c \tag{2.57}$$

confirming Eq. (2.16) and consistency across the loop-frequency–wavenumber pair.

2.7 Planck Mass

Titus definitions

$$m_P = \frac{E_P}{c^2} \tag{2.58}$$

With $E_P t_P = h$ Eq. (2.26) and $c = \ell_P/t_P$ Eq. (2.16), this becomes the primary Titus definition of inertial mass in the lattice.

Loop-to-length–time form. From Eq. (2.58) together with $E_P = h/t_P$ and $c = \ell_P/t_P$,

$$m_P = \frac{E_P}{c^2} = \frac{\frac{h}{t_P}}{\left(\frac{\ell_P}{t_P}\right)^2} = \frac{h\,t_P}{\ell_P^2} = \frac{E_P t_P^2}{\ell_P^2} \tag{2.59}$$

$$m_{P,\text{std}} = \frac{\hbar\,t_P}{\ell_P^2} = \frac{m_P}{2\pi} \tag{2.60}$$

where the reduced form follows trivially from $h = 2\pi\hbar$.

Numeric (CODATA 2022)

$$m_P = \frac{6.626\,070 \times 10^{-34}\,\text{J s}\; 5.391\,247 \times 10^{-44}\,\text{s}}{(1.616\,255 \times 10^{-35}\,\text{m})^2} = 1.367\,494 \times 10^{-7}\,\text{kg} \tag{2.61}$$

$$m_{P,\text{std}} = \frac{1.054\,572 \times 10^{-34}\,\text{J s}\; 5.391\,247 \times 10^{-44}\,\text{s}}{(1.616\,255 \times 10^{-35}\,\text{m})^2} = 2.176\,434 \times 10^{-8}\,\text{kg} \tag{2.62}$$

Einstein relation and $E = mc^2$ equivalence. Using $\hbar = m_e v_B a_0$ and $h = 2\pi m_e v_B a_0$, the Bohr-linked expression for Planck mass becomes

$$m_P = \frac{E_P}{c^2} = \frac{2\pi\, m_e v_B a_0}{t_P\, c^2} \tag{2.63}$$

Then the Einstein relation reproduces Titus energy directly:

$$E_P = m_P c^2 = \left(\frac{2\pi\, m_e v_B a_0}{t_P\, c^2}\right) c^2 = \frac{2\pi\, m_e v_B a_0}{t_P} = \frac{h}{t_P} \tag{2.64}$$

Lattice form of the $E = mc^2$ identity. Starting from $m_P = h t_P/\ell_P^2$ Eq. (2.59) and $c = \ell_P/t_P$ Eq. (2.16),

$$E_P = m_P c^2 = \left(\frac{h\,t_P}{\ell_P^2}\right)\left(\frac{\ell_P}{t_P}\right)^2 = \frac{h}{t_P} \tag{2.65}$$

identical to the loop-action definition Eq. (2.26). The expression remains invariant when expressed in terms of angular frequency and wavenumber, using $\omega_P/k_P = c$:

$$E_P = \left(\frac{h\,t_P}{\ell_P^2}\right)\left(\frac{\ell_P}{t_P}\right)\left(\frac{\omega_P}{k_P}\right) = \frac{h\,\omega_P}{\ell_P\, k_P} \tag{2.66}$$

Reduced (Std) versions. Likewise for the reduced-action forms,

$$m_{P,\text{std}} = \frac{m_e v_B a_0}{t_P\, c^2} \qquad E_{P,\text{std}} = m_{P,\text{std}} c^2 = \frac{m_e v_B a_0}{t_P} = \frac{\hbar}{t_P} \tag{2.67}$$

This confirms the uniform mapping $E_P = 2\pi\, E_{P,\text{std}}$.

Clarification on the standard ($\hbar$-based) form. The "standard" Planck energy $E_{P,\text{std}}$ is the same definition with $\hbar$ substituted for h:

$$E_{P,\text{std}} = \frac{\hbar}{t_P} = \frac{\hbar\,\omega_P}{\ell_P\,k_P} \tag{2.68}$$

so that $E_P = 2\pi\,E_{P,\text{std}}$ follows trivially from $h = 2\pi\,\hbar$. Both yield the same numeric values when evaluated through $E_P t_P = h$.

Action-over-Planck-time viewpoint. Let $S_{\text{orb}} = mrv$ be an orbital action ($S_{\text{orb}} = \hbar$ for the Bohr ground state). Then

$$E = \left(\frac{\vartheta}{t}\right)\hbar \quad \text{(reduced action; } \vartheta \text{ is phase)} \tag{2.69}$$

$$E = \frac{2\pi}{t_P}\,\hbar = \frac{h}{t_P} \quad \text{(full-loop Titus form at } t = t_P\text{)} \tag{2.70}$$

Numeric cross-check (CODATA 2022)

$$m_P = \frac{E_P}{c^2} = \frac{1.229\,042 \times 10^{10}\ \text{J}}{(2.997\,925 \times 10^{8}\ \text{m}\,\text{s}^{-1})^2} = 1.367\,494 \times 10^{-7}\,\text{kg} \tag{2.71}$$

$$m_{P,\text{std}} = \frac{m_P}{2\pi} = 2.176\,434 \times 10^{-8}\,\text{kg} \tag{2.72}$$

2.8 Planck Momentum

Titus definition

$$p_P = \frac{E_P}{c} \tag{2.73}$$

Using $E_P = h/t_P$ and $c = \ell_P/t_P$ gives

$$p_P = \frac{E_P}{c} = \frac{\frac{h}{t_P}}{\frac{\ell_P}{t_P}} = \frac{h}{\ell_P} = \frac{E_P t_P}{\ell_P} \tag{2.74}$$

Loop-to-wavenumber form. Using $\hbar = h/2\pi$ and $k_P = 2\pi/\ell_P$,

$$p_P = \frac{h}{\ell_P} = \frac{2\pi\,\hbar}{\ell_P} = \hbar\,k_P \tag{2.75}$$

Bohr-anchored form. From $h = 2\pi m_e v_B a_0$,

$$p_P = \frac{E_P}{c} = \frac{2\pi\,m_e v_B a_0}{t_P\,c} = \frac{2\pi\,m_e v_B a_0}{\ell_P} \tag{2.76}$$

Numeric (CODATA 2022). All formulations yield the same value:

$$p_P = \frac{E_P}{c} = \frac{1.229\,042 \times 10^{10}\,\mathrm{J}}{2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}} = 4.099\,644 \times 10^{1}\,\mathrm{kg\,m\,s^{-1}} \tag{2.77}$$

$$= \frac{h}{\ell_P} = \frac{6.626\,070 \times 10^{-34}\,\mathrm{J\,s}}{1.616\,255 \times 10^{-35}\,\mathrm{m}} = 4.099\,644 \times 10^{1}\,\mathrm{kg\,m\,s^{-1}} \tag{2.78}$$

$$= \hbar\, k_P = \left(1.054\,572 \times 10^{-34}\,\mathrm{J\,s}\right)\left(3.887\,496 \times 10^{35}\,\mathrm{m^{-1}}\right) = 4.099\,644 \times 10^{1}\,\mathrm{kg\,m\,s^{-1}} \tag{2.79}$$

Reduced (Std) forms

$$p_{P,\mathrm{std}} = \sqrt{\frac{\hbar c^3}{G}} = \frac{p_P}{2\pi} \tag{2.80}$$

Substituting $\hbar = E_P \ell_P/(2\pi c)$ and $G = 2\pi \ell_P c^4/E_P$,

$$p_{P,\mathrm{std}} = \sqrt{\frac{\left(\frac{E_P \ell_P}{2\pi c}\right) c^3}{\frac{2\pi\,\ell_P c^4}{E_P}}} = \sqrt{\frac{E_P^2}{(2\pi)^2 c^2}} = \frac{E_P}{2\pi c} = \frac{p_P}{2\pi} \tag{2.81}$$

Numeric (CODATA 2022)

$$p_P = 4.099\,644 \times 10^{1}\,\mathrm{kg\,m\,s^{-1}} \qquad p_{P,\mathrm{std}} = \frac{p_P}{2\pi} = 6.524\,785\,\mathrm{kg\,m\,s^{-1}} \tag{2.82}$$

2.9 Planck Force

Definition (spatial derivative of energy). The Planck force is the ratio of Planck energy to Planck length:

$$F_P = \frac{E_P}{\ell_P} = \frac{h}{t_P\,\ell_P} \tag{2.83}$$

This represents the maximal deterministic force transmissible through a single lattice link per Planck tick.

Bohr-anchored form. Using $h = 2\pi m_e v_B a_0$ and $\ell_P = c t_P$,

$$F_P = \frac{2\pi\, m_e v_B a_0\, c}{\ell_P^2} \tag{2.84}$$

Numeric (CODATA 2022)

$$F_P = \frac{E_P}{\ell_P} = \frac{1.229\,042 \times 10^{10}\,\mathrm{J}}{1.616\,255 \times 10^{-35}\,\mathrm{m}} = 7.604\,259 \times 10^{44}\,\mathrm{N} \tag{2.85}$$

2.10 Planck Acceleration

Definition (temporal derivative of velocity). The Planck acceleration defines the rate of change of velocity per lattice tick:

$$a_P = \frac{c}{t_P} = \frac{\ell_P}{t_P^2} = \frac{\omega_P}{k_P t_P} \tag{2.86}$$

It represents the deterministic upper bound on coherent acceleration within the Titus lattice.

Bohr-anchored form. Using $v_B = \alpha c$ and substituting $c = v_B/\alpha$ gives

$$a_P = \frac{c}{t_P} = \frac{v_B/\alpha}{t_P} = \frac{v_B}{\alpha\, t_P} \tag{2.87}$$

where α and v_B define the Bohr reference coupling.

Numeric (CODATA 2022)

$$a_P = \frac{c}{t_P} = \frac{2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}}{5.391\,247 \times 10^{-44}\,\mathrm{s}} = 5.560\,726 \times 10^{51}\,\mathrm{m\,s^{-2}} \tag{2.88}$$

Bohr-anchored check with $v_B = \alpha c$ *and* $a_P = \dfrac{v_B}{\alpha t_P}$:

$$v_B = \alpha\, c = (7.297\,353 \times 10^{-3})\left(2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}\right) = 2.187\,691 \times 10^{6}\,\mathrm{m\,s^{-1}} \tag{2.89}$$

$$a_P = \frac{v_B}{\alpha\, t_P} = \frac{2.187\,691 \times 10^{6}\,\mathrm{m\,s^{-1}}}{(7.297\,353 \times 10^{-3})\, 5.391\,247 \times 10^{-44}\,\mathrm{s}} = 5.560\,726 \times 10^{51}\,\mathrm{m\,s^{-2}} \tag{2.90}$$

2.11 Planck Power

Definition (loop rate of energy). The Planck power P_P represents the rate at which one loop quantum of energy E_P is transferred per single lattice tick t_P:

$$P_P = \frac{E_P}{t_P} = \frac{h}{t_P^2} \tag{2.91}$$

This defines the maximal coherent power flux achievable within one Planck time interval—the fundamental limit of energy flow in the Titus lattice.

Lattice interpretation. In the deterministic lattice, P_P corresponds to the complete conversion of one phase loop of action h per temporal tick t_P, such that

$$P_P = \frac{h}{t_P^2} = E_P\, f_P \tag{2.92}$$

where $f_P = 1/t_P$ is the Planck frequency. Thus each Planck tick delivers one quantum of action, maintaining perfect phase continuity between adjacent nodes.

Relations to other Planck quantities. By substitution from $E_P = h/t_P$ and $F_P = E_P/\ell_P$, the Planck power links directly to force and velocity:

$$P_P = F_P\, c = \frac{h}{\ell_P t_P}\, c = \frac{hc}{\ell_P t_P} \tag{2.93}$$

It also satisfies the identity

$$P_P \;=\; u_P\, \ell_P^2\, c \tag{2.94}$$

where $u_P = h/(\ell_P^3 t_P)$ is the Planck energy density, showing that power represents the volumetric energy transport across one lattice face per Planck tick.

Numeric (CODATA 2022)

$$P_P = \frac{h}{t_P^2} = \frac{6.626\,070 \times 10^{-34}\,\mathrm{J\,s}}{(5.391\,247 \times 10^{-44}\,\mathrm{s})^2} = 2.279\,699 \times 10^{53}\,\mathrm{W} \tag{2.95}$$

This represents the deterministic maximum instantaneous power per phase tick in the Titus lattice.

2.12 Planck Angular Momentum

Definition (action quantum). The Planck angular momentum is defined as the reduced form of the loop action:

$$L_P \equiv \hbar = \frac{h}{2\pi} \tag{2.96}$$

In the Titus lattice, $\hbar$ represents the angular phase quantum—the elementary unit of rotational action corresponding to one full 2π phase cycle of a lattice node.

Equivalent forms (lattice, Bohr, and EM). Using the loop and frequency identity $\hbar = E_P/\omega_P$ Eq. (1.21), the Bohr relation $\hbar = m_e v_B a_0$ Eq. (1.24), and the electromagnetic form $\hbar = (Z_0/4\pi)\, q_P^2$ Eq. (1.22), the Planck angular momentum can be expressed equivalently as

$$L_P = \frac{E_P}{\omega_P} = m_e v_B a_0 = \frac{Z_0}{4\pi}\, q_P^2 \tag{2.97}$$

Each representation describes the same deterministic coupling between action, frequency, charge, and geometry within the Titus framework.

Bohr identity and mechanical equivalence. The classical mechanical definition of angular momentum, $L = mvr$, gives

$$L = m_e v_B a_0 = \hbar \tag{2.98}$$

demonstrating that the reduced action $\hbar$ is itself the mechanical angular momentum of one quantum orbital. In Titus lattice terms, this equality shows that angular momentum is the direct geometric product of a phase mass, a phase velocity, and a phase radius—a single closed loop of deterministic phase motion. Thus the loop action and the mechanical rotation are not distinct quantities; $\hbar$ is both the angular momentum and the elementary action of a one-phase rotation.

Wave–momentum closure. Using the Planck wavenumber $k_P = 2\pi/\ell_P$ and Planck momentum $p_P = h/\ell_P$, the ratio gives

$$\frac{p_P}{k_P} = \frac{h/\ell_P}{2\pi/\ell_P} = \frac{h}{2\pi} = \hbar = L_P \tag{2.99}$$

and is consistent with the relation $\omega_P/k_P = c$ and with the energy–momentum identities $E_P = \hbar\omega_P$ and $p_P = \hbar\, k_P$.

Numeric (CODATA 2022)

$$L_P = \hbar = \frac{h}{2\pi} = 1.054\,572 \times 10^{-34}\,\mathrm{J\,s} \tag{2.100}$$

2.13 Planck Energy Density

Definition (energy per volume). The Planck energy density or mechanical pressure is given by

$$u_P = \frac{E_P}{\ell_P^3} = \frac{h}{\ell_P^3 t_P} \tag{2.101}$$

This defines the intrinsic energy compression of one Planck cube per phase tick.

Numeric (CODATA 2022)

$$u_P = \frac{E_P}{\ell_P^3} = \frac{1.229\,042 \times 10^{10}\,\mathrm{J}}{(1.616\,255 \times 10^{-35}\,\mathrm{m})^3} = 2.910\,966 \times 10^{114}\,\mathrm{J\,m^{-3}} \tag{2.102}$$

2.14 Planck Mass Density

Definition (mass per volume). The Planck mass density follows as

$$\rho_P = \frac{m_P}{\ell_P^3} = \frac{h t_P}{\ell_P^5} \tag{2.103}$$

This expresses the deterministic mass density of one Planck volume.

Numeric (CODATA 2022)

$$\rho_P = \frac{m_P}{\ell_P^3} = \frac{1.367\,494 \times 10^{-7}\,\mathrm{kg}}{(1.616\,255 \times 10^{-35}\,\mathrm{m})^3} = 3.238\,887 \times 10^{97}\,\mathrm{kg\,m^{-3}} \tag{2.104}$$

Closure and relations. Using $m_P = E_P/c^2$ and the definitions above,

$$u_P = \frac{E_P}{\ell_P^3} = \frac{m_P c^2}{\ell_P^3} \qquad \Rightarrow \qquad \boxed{u_P = \rho_P\, c^2} \tag{2.105}$$

From the phase-density definition $\Phi_P = 1/(\ell_P^3 t_P)$ (Eq. (2.108)), Eq. (2.101) reads

$$u_P = \frac{h}{\ell_P^3 t_P} = h\,\Phi_P \tag{2.106}$$

consistent with the lattice relation summarized in Eq. (2.111).

Pressure remark (Titus-native). Here "mechanical pressure" is the work-per-volume exchanged by a single phase tick in the Titus lattice. Let u denote the energy per spatial cell (ℓ_P^3) per tick t_P. The physical pressure is defined kinematically by

$$p \equiv \left(\frac{\Delta E}{\Delta V}\right)_{t=t_P} \tag{2.107}$$

so once a medium's *phase-transport rule* is specified, one obtains an effective equation of state $p = w\,u$, with w fixed by that rule (not by GR). Typical cases in this lattice picture are: (i) isotropic, freely propagating phase flux (radiation-like) $\Rightarrow w = 1/3$; (ii) coherent, nearly incompressible phase compression $\Rightarrow w \simeq 1$; (iii) static phase offset with negligible transport (vacuum-like) $\Rightarrow w \simeq -1$. In this section u_P serves only as the energy-density scale; no tensor machinery is invoked.

2.15 Planck Phase Density

Definition and geometric origin. From the geometric relations in Eq. (2.112), the Planck 4-volume tick $\mathcal{V}_4 = \ell_P^3 t_P$ defines the fundamental spacetime cell of the Titus lattice. The phase density Φ_P represents the number of such ticks per unit 4-volume:

$$\Phi_P = \frac{1}{\mathcal{V}_4} = \frac{1}{\ell_P^3 t_P} \tag{2.108}$$

Interpretation. Φ_P quantifies the intrinsic phase resolution of spacetime itself—the total number of deterministic phase ticks per cubic meter per second in the Titus lattice. It is therefore the measure of spacetime granularity at the Planck scale: every increment of physical action corresponds to one Planck phase tick, such that Φ_P serves as the lattice's phase density constant.

Relation to frequency and energy density. Using $\omega_P = 2\pi/t_P$ Eq. (2.49) and $\ell_P = c\,t_P$ Eq. (2.16), one may express Φ_P as

$$\begin{aligned}
\Phi_P &= \frac{1}{\ell_P^3\, t_P} && \text{(definition)} \\
&= \frac{f_P}{\ell_P^3} && \text{(using } f_P = 1/t_P\text{)} \\
&= \frac{\omega_P}{2\pi\,\ell_P^3} && \text{(using } \omega_P = 2\pi f_P\text{)} \\
&= \frac{f_P}{c^3\, t_P^3} && \text{(substituting } \ell_P = ct_P\text{)} \\
&= \frac{\omega_P}{2\pi\, c^3\, t_P^3} && \text{(frequency form)} \\
&= \frac{1}{c^3\, t_P^4} && \text{(fully expanded form)} \qquad (2.109)
\end{aligned}$$

showing that Φ_P scales directly with angular frequency and inversely with the cube of the spatial and temporal lattice spacing. The energy density of one tick is then $u_P = E_P/\ell_P^3 = h/(\ell_P^3 t_P) = h\,\Phi_P$, linking mechanical energy density directly to phase density through the loop action h.

Numeric (CODATA 2022)

$$\Phi_P = \frac{1}{\left(1.616\,255 \times 10^{-35}\,\mathrm{m}\right)^3 5.391\,247 \times 10^{-44}\,\mathrm{s}} = 4.393\,202 \times 10^{147}\,\mathrm{m}^{-3}\,\mathrm{s}^{-1} \tag{2.110}$$

This value represents the deterministic phase density of the Titus lattice—that is, approximately 4.39×10^{147} discrete Planck phase ticks per cubic meter per second.

Summary. The Planck phase density forms the bridge between geometry and energy:

$$u_P = h\,\Phi_P \tag{2.111}$$

showing that the loop action h couples directly to the four-dimensional tick rate of spacetime, linking the discrete geometry of the lattice to the continuous energy field.

2.16 Geometric Basis for Phase Density

Area, volume, and 4-volume tick. The fundamental geometric scalars of the Titus lattice describe the dimensional structure of a single Planck cell:

$$A_P = \ell_P^2 \qquad V_P = \ell_P^3 \qquad \mathcal{V}_4 = \ell_P^3 t_P \qquad \Phi_P = \frac{1}{\mathcal{V}_4} \tag{2.112}$$

Here:

- A_P is the *Planck area*, representing one lattice face or 2D boundary element.
- V_P is the *Planck spatial volume*, defining the 3D cell of a single lattice node.
- $\mathcal{V}_4$ is the *Planck 4-volume tick*, the full spacetime cell volume representing one complete phase tick in the 4D lattice.
- $\Phi_P = 1/\mathcal{V}_4$ is the *Planck phase density*, the number of phase ticks per unit spacetime volume, introduced formally in the next section.

These geometric quantities provide the dimensional basis for constructing all lattice densities and fluxes that follow. They relate spatial extent (ℓ_P) and temporal extent (t_P) to a discrete 4D phase cell, ensuring a deterministic volumetric definition of Planck-scale structure.

2.17 Planck Temperature

Definition (loop and lattice forms). The Titus Planck temperature is defined as the loop energy per Boltzmann quantum,

$$T_P = \frac{E_P}{k_B} = \frac{h}{t_P\, k_B} \tag{2.113}$$

linking the Planck time and Boltzmann constant through the full Titus loop action. Using $\omega_P/k_P = c$ Eq. (2.55) and $\ell_P/t_P = c$ Eq. (2.16), this can equivalently be expressed as

$$T_P = \frac{h}{k_B\, t_P} = \frac{h\,\omega_P}{k_B\, k_P\, \ell_P} \tag{2.114}$$

This expresses the Planck temperature directly in angular–spatial lattice form, showing that thermal energy corresponds to a phase oscillation frequency within the Planck coherence cell of size (ℓ_P, t_P).

Reduced (Std) form. The conventional form employs the reduced loop energy $E_{P,\text{std}} = \hbar/t_P$:

$$T_{P,\text{std}} = \frac{E_{P,\text{std}}}{k_B} = \frac{1}{k_B}\sqrt{\frac{\hbar c^5}{G}} \tag{2.115}$$

which satisfies

$$T_P = 2\pi\, T_{P,\text{std}} \qquad T_{P,\text{std}} = \frac{E_P}{2\pi\, k_B} \tag{2.116}$$

Std → Titus via lattice substitution. Substituting $\hbar = \dfrac{E_P \ell_P}{2\pi c}$ and $G = \dfrac{2\pi\, \ell_P c^4}{E_P}$ into Eq. (2.115) gives

$$T_{P,\text{std}} = \frac{1}{k_B}\sqrt{\frac{\left(\dfrac{E_P \ell_P}{2\pi c}\right) c^5}{\dfrac{2\pi\, \ell_P c^4}{E_P}}} = \frac{E_P}{2\pi\, k_B} \tag{2.117}$$

which implies Eq. (2.116) and confirms the loop-energy form Eq. (2.113).

Bohr-anchored forms. With $\hbar = m_e v_B a_0$ and $h = 2\pi\, m_e v_B a_0$,

$$T_{P,\text{std}} = \frac{m_e v_B a_0}{t_P\, k_B} \qquad T_P = \frac{2\pi\, m_e v_B a_0}{t_P\, k_B} = 2\pi\, T_{P,\text{std}} \tag{2.118}$$

Numeric (CODATA 2022) Constants from Table 1.

$$E_{P,\mathrm{std}} = \frac{E_P}{2\pi} = \frac{1.229\,042 \times 10^{10}\,\mathrm{J}}{2\pi} = 1.956\,081 \times 10^{9}\,\mathrm{J} \tag{2.119}$$

$$T_{P,\mathrm{std}} = \frac{E_{P,\mathrm{std}}}{k_B} = \frac{1.956\,081 \times 10^{9}\,\mathrm{J}}{1.380\,649 \times 10^{-23}\,\mathrm{J\,K^{-1}}} = 1.416\,784 \times 10^{32}\,\mathrm{K} \tag{2.120}$$

$$T_P = \frac{E_P}{k_B} = \frac{1.229\,042 \times 10^{10}\,\mathrm{J}}{1.380\,649 \times 10^{-23}\,\mathrm{J\,K^{-1}}} = 8.901\,917 \times 10^{32}\,\mathrm{K} \tag{2.121}$$

Phase increment per Kelvin (modular mapping). Energy–phase mapping is given by

$$\vartheta(E) = 2\pi\,\frac{E}{E_P} \pmod{2\pi} \tag{2.122}$$

so for $E = k_B\,\Delta T$, the per-Kelvin phase increment is

$$\vartheta_{1\mathrm{K}}^{(\mathrm{Titus})} = \frac{2\pi}{T_P} = 7.058\,239 \times 10^{-33}\,\mathrm{rad\,K^{-1}} \tag{2.123}$$

and for the reduced form ($T_{P,\mathrm{std}} = T_P/2\pi$),

$$\vartheta_{1\mathrm{K}}^{(\mathrm{std})} = \frac{2\pi}{T_{P,\mathrm{std}}} = 4.434\,822 \times 10^{-32}\,\mathrm{rad\,K^{-1}} \tag{2.124}$$

These correspond to the identities $\vartheta_{1\mathrm{K}}^{(\mathrm{Titus})} = 2\pi k_B/E_P$ and $\vartheta_{1\mathrm{K}}^{(\mathrm{std})} = 2\pi k_B/E_{P,\mathrm{std}}$.

Summary

$$\boxed{T_P = \frac{E_P}{k_B} = \frac{h}{t_P\,k_B} = \frac{h\,\omega_P}{\ell_P\,k_P\,k_B} \qquad T_{P,\mathrm{std}} = \frac{E_P}{2\pi\,k_B} = \frac{1}{k_B}\sqrt{\frac{\hbar c^5}{G}} = \frac{m_e v_B a_0}{t_P\,k_B}}$$

$$\boxed{\vartheta_{1\mathrm{K}}^{(\mathrm{Titus})} = \frac{2\pi}{T_P} \qquad \vartheta_{1\mathrm{K}}^{(\mathrm{std})} = \frac{2\pi}{T_{P,\mathrm{std}}}}$$

3 Charge and Electromagnetism

3.1 Planck Charge and Fine Structure

Definitions and Titus / Std equivalences. Starting from the Titus loop identity $E_P t_P = h$ and the vacuum relations $Z_0 = \mu_0 c = 1/(\epsilon_0 c)$, the Planck charge follows in several algebraically equivalent forms:

$$q_P^2 = \frac{4\pi}{Z_0}\,m_e v_B a_0 = \frac{2E_P t_P}{Z_0} = 4\pi\,\epsilon_0\,\hbar\,c = \frac{4\pi\hbar}{Z_0} = \frac{2\pi\hbar}{Z_P} = \frac{I_P h}{V_P} = \frac{I_P E_P t_P}{V_P} \tag{3.1}$$

$$q_P = \sqrt{q_P^2} \tag{3.2}$$

Equivalence of electromagnetic forms. To see the equivalence explicitly, use $\hbar = m_e v_B a_0$ and $1/Z_0 = \epsilon_0 c$:

$$\frac{4\pi}{Z_0}\, m_e v_B a_0 = 4\pi\, \frac{\hbar}{Z_0} = 4\pi\, \hbar\, (\epsilon_0 c) = 4\pi\, \epsilon_0\, \hbar\, c \tag{3.3}$$

$$\frac{2E_P t_P}{Z_0} = \frac{2h}{Z_0} = 2(2\pi\hbar)(\epsilon_0 c) = 4\pi\, \epsilon_0\, \hbar\, c \tag{3.4}$$

demonstrating that both expressions yield the same electromagnetic invariant.

Deterministic charge (Titus electromagnetic closure). Starting from the invariant $q_P^2 = 4\pi\, \epsilon_0\, \hbar\, c$ and using $Z_0 = 1/(\epsilon_0 c)$, we have

$$q_P^2 = \frac{4\pi\, \hbar}{Z_0} \tag{3.5}$$

Since $Z_P = Z_0/2$, substitution gives

$$q_P^2 = \frac{4\pi\, \hbar}{2Z_P} = \frac{2\pi\, \hbar}{Z_P} \tag{3.6}$$

Using $Z_P = V_P/I_P$, the reciprocal relation $\frac{1}{Z_P} = \frac{I_P}{V_P}$ yields

$$q_P^2 = \frac{2\pi\, \hbar\, I_P}{V_P} = \frac{h\, I_P}{V_P} \qquad \boxed{q_P = \sqrt{\frac{h\, I_P}{V_P}}} \tag{3.7}$$

This expresses the Planck charge deterministically in terms of Planck current, Planck voltage, and the loop action h. It confirms that charge, like energy, momentum, and power, emerges directly from coherent lattice phase motion. (Units check: $[hI_P/V_P] = \mathrm{C}^2$.)

Fine-structure constant: Titus and Std routes to v_B/c (full cancellations). *Titus route (via $E_P t_P = h$).*

$$\alpha = \frac{e^2}{q_P^2}, \qquad q_P^2 = \frac{4\pi}{Z_0}\, m_e v_B a_0 = \frac{2E_P t_P}{Z_0} \tag{3.8}$$

$$\alpha = \frac{e^2 Z_0}{2E_P t_P} = \frac{e^2 Z_0}{2\left(\frac{2\pi m_e v_B a_0}{t_P}\right) t_P} = \frac{e^2 Z_0}{4\pi\, m_e v_B a_0} \tag{3.9}$$

Auxiliary identity for e^2. Using $c = \ell_P/t_P$,

$$e^2 = \alpha\, q_P^2 = \frac{v_B}{c} \cdot \frac{4\pi}{Z_0}\, m_e v_B a_0 = \frac{4\pi\, t_P\, m_e v_B a_0^2}{Z_0\, \ell_P} \tag{3.10}$$

Titus $\rightarrow v_B/c$ by direct cancellation using $\ell_P = t_P \cdot c$. Insert (3.10) into (3.9):

$$\alpha = \frac{\left(\frac{4\pi\, t_P\, m_e v_B a_0^2}{Z_0\, \ell_P}\right) Z_0}{4\pi\, m_e v_B a_0} = \frac{t_P\, v_B}{\ell_P} = \frac{t_P\, v_B}{t_P \cdot c} = \frac{v_B}{c} \tag{3.11}$$

Std route (start from the canonical definition).

$$\alpha = \frac{e^2}{4\pi\,\epsilon_0\,\hbar\,c} \tag{3.12}$$

Use Titus substitutions $\epsilon_0 = \frac{t_P}{Z_0 \ell_P}$, $\hbar = m_e v_B a_0$, and $\ell_P = t_P c$:

$$\alpha = \frac{e^2}{4\pi} \cdot \frac{Z_0 \ell_P}{t_P\,\hbar\,c} = \frac{e^2 Z_0}{4\pi\, m_e v_B a_0} \cdot \frac{\ell_P}{t_P c} = \frac{e^2 Z_0}{4\pi\, m_e v_B a_0} \tag{3.13}$$

Now reuse (3.10) to reach the same end: $\alpha = \dfrac{t_P v_B}{\ell_P} = \dfrac{v_B}{c}$

Numeric validations (CODATA 2022)

$$\alpha = \frac{v_B}{c} = \frac{2.187\,691 \times 10^6}{2.997\,925 \times 10^8} = 7.297\,353 \times 10^{-3} \tag{3.14}$$

$$\begin{aligned}\alpha &= \frac{e^2 Z_0}{4\pi\, m_e v_B a_0} \\ &= \frac{(1.602\,177 \times 10^{-19})^2}{9.109\,384 \times 10^{-31}\, 5.291\,772 \times 10^{-11}\, 2.187\,691 \times 10^6} \cdot \frac{3.767\,303 \times 10^2}{4\pi} \\ &= 7.297\,353 \times 10^{-3}\end{aligned} \tag{3.15}$$

Units check (dimensionless). $e^2 Z_0$ has units $\mathrm{C}^2 \cdot \Omega = \mathrm{kg\,m^2/s}$, while $m_e v_B a_0$ has units $\mathrm{kg\,m^2/s}$; hence their ratio is unitless, as required for α.

Numeric validations (CODATA 2022): Planck charge. *Titus (loop/impedance):*

$$q_P^2 = \frac{2h}{Z_0} = \frac{2\,6.626\,070 \times 10^{-34}\,\mathrm{J\,s}}{3.767\,303 \times 10^2\,\Omega} = 3.517\,673 \times 10^{-36}\,\mathrm{C}^2 \tag{3.16}$$

$$q_P = \sqrt{q_P^2} = 1.875\,546 \times 10^{-18}\,\mathrm{C} \tag{3.17}$$

Std/Bohr (capacitance route):

$$\begin{aligned}q_P^2 &= 4\pi\,\epsilon_0\,\hbar\,c \\ &= 4\pi\,(8.854\,188 \times 10^{-12})\,(1.054\,572 \times 10^{-34}\,\mathrm{J\,s})\,(2.997\,925 \times 10^8\,\mathrm{m\,s^{-1}})\,\mathrm{C}^2 \\ &= 3.517\,673 \times 10^{-36}\,\mathrm{C}^2\end{aligned} \tag{3.18}$$

The tiny last-digit difference between (3.16) and (3.18) comes from CODATA uncertainties/rounding in ϵ_0 (and thus Z_0). Both routes are algebraically identical via $h = 2\pi\hbar$ and $1/Z_0 = \epsilon_0 c$.

Velocity ratios for α. From the Bohr identity $\hbar = m_e v_B a_0$ and the CODATA relation $\alpha = v_B/c$ shown earlier in Eq. (1.15), and using $f_P \ell_P = c$ from Eqs. (2.41) and (2.16), we obtain the useful equivalents

$$\alpha = \frac{v_B}{c} = \frac{v_B}{f_P\,\ell_P} = \frac{v_B}{f\,\lambda} \quad \text{(for any EM mode in vacuum, where } f\lambda = c\text{)} \tag{3.19}$$

3.2 Vacuum Permeability and Permittivity

Titus identities (from $Z_0 = \mu_0 c = 1/(\epsilon_0 c)$ and $c = \ell_P/t_P$).

$$\mu_0 = \frac{Z_0}{c} = Z_0\,\frac{t_P}{\ell_P} \tag{3.20}$$

$$\epsilon_0 = \frac{1}{Z_0 c} = \frac{t_P}{Z_0\,\ell_P} \tag{3.21}$$

The second equalities use $c = \ell_P/t_P$ Eq. (2.16).

Numeric (length–time form, Titus)

$$\begin{aligned}\mu_0 &\approx Z_0\,\frac{t_P}{\ell_P}\\ &\approx \left(3.767\,303\times10^{2}\,\Omega\right)\left(\frac{5.391\,247\times10^{-44}\,\mathrm{s}}{1.616\,255\times10^{-35}\,\mathrm{m}}\right)\\ &\approx 1.256\,637\times10^{-6}\,\mathrm{N\,A^{-2}}\end{aligned} \tag{3.22}$$

$$\begin{aligned}\epsilon_0 &\approx \frac{t_P}{Z_0\,\ell_P}\\ &\approx \frac{5.391\,247\times10^{-44}\,\mathrm{s}}{3.767\,303\times10^{2}\,\Omega\,1.616\,255\times10^{-35}\,\mathrm{m}}\\ &\approx 8.854\,188\times10^{-12}\,\mathrm{F\,m^{-1}}\end{aligned} \tag{3.23}$$

These agree with the direct Z_0–c evaluations in (3.28) and (3.29).

Minimal Titus/Bohr relations. Using $Z_P = \dfrac{h}{q_P^2} = \dfrac{Z_0}{2}$ and $h = 2\pi m_e v_B a_0$:

$$Z_0 = \frac{2h}{q_P^2} = \frac{4\pi\,m_e v_B a_0}{q_P^2} \tag{3.24}$$

$$\mu_0 = \frac{Z_0}{c} = \frac{4\pi\,m_e v_B a_0}{q_P^2\,c} \tag{3.25}$$

$$\epsilon_0 = \frac{1}{Z_0 c} = \frac{q_P^2}{4\pi\,m_e v_B a_0\,c} \tag{3.26}$$

These express Z_0, μ_0, ϵ_0 directly in terms of Titus lattice quantities (E_P, t_P, q_P) and the Bohr triplet (m_e, a_0, v_B), without introducing $\hbar$ or α.

Consistency check. Using (3.20)–(3.21),

$$\mu_0\,\epsilon_0\,c^2 = \left(\frac{Z_0}{c}\right)\left(\frac{1}{Z_0 c}\right)c^2 = 1 \tag{3.27}$$

Numeric (CODATA 2022) Using Z_0 and c from Table 1 [20, 13]:

$$\mu_0 = \frac{3.767\,303 \times 10^2\,\Omega}{2.997\,925 \times 10^8\,\mathrm{m\,s^{-1}}} = 1.256\,637 \times 10^{-6}\,\mathrm{N\,A^{-2}} \tag{3.28}$$

$$\epsilon_0 = \frac{1}{(3.767\,303 \times 10^2\,\Omega)\,(2.997\,925 \times 10^8\,\mathrm{m\,s^{-1}})} = 8.854\,188 \times 10^{-12}\,\mathrm{F\,m^{-1}} \tag{3.29}$$

Alternate Bohr-anchored numeric (using e, α, h, c). With e exact, h exact, and α from CODATA 2022:

$$\begin{aligned}\epsilon_0 &= \frac{e^2}{2\alpha hc} = \frac{(1.602\,177 \times 10^{-19}\,\mathrm{C})^2}{2\,(7.297\,353 \times 10^{-3})\,6.626\,070 \times 10^{-34}\,\mathrm{J\,s}\,2.997\,925 \times 10^8\,\mathrm{m\,s^{-1}}} \\ &= 8.854\,188 \times 10^{-12}\,\mathrm{F\,m^{-1}}\end{aligned} \tag{3.30}$$

agreeing with (3.29). (Then $\mu_0 = 1/(\epsilon_0 c^2)$ reproduces (3.28).

4 Electromagnetic Quartet

4.1 Planck Power, Voltage, and Current

We collect the Planck–electromagnetic "circuit" quantities $\{V_P, I_P, P_P\}$ and derive them from the Titus lattice (E_P, t_P), using the loop identity $E_P t_P = h$ (with $h = 2\pi\hbar$). For impedance and admittance, see the next subsection.

Power (Titus vs. Std). From $E_P t_P = h$,

$$P_P^{(\mathrm{Titus})} \equiv \frac{E_P}{t_P} = \frac{h}{t_P^2} = \frac{2\pi\,\hbar}{t_P^2} \tag{4.1}$$

while the Std (reduced) form uses $E_{P,\mathrm{std}} = \hbar/t_P$:

$$P_{P,\mathrm{std}} \equiv \frac{E_{P,\mathrm{std}}}{t_P} = \frac{\hbar}{t_P^2} \tag{4.2}$$

Hence $P_P^{(\mathrm{Titus})} = 2\pi\,P_{P,\mathrm{std}}$. With $\hbar = m_e v_B a_0$,

$$P_{P,\mathrm{std}} = \frac{m_e v_B a_0}{t_P^2} \qquad P_P^{(\mathrm{Titus})} = \frac{2\pi\,m_e v_B a_0}{t_P^2} \tag{4.3}$$

Voltage–current closure (using Z_P). With $Z_P = \frac{Z_0}{2}$ (derived in Eq. (4.11) below), the quartet closure fixes the magnitudes

$$V_P = \sqrt{P_P Z_P} = \sqrt{\frac{h}{t_P^2}\,\frac{Z_0}{2}} \tag{4.4}$$

$$I_P = \sqrt{\frac{P_P}{Z_P}} = \sqrt{\frac{h}{t_P^2}\,\frac{2}{Z_0}} \tag{4.5}$$

so that $V_P I_P = P_P$ and $\frac{V_P}{I_P} = Z_P$ identically.

Equivalences via q_P (no re-derivation). Using $q_P^2 = \frac{2E_P t_P}{Z_0}$ and $E_P = h/t_P$ one finds

$$\begin{aligned} I_P &= \frac{q_P}{t_P} \\ V_P &= \frac{E_P}{q_P} \end{aligned} \tag{4.6}$$

These are algebraically equivalent to Eqs. (4.5)–(4.4) after substituting q_P.

Numeric (CODATA 2022)

$$P_P^{(\text{Titus})} = \frac{E_P}{t_P} = \frac{1.229\,042 \times 10^{10}}{5.391\,247 \times 10^{-44}}\ \text{W} = 2.279\,699 \times 10^{53}\ \text{W} \tag{4.7}$$

$$V_P = \sqrt{P_P Z_P} = \sqrt{2.279\,699 \times 10^{53}\ \text{W} \ \cdot\ 1.883\,652 \times 10^{2}\ \Omega} = 6.552\,983 \times 10^{27}\ \text{V} \tag{4.8}$$

$$I_P = \sqrt{\frac{P_P}{Z_P}} = \sqrt{\frac{2.279\,699 \times 10^{53}\ \text{W}}{1.883\,652 \times 10^{2}\ \Omega}} = 3.478\,872 \times 10^{25}\ \text{A} \tag{4.9}$$

(Here $Z_P = \frac{Z_0}{2}$ with $Z_0 = 3.767\,303 \times 10^{2}\ \Omega$.)

4.2 Planck Impedance and Admittance

Impedance (two derivations). From the voltage/current magnitudes above,

$$Z_P \equiv \frac{V_P}{I_P} = \frac{\frac{1}{t_P}\sqrt{\pi\, Z_0\, m_e v_B a_0}}{\frac{1}{t_P}\sqrt{(4\pi/Z_0)\, m_e v_B a_0}} = \sqrt{\frac{\pi\, Z_0}{4\pi/Z_0}} = \frac{Z_0}{2} \tag{4.10}$$

Alternatively, use the action–charge relation $Z_P = \dfrac{h}{q_P^2}$ with $h = 2\pi m_e v_B a_0$ and $q_P^2 = \dfrac{4\pi}{Z_0} m_e v_B a_0$:

$$Z_P = \frac{2\pi m_e v_B a_0}{(4\pi/Z_0)\, m_e v_B a_0} = \frac{Z_0}{2} \tag{4.11}$$

Admittance (reciprocal)

$$Y_P \equiv \frac{1}{Z_P} = \frac{2}{Z_0} = \frac{q_P^2}{h} = \frac{\frac{4\pi}{Z_0} m_e v_B a_0}{2\pi m_e v_B a_0} \tag{4.12}$$

showing exact reciprocal closure with Eq. (4.11).

Power identity (reference). The quartet relations hold in either representation,

$$P_P = V_P I_P = \frac{V_P^2}{Z_P} = I_P^2 Z_P = V_P^2 Y_P \tag{4.13}$$

consistent with Eqs. (4.1)–(4.2).

Numeric (CODATA 2022) With $Z_0 = 3.767\,303 \times 10^2\,\Omega$,

$$Z_P = \frac{Z_0}{2} = 1.883\,652 \times 10^2\,\Omega \tag{4.14}$$

$$Y_P = \frac{2}{Z_0} = 5.308\,837 \times 10^{-3}\,\mathrm{S} \tag{4.15}$$

satisfying $Z_P Y_P = 1$ within CODATA precision.

Dimensional meaning. Z_P (in Ω) is the Planck-scale opposition to EM action flow; Y_P (in S) is the corresponding phase admittance (conductance) of the Titus lattice.

4.3 Planck Electromagnetic Field Strengths and Densities

Energy–field basis. The *Planck energy density* u_P is defined as the Planck energy E_P stored in a single Planck spatial cell of volume ℓ_P^3 per Planck tick t_P of the lattice:

$$u_P = \frac{E_P}{\ell_P^3} = \frac{h}{\ell_P^3 t_P} \tag{4.16}$$

Here h is Planck's constant (exact in SI), ℓ_P is the Planck length, and t_P is the Planck time. Physically, u_P represents "one loop of action" worth of energy compressed into one Planck voxel per tick. In classical electrodynamics, the (time-averaged) electromagnetic energy densities are

$$u_E = \tfrac{1}{2}\epsilon_0 E^2 \qquad u_B = \tfrac{B^2}{2\mu_0} \tag{4.17}$$

where ϵ_0 is the vacuum permittivity $[\mathrm{F\,m^{-1}}]$, μ_0 is the vacuum permeability $[\mathrm{N\,A^{-2}} = \mathrm{H\,m^{-1}}]$, E is the electric-field magnitude $[\mathrm{V\,m^{-1}}]$, and B is the magnetic-flux density [T]. Equating $u_P = u_E = u_B$ yields the corresponding Planck-scale field magnitudes.

Planck electric and magnetic fields.

$$E_P^{(\text{field})} = \sqrt{\frac{2u_P}{\epsilon_0}} = \sqrt{\frac{2}{\epsilon_0}\,\frac{h}{\ell_P^3 t_P}} \tag{4.18}$$

$$B_P^{(\text{field})} = \sqrt{2\mu_0\,u_P} = \sqrt{2\mu_0\,\frac{h}{\ell_P^3 t_P}} \tag{4.19}$$

The ratio of these Planck field magnitudes reproduces the free-space wave impedance relation,

$$\frac{E_P^{(\text{field})}}{B_P^{(\text{field})}} = \frac{1}{\sqrt{\epsilon_0\mu_0}} = c \tag{4.20}$$

consistent with the Titus lattice wave identity $\omega_P/k_P = c$ (Eq. (2.55)).

Titus vs. Std forms. Using $h = 2\pi\hbar$ converts to the reduced ($\hbar$-based, "Std") field strengths,

$$E_{P,\mathrm{std}}^{(\mathrm{field})} = \sqrt{\frac{2}{\epsilon_0}\,\frac{\hbar}{\ell_P^3 t_P}} = \frac{E_P^{(\mathrm{field})}}{\sqrt{2\pi}} \tag{4.21}$$

$$B_{P,\mathrm{std}}^{(\mathrm{field})} = \sqrt{2\mu_0\,\frac{\hbar}{\ell_P^3 t_P}} = \frac{B_P^{(\mathrm{field})}}{\sqrt{2\pi}} \tag{4.22}$$

showing the uniform 2π mapping between Titus (loop) and Std (reduced) conventions.

What the densities mean (and where J_P and $n_P^{(\mathrm{charge})}$ arise from). Beyond field magnitudes, we will also use:

- *Planck current* I_P [A]: the current scale in the EM quartet defined earlier via $P_P = V_P I_P$ and $Z_P = V_P/I_P$ with $Z_P = Z_0/2$.
- *Planck current density* J_P [$\mathrm{A\,m^{-2}}$]: current per unit area when one Planck current crosses one Planck face; by geometry,

  $$J_P \equiv \frac{I_P}{\ell_P^2} \tag{4.23}$$

  It answers "how much Planck current threads a single Planck-area face."
- *Planck charge* q_P [C]: the EM charge scale defined earlier (e.g., $q_P^2 = 4\pi\,\epsilon_0\,\hbar c = 2h/Z_0$).
- *Planck charge density* $n_P^{(\mathrm{charge})}$ [$\mathrm{C\,m^{-3}}$]: charge per unit volume when one Planck charge occupies one Planck voxel; by geometry,

  $$n_P^{(\mathrm{charge})} \equiv \frac{q_P}{\ell_P^3} \tag{4.24}$$

  It answers "how much charge resides in a single Planck-volume cell."

These densities are purely geometric lifts of previously defined quartet quantities (I_P, q_P) onto the Planck area/volume elements (ℓ_P^2, ℓ_P^3).

Numerics (CODATA 2022) Using constants h, ℓ_P, t_P, ϵ_0, μ_0 from Table 1,

$$u_P = \frac{h}{\ell_P^3 t_P} = \frac{6.626\,070 \times 10^{-34}}{(1.616\,255 \times 10^{-35})^3 (5.391\,247 \times 10^{-44})} \tag{4.25}$$
$$= 2.910\,966 \times 10^{114}\,\mathrm{J\,m^{-3}} \tag{4.26}$$

$$E_P^{(\text{field})} = \sqrt{\frac{2u_P}{\epsilon_0}} = \sqrt{\frac{2(2.910\,966 \times 10^{114}\,\mathrm{J\,m^{-3}})}{8.854\,188 \times 10^{-12}\,\mathrm{F\,m^{-1}}}} \tag{4.27}$$
$$= 8.108\,849 \times 10^{62}\,\mathrm{V\,m^{-1}} \tag{4.28}$$

$$B_P^{(\text{field})} = \sqrt{2\mu_0\, u_P} = \sqrt{2(1.256\,637 \times 10^{-6}\,\mathrm{N\,A^{-2}})(2.910\,966 \times 10^{114}\,\mathrm{J\,m^{-3}})} \tag{4.29}$$
$$= 2.704\,821 \times 10^{54}\,\mathrm{T} \tag{4.30}$$

$$J_P = \frac{I_P}{\ell_P^2} = \frac{3.478\,872 \times 10^{25}\,\mathrm{A}}{(1.616\,255 \times 10^{-35})^2} \tag{4.31}$$
$$= 1.331\,738 \times 10^{95}\,\mathrm{A\,m^{-2}} \tag{4.32}$$

$$n_P^{(\text{charge})} = \frac{q_P}{\ell_P^3} = \frac{1.875\,546 \times 10^{-18}\,\mathrm{C}}{(1.616\,255 \times 10^{-35})^3} \tag{4.33}$$
$$= 4.442\,200 \times 10^{86}\,\mathrm{C\,m^{-3}} \tag{4.34}$$

All values satisfy the free-space consistency $E_P/B_P = c$ and the energy identity $u_P = \frac{1}{2}(\epsilon_0 E_P^2 + B_P^2/\mu_0)$ within CODATA precision.

Dimensional meaning. $E_P^{(\text{field})}$ and $B_P^{(\text{field})}$ are the Planck-scale field intensities that yield one Planck energy per Planck volume. J_P is the Planck current *per Planck area* (flux of the Planck current through a lattice face), and $n_P^{(\text{charge})}$ is the Planck charge *per Planck volume.* Together these quantify, at the lattice scale, how EM action density (u_P) relates to field strengths and to the flow/packing of charge.

Summary

$$\boxed{E_P^{(\text{field})} = \sqrt{\frac{2h}{\epsilon_0 \ell_P^3 t_P}} \quad B_P^{(\text{field})} = \sqrt{2\mu_0 \frac{h}{\ell_P^3 t_P}} \quad \frac{E_P^{(\text{field})}}{B_P^{(\text{field})}} = c}$$

$$\boxed{J_P = \frac{I_P}{\ell_P^2} \quad n_P^{(\text{charge})} = \frac{q_P}{\ell_P^3} \quad u_P = \frac{h}{\ell_P^3 t_P}}$$

These define the maximum electromagnetic field intensities and densities consistent with the Planck–Titus lattice structure and tie them explicitly to the Planck current I_P and Planck charge q_P via geometric area/volume factors.

5 Bohr Radius (Consistency and Evaluation)

Canonical and Titus–consistent forms. Starting from the canonical definition,

$$a_0 = \frac{\hbar}{m_e c\, \alpha} \tag{5.1}$$

the Titus loop identity $E_P t_P = h = 2\pi\hbar$ implies

$$a_0 = \frac{E_P t_P}{2\pi\, m_e c\, \alpha} \tag{5.2}$$

and using $c = \ell_P / t_P$ (i.e. $\ell_P = c\, t_P$; Eq. (2.16) gives the length–time form

$$a_0 = \frac{E_P\, t_P^2}{2\pi\, m_e\, \alpha\, \ell_P} \tag{5.3}$$

Equations (5.1), (5.2), and (5.3) are algebraically identical under the Titus substitutions.

Bohr identity $\Rightarrow$ $\alpha = v_B/c$ (full cancellation). Insert the Bohr action identity $\hbar = m_e v_B a_0$ into (5.1):

$$a_0 = \frac{m_e v_B a_0}{m_e c\, \alpha} = a_0\, \frac{v_B}{c\, \alpha} \;\Rightarrow\; 1 = \frac{v_B}{c\, \alpha} \;\Rightarrow\; \boxed{\alpha = \frac{v_B}{c}} \tag{5.4}$$

Equivalently, starting from (5.2) and using $h = 2\pi m_e v_B a_0$ and $\ell_P = c\, t_P$ leads to the same result.

Units checks. From (5.1): $[\hbar] = \mathrm{J\,s} = \mathrm{kg\,m^2\,s^{-1}}$, and $[m_e c\alpha] = (\mathrm{kg})(\mathrm{m\,s^{-1}})(1) = \mathrm{kg\,m\,s^{-1}}$. Hence $[a_0] = \mathrm{m}$. From (5.3): $[E_P t_P^2] = (\mathrm{J})(\mathrm{s^2}) = \mathrm{kg\,m^2}$ and $[m_e \alpha \ell_P] = (\mathrm{kg})(1)(\mathrm{m}) = \mathrm{kg\,m}$, so $[a_0] = \mathrm{m}$.

Numerical validation (CODATA 2022) All three routes yield the same value.

Canonical ($\hbar$, m_e, c, α):

$$\begin{aligned} a_0 &= \frac{1.054\,572 \times 10^{-34}\,\mathrm{J\,s}}{(9.109\,384 \times 10^{-31}\,\mathrm{kg})(2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}})\,(7.297\,353 \times 10^{-3})} \\ &= 5.291\,772 \times 10^{-11}\,\mathrm{m} \end{aligned} \tag{5.5}$$

Titus loop form (E_P, t_P, m_e, c, α):

$$\begin{aligned} a_0 &= \frac{1.229\,042 \times 10^{10}\,\mathrm{J}\,(5.391\,247 \times 10^{-44}\,\mathrm{s})}{2\pi\,(9.109\,384 \times 10^{-31}\,\mathrm{kg})(2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}})\,(7.297\,353 \times 10^{-3})} \\ &= 5.291\,772 \times 10^{-11}\,\mathrm{m} \end{aligned} \tag{5.6}$$

Length–time form (E_P, t_P, ℓ_P, m_e, α):

$$\begin{aligned} a_0 &= \frac{1.229\,042 \times 10^{10}\,\mathrm{J}\,(5.391\,247 \times 10^{-44}\,\mathrm{s})^2}{2\pi\,(9.109\,384 \times 10^{-31}\,\mathrm{kg})\,(7.297\,353 \times 10^{-3})\,1.616\,255 \times 10^{-35}\,\mathrm{m}} \\ &= 5.291\,772 \times 10^{-11}\,\mathrm{m} \end{aligned} \tag{5.7}$$

Velocity check (for completeness). From (5.4), $v_B = \alpha c$ gives

$$v_B = (7.297\,353 \times 10^{-3})\,(2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}) = 2.187\,691 \times 10^{6}\,\mathrm{m\,s^{-1}} \tag{5.8}$$

consistent with earlier evaluations and confirming $\alpha = v_B/c$.

6 Elementary Charge (Closed Loop)

Closed-loop identities (Titus and Std, step by step). Starting from the Titus/Bohr and vacuum relations Eq. (3.1),

$$\begin{aligned} q_P^2 &= \frac{4\pi}{Z_0}\, m_e a_0 v_B \\ &= \frac{2E_P t_P}{Z_0} \\ &= 4\pi\,\epsilon_0\,\hbar\,c \end{aligned} \tag{6.1}$$

and the definition $\alpha = \dfrac{e^2}{q_P^2}$, we obtain the equivalent forms for the elementary charge:

$$e^2 = \alpha\, q_P^2 \tag{6.2}$$

$$= \alpha\,\frac{4\pi}{Z_0}\, m_e v_B a_0 \tag{6.3}$$

$$= \alpha\,\frac{2E_P t_P}{Z_0} \tag{6.4}$$

$$= 4\pi\,\epsilon_0\,\hbar\,c\,\alpha \tag{6.5}$$

Equivalences: (6.3)→(6.5) uses $1/Z_0 = \epsilon_0 c$ and $\hbar = m_e v_B a_0$; (6.4)→(6.5) uses $c = \ell_P/t_P$ and $E_P t_P = 2\pi\hbar$.

Length–time (lattice) variants. Using $\alpha = \dfrac{v_B}{c}$ and $c = \dfrac{\ell_P}{t_P}$,

$$e^2 = \alpha\,\frac{4\pi}{Z_0}\, m_e v_B a_0 = \frac{4\pi\, m_e v_B a_0^2}{Z_0\, c} = \boxed{\frac{4\pi\, t_P\, m_e v_B a_0^2}{Z_0\,\ell_P}} \tag{6.6}$$

$$e^2 = \alpha\,\frac{2E_P t_P}{Z_0} = \frac{2E_P t_P}{Z_0}\,\frac{v_B}{c} = \boxed{\frac{2E_P\, t_P^2\, v_B}{Z_0\,\ell_P}} \tag{6.7}$$

These show e^2 directly in terms of the Titus lattice triplet (E_P, t_P, ℓ_P) and the Bohr triplet (m_e, a_0, v_B).

Units checks. From (6.4): $[\alpha] = 1$ and $[q_P^2] = \mathrm{C}^2 \Rightarrow [e^2] = \mathrm{C}^2$. From (6.5): $[\epsilon_0] = \mathrm{C/(V\,m)}$, $[\hbar] = \mathrm{J\,s} = \mathrm{V\,C\,s}$, $[c] = \mathrm{m/s} \Rightarrow [\epsilon_0\hbar c] = \mathrm{C}^2$; with $[\alpha] = 1$, $[e^2] = \mathrm{C}^2$.

Numeric validations (CODATA 2022) Two independent routes give the same value (within the small uncertainties of non-exact constants). We keep numbers compact to fit line width.

Titus route (using (6.4)*):*

$$\begin{aligned} e^2 &= \alpha\,\frac{2E_P t_P}{Z_0} \\ &= \left(7.297\,353\times10^{-3}\right)\frac{2\,(1.229\,042\times10^{10})\,(5.391\,247\times10^{-44})}{3.767\,303\times10^{2}}\ \mathrm{C}^2 \\ &= 2.566\,970\times10^{-38}\ \mathrm{C}^2 \end{aligned} \tag{6.8}$$

Std route (using (6.5)*):*

$$\begin{aligned} e^2 &= 4\pi\,\epsilon_0\,\hbar\,c\,\alpha \\ &= 4\pi\left(8.854\,188\times10^{-12}\right)\left(1.054\,572\times10^{-34}\right)\left(2.997\,925\times10^{8}\right)\left(7.297\,353\times10^{-3}\right)\mathrm{C}^2 \\ &= 2.566\,970\times10^{-38}\ \mathrm{C}^2 \end{aligned} \tag{6.9}$$

Exact comparison from the SI definition of e:

$$e^2_{\text{exact}} = \left(1.602\,177\times10^{-19}\ \mathrm{C}\right)^2 = 2.566\,970\times10^{-38}\ \mathrm{C}^2 \tag{6.10}$$

The tiny difference between (6.8)–(6.9) and (6.10) reflects the experimental uncertainty in α and for the Std route in ϵ_0 as inferred via α, whereas e is exact in the SI.

7 Unified Planck-Scale Two-Plane Throttling, Phase–Time Reciprocity, and Applications

We model the lattice with two planes: *time* and *length*. Gravity throttles both planes equally (two-plane), while velocity dilates the time plane only (one-plane). Planck's constant h is invariant as the action per full 2π of phase; frequencies/energies shift because the lattice dilates the tick. We derive the orbit-lock condition, map energy to phase/time, and work through orbital and neutron-star examples, plus impedance/atomic checks.

7.1 Lattice Throttling Framework (Two-Plane) and Redshift

In the Titus framework, gravitational potential throttles the local phase channels (time tick t_P and spatial step ℓ_P) while preserving $c = \ell_P/t_P$. Define the dimensionless potential parameter

$$\theta(r) \;\equiv\; \frac{2\,G\,M}{r\,c^2} \;=\; \frac{r_s}{r} \tag{7.1}$$

where r_s is the Schwarzschild radius. This dimensionless form is used throughout; the full angular representation $\theta = (|\phi_g|/c^2)\,2\pi$ appears in [25].

The **throttle (phase index)** is

$$k(r) \;=\; \frac{1}{\sqrt{1-\theta(r)}} \;=\; 1 + \frac{\theta}{2} + \mathcal{O}(\theta^2) \tag{7.2}$$

At each lattice node,

$$t'_P(r) = k\, t_P \qquad \ell'_P(r) = k\, \ell_P \qquad \frac{\ell'_P}{t'_P} = c \quad \text{(invariant)} \tag{7.3}$$

For light emitted at radius r and received at infinity, the Schwarzschild redshift gives [5, 6, 7]

$$f_\infty = f_0 \sqrt{1-\theta(r)} = \frac{f_0}{k(r)} \tag{7.4}$$

and, because $c = f\lambda$ is invariant,

$$\lambda_\infty = k(r)\, \lambda_0 \tag{7.5}$$

Defining the gravitational redshift parameter,

$$z \;\equiv\; \frac{\lambda_\infty - \lambda_0}{\lambda_0} = k - 1 \tag{7.6}$$

the first-order (weak-field) relations follow directly:

$$\frac{\Delta t}{t} = \frac{\Delta \ell}{\ell} = +\frac{\theta}{2} \qquad \frac{\Delta f}{f} = -\frac{\theta}{2} \qquad \frac{\Delta \lambda}{\lambda} = +\frac{\theta}{2} \tag{7.7}$$

7.2 Action–Phase Mapping and Cross-Field Comparison (One-Plane vs Two-Plane)

Planck anchors and constants

$$c = 2.997\,925 \times 10^{8}\,\text{m/s} \quad \text{(exact)} \qquad h = 6.626\,070 \times 10^{-34}\,\text{J s} \qquad \hbar = \frac{h}{2\pi} \tag{7.8}$$

$$t_P = 5.391\,247 \times 10^{-44}\,\text{s} \qquad f_P = \frac{1}{t_P} = 1.854\,858 \times 10^{43}\,\text{Hz} \qquad E_P = \frac{h}{t_P} \tag{7.9}$$

Planck energy identity:

$$E_P \;=\; \frac{2\pi\,\hbar}{t_P} \;=\; \hbar\omega_P \;=\; \frac{h}{t_P} \tag{7.10}$$

Energy from phase per time. Let ϑ be the phase angle accumulated over interval t':

$$E \;=\; \hbar\,\frac{\vartheta}{t'} \tag{7.11}$$

so per baseline tick t_P,

$$\frac{E}{E_P} \;=\; \frac{\vartheta}{2\pi}\,\frac{t_P}{t'} \tag{7.12}$$

Cross-field (and kinematic) comparison. Let the emitter have $(k_g, k_v)_{\text{em}}$ and the observer $(k_g, k_v)_{\text{obs}}$, where the gravitational throttle is two-plane and the (transverse) kinematic dilation is one-plane:

$$\frac{f_{\text{obs}}}{f_{\text{em}}} \;=\; \frac{k_{g,\text{obs}}}{k_{g,\text{em}}} \cdot \frac{k_{v,\text{obs}}}{k_{v,\text{em}}} \qquad \frac{E_{\text{obs}}}{E_{\text{em}}} = \frac{f_{\text{obs}}}{f_{\text{em}}} \qquad \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} = \left(\frac{f_{\text{obs}}}{f_{\text{em}}}\right)^{-1} \tag{7.13}$$

If the local tick dilates to $t' = k_{\text{tot}}\, t_P$ with

$$k_{\text{tot}} \;=\; k_g\, k_v \tag{7.14}$$

then keeping the *local* energy/action invariant requires

$$\vartheta' \;=\; k_{\text{tot}}\, \vartheta_0 \qquad E_{\text{local}} \;=\; \hbar\, \frac{\vartheta'}{t'} \;=\; \hbar\, \frac{\vartheta_0}{t_P} \;=\; E_{\text{em}} \tag{7.15}$$

and a different observer measures

$$\frac{E_{\text{obs}}}{E_P} \;=\; \frac{f_{\text{em}} t_P}{k_{\text{tot}}} \;=\; \frac{E_{\text{em}}}{E_P}\, \frac{1}{k_{\text{tot}}} \tag{7.16}$$

7.3 Orbit Lock: Two-Plane vs One-Plane Balance

Let $\beta = v/c$, $\gamma_v = 1/\sqrt{1-\beta^2}$. Velocity spreads phase only in the time plane; gravity stretches both tick and step. In the weak-limit, a circular orbit corresponds to a constant phase density along the path:

$$\delta_g \simeq 2\, \delta_v \tag{7.17}$$

with $\delta_g \simeq \theta/2$ and $\delta_v \simeq \beta^2/2$, giving

$$\frac{\theta}{2} \simeq 2\, \frac{\beta^2}{2} \quad \Rightarrow \quad \boxed{\theta \;=\; 2\beta^2} \tag{7.18}$$

Using (7.1),

$$\boxed{v^2 = \frac{GM}{r}} \tag{7.19}$$

i.e. the standard circular condition; the factor of two reflects gravity acting on both planes while velocity acts on one [5, 6, 7].

7.4 Constants and Planck/Titus anchors

Unless otherwise specified, CODATA 2022 [1]:

$$G = 6.674\,300 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2} \qquad c = 2.997\,925 \times 10^{8}\,\text{m/s (exact)} \tag{7.20}$$

$$M_\oplus = 5.972\,000 \times 10^{24}\,\text{kg} \qquad M_\odot = 1.988\,470 \times 10^{30}\,\text{kg} \tag{7.21}$$

Mean orbital radii:

$$r_{\text{Moon}\leftarrow\oplus} = 3.844\,000 \times 10^{8}\,\text{m} \quad r_{\oplus\leftarrow\odot} = 1.495\,979 \times 10^{11}\,\text{m} \quad r_{\text{Mars}\leftarrow\odot} = 2.279\,392 \times 10^{11}\,\text{m} \tag{7.22}$$

Titus Planck-frequency anchor:

$$f_P = 1.854\,858 \times 10^{43}\,\text{Hz} \quad t_P = \frac{1}{f_P} = 5.391\,247 \times 10^{-44}\,\text{s} \quad \ell_P = c\,t_P = 1.616\,255 \times 10^{-35}\,\text{m} \tag{7.23}$$

7.5 Worked Examples: Moon, Earth, and Mars (Circular)

For each case, $v = \sqrt{GM/r}$, $\beta^2 = (v/c)^2$, $\theta = 2GM/(rc^2)$, $k_g = 1/\sqrt{1-\theta}$, $k_v = 1/\sqrt{1-\beta^2}$, $\delta_g = k_g - 1$, $\delta_v = k_v - 1$, and per-tick dilations $R_{t,g} = \delta_g t_P$, $R_{t,v} = \delta_v t_P$, $\Delta\ell_g = \delta_g \ell_P$, $\Delta\ell_v = \delta_v \ell_P$. We report $\delta_g/(2\delta_v)$.

(1) Moon about Earth

$$r = 3.844\,000 \times 10^{8}\,\text{m} \qquad v = \sqrt{\frac{GM_\oplus}{r}} = 1.018\,289 \times 10^{3}\,\text{m/s} \tag{7.24}$$

$$\beta^2 = 1.153\,721 \times 10^{-11} \qquad \theta = 2.307\,442 \times 10^{-11} \tag{7.25}$$

$$k_g = 1.000\,000 \qquad k_v = 1.000\,000 \tag{7.26}$$

$$\delta_g = 1.153\,722 \times 10^{-11} \qquad \delta_v = 5.768\,497 \times 10^{-12} \tag{7.27}$$

$$\frac{\delta_g}{2\delta_v} = 1.000\,019 \tag{7.28}$$

$$R_{t,g} = 6.219\,523 \times 10^{-55}\,\text{s} \qquad R_{t,v} = 3.109\,702 \times 10^{-55}\,\text{s} \tag{7.29}$$

$$\Delta\ell_g = 1.864\,566 \times 10^{-46}\,\text{m} \qquad \Delta\ell_v = 9.322\,651 \times 10^{-47}\,\text{m} \tag{7.30}$$

(2) Earth about the Sun (near 1 AU)

$$r = 1.495\,979 \times 10^{11}\,\text{m} \qquad v = \sqrt{\frac{GM_\odot}{r}} = 2.978\,514 \times 10^{4}\,\text{m/s} \tag{7.31}$$

$$\beta^2 = 9.870\,927 \times 10^{-9} \qquad \theta = 1.974\,185 \times 10^{-8} \tag{7.32}$$

$$k_g = 1.000\,000 \qquad k_v = 1.000\,000 \tag{7.33}$$

$$\delta_g = 9.870\,927 \times 10^{-9} \qquad \delta_v = 4.935\,464 \times 10^{-9} \tag{7.34}$$

$$\frac{\delta_g}{2\delta_v} = 1.000\,000 \tag{7.35}$$

$$R_{t,g} = 5.321\,255 \times 10^{-52}\,\text{s} \qquad R_{t,v} = 2.660\,627 \times 10^{-52}\,\text{s} \tag{7.36}$$

$$\Delta\ell_g = 1.595\,272 \times 10^{-43}\,\text{m} \qquad \Delta\ell_v = 7.976\,360 \times 10^{-44}\,\text{m} \tag{7.37}$$

(3) Mars about the Sun

$$r = 2.279\,392 \times 10^{11}\,\mathrm{m} \qquad v = \sqrt{\frac{GM_\odot}{r}} = 2.412\,975 \times 10^{4}\,\mathrm{m/s} \tag{7.38}$$

$$\beta^2 = 6.478\,349 \times 10^{-9} \qquad \theta = 1.295\,670 \times 10^{-8} \tag{7.39}$$

$$k_g = 1.000\,000 \qquad k_v = 1.000\,000 \tag{7.40}$$

$$\delta_g = 6.478\,349 \times 10^{-9} \qquad \delta_v = 3.239\,174 \times 10^{-9} \tag{7.41}$$

$$\frac{\delta_g}{2\delta_v} = 1.000\,000 \tag{7.42}$$

$$R_{t,g} = 3.492\,371 \times 10^{-52}\,\mathrm{s} \qquad R_{t,v} = 1.746\,186 \times 10^{-52}\,\mathrm{s} \tag{7.43}$$

$$\Delta\ell_g = 1.046\,987 \times 10^{-43}\,\mathrm{m} \qquad \Delta\ell_v = 5.234\,933 \times 10^{-44}\,\mathrm{m} \tag{7.44}$$

7.6 540 nm Test Line: Earth, Sun, and Neutron Stars

Constants used (CODATA 2022) [1]:

$$c = 2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}} \qquad \text{(exact)}$$
$$G = 6.674\,300 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$$
$$M_\oplus = 5.972\,000 \times 10^{24}\,\mathrm{kg} \qquad R_\oplus = 6.371\,000 \times 10^{6}\,\mathrm{m}$$
$$M_\odot = 1.988\,470 \times 10^{30}\,\mathrm{kg} \qquad R_\odot = 6.963\,400 \times 10^{8}\,\mathrm{m}$$

Baseline $\lambda_0 = 5.400\,000 \times 10^{2}\,\mathrm{nm} = 5.400\,000 \times 10^{-7}\,\mathrm{m}$ gives

$$f_0 = \frac{c}{\lambda_0} = \frac{2.997\,925 \times 10^{8}\,\mathrm{m/s}}{5.400\,000 \times 10^{-7}\,\mathrm{m}} = 5.551\,712 \times 10^{14}\,\mathrm{Hz} \tag{7.45}$$

Earth surface → infinity

$$\theta_\oplus = \frac{2GM_\oplus}{R_\oplus c^2} = 1.392\,216 \times 10^{-9} \tag{7.46}$$

$$k_\oplus = \frac{1}{\sqrt{1-\theta_\oplus}} = 1.000\,000 \tag{7.47}$$

Using (7.4)–(7.5):

$$f_\infty = \frac{f_0}{k_\oplus} = 5.551\,708 \times 10^{14}\,\mathrm{Hz} \qquad \Delta f = -3.864\,590 \times 10^{5}\,\mathrm{Hz} \tag{7.48}$$

$$\lambda_\infty = k_\oplus\,\lambda_0 = 5.400\,000 \times 10^{2}\,\mathrm{nm} \qquad \Delta\lambda = 3.758\,980 \times 10^{-7}\,\mathrm{nm} \tag{7.49}$$

Sun photosphere → infinity

$$\theta_\odot = \frac{2GM_\odot}{R_\odot c^2} = 4.241\,232 \times 10^{-6} \qquad k_\odot = \frac{1}{\sqrt{1-\theta_\odot}} = 1.000\,002 \tag{7.50}$$

$$f_\infty = \frac{f_0}{k_\odot} = 5.551\,700 \times 10^{14}\,\mathrm{Hz} \qquad \lambda_\infty = k_\odot\,\lambda_0 = 5.400\,011 \times 10^{2}\,\mathrm{nm} \tag{7.51}$$

Neutron stars (surface → infinity). For $M = \{1.4, 2.0\}\,M_\odot$ and $R = \{10, 12\}$ km:

Case	θ	k	z	f_∞ [THz]	λ_∞ [nm]	$\Delta\lambda$ [nm]
M=1.4 $M_\odot$, R=1.200 000 × 10^1 km	0.3446	1.2352	0.2352	449.46	667.0	+127.0
M=1.4 $M_\odot$, R=1.000 000 × 10^1 km	0.4135	1.3057	0.3057	425.18	705.1	+165.1
M=2.0 $M_\odot$, R=1.200 000 × 10^1 km	0.4922	1.4033	0.4033	395.61	757.8	+217.8
M=2.0 $M_\odot$, R=1.000 000 × 10^1 km	0.5907	1.5630	0.5630	355.19	844.0	+304.0

Force from throttle gradient. Interpreting $n \equiv k$ as a phase index, the weak-field geodesic acceleration recovers Newton's law:

$$\mathbf{a} = c^2 \nabla \ln n \;\Rightarrow\; a_r = c^2 \frac{d}{dr} \ln\!\left(\frac{1}{\sqrt{1-\theta}}\right) = -\frac{GM}{r^2} \tag{7.52}$$

7.7 Neutron-Star Redshift with Local Phase Compensation (540 nm)

Using the same line, $\lambda_{\rm em} = 5.400\,000 \times 10^2$ nm gives $f_{\rm em} = c/\lambda_{\rm em} = 5.551\,712 \times 10^{14}$ Hz, $E_{\rm em} = h f_{\rm em} = 3.678\,603 \times 10^{-19}$ J, and $E_{\rm em}/E_P = f_{\rm em} t_P = 2.992\,837 \times 10^{-29}$. The baseline phase per tick is

$$\vartheta_0 \;=\; 2\pi \frac{E_{\rm em}}{E_P} \;=\; 1.880\,455 \times 10^{-28} \text{ rad per } t_P \tag{7.53}$$

Gravity only: $M = 1.4\,M_\odot,\;\; R = 1.200\,000 \times 10^1$ **km.**

$$\theta = \frac{2GM}{Rc^2} = 3.445\,563 \times 10^{-1} \qquad\qquad k_g = \frac{1}{\sqrt{1-\theta}} = 1.235\,186 \tag{7.54}$$

Observer at infinity ($k_{g,\rm obs} = 1$):

$$f_\infty = \frac{f_{\rm em}}{k_g} = 4.494\,637 \times 10^{14}\ \text{Hz} \qquad \lambda_\infty = k_g\,\lambda_{\rm em} = 6.670\,000 \times 10^2\ \text{nm} \tag{7.55}$$

$$E_\infty = \frac{E_{\rm em}}{k_g} = 2.978\,178 \times 10^{-19}\ \text{J} \qquad \frac{E_\infty}{E_P} = \frac{E_{\rm em}}{E_P} \cdot \frac{1}{k_g} = 2.422\,985 \times 10^{-29} \tag{7.56}$$

Local phase compensation (source frame) keeping $E_{\rm em}$ fixed:

$$\vartheta'_{\rm NS} \;=\; k_g\,\vartheta_0 \;=\; 2.322\,711 \times 10^{-28} \text{ rad per } t_P \qquad t'_P = k_g\,t_P \tag{7.57}$$

Gravity + velocity (equatorial spin $f_{\rm spin} = 3.000\,000 \times 10^2$ Hz). Equatorial speed $v = 2\pi R f_{\rm spin} = 2.262\,000 \times 10^7$ m/s, $\beta = 7.545\,040 \times 10^{-2}$, $k_v = \gamma = 1.002\,859$.

$$k_{\rm tot} = k_g\,k_v = 1.238\,717 \tag{7.58}$$

$$f_\infty = \frac{f_{\rm em}}{k_{\rm tot}} = 4.482\,060 \times 10^{14}\ \text{Hz} \qquad \lambda_\infty = k_{\rm tot}\,\lambda_{\rm em} = 6.689\,070 \times 10^2\ \text{nm} \tag{7.59}$$

Local compensation:

$$\vartheta'_{\rm NS+spin} \;=\; k_{\rm tot}\,\vartheta_0 \;=\; 2.329\,000 \times 10^{-28} \text{ rad per } t_P \qquad t'_P \;=\; k_{\rm tot}\,t_P \tag{7.60}$$

7.8 Fine-Structure Constant Validation via Lattice Throttling

The fine-structure constant couples the Bohr velocity to the Planck lattice ratio:

$$\alpha = \frac{v_B}{c} = v_B \, \frac{t_P}{\ell_P} \tag{7.61}$$

Because $c = \ell_P/t_P$, throttling t_P and ℓ_P by the same factor k leaves their ratio and therefore α invariant:

$$\alpha' = v'_B \frac{t'_P}{\ell'_P} = v'_B \frac{k_t}{k_\ell} \frac{t_P}{\ell_P} \;\Rightarrow\; v'_B = v_B \, \frac{k_\ell}{k_t} \tag{7.62}$$

If time dilates slightly more than length ($k_t > k_\ell$), the local orbital velocity v'_B decreases to preserve α, producing the same fractional frequency shift as gravitational redshift.

Numerical validation. Since $t_P/\ell_P = 1/c$, the Bohr radius a_0 tests the relation:

$$a_0 \, \frac{t_P}{\ell_P} = \frac{a_0}{c} \tag{7.63}$$

Using CODATA 2022 constants ($a_0 = 5.29177210903 \times 10^{-11}\,\mathrm{m}$, $c = 2.99792458 \times 10^{8}\,\mathrm{m/s}$)

$$\begin{aligned} \frac{1}{c} &= 3.33564095198 \times 10^{-9} \text{ s/m} \\ \frac{a_0}{c} &= (5.29177210903 \times 10^{-11})(3.33564095198 \times 10^{-9}) = 1.7651451755 \times 10^{-19} \text{ s} \end{aligned} \tag{7.64}$$

Hence

$$\boxed{a_0 \, (t_P/\ell_P) = 1.76515 \times 10^{-19}\,\mathrm{s}}$$

the light-travel time across one Bohr radius (0.1765 as).

7.9 Lattice Impedance and Coherent Energy Throughput (Planck Application)

Classically, the impedance of free space is

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \mu_0 c = \frac{1}{\varepsilon_0 c} \tag{7.65}$$

with $c = 1/\sqrt{\mu_0 \varepsilon_0}$ ensuring $u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2} B^2/\mu_0 = u_B$, so $E/H = Z_0 \approx 3.767\,303 \times 10^2\,\Omega$. Within the Titus framework, the Planck loop $E_P t_P = h$ and the Planck charge [1]

$$q_P^2 = \frac{2h}{Z_0} \tag{7.66}$$

lead to a lattice (Planck) impedance

$$Z_P \equiv \frac{h}{q_P^2} = \frac{Z_0}{2} \tag{7.67}$$

Thus the effective vacuum impedance during complete lattice coherence is one-half the Maxwell value.

Power flow and energy partition. The time-averaged Poynting vector is

$$S = \frac{E^2}{Z_{\text{eff}}} \tag{7.68}$$

so at $Z_{\text{eff}} = Z_0/2$

$$S_P = \frac{E^2}{(Z_0/2)} = 2\,\frac{E^2}{Z_0} = 2S_0 \tag{7.69}$$

Energy-density ratio follows from

$$\frac{u_E}{u_B} = \frac{Z_{\text{eff}}^2}{Z_0^2} \quad \Rightarrow \quad \frac{u_E}{u_B}\bigg|_{Z_0/2} = \frac{1}{4} \tag{7.70}$$

Numeric check. With $Z_0 = 3.767\,303 \times 10^2\,\Omega$ [1],

$$Z_P = \frac{Z_0}{2} = 1.883\,652 \times 10^2\,\Omega \tag{7.71}$$

At $E = 1.000\,000\,\text{V/m}$:

$$S_0 = \frac{1}{376.730313} = 2.655 \times 10^{-3}\ \text{W/m}^2 \tag{7.72}$$

$$S_P = \frac{1}{188.3651565} = 5.310 \times 10^{-3}\ \text{W/m}^2 \tag{7.73}$$

so $S_P/S_0 = 2.000$, and $(u_E/u_B)_{Z_0/2} = 0.25$.

7.10 Selected Dimensionless Checks

- Sensitivity of magnetic permeability to α: $\dfrac{d\mu_0}{d\alpha} = 1.722 \times 10^{-4}$, from $Z_0 = 2\alpha R_K$ and $2R_K/c$ [1].
- Mass ratio $\delta = m_e/m_p = 5.446 \times 10^{-4}$, using $m_p = 1.67262192595 \times 10^{-27}\,\text{kg}$ (NIST) [23].
- Dilation parameter reminder: $\theta = 2GM/(rc^2) = r_s/r$ (dimensionless) used here; the full angular-phase form $\theta = (|\phi_g|/c^2)\,2\pi$ is discussed in [25].

7.11 Interpretation and Scope

Across orbital examples, $\delta_g \approx 2\delta_v$ within rounding: the two-plane gravitational throttle offsets the one-plane velocity phase-spread so that phase density along the path is locked and the trajectory closes—the circular-orbit state. In strong fields, gravity dilates time (and length), and local phase per tick grows by the same factor to keep the local action/energy invariant; a distant observer measures multiplicative redshifts set by the k-ratio law (7.13). Classic factor-of-two effects (e.g., light bending) likewise reflect the sum of time- and length-plane contributions [5, 6, 7]. For precision timing (e.g., GNSS), gravitational and kinematic dilations combine as observed [8]. Atomic-scale checks (Sec. 7.8) and impedance scaling (Sec. 7.9) numerically align with the Planck-loop $E_P t_P = h$ and the two-plane throttling picture.

Appendix: Raw Numeric Values

Using Table 1. Titus ("loop") values use $E_P = h/t_P$; Std ("reduced") values use $E_{P,\text{std}} = \hbar/t_P$.

Energy and Frequency

$$E_P = 1.229\,042 \times 10^{10}\,\text{J} \qquad E_{P,\text{std}} = 1.956\,081 \times 10^{9}\,\text{J}$$
$$f_P = 1.854\,858 \times 10^{43}\,\text{Hz} \qquad \omega_P = 1.165\,442 \times 10^{44}\,\text{s}^{-1}$$

Wavenumber and ω/k Check

$$k_P = 3.887\,496 \times 10^{35}\,\text{m}^{-1} \qquad \frac{\omega_P}{k_P} = 2.997\,925 \times 10^{8}\,\text{m}\,\text{s}^{-1} \;=\; c$$

Mass and Momentum

$$m_P = 1.367\,494 \times 10^{-7}\,\text{kg} \qquad p_P = 4.099\,644 \times 10^{1}\,\text{kg}\,\text{m}\,\text{s}^{-1}$$

Mass Identities: $m_P = h\,t_P/\ell_P^2, \quad m_{P,\text{std}} = \hbar\,t_P/\ell_P^2$
Momentum Identities: $p_P = h/\ell_P = \hbar\,k_P = 4.099\,644 \times 10^{1}\,\text{kg}\,\text{m}\,\text{s}^{-1}$

Force and Densities

$$F_P = 7.604\,259 \times 10^{44}\,\text{N} \qquad u_P \equiv \frac{E_P}{\ell_P^3} = 2.910\,966 \times 10^{114}\,\text{Pa}$$
$$\rho_P = 3.238\,887 \times 10^{97}\,\text{kg}\,\text{m}^{-3} \qquad \Phi_P \equiv \frac{1}{\ell_P^3 t_P} = 4.393\,202 \times 10^{147}\,\text{m}^{-3}\,\text{s}^{-1}$$

Temperature and Phase-per-K

$$T_P = 8.901\,917 \times 10^{32}\,\text{K} \qquad T_{P,\text{std}} = 1.416\,784 \times 10^{32}\,\text{K}$$
$$\vartheta_{1\text{K}}^{(\text{Titus})} = 7.058\,239 \times 10^{-33}\,\text{rad}\,\text{K}^{-1} \qquad \vartheta_{1\text{K}}^{(\text{std})} = 4.434\,822 \times 10^{-32}\,\text{rad}\,\text{K}^{-1}$$

EM Constants and Quartet

$$Z_0 = 3.767\,303 \times 10^{2}\,\Omega \qquad Z_P = 1.883\,652 \times 10^{2}\,\Omega$$
$$\mu_0 = 1.256\,637 \times 10^{-6}\,\text{N}\,\text{A}^{-2} \qquad \epsilon_0 = 8.854\,188 \times 10^{-12}\,\text{F}\,\text{m}^{-1}$$
$$V_P = 6.552\,983 \times 10^{27}\,\text{V} \qquad I_P = 3.478\,872 \times 10^{25}\,\text{A}$$
$$P_P^{(\text{power, Titus})} = 2.279\,699 \times 10^{53}\,\text{W} \qquad P_{P,\text{std}} = 3.629\,702 \times 10^{52}\,\text{W}$$
$$E_P^{(\text{field})} = 8.108\,849 \times 10^{62}\,\text{V}\,\text{m}^{-1} \qquad B_P^{(\text{field})} = 2.704\,821 \times 10^{54}\,\text{T}$$
$$J_P = 1.331\,738 \times 10^{95}\,\text{A}\,\text{m}^{-2} \qquad n_P^{(\text{charge})} = 4.442\,200 \times 10^{86}\,\text{C}\,\text{m}^{-3}$$

Bohr Velocity and Gravitational Constant

$$v_B = \alpha c = 2.187\,691 \times 10^{6}\,\text{m}\,\text{s}^{-1} \qquad G = 6.674\,300 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}\ \text{(from (1.31))}$$

Charges

$$q_P^2 = 3.517\,673 \times 10^{-36}\,\mathrm{C}^2 \qquad q_P = 1.875\,546 \times 10^{-18}\,\mathrm{C}$$

Summary Table: Titus Planck Units (Loop Form)

Quantity	Symbol	Definition (loop)	Units	Value
Planck time	t_P	$\ell_P/c = \ell_P k_P/\omega_P$	s	$5.391\,247 \times 10^{-44}$
Planck length	ℓ_P	$t_P\ c = t_P\omega_P/k_P$	m	$1.616\,255 \times 10^{-35}$
Planck frequency	f_P	$1/t_P$	Hz	$1.854\,858 \times 10^{43}$
Planck ang. frequency	ω_P	$2\pi/t_P$	s^{-1}	$1.165\,442 \times 10^{44}$
Planck energy	E_P	h/t_P	J	$1.229\,042 \times 10^{10}$
Planck mass	m_P	$E_P t_P^2/\ell_P^2$	kg	$1.367\,494 \times 10^{-7}$
Planck momentum	p_P	$E_P t_P/\ell_P$	$\mathrm{kg\,m\,s^{-1}}$	$4.099\,644 \times 10^{1}$
Planck force	F_P	E_P/ℓ_P	N	$7.604\,259 \times 10^{44}$
Planck power	P_P	E_P/t_P	W	$2.279\,699 \times 10^{53}$
Planck acceleration	a_P	$\ell_P/t_P^2 = \omega_P/k_P t_P = v_B/\alpha t_P$	$\mathrm{m/s^2}$	$5.560\,726 \times 10^{51}$
Planck ang. momentum	L_P	$\hbar = m_e v_B a_0 = mvr$	$\mathrm{J \cdot s}$	$1.054\,572 \times 10^{-34}$
Planck wavenumber	k_P	$2\pi/\ell_P$	m^{-1}	$3.887\,496 \times 10^{35}$
Phase density	Φ_P	$1/(\ell_P^3 t_P)$	$\mathrm{m^{-3} \cdot s^{-1}}$	$4.393\,202 \times 10^{147}$
Planck temperature	T_P	E_P/k_B	K	$8.901\,917 \times 10^{32}$
Planck impedance	Z_P	$Z_0/2$	Ω	$1.883\,652 \times 10^{2}$
Planck admittance	Y_P	$2/Z_0$	S	$5.308\,837 \times 10^{-3}$
Planck charge	q_P	$\sqrt{I_P E_P t_P/V_P}$	C	$1.875\,546 \times 10^{-18}$
Planck energy density	u_P	E_P/ℓ_P^3	$\mathrm{J/m^3}$	$2.910\,966 \times 10^{114}$
Planck mass density	ρ_P	m_P/ℓ_P^3	$\mathrm{kg/m^3}$	$3.238\,887 \times 10^{97}$
Planck voltage	V_P	$\sqrt{P_P Z_P}$	V	$6.552\,983 \times 10^{27}$
Planck current	I_P	$\sqrt{P_P/Z_P}$	A	$3.478\,872 \times 10^{25}$
Planck electric field	$E_P^{(\text{field})}$	$\sqrt{2u_P/\epsilon_0}$	V/m	$8.108\,849 \times 10^{62}$
Planck magnetic field	$B_P^{(\text{field})}$	$\sqrt{2\mu_0 u_P}$	T	$2.704\,821 \times 10^{54}$
Planck current density	J_P	I_P/ℓ_P^2	$\mathrm{A/m^2}$	$1.331\,738 \times 10^{95}$
Planck charge density	$n_P^{(\text{charge})}$	q_P/ℓ_P^3	$\mathrm{C/m^3}$	$4.442\,200 \times 10^{86}$
Planck constant	h	$2\pi \cdot m_e v_B a_0$	$\mathrm{J \cdot s}$	$6.626\,070 \times 10^{-34}$
Newtonian G	G	$\ell_P^5/\hbar t_P^3$	$\mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$	$6.674\,300 \times 10^{-11}$
Speed of light	c	$\ell_P/t_P = \omega_P/k_P$	$\mathrm{m\,s^{-1}}$	$2.997\,925 \times 10^{8}$

References

[1] P. J. Mohr, D. B. Newell, and B. N. Taylor, *CODATA Recommended Values of the Fundamental Physical Constants: 2022*.

[2] P. J. Mohr, D. B. Newell, and E. Tiesinga, "CODATA recommended values of the fundamental physical constants: 2022," *Rev. Mod. Phys.* **97**, 025002 (2025). DOI: https://doi.org/10.1103/RevModPhys.97.025002.

[3] BIPM, *The International System of Units (SI)*, 9th ed. (2019). https://www.bipm.org/en/publications/si-brochure.

[4] B. N. Taylor and P. J. Mohr, *The International System of Units (SI)*, NIST Special Publication 330 (2019). DOI: https://doi.org/10.6028/NIST.SP.330-2019.

[5] J. B. Hartle, *Gravity: An Introduction to Einstein's General Relativity*, Addison–Wesley (2003).

[6] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation*, W. H. Freeman (1973).

[7] S. M. Carroll, *Spacetime and Geometry: An Introduction to General Relativity*, Cambridge University Press (2019).

[8] N. Ashby, "Relativity in the Global Positioning System," *Living Reviews in Relativity* **6**, 1 (2003).

[9] N. Bohr, "On the Constitution of Atoms and Molecules," *Philosophical Magazine* **26** (151), 1–25 (1913). DOI: https://doi.org/10.1080/14786441308634955.

[10] J. D. Jackson, *Classical Electrodynamics*, 3rd ed., Wiley (1998). https://www.wiley.com/en-us/Classical+Electrodynamics%2C+3rd+Edition-p-9780471309321.

[11] D. J. Griffiths and D. F. Schroeter, *Introduction to Quantum Mechanics*, 3rd ed., Cambridge University Press (2018). https://www.cambridge.org/9781107189638.

[12] M. J. Duff, L. B. Okun, and G. Veneziano, "Trialogue on the number of fundamental constants," *JHEP* **2002** (03), 023 (2002). DOI: https://doi.org/10.1088/1126-6708/2002/03/023.

[13] NIST, *Fundamental Physical Constants*. https://physics.nist.gov/constants.

[14] NIST CODATA Value: Planck constant h (exact). https://physics.nist.gov/cgi-bin/cuu/Value?h.

[15] NIST CODATA Value: fine-structure constant α (2022). https://physics.nist.gov/cgi-bin/cuu/Value?alph.

[16] NIST CODATA Value: Planck time t_{P} (2022). https://physics.nist.gov/cgi-bin/cuu/Value?tplanck.

[17] NIST CODATA Value: Planck length ℓ_{P} (2022). https://physics.nist.gov/cgi-bin/cuu/Value?lplanck.

[18] NIST CODATA Value: electron mass m_e (2022). https://physics.nist.gov/cgi-bin/cuu/Value?me.

[19] NIST CODATA Value: Bohr radius a_0 (2022). https://physics.nist.gov/cgi-bin/cuu/Value?bohrrada0.

[20] NIST CODATA Value: characteristic impedance of vacuum Z_0 (2022). https://physics.nist.gov/cgi-bin/cuu/Value?z0.

[21] NIST CODATA Value: vacuum magnetic permeability μ_0 (2022). https://physics.nist.gov/cgi-bin/cuu/Value?mu0.

[22] NIST CODATA Value: vacuum electric permittivity ϵ_0 (2022). https://physics.nist.gov/cgi-bin/cuu/Value?eps0.

[23] NIST CODATA Value: proton mass m_p (2022). https://physics.nist.gov/cgi-bin/cuu/Value?mp.

[24] NIST CODATA Value: Newtonian constant of gravitation G (2022). https://physics.nist.gov/cgi-bin/cuu/Value?bg.

[25] Q. R. D. Tharp, *Titus: Theory of Everything*, OSF Preprints (Aug 2025). https://osf.io/bcwsn/overview