Titus Planck Units: Derivations & Numerical Evaluations

Quinton R. D. Tharp

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Abstract

This paper presents a detailed and foundational analysis of Planck-scale quantities within the Quantum Phase Lattice Model (Titus framework). Unlike conventional treatments, which often introduce Planck units as derived dimensional combinations, the Titus approach begins from a deterministic lattice interpretation of energy and time, expressed through the Planck loop condition $E_P t_P = h$. This definition implies $E_P = h/t_P$, a form larger than the conventional ("$\hbar$-based") Planck energy by a factor of 2π. By anchoring all substitutions to this loop identity, the derivations remain internally consistent, deterministic, and directly linked to observable quantities.

We provide symbolic derivations, numerical evaluations, and explicit unit checks for the full set of Planck units, using CODATA 2022 values. Each equation is cross-referenced with NIST standards to ensure clarity and reproducibility. The electromagnetic quartet (Planck charge, voltage, current, and power), together with Planck impedance, are shown to emerge transparently from the Titus substitutions. These quantities demonstrate an algebraic structure, making transparent that the Planck impedance equals half the vacuum impedance ($Z_P = Z_0/2$).

The analysis highlights how the Titus framework unifies Planck-scale physics with atomic-scale structure: the Bohr identities $h = 2\pi m_e a_0 v_B$ and $\hbar = m_e a_0 v_B$ arise naturally and consistently from the lattice formalism. By presenting all Planck units within this structure, the work establishes a rigorous foundation for later sections of the Quantum Phase Lattice Model, where phase-coherence mechanics and cosmological implications can be developed upon this Planck-scale base.

1 Introduction

Planck units are often presented as dimensional combinations of $\hbar$, c, and G that set a natural scale for quantum gravity. In this paper we take a different starting point, motivated by the Quantum Phase Lattice ("Titus") framework: the *loop* action identity

$$E_P t_P = h \tag{1.1}$$

which we adopt as fundamental.[1] Equation (1.1) implies $E_P = h/t_P$ and leads to a systematic 2π enhancement relative to the reduced-action (Std) convention $E_{P,\mathrm{std}} = \hbar/t_P$, namely $E_P = 2\pi\, E_{P,\mathrm{std}}$. We show that this bookkeeping choice propagates consistently and transparently across the Planck set, electromagnetism, and Bohr-anchored atomic identities.

Perspective and inputs. We treat t_P (and ℓ_P) as primitive lattice spacings linked by

$$c = \frac{\ell_P}{t_P} \tag{1.2}$$

so that space and time are set by a common cycle. The analysis uses only SI-defined exact constants (h, e, k_B, c) and CODATA 2022 recommended values for the remaining quantities; after the 2019 SI redefinition, h and e are exact, while μ_0, ϵ_0, and Z_0 are inferred and inherit the small uncertainty mainly from α. All numerical evaluations are carried out with CODATA 2022 and are cross-checked for consistency.

Unifying Bohr, EM, and Planck structure. A key bridge is the Bohr action identity

$$\hbar = m_e a_0 v_B \tag{1.3}$$

from which the fine-structure constant emerges as a pure velocity ratio,

$$\alpha = \frac{v_B}{c} \tag{1.4}$$

and the Planck charge admits equivalent Titus/EM forms,

$$q_P^2 = \frac{2E_P t_P}{Z_0} = \frac{4\pi}{Z_0}\, m_e a_0 v_B = 4\pi\, \epsilon_0\, \hbar\, c \tag{1.5}$$

These identities culminate in a compact electromagnetic *quartet*

$$\{V_P,\ I_P,\ P_P,\ Z_P\}$$

with closure relations $P_P = V_P I_P = V_P^2/Z_P = I_P^2 Z_P$ and the notably simple impedance result

$$Z_P = \frac{h}{q_P^2} = \frac{Z_0}{2} \tag{1.6}$$

The 2π map. Using the loop action h rather than $\hbar$ produces a uniform correspondence

$$X \ = \ 2\pi\, X_{\mathrm{std}} \quad \text{for} \quad X \in \{E_P,\ m_P,\ p_P,\ F_P,\ u_P,\ \rho_P,\ T_P,\ P_P\}$$

while leaving dimensionless quantities (e.g. α) and vacuum-impedance relations unchanged. We emphasize that this is a matter of algebraic bookkeeping, not a change to SI definitions; the two conventions are related by explicit, exact factors of 2π.

[1] Throughout, we tag the conventional $\hbar$-*based* Planck definitions with Std to avoid confusion with the particle-physics $\hbar$-based convention.

Contributions.

- A closed, minimal dictionary linking lattice (E_P, t_P, ℓ_P), EM (Z_0, μ_0, ϵ_0), and Bohr (m_e, a_0, v_B) quantities with exact cancellations.
- Derivations of the electromagnetic quartet and the Planck impedance $Z_P = Z_0/2$ directly from lattice and Bohr inputs.
- Parallel Titus (loop) and Std (reduced) forms for all major Planck units, exposing a transparent 2π mapping.
- Length–time representations that eliminate G and $\hbar$ where possible, clarifying which identities are definitional versus inferred.
- Comprehensive numerical validations (CODATA 2022) and compact unit checks for reproducibility.

Roadmap. Table 1 fixes notation and inputs. We then establish the action constants and gravitational-consistency forms, followed by the core Planck set (time, length, energy; frequency/angle; mass/momentum; forces and densities; temperature). The charge and EM section derives q_P, $\alpha = v_B/c$, and the vacuum relations. The electromagnetic quartet consolidates V_P, I_P, P_P, and Z_P. We confirm the Bohr radius and close the elementary-charge loop. A brief note summarizes the induced 2π scalings, followed by compact unit checks and an appendix of raw numerics.

Unless stated otherwise, numerical values are from CODATA 2022 (NIST) [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 1]. Exact SI definitions after the 2019 redefinition are noted.

Table 1: CODATA 2022 constants used.

Quantity	Symbol	Value (CODATA 2022)	Source
Speed of light (exact)	c	$2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}$	[13]
Planck constant (exact)	h	$6.626\,070 \times 10^{-34}\,\mathrm{J\,s}$	[14]
Reduced Planck constant	$\hbar$	$h/2\pi$ (exact from h)	[14]
Elementary charge (exact)	e	$1.602\,177 \times 10^{-19}\,\mathrm{C}$	[13]
Boltzmann constant (exact)	k_B	$1.380\,649 \times 10^{-23}\,\mathrm{J\,K^{-1}}$	[13]
Fine-structure constant	α	$7.297\,353 \times 10^{-3}$	[15, 1]
Planck time	t_P	$5.391\,247 \times 10^{-44}\,\mathrm{s}$	[16]
Planck length	ℓ_P	$1.616\,255 \times 10^{-35}\,\mathrm{m}$	[17]
Electron mass	m_e	$9.109\,384 \times 10^{-31}\,\mathrm{kg}$	[18]
Bohr radius	a_0	$5.291\,772 \times 10^{-11}\,\mathrm{m}$	[19]
Vacuum impedance	Z_0	$3.767\,303 \times 10^{2}\,\Omega$	[20]
Vacuum permeability	μ_0	$1.256\,637 \times 10^{-6}\,\mathrm{N\,A^{-2}}$	[21, 1]
Vacuum permittivity	ϵ_0	$8.854\,188 \times 10^{-12}\,\mathrm{F\,m^{-1}}$	[22, 1]
Newtonian constant of gravitation	G	$6.674\,300 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$	[24, 1]

Numerical evaluations (CODATA 2022). Using the constants in Table 1:

$$\begin{aligned} E_P t_P &= h \\ &\Rightarrow E_P = \frac{h}{t_P} \\ &= \frac{6.626\,070 \times 10^{-34}\,\mathrm{J\,s}}{5.391\,247 \times 10^{-44}\,\mathrm{s}} \\ &= 1.229\,042 \times 10^{10}\,\mathrm{J} \end{aligned} \tag{1.7}$$

$$\begin{aligned} v_B &= \alpha\, c \\ &= (7.297\,353 \times 10^{-3})(2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}) \\ &= 2.187\,691 \times 10^{6}\,\mathrm{m\,s^{-1}} \end{aligned} \tag{1.8}$$

$$\begin{aligned} h &= 2\pi\, m_e a_0 v_B \\ &= 2\pi\,(9.109\,384 \times 10^{-31}\,\mathrm{kg})(5.291\,772 \times 10^{-11}\,\mathrm{m})(2.187\,691 \times 10^{6}\,\mathrm{m\,s^{-1}}) \\ &= 6.626\,070 \times 10^{-34}\,\mathrm{J\,s} \end{aligned} \tag{1.9}$$

$$\begin{aligned}\hbar &= m_e a_0 v_B \\ &= (9.109\,384 \times 10^{-31}\,\mathrm{kg})(5.291\,772 \times 10^{-11}\,\mathrm{m})(2.187\,691 \times 10^{6}\,\mathrm{m\,s^{-1}}) \\ &= 1.054\,572 \times 10^{-34}\,\mathrm{J\,s}\end{aligned} \tag{1.10}$$

$$\begin{aligned}h &= 2\pi\,\hbar \\ &= 2\pi \times 1.054\,572 \times 10^{-34}\,\mathrm{J\,s} \\ &= 6.626\,070 \times 10^{-34}\,\mathrm{J\,s} \quad \text{(consistent with CODATA value)}\end{aligned} \tag{1.11}$$

$$\begin{aligned}c &= \frac{\ell_P}{t_P} \\ &= \frac{1.616\,255 \times 10^{-35}\,\mathrm{m}}{5.391\,247 \times 10^{-44}\,\mathrm{s}} \\ &= 2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}} \quad \text{(consistent with CODATA value)}\end{aligned} \tag{1.12}$$

$$\begin{aligned}Z_0 &= \mu_0 c \\ &= (1.256\,637 \times 10^{-6}\,\mathrm{N\,A^{-2}})(2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}) \\ &= 3.767\,303 \times 10^{2}\,\Omega\end{aligned} \tag{1.13}$$

$$\begin{aligned}Z_0 &= \frac{1}{\epsilon_0 c} \\ &= \frac{1}{(8.854\,188 \times 10^{-12}\,\mathrm{F\,m^{-1}})(2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}})} \\ &= 3.767\,303 \times 10^{2}\,\Omega\end{aligned} \tag{1.14}$$

Bohr velocity and fine-structure constant. From Eq. (1.8), the ratio of Bohr velocity to c gives

$$\begin{aligned}\frac{v_B}{c} &= \frac{2.187\,691 \times 10^{6}\,\mathrm{m\,s^{-1}}}{2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}} \\ &= 7.297\,353 \times 10^{-3} \\ &= \alpha\end{aligned} \tag{1.15}$$

which matches the CODATA 2022 fine-structure constant value.

1.1 Action constants

Using Eq. (1.1) together with the Bohr identities $\hbar = m_e a_0 v_B$ and $h = 2\pi\hbar$,

$$h = E_P t_P = \frac{E_P}{f_P} = 2\pi\, m_e a_0 v_B \tag{1.16}$$

$$\hbar = \frac{E_P}{\omega_P} = \frac{h}{2\pi} = m_e a_0 v_B \tag{1.17}$$

Titus lattice forms for h. Directly from the lattice parameters ($E_P = \frac{h}{t_P}$, $c = \frac{\ell_P}{t_P}$):

$$h = E_P\, t_P \tag{1.18}$$

$$= E_P\, \frac{\ell_P}{c} \tag{1.19}$$

An electromagnetic expression $h = \frac{Z_0}{2}\, q_P^2$ is shown later, after establishing $Z_P = h/q_P^2 = Z_0/2$.

Titus lattice forms for $\hbar$. Using the angular pairing ($E_P = \hbar\,\omega_P$) and the frequency identities:

$$\hbar = \frac{h}{2\pi} \tag{1.20}$$

$$= \frac{E_P\, t_P}{2\pi} = \frac{E_P}{\omega_P} \tag{1.21}$$

$$= \frac{Z_0}{4\pi}\, q_P^2 \tag{1.22}$$

The last line is equivalent to $q_P^2 = 4\pi\,\epsilon_0\,\hbar\,c$ together with $Z_0 = 1/(\epsilon_0 c)$ and matches (3.1).

Bohr forms (kept explicitly).

$$E_P^{\text{Titus}}/f_P = h = 2\pi\, m_e a_0 v_B \tag{1.23}$$

$$E_P^{\text{Titus}}/\omega_P = \hbar = m_e a_0 v_B \tag{1.24}$$

Numeric validations (CODATA 2022). All forms below evaluate to the same values for h and $\hbar$.

Planck loop:

$$\begin{aligned} h = E_P t_P &= \left(1.229\,042 \times 10^{10}\,\mathrm{J}\right)\left(5.391\,247 \times 10^{-44}\,\mathrm{s}\right) \\ &= 6.626\,070 \times 10^{-34}\,\mathrm{J\,s} \quad \text{(consistent with CODATA value [14])} \end{aligned} \tag{1.25}$$

Length–time lattice:

$$\begin{aligned} h = E_P\, \frac{\ell_P}{c} &= \frac{\left(1.229\,042 \times 10^{10}\,\mathrm{J}\right)\left(1.616\,255 \times 10^{-35}\,\mathrm{m}\right)}{2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}} \\ &= 6.626\,070 \times 10^{-34}\,\mathrm{J\,s} \end{aligned} \tag{1.26}$$

Impedance/charge lattice:

$$h = \frac{Z_0}{2}\,q_P^2 = \frac{3.767\,303 \times 10^2\,\Omega}{2}\left(3.517\,673 \times 10^{-36}\,\mathrm{C}^2\right)$$
$$= 6.626\,070 \times 10^{-34}\,\mathrm{J\,s} \tag{1.27}$$

using q_P^2 from (3.1) (numeric value quoted in the charge section).

Angular (frequency) pairing:

$$\hbar = \frac{E_P}{\omega_P} = \frac{1.229\,042 \times 10^{10}\,\mathrm{J}}{1.165\,442 \times 10^{44}\,\mathrm{s}^{-1}}$$
$$= 1.054\,572 \times 10^{-34}\,\mathrm{J\,s} \quad \text{(exact from } h/2\pi \text{ [14])} \tag{1.28}$$

Summary. Across lattice (loop and length–time), electromagnetic (impedance/charge), angular (frequency), and Bohr constructions, we obtain the same action constants:

$$\boxed{h = E_P t_P = \frac{E_P \ell_P}{c} = \frac{Z_0}{2}\,q_P^2 = 2\pi m_e a_0 v_B} \tag{1.29}$$

$$\boxed{\hbar = \frac{h}{2\pi} = \frac{E_P}{\omega_P} = \frac{Z_0}{4\pi}\,q_P^2 = m_e a_0 v_B} \tag{1.30}$$

1.2 Gravitational constant consistency

In the Titus framework, the gravitational constant G admits several equivalent forms, depending on whether one emphasizes time or length as the scaling basis. A convenient starting point is the Planck–Einstein identity

$$G = \frac{2\pi\,\ell_P\,c^4}{E_P} \tag{1.31}$$

obtained from the lattice relations $E_P = \frac{h}{t_P}$ and $c = \frac{\ell_P}{t_P}$.

Temporal form (with $\hbar$). Using $t_P^2 = \dfrac{\hbar G}{c^5}$ gives

$$G_{\text{temporal}} = \frac{c^5 t_P^2}{\hbar} \tag{1.32}$$

Temporal form (Bohr-anchored). With the Bohr identity $\hbar = m_e a_0 v_B$ one has

$$G_{\text{temporal,Bohr}} = \frac{c^5 t_P^2}{m_e a_0 v_B} \tag{1.33}$$

Length–time (ℓ/t) form (Bohr-anchored). Eliminating c with $c = \ell_P/t_P$ and using $\hbar = m_e a_0 v_B$,

$$G_{\ell/t} = \frac{\ell_P^5}{t_P^3\, m_e a_0 v_B} \tag{1.34}$$

Equivalence. From $c = \ell_P/t_P$,

$$\frac{c^5 t_P^2}{m_e a_0 v_B} = \frac{\left(\frac{\ell_P}{t_P}\right)^5 t_P^2}{m_e a_0 v_B} = \frac{\ell_P^5}{t_P^3\, m_e a_0 v_B}$$

so (1.33) and (1.34) are identical; all representations are mutually consistent.

Numeric (from Titus form). Using ℓ_P (CODATA 2022) [17], c (exact) [13], and E_P from Eq. (1.7),

$$G = \frac{2\pi \left(1.616\,255 \times 10^{-35}\,\mathrm{m}\right) \left(2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}\right)^4}{1.229\,042 \times 10^{10}\,\mathrm{J}} = 6.674\,300 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} \tag{1.35}$$

Numeric (temporal, with $\hbar$). Using t_P and $\hbar$ (CODATA 2022) [16, 14, 13],

$$G_{\text{temporal}} = \frac{c^5 t_P^2}{\hbar} \tag{1.36}$$

$$= \frac{\left(2.997\,925 \times 10^{8}\right)^5 \left(5.391\,247 \times 10^{-44}\right)^2}{1.054\,572 \times 10^{-34}}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} \tag{1.37}$$

$$= 6.674\,300 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} \tag{1.38}$$

Numeric (temporal, Bohr-anchored). With $v_B = \alpha c$ see Eq. (1.8); CODATA 2022 α, c) [15, 13],

$$G_{\text{temporal,Bohr}} = \frac{c^5 t_P^2}{m_e a_0 v_B} \tag{1.39}$$

$$= \frac{\left(2.997\,925 \times 10^{8}\right)^5 \left(5.391\,247 \times 10^{-44}\right)^2}{\left(9.109\,384 \times 10^{-31}\right)\left(5.291\,772 \times 10^{-11}\right)\left(2.187\,691 \times 10^{6}\right)}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} \tag{1.40}$$

$$= 6.674\,300 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} \tag{1.41}$$

Numeric (ℓ/t form, Bohr-anchored). Using v_B from Eq. (1.8),

$$G_{\ell/t} = \frac{\ell_P^5}{t_P^3\, m_e a_0 v_B} \tag{1.42}$$

$$= \frac{\left(1.616\,255 \times 10^{-35}\right)^5}{\left(5.391\,247 \times 10^{-44}\right)^3} \frac{1}{\left(9.109\,384 \times 10^{-31}\right)\left(5.291\,772 \times 10^{-11}\right)} \tag{1.43}$$

$$\times \frac{1}{\left(2.187\,691 \times 10^{6}\right)}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} \tag{1.44}$$

$$= 6.674\,300 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} \tag{1.45}$$

Bohr identity closure. Substituting any of (1.31), (1.32), or (1.34) into

$$t_P^2 = \frac{\hbar G}{c^5} \qquad \ell_P^2 = \frac{\hbar G}{c^3} \tag{1.46}$$

yields $\hbar = m_e a_0 v_B$, confirming that the Titus and Bohr-anchored forms are algebraically closed and consistent with CODATA 2022.

2 Core Planck set

2.1 Planck time

The Planck time t_P represents the fundamental unit of duration in the Planck system of natural units. It is defined as the time required for light to travel one Planck length in vacuum. In conventional form, it is derived by combining $\hbar$, G, and c:

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \tag{2.1}$$

In the Titus framework, t_P is treated as a primitive lattice unit of time, directly linked to the Planck loop relation ($E_P t_P = h$, Eq. (1.1)). Thus, t_P is the base temporal spacing of the Einstein lattice, from which other quantities are deterministically constructed.

Square-root check (Std form). Starting from the standard Planck combination,

$$\begin{aligned}
t_P &= \sqrt{\left(\frac{E_P\,\ell_P}{2\pi c}\right)\left(\frac{2\pi \ell_P c^4}{E_P}\right)\frac{1}{c^5}} && (2.2)\\
&= \sqrt{\frac{\ell_P^2 c^3}{c^5}} \quad (\text{cancel } E_P \text{ and } 2\pi;\ c\cdot c^4 = c^5) && (2.3)\\
&= \sqrt{\frac{\ell_P^2}{c^2}} && (2.4)\\
&= \frac{\ell_P}{c} && (2.5)\\
&= t_P \quad \text{since } \ell_P = c\,t_P \text{ by (2.14).} && (2.6)
\end{aligned}$$

Square-root check (Bohr form). Using the Bohr identity $\hbar = m_e a_0 v_B$ and the equivalent form

$$G = \frac{\ell_P^5}{(\ell_P/c)^3\,(m_e a_0 v_B)} = \frac{\ell_P^2 c^3}{m_e a_0 v_B}$$

we have

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \tag{2.7}$$

$$= \sqrt{\frac{(m_e a_0 v_B)\left(\frac{\ell_P^5}{(\ell_P/c)^3\,(m_e a_0 v_B)}\right)}{c^5}} \tag{2.8}$$

$$= \sqrt{\frac{\ell_P^5}{(\ell_P/c)^3\,c^5}} \tag{2.9}$$

$$= \sqrt{\frac{\ell_P^5}{(\ell_P^3/c^3)\,c^5}} = \sqrt{\frac{\ell_P^5}{\ell_P^3\,c^2}} \tag{2.10}$$

$$= \sqrt{\frac{\ell_P^2}{c^2}} = \frac{\ell_P}{c} = t_P \tag{2.11}$$

Numeric:

$$t_P = \sqrt{\frac{(1.054\,572 \times 10^{-34}\,\mathrm{J\,s})(6.674\,300 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}})}{(2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}})^5}}$$
$$= 5.391\,247 \times 10^{-44}\,\mathrm{s} \tag{2.12}$$

This matches the CODATA 2022 recommended value [16], and both the Std and Bohr routes collapse consistently to the same result $t_P = \ell_P/c$.

2.2 Planck length

The Planck length ℓ_P represents the fundamental unit of spatial distance in the Planck system of natural units. It is defined as the distance light travels in one Planck time. In conventional form, it is derived by combining $\hbar$, G, and c:

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \tag{2.13}$$

In the Titus framework, ℓ_P is treated as a primitive lattice unit of length, directly linked to the temporal spacing t_P by

$$\ell_P = c\,t_P \tag{2.14}$$

so that space and time emerge coherently from the same lattice cycle.

Square-root check (Std form). Starting from

$$\ell_P = \sqrt{\left(\frac{E_P\,(t_P c)}{2\pi c}\right)\left(\frac{2\pi\,(t_P c)\,c^4}{E_P}\right)\frac{1}{c^3}} \tag{2.15}$$

$$= \sqrt{\left(\frac{t_P c}{c}\right)\,(t_P c)\,c^4 \cdot \frac{1}{c^3}} \tag{2.16}$$

$$= \sqrt{t_P \cdot (t_P c) \cdot c} \tag{2.17}$$

$$= \sqrt{t_P^2 c^2} \tag{2.18}$$

$$= t_P c \tag{2.19}$$

Square-root check (Bohr form). Using the Bohr identity $\hbar = m_e a_0 v_B$,

$$\ell_P = \sqrt{\Big(m_e a_0 v_B\Big)\left(\frac{(t_P c)^5}{t_P^3\, m_e a_0 v_B}\right)\frac{1}{c^3}} \tag{2.20}$$

Inside the root,

$$\sqrt{\Big(m_e a_0 v_B\Big)\left(\frac{(t_P c)^5}{t_P^3\, m_e a_0 v_B}\right)\frac{1}{c^3}} = \sqrt{\Big(m_e a_0 v_B\Big)\left(\frac{t_P^2 c^5}{m_e a_0 v_B}\right)\frac{1}{c^3}} \tag{2.21}$$

so multiplying by $m_e a_0 v_B$ cancels it cleanly, leaving

$$\ell_P = \sqrt{t_P^2 c^5 \cdot \frac{1}{c^3}} = \sqrt{t_P^2 c^2} = t_P c \tag{2.22}$$

Numeric:

$$\ell_P = (2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}})(5.391\,247 \times 10^{-44}\,\mathrm{s}) = 1.616\,255 \times 10^{-35}\,\mathrm{m} \tag{2.23}$$

which matches the CODATA 2022 recommended value [17]; constants c and t_P from [13, 16].

2.3 Planck energy

The Planck energy E_P represents the characteristic loop energy of a single Planck cycle of duration t_P. In the Titus framework it is

$$E_P = \frac{h}{t_P} \tag{2.24}$$

with the Planck loop relation $E_P t_P = h$ defining h as the energy–frequency exchange constant of the lattice [14, 16].

Square-root form ($\hbar$-based (Std). The conventional expression is

$$E_{P,\text{std}} = \sqrt{\frac{\hbar c^5}{G}} \tag{2.25}$$

Using $t_P^2 = \hbar G/c^5$ (CODATA) [16], one may eliminate G:

$$E_{P,\text{std}} = \sqrt{\frac{\hbar c^5}{\frac{t_P^2 c^5}{\hbar}}} = \sqrt{\frac{\hbar^2}{t_P^2}} = \frac{\hbar}{t_P} \tag{2.26}$$

With the Bohr identity

$$\hbar = m_e a_0 v_B \qquad v_B = \alpha\, c \tag{2.27}$$

one obtains

$$E_{P,\text{std}} = \frac{m_e a_0 v_B}{t_P} \tag{2.28}$$

where m_e, a_0, α, and c are CODATA 2022 [18, 19, 15, 13].

Titus form and the 2π enhancement. In the Titus framework, the loop relation uses h rather than $\hbar$:

$$E_P = \frac{h}{t_P} = \frac{2\pi\hbar}{t_P} \tag{2.29}$$

Comparing (2.26) and (2.29),

$$\boxed{E_P = 2\pi\, E_{P,\text{std}}} \tag{2.30}$$

Thus, the Titus definition is a full 2π increase over the Std value, reflecting the use of the complete loop action h rather than the reduced action $\hbar$.

Bohr form (Titus). Using Bohr's action quantization,

$$\hbar = m_e a_0 v_B \qquad h = 2\pi\,\hbar \tag{2.31}$$

write energy as an explicit phase–rate product,

$$E = \left(\frac{\vartheta}{t}\right)\hbar \tag{2.32}$$

$$E = \omega\,\hbar \qquad \omega = \frac{\vartheta}{t} \tag{2.33}$$

Setting $\vartheta = 2\pi$ and $t = t_P$ reproduces the Titus loop relation,

$$E_P = \left(\frac{2\pi}{t_P}\right)\hbar = \omega_P\,\hbar \quad (\omega_P = 2\pi/t_P) = \frac{h}{t_P} = \frac{2\pi\, m_e a_0 v_B}{t_P} \tag{2.34}$$

Numeric (CODATA 2022, Titus value):

$$E_P = \frac{h}{t_P} = \frac{6.626\,070 \times 10^{-34}\,\mathrm{J\,s}}{5.391\,247 \times 10^{-44}\,\mathrm{s}} = 1.229\,042 \times 10^{10}\,\mathrm{J} \tag{2.35}$$

using exact h and t_P from CODATA [14, 16].

Equivalently, using the Bohr form with the constants from Table 1 and $v_B = \alpha c$ [15, 13]:

$$\begin{aligned} E_P &= \frac{2\pi\,9.109\,384 \times 10^{-31}\,\mathrm{kg}\,5.291\,772 \times 10^{-11}\,\mathrm{m}\,2.187\,691 \times 10^{6}\,\mathrm{m/s}}{5.391\,247 \times 10^{-44}\,\mathrm{s}} \\ &= 1.229\,042 \times 10^{10}\,\mathrm{J} \end{aligned} \tag{2.36}$$

which agrees with (1.7) within the precision of the tabulated α digits.

Numeric (CODATA 2022, Std value):

$$\begin{aligned} E_{P,\mathrm{std}} &= \frac{m_e a_0 v_B}{t_P} \\ &= \frac{9.109\,384 \times 10^{-31}\,\mathrm{kg}\,5.291\,772 \times 10^{-11}\,\mathrm{m}\,2.187\,691 \times 10^{6}\,\mathrm{m/s}}{5.391\,247 \times 10^{-44}\,\mathrm{s}} \\ &= 1.956\,081 \times 10^{9}\,\mathrm{J} \end{aligned} \tag{2.37}$$

Summary.

$$\boxed{E_P = \frac{h}{t_P} = \frac{2\pi\, m_e a_0 v_B}{t_P} = 2\pi\, E_{P,\mathrm{std}} = 2\pi \left(\frac{m_e a_0 v_B}{t_P}\right)} \tag{2.38}$$

2.4 Planck frequency and angular frequency

$$f_P = \frac{1}{t_P} \tag{2.39}$$

$$\omega_P = \frac{2\pi}{t_P} \tag{2.40}$$

By the Titus loop identity $E_P t_P = h$ Eq. (1.1), the linear and angular pairings are equivalent:

$$E_P = h\, f_P = \hbar\, \omega_P \tag{2.41}$$

$\hbar$-based (Std) square-root route. From $t_P^2 = \hbar G/c^5$ we have

$$f_P = \frac{1}{t_P} = \sqrt{\frac{c^5}{\hbar G}} \tag{2.42}$$

Using the Titus forms $\hbar = \dfrac{E_P t_P}{2\pi} = \dfrac{E_P \ell_P}{2\pi c}$ (with $\ell_P = c\,t_P$; Eq. (2.14) and $G = \dfrac{2\pi\,\ell_P c^4}{E_P}$ Eq. (1.31),

$$f_P = \sqrt{\frac{c^5}{\left(\frac{E_P \ell_P}{2\pi c}\right)\left(\frac{2\pi\,\ell_P c^4}{E_P}\right)}} \tag{2.43}$$

$$= \sqrt{\frac{c^5}{\ell_P^2 c^3}} \tag{2.44}$$

$$= \sqrt{\frac{c^2}{\ell_P^2}} = \frac{c}{\ell_P} \tag{2.45}$$

Since $c = \ell_P/t_P$ Eq. (2.14), this gives $f_P = c/\ell_P = 1/t_P$, consistent with (2.39). The angular frequency follows as

$$\omega_P = 2\pi f_P = \frac{2\pi}{t_P} = 2\pi\sqrt{\frac{c^5}{\hbar G}} \tag{2.46}$$

Std square-root (Bohr-anchored) cancellation. Using $\hbar = m_e a_0 v_B$ and the temporal form $G = \dfrac{c^5 t_P^2}{\hbar}$ Eq. (1.33) with $\hbar = m_e a_0 v_B$),

$$f_P = \sqrt{\frac{c^5}{\hbar G}} \tag{2.47}$$

$$= \sqrt{\frac{c^5}{(m_e a_0 v_B)\left(\dfrac{c^5 t_P^2}{\hbar}\right)}} \tag{2.48}$$

$$= \sqrt{\frac{c^5}{(m_e a_0 v_B)\left(\dfrac{c^5 t_P^2}{m_e a_0 v_B}\right)}} \tag{2.49}$$

$$= \sqrt{\frac{c^5}{c^5 t_P^2}} = \sqrt{\frac{1}{t_P^2}} = \frac{1}{t_P} \tag{2.50}$$

so again $f_P = 1/t_P$ and $\omega_P = 2\pi/t_P$.

Numeric (CODATA 2022).

$$f_P = \frac{1}{5.391\,247 \times 10^{-44}\,\mathrm{s}} = 1.854\,858 \times 10^{43}\,\mathrm{Hz} \tag{2.51}$$

$$\omega_P = \frac{2\pi}{5.391\,247 \times 10^{-44}\,\mathrm{s}} = 1.165\,442 \times 10^{44}\,\mathrm{s}^{-1} \tag{2.52}$$

Cross-checks with h and $\hbar$:

$$h\,f_P = \left(6.626\,070 \times 10^{-34}\,\mathrm{J\,s}\right)\left(1.854\,858 \times 10^{43}\,\mathrm{s}^{-1}\right) = 1.229\,042 \times 10^{10}\,\mathrm{J} \tag{2.53}$$

$$\hbar\,\omega_P = \left(\frac{6.626\,070 \times 10^{-34}\,\mathrm{J\,s}}{2\pi}\right)\left(1.165\,442 \times 10^{44}\,\mathrm{s}^{-1}\right) = 1.229\,042 \times 10^{10}\,\mathrm{J} \tag{2.54}$$

which match E_P from Eq. (2.24). (Constants t_P, h, c from [16, 14, 13].)

2.5 Planck mass, momentum

Titus definitions.

$$m_P = \frac{E_P}{c^2} \tag{2.55}$$

$$p_P = \frac{E_P}{c} \tag{2.56}$$

With $E_P t_P = h$ Eq. (2.24) and $c = \ell_P/t_P$ Eq. (2.14), these are the primary Titus definitions.

$\hbar$-based (Std) square-root forms.

$$m_{P,\mathrm{std}} = \sqrt{\frac{\hbar c}{G}} \tag{2.57}$$

$$p_{P,\mathrm{std}} = \sqrt{\frac{\hbar c^3}{G}} \tag{2.58}$$

Using the Titus substitutions $\hbar = \dfrac{E_P t_P}{2\pi} = \dfrac{E_P \ell_P}{2\pi c}$ and $G = \dfrac{2\pi\,\ell_P c^4}{E_P}$ Eq. (1.21) and (1.31), we obtain

$$m_{P,\mathrm{std}} = \sqrt{\frac{\left(\frac{E_P\ell_P}{2\pi c}\right)c}{\frac{2\pi\,\ell_P c^4}{E_P}}} = \sqrt{\frac{E_P^2}{(2\pi)^2 c^4}} = \frac{E_P}{2\pi c^2} = \frac{m_P}{2\pi} \tag{2.59}$$

$$p_{P,\mathrm{std}} = \sqrt{\frac{\left(\frac{E_P\ell_P}{2\pi c}\right)c^3}{\frac{2\pi\,\ell_P c^4}{E_P}}} = \sqrt{\frac{E_P^2}{(2\pi)^2 c^2}} = \frac{E_P}{2\pi c} = \frac{p_P}{2\pi} \tag{2.60}$$

Hence the explicit 2π relations:

$$\boxed{m_P = 2\pi\, m_{P,\mathrm{std}}} \qquad \boxed{p_P = 2\pi\, p_{P,\mathrm{std}}} \tag{2.61}$$

Bohr-anchored forms and the $E = mc^2$ bridge. Using $\hbar = m_e a_0 v_B$ and $h = 2\pi m_e a_0 v_B$,

$$m_P = \frac{E_P}{c^2} = \frac{2\pi\, m_e a_0 v_B}{t_P\, c^2} \tag{2.62}$$

$$p_P = \frac{E_P}{c} = \frac{2\pi\, m_e a_0 v_B}{t_P\, c} = \frac{2\pi\, m_e a_0 v_B}{\ell_P} \tag{2.63}$$

Then the Einstein relation reproduces Titus energy directly:

$$E_P = m_P c^2 = \left(\frac{2\pi\, m_e a_0 v_B}{t_P\, c^2}\right)c^2 = \frac{2\pi\, m_e a_0 v_B}{t_P} = \frac{h}{t_P} \tag{2.64}$$

Likewise for the Std (reduced) versions,

$$m_{P,\mathrm{std}} = \frac{m_e a_0 v_B}{t_P\, c^2} \qquad E_{P,\mathrm{std}} = m_{P,\mathrm{std}} c^2 = \frac{m_e a_0 v_B}{t_P} = \frac{\hbar}{t_P} \tag{2.65}$$

consistently showing $E_P = 2\pi E_{P,\mathrm{std}}$ Eq. (2.30).

Action-over-Planck-time viewpoint. Let $S_{\rm orb} = m\,r\,v$ be an orbital action (ground state $S_{\rm orb} = \hbar$ in the Bohr model). Then

$$E = \left(\frac{\vartheta}{t}\right)\hbar \quad \text{(reduced action; } \vartheta \text{ is phase)} \tag{2.66}$$

$$E = \frac{2\pi}{t_P}\,\hbar \;=\; \frac{h}{t_P} \quad \text{(full-loop Titus form at } t = t_P\text{)}. \tag{2.67}$$

so keeping $S_{\rm orb}$ fixed fixes E at fixed t_P; varying m, r, v while preserving $mrv = \hbar$ (or $= h/2\pi$) keeps E unchanged.

Numeric (CODATA 2022).

$$m_P = \frac{E_P}{c^2} = \frac{1.229\,042 \times 10^{10}\,\mathrm{J}}{(2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}})^2} = 1.367\,494 \times 10^{-7}\,\mathrm{kg} \tag{2.68}$$

$$p_P = \frac{E_P}{c} = \frac{1.229\,042 \times 10^{10}\,\mathrm{J}}{2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}} = 4.099\,644 \times 10^{1}\,\mathrm{kg\,m\,s^{-1}} \tag{2.69}$$

Std counterparts (explicit 2π contrast):

$$m_{P,\rm std} = \frac{m_P}{2\pi} = 2.176\,434 \times 10^{-8}\,\mathrm{kg} \tag{2.70}$$

$$p_{P,\rm std} = \frac{p_P}{2\pi} = 6.524\,785\,\mathrm{kg\,m\,s^{-1}} \tag{2.71}$$

(Here E_P from (1.7), c from [13], and m_e, a_0, α from Table 1 [18, 19, 15].)

2.6 Planck force, energy density (pressure), and mass density

Titus definitions (from $E_P = \frac{h}{t_P}$ and $c = \frac{\ell_P}{t_P}$).

$$F_P \equiv \frac{E_P}{\ell_P} \tag{2.72}$$

$$u_P \equiv \frac{E_P}{\ell_P^3} \tag{2.73}$$

$$\rho_P \equiv \frac{m_P}{\ell_P^3} = \frac{E_P}{c^2\,\ell_P^3} \tag{2.74}$$

Substituting $E_P = \frac{h}{t_P}$ Eq. (2.24) and $c = \frac{\ell_P}{t_P}$ Eq. (2.14) gives

$$F_P = \frac{h}{t_P\,\ell_P} = \frac{h\,c}{\ell_P^2} \tag{2.75}$$

$$u_P = \frac{h}{t_P\,\ell_P^3} = \frac{h\,c}{\ell_P^4} \tag{2.76}$$

$$\rho_P = \frac{E_P}{c^2\,\ell_P^3} = \frac{u_P}{c^2} = \frac{h}{c\,\ell_P^4} \tag{2.77}$$

Bohr-anchored Titus forms (still no $\hbar$, no G). Using $h = 2\pi m_e a_0 v_B$ and $\ell_P = c\,t_P$,

$$F_P = \frac{2\pi\, m_e a_0 v_B}{t_P\, \ell_P} = \frac{2\pi\, m_e a_0 v_B\, c}{\ell_P^2} \tag{2.78}$$

$$u_P = \frac{2\pi\, m_e a_0 v_B}{t_P\, \ell_P^3} = \frac{2\pi\, m_e a_0 v_B\, c}{\ell_P^4} \tag{2.79}$$

$$\rho_P = \frac{u_P}{c^2} = \frac{2\pi\, m_e a_0 v_B}{c\, \ell_P^4} \tag{2.80}$$

Std forms with Titus substitutions (the 2π trend from G). For comparison, start from the $\hbar$-based (Std) expressions that place G in the denominator, then apply the Titus dictionary $\hbar = \dfrac{E_P \ell_P}{2\pi c}$ and $G = \dfrac{2\pi\, \ell_P c^4}{E_P}$ Eq. (1.31).

Force:

$$F_{P,\mathrm{std}} = \frac{c^4}{G} \tag{2.81}$$

$$= \frac{c^4}{\frac{2\pi\, \ell_P c^4}{E_P}} = \frac{E_P}{2\pi\, \ell_P} = \frac{F_P}{2\pi} \tag{2.82}$$

Energy density / pressure:

$$u_{P,\mathrm{std}} = \frac{c^7}{\hbar\, G^2} \tag{2.83}$$

$$= \frac{c^7}{\left(\frac{E_P \ell_P}{2\pi c}\right)\left(\frac{2\pi\, \ell_P c^4}{E_P}\right)^2} = \frac{E_P}{2\pi\, \ell_P^3} = \frac{u_P}{2\pi} \tag{2.84}$$

Mass density:

$$\rho_{P,\mathrm{std}} = \frac{c^5}{\hbar\, G^2} \tag{2.85}$$

$$= \frac{c^5}{\left(\frac{E_P \ell_P}{2\pi c}\right)\left(\frac{2\pi\, \ell_P c^4}{E_P}\right)^2} = \frac{E_P}{2\pi\, c^2\, \ell_P^3} = \frac{\rho_P}{2\pi} \tag{2.86}$$

Thus, in each case,

$$\boxed{F_P = 2\pi\, F_{P,\mathrm{std}}, \quad u_P = 2\pi\, u_{P,\mathrm{std}}, \quad \rho_P = 2\pi\, \rho_{P,\mathrm{std}}} \tag{2.87}$$

i.e., introducing G in the Std denominator produces a systematic $1/(2\pi)$ reduction relative to the Titus forms.

Units sanity. $[F_P] = \mathrm{J/m} = \mathrm{N}$, $[u_P] = \mathrm{J/m^3} = \mathrm{Pa}$, $[\,\rho_P\,] = \mathrm{kg/m^3}$, and

$$u_P = c^2\, \rho_P \tag{2.88}$$

consistent with Eqs (2.75), (2.76), and (2.77).

Numeric (CODATA 2022). Using E_P from Eq. (1.7) and ℓ_P, c from [17, 13]:

$$F_P = \frac{E_P}{\ell_P} = \frac{1.229\,042 \times 10^{10}\,\mathrm{J}}{1.616\,255 \times 10^{-35}\,\mathrm{m}} = 7.604\,259 \times 10^{44}\,\mathrm{N} \tag{2.89}$$

$$u_P = \frac{E_P}{\ell_P^3} = \frac{1.229\,042 \times 10^{10}\,\mathrm{J}}{(1.616\,255 \times 10^{-35}\,\mathrm{m})^3} = 2.910\,966 \times 10^{114}\,\mathrm{Pa} \tag{2.90}$$

$$\rho_P = \frac{m_P}{\ell_P^3} = \frac{1.367\,494 \times 10^{-7}\,\mathrm{kg}}{(1.616\,255 \times 10^{-35}\,\mathrm{m})^3} = 3.238\,887 \times 10^{97}\,\mathrm{kg\,m^{-3}} \tag{2.91}$$

A consistency check with Eq. (2.88):

$$\begin{aligned} c^2\,\rho_P &= (2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}})^2 \times 3.238\,887 \times 10^{97}\,\mathrm{kg\,m^{-3}} \\ &= 2.910\,966 \times 10^{114}\,\mathrm{Pa} = u_P \end{aligned} \tag{2.92}$$

as required.

2.7 Planck temperature

Definitions (Titus vs. Std). Titus temperature is defined by the loop energy per Boltzmann quantum,

$$T_P = \frac{E_P}{k_B} \tag{2.93}$$

whereas the Std form uses the reduced-loop energy

$$T_{P,\mathrm{std}} = \frac{E_{P,\mathrm{std}}}{k_B} = \frac{1}{k_B}\sqrt{\frac{\hbar c^5}{G}} \tag{2.94}$$

Since $E_P = 2\pi\,E_{P,\mathrm{std}}$ Eq. (2.30),

$$T_{P,\mathrm{std}} = \frac{E_P}{2\pi\,k_B} \qquad T_P = 2\pi\,T_{P,\mathrm{std}} \tag{2.95}$$

Std → Titus via lattice substitutions (single proof). Insert $\hbar = \dfrac{E_P \ell_P}{2\pi c}$ and $G = \dfrac{2\pi\,\ell_P c^4}{E_P}$ into (2.94):

$$T_{P,\mathrm{std}} = \frac{1}{k_B}\sqrt{\frac{\left(\dfrac{E_P \ell_P}{2\pi c}\right) c^5}{\dfrac{2\pi\,\ell_P c^4}{E_P}}} = \frac{1}{k_B}\sqrt{\frac{E_P^2}{(2\pi)^2}} = \frac{E_P}{2\pi\,k_B} \tag{2.96}$$

which implies (2.95) and the Titus form (2.93).

Bohr-anchored forms (concise). With $\hbar = m_e a_0 v_B$ and $h = 2\pi\hbar$,

$$T_{P,\mathrm{std}} = \frac{m_e a_0 v_B}{t_P\, k_B} \qquad T_P = \frac{2\pi\, m_e a_0 v_B}{t_P\, k_B} = 2\pi\, T_{P,\mathrm{std}} \tag{2.97}$$

Numeric (CODATA 2022) Constants from Table 1.

$$E_{P,\mathrm{std}} = \frac{E_P}{2\pi} = \frac{1.229\,042\times 10^{10}\,\mathrm{J}}{2\pi} = 1.956\,081\times 10^{9}\,\mathrm{J} \tag{2.98}$$

$$T_{P,\mathrm{std}} = \frac{E_{P,\mathrm{std}}}{k_B} = \frac{1.956\,081\times 10^{9}\,\mathrm{J}}{1.380\,649\times 10^{-23}\,\mathrm{J\,K^{-1}}} = 1.416\,784\times 10^{32}\,\mathrm{K} \tag{2.99}$$

$$T_P = \frac{E_P}{k_B} = \frac{1.229\,042\times 10^{10}\,\mathrm{J}}{1.380\,649\times 10^{-23}\,\mathrm{J\,K^{-1}}} = 8.901\,917\times 10^{32}\,\mathrm{K} \tag{2.100}$$

Phase increment per Kelvin (modular mapping). Energy→phase map:

$$\vartheta(E) = 2\pi\,\frac{E}{E_P} \pmod{2\pi} \tag{2.101}$$

For $E = k_B\,\Delta T$, the per-Kelvin phase step is

$$\vartheta_{1\mathrm{K}}^{(\mathrm{Titus})} = \frac{2\pi}{T_P} = 7.058\,239\times 10^{-33}\,\mathrm{rad\,K^{-1}} \tag{2.102}$$

and in Std units ($T_{P,\mathrm{std}} = T_P/2\pi$)

$$\vartheta_{1\mathrm{K}}^{(\mathrm{std})} = \frac{2\pi}{T_{P,\mathrm{std}}} = 4.434\,822\times 10^{-32}\,\mathrm{rad\,K^{-1}} \tag{2.103}$$

(Identity: $\vartheta_{1\mathrm{K}}^{(\mathrm{Titus})} = 2\pi k_B/E_P$ and $\vartheta_{1\mathrm{K}}^{(\mathrm{std})} = 2\pi k_B/E_{P,\mathrm{std}}$)

Summary.

$$\boxed{T_{P,\mathrm{std}} = \frac{1}{k_B}\sqrt{\frac{\hbar c^5}{G}} = \frac{E_P}{2\pi\, k_B} = \frac{m_e a_0 v_B}{t_P\, k_B} \quad T_P = \frac{E_P}{k_B} = \frac{2\pi\, m_e a_0 v_B}{t_P\, k_B} = 2\pi\, T_{P,\mathrm{std}}}$$

$$\boxed{\vartheta_{1\mathrm{K}}^{(\mathrm{Titus})} = \frac{2\pi}{T_P} \quad \vartheta_{1\mathrm{K}}^{(\mathrm{std})} = \frac{2\pi}{T_{P,\mathrm{std}}}}$$

3 Charge and electromagnetism

3.1 Planck charge and fine structure

Definitions and Titus / Std equivalences. Starting from the Titus loop identity $E_P t_P = h$ and the vacuum relations $Z_0 = \mu_0 c = 1/(\epsilon_0 c)$, the Planck charge follows in several algebraically equivalent forms:

$$q_P^2 = \frac{4\pi}{Z_0}\, m_e a_0 v_B = \frac{2E_P t_P}{Z_0} = 4\pi\, \epsilon_0\, \hbar\, c \tag{3.1}$$

$$q_P = \sqrt{q_P^2} \tag{3.2}$$

To see the equivalence explicitly, use $\hbar = m_e a_0 v_B$ and $1/Z_0 = \epsilon_0 c$:

$$\frac{4\pi}{Z_0}\, m_e a_0 v_B = 4\pi\, \frac{\hbar}{Z_0} = 4\pi\, \hbar\, (\epsilon_0 c) = 4\pi\, \epsilon_0\, \hbar\, c$$

and

$$\frac{2E_P t_P}{Z_0} = \frac{2h}{Z_0} = 2(2\pi\hbar)(\epsilon_0 c) = 4\pi\, \epsilon_0\, \hbar\, c$$

Fine-structure constant: Titus and Std routes to v_B/c (full cancellations). *Titus route (via $E_P t_P = h$).*

$$\alpha = \frac{e^2}{q_P^2}, \qquad q_P^2 = \frac{4\pi}{Z_0}\, m_e a_0 v_B = \frac{2E_P t_P}{Z_0} \tag{3.3}$$

$$\alpha = \frac{e^2 Z_0}{2E_P t_P} = \frac{e^2 Z_0}{2\left(\frac{2\pi m_e a_0 v_B}{t_P}\right) t_P} = \frac{e^2 Z_0}{4\pi\, m_e a_0 v_B} \tag{3.4}$$

Auxiliary identity for e^2. Using $c = \ell_P / t_P$,

$$e^2 = \alpha\, q_P^2 = \frac{v_B}{c} \cdot \frac{4\pi}{Z_0}\, m_e a_0 v_B = \frac{4\pi\, t_P\, m_e a_0 v_B^2}{Z_0\, \ell_P} \tag{3.5}$$

Titus $\to v_B/c$ by direct cancellation using $\ell_P = t_P \cdot c$. Insert (3.5) into (3.4):

$$\alpha = \frac{\left(\frac{4\pi\, t_P\, m_e a_0 v_B^2}{Z_0\, \ell_P}\right) Z_0}{4\pi\, m_e a_0 v_B} = \frac{t_P\, v_B}{\ell_P} = \frac{t_P\, v_B}{t_P \cdot c} = \frac{v_B}{c} \tag{3.6}$$

Std route (start from the canonical definition).

$$\alpha = \frac{e^2}{4\pi\, \epsilon_0\, \hbar\, c} \tag{3.7}$$

Use Titus substitutions $\epsilon_0 = \frac{t_P}{Z_0 \ell_P}$, $\hbar = m_e a_0 v_B$, and $\ell_P = t_P c$:

$$\alpha = \frac{e^2}{4\pi} \cdot \frac{Z_0 \ell_P}{t_P\, \hbar\, c} = \frac{e^2 Z_0}{4\pi\, m_e a_0 v_B} \cdot \frac{\ell_P}{t_P c} = \frac{e^2 Z_0}{4\pi\, m_e a_0 v_B} \tag{3.8}$$

Now reuse (3.5) to reach the same end: $\alpha = \dfrac{t_P v_B}{\ell_P} = \dfrac{v_B}{c}$

Numeric validations (CODATA 2022).

$$\alpha = \frac{v_B}{c} = \frac{2.187\,691 \times 10^6}{2.997\,925 \times 10^8} = 7.297\,353 \times 10^{-3} \tag{3.9}$$

$$\begin{aligned}\alpha &= \frac{e^2 Z_0}{4\pi\, m_e a_0 v_B} \\ &= \frac{(1.602\,177 \times 10^{-19})^2}{9.109\,384 \times 10^{-31}\, 5.291\,772 \times 10^{-11}\, 2.187\,691 \times 10^6} \cdot \frac{3.767\,303 \times 10^2}{4\pi} \\ &= 7.297\,353 \times 10^{-3}\end{aligned} \tag{3.10}$$

Units check (dimensionless). $e^2 Z_0$ has units $\mathrm{C}^2 \cdot \Omega = \mathrm{kg\,m^2/s}$, while $m_e a_0 v_B$ has units $\mathrm{kg\,m^2/s}$; hence their ratio is unitless, as required for α.

Numeric validations (CODATA 2022): Planck charge. *Titus (loop/impedance):*

$$q_P^2 = \frac{2h}{Z_0} = \frac{2\, 6.626\,070 \times 10^{-34}\,\mathrm{J\,s}}{3.767\,303 \times 10^2\,\Omega} = 3.517\,673 \times 10^{-36}\,\mathrm{C}^2 \tag{3.11}$$

$$q_P = \sqrt{q_P^2} = 1.875\,546 \times 10^{-18}\,\mathrm{C} \tag{3.12}$$

Std/Bohr (capacitance route):

$$\begin{aligned}q_P^2 &= 4\pi\,\epsilon_0\,\hbar\,c \\ &= 4\pi\,\big(8.854\,188 \times 10^{-12}\big)\,\big(1.054\,572 \times 10^{-34}\,\mathrm{J\,s}\big)\,\big(2.997\,925 \times 10^8\,\mathrm{m\,s^{-1}}\big)\,\mathrm{C}^2 \\ &= 3.517\,673 \times 10^{-36}\,\mathrm{C}^2\end{aligned} \tag{3.13}$$

The tiny last-digit difference between (3.11) and (3.13) comes from CODATA uncertainties/rounding in ϵ_0 (and thus Z_0). Both routes are algebraically identical via $h = 2\pi\hbar$ and $1/Z_0 = \epsilon_0 c$.

Velocity ratios for α. From the Bohr identity $\hbar = m_e a_0 v_B$ and the CODATA relation $\alpha = v_B/c$ shown earlier in Eq. (1.15), and using $f_P \ell_P = c$ from Eqs. (2.39) and (2.14), we obtain the useful equivalents

$$\alpha = \frac{v_B}{c} = \frac{v_B}{f_P\,\ell_P} = \frac{v_B}{f\,\lambda} \quad \text{(for any EM mode in vacuum, where } f\lambda = c\text{)} \tag{3.14}$$

3.2 Vacuum permeability and permittivity

Titus identities (from $Z_0 = \mu_0 c = 1/(\epsilon_0 c)$ and $c = \ell_P/t_P$).

$$\mu_0 = \frac{Z_0}{c} = Z_0\,\frac{t_P}{\ell_P} \tag{3.15}$$

$$\epsilon_0 = \frac{1}{Z_0 c} = \frac{t_P}{Z_0\,\ell_P} \tag{3.16}$$

The second equalities use $c = \ell_P/t_P$ Eq. (2.14).

Numeric (length–time form, Titus). Using $c = \ell_P/t_P$ explicitly in (3.15)–(3.16):

$$\begin{aligned}
\mu_0 &= Z_0 \frac{t_P}{\ell_P} \\
&= \left(3.767\,303 \times 10^2\,\Omega\right)\left(\frac{5.391\,247 \times 10^{-44}\,\mathrm{s}}{1.616\,255 \times 10^{-35}\,\mathrm{m}}\right) \\
&= 1.256\,637 \times 10^{-6}\,\mathrm{H\,m^{-1}} \\
&= 1.256\,637 \times 10^{-6}\,\mathrm{N\,A^{-2}}
\end{aligned} \tag{3.17}$$

$$\begin{aligned}
\epsilon_0 &= \frac{t_P}{Z_0\,\ell_P} \\
&= \frac{5.391\,247 \times 10^{-44}\,\mathrm{s}}{3.767\,303 \times 10^2\,\Omega\,1.616\,255 \times 10^{-35}\,\mathrm{m}} \\
&= 8.854\,188 \times 10^{-12}\,\mathrm{F\,m^{-1}}
\end{aligned} \tag{3.18}$$

These agree with the direct Z_0–c evaluations in (3.23) and (3.24).

Minimal Titus/Bohr relations. Using $Z_P = \dfrac{h}{q_P^2} = \dfrac{Z_0}{2}$ and $h = 2\pi m_e a_0 v_B$:

$$Z_0 = \frac{2h}{q_P^2} = \frac{4\pi\, m_e a_0 v_B}{q_P^2} \tag{3.19}$$

$$\mu_0 = \frac{Z_0}{c} = \frac{4\pi\, m_e a_0 v_B}{q_P^2\, c} \tag{3.20}$$

$$\epsilon_0 = \frac{1}{Z_0 c} = \frac{q_P^2}{4\pi\, m_e a_0 v_B\, c} \tag{3.21}$$

These express Z_0, μ_0, ϵ_0 directly in terms of Titus lattice quantities (E_P, t_P, q_P) and the Bohr triplet (m_e, a_0, v_B), without introducing $\hbar$ or α.

Consistency check. Using (3.15)–(3.16),

$$\mu_0\,\epsilon_0\,c^2 = \left(\frac{Z_0}{c}\right)\left(\frac{1}{Z_0 c}\right)c^2 = 1 \tag{3.22}$$

Numeric (CODATA 2022). Using Z_0 and c from Table 1 [20, 13]:

$$\mu_0 = \frac{3.767\,303 \times 10^2\,\Omega}{2.997\,925 \times 10^8\,\mathrm{m\,s^{-1}}} = 1.256\,637 \times 10^{-6}\,\mathrm{N\,A^{-2}} \tag{3.23}$$

$$\epsilon_0 = \frac{1}{(3.767\,303 \times 10^2\,\Omega)\,(2.997\,925 \times 10^8\,\mathrm{m\,s^{-1}})} = 8.854\,188 \times 10^{-12}\,\mathrm{F\,m^{-1}} \tag{3.24}$$

Alternate Bohr-anchored numeric (using e, α, h, c). With e exact, h exact, and α from CODATA 2022:

$$\epsilon_0 = \frac{e^2}{2\alpha hc} = \frac{(1.602\,177\times 10^{-19}\,\mathrm{C})^2}{2\,(7.297\,353\times 10^{-3})\,6.626\,070\times 10^{-34}\,\mathrm{J\,s}\,2.997\,925\times 10^{8}\,\mathrm{m\,s^{-1}}}$$
$$= 8.854\,188\times 10^{-12}\,\mathrm{F\,m^{-1}} \tag{3.25}$$

agreeing with (3.24). (Then $\mu_0 = 1/(\epsilon_0 c^2)$ reproduces (3.23).

4 Electromagnetic Quartet

4.1 Voltage, current, power, impedance

We collect the four Planck–electromagnetic "circuit" quantities $\{V_P,\ I_P,\ P_P,\ Z_P\}$ and derive each from the Titus lattice (E_P, t_P, q_P) with explicit substitutions. Throughout we *plug in the Titus Planck charge*

$$q_P = \sqrt{\frac{4\pi}{Z_0}\,m_e a_0 v_B} \quad \text{(equivalently } q_P^2 = \tfrac{4\pi}{Z_0} m_e a_0 v_B\text{, Eq. (3.1)}$$

and use the loop identity $E_P t_P = h = 2\pi\hbar$ with $\hbar = m_e a_0 v_B$. The quartet is tightly coupled: $P_P = V_P I_P = V_P^2/Z_P = I_P^2 Z_P$ and $Z_P = Z_0/2$.

Power (Titus vs. Std step by step). From $E_P t_P = h$,

$$P_P^{(\mathrm{Titus})} \equiv \frac{E_P}{t_P} = \frac{h}{t_P^2} = \frac{2\pi\,\hbar}{t_P^2} \tag{4.1}$$

while the Std (reduced) form uses $E_{P,\mathrm{std}} = \hbar/t_P$:

$$P_{P,\mathrm{std}} \equiv \frac{E_{P,\mathrm{std}}}{t_P} = \frac{\hbar}{t_P^2} \tag{4.2}$$

Hence $P_P^{(\mathrm{Titus})} = 2\pi\,P_{P,\mathrm{std}}$. With $\hbar = m_e a_0 v_B$,

$$P_{P,\mathrm{std}} = \frac{m_e a_0 v_B}{t_P^2} \qquad P_P^{(\mathrm{Titus})} = \frac{2\pi\,m_e a_0 v_B}{t_P^2} \tag{4.3}$$

Current (plug in Titus q_P, shown step by step). Start from the Titus Planck charge Eq. (3.1):

$$q_P^2 = \frac{4\pi}{Z_0}\,m_e a_0 v_B \tag{4.4}$$

$$q_P = \sqrt{\frac{4\pi}{Z_0}\,m_e a_0 v_B} \tag{4.5}$$

Then use the definition $I_P \equiv q_P/t_P$:

$$I_P \equiv \frac{q_P}{t_P} = \frac{1}{t_P}\sqrt{\frac{4\pi}{Z_0}\,m_e a_0 v_B} \tag{4.6}$$

Equivalently, start from the lattice relation $q_P^2 = \dfrac{2E_P t_P}{Z_0}$ and take the square root:

$$q_P = \sqrt{\frac{2E_P t_P}{Z_0}} \tag{4.7}$$

$$I_P = \frac{q_P}{t_P} = \frac{1}{t_P}\sqrt{\frac{2E_P t_P}{Z_0}} = \sqrt{\frac{2E_P}{Z_0\,t_P}} \tag{4.8}$$

To see that (4.8) matches (4.6) explicitly, insert the Titus energy $E_P = \dfrac{2\pi\,m_e a_0 v_B}{t_P}$:

$$\sqrt{\frac{2E_P}{Z_0\,t_P}} = \sqrt{\frac{2}{Z_0\,t_P}\cdot\frac{2\pi\,m_e a_0 v_B}{t_P}} = \sqrt{\frac{4\pi}{Z_0}\,\frac{m_e a_0 v_B}{t_P^2}} = \frac{1}{t_P}\sqrt{\frac{4\pi}{Z_0}\,m_e a_0 v_B} \tag{4.9}$$

Units check (numberless): $(4\pi/Z_0)\,m_e a_0 v_B$ has units of q_P^2 (C^2); the square root gives q_P (C); dividing by t_P (s) yields I_P in amperes.

Numeric (CODATA 2022).

$$I_P = \frac{q_P}{t_P} = \frac{1.875\,546\times10^{-18}}{5.391\,247\times10^{-44}}\ \mathrm{A} = 3.478\,872\times10^{25}\ \mathrm{A} \tag{4.10}$$

Voltage (plug in Titus q_P). Begin with $V_P \equiv E_P/q_P$ and substitute q_P:

$$V_P = \frac{E_P}{\sqrt{(4\pi/Z_0)\,m_e a_0 v_B}} = \sqrt{\frac{E_P^2}{(4\pi/Z_0)\,m_e a_0 v_B}} \tag{4.11}$$

Insert the Titus energy $E_P = \dfrac{2\pi m_e a_0 v_B}{t_P}$ and simplify:

$$\begin{aligned} V_P &= \sqrt{\frac{\left(\frac{2\pi m_e a_0 v_B}{t_P}\right)^2}{(4\pi/Z_0)\,m_e a_0 v_B}} = \frac{1}{t_P}\sqrt{\frac{(2\pi)^2(m_e a_0 v_B)^2}{4\pi/Z_0}\cdot\frac{1}{m_e a_0 v_B}} \\ &= \frac{1}{t_P}\sqrt{\pi\,Z_0\,m_e a_0 v_B}, \quad \text{since } \frac{(2\pi)^2}{4\pi} = \pi \end{aligned} \tag{4.12}$$

Equivalently, combining $q_P^2 = \dfrac{2E_P t_P}{Z_0}$ with $V_P = E_P/q_P$ gives

$$V_P = \sqrt{\frac{E_P Z_0}{2t_P}} \tag{4.13}$$

which is the same result as (4.12) after inserting E_P.

Impedance (plug in Titus q_P; two ways). Using the current and voltage just obtained,

$$Z_P \equiv \frac{V_P}{I_P} = \frac{\frac{1}{t_P}\sqrt{\pi\, Z_0\, m_e a_0 v_B}}{\frac{1}{t_P}\sqrt{(4\pi/Z_0)\, m_e a_0 v_B}} = \sqrt{\frac{\pi\, Z_0}{4\pi/Z_0}} = \sqrt{\frac{Z_0^2}{4}} = \frac{Z_0}{2} \tag{4.14}$$

Alternatively, use $Z_P = \dfrac{h}{q_P^2}$ and substitute $h = 2\pi m_e a_0 v_B$, $q_P^2 = \dfrac{4\pi}{Z_0} m_e a_0 v_B$:

$$Z_P = \frac{2\pi m_e a_0 v_B}{(4\pi/Z_0)\, m_e a_0 v_B} = \frac{Z_0}{2} \tag{4.15}$$

The power identities then follow:

$$P_P^{(\text{Titus})} = V_P I_P = \frac{V_P^2}{Z_P} = I_P^2 Z_P \tag{4.16}$$

and explicitly,

$$\begin{aligned} V_P I_P &= \left(\frac{1}{t_P}\sqrt{\pi\, Z_0\, m_e a_0 v_B}\right)\left(\frac{1}{t_P}\sqrt{\frac{4\pi}{Z_0}\, m_e a_0 v_B}\right) \\ &= \frac{1}{t_P^2}\sqrt{\pi\, Z_0\, m_e a_0 v_B \cdot \frac{4\pi}{Z_0}\, m_e a_0 v_B} = \frac{1}{t_P^2}\sqrt{4\pi^2 (m_e a_0 v_B)^2} \\ &= \frac{2\pi\, m_e a_0 v_B}{t_P^2} = P_P^{(\text{Titus})} \end{aligned} \tag{4.17}$$

consistent with (4.3).

Numeric (CODATA 2022, all routes agree). *Voltage:*

$$V_P = \frac{E_P}{q_P} = \frac{1.229\,042 \times 10^{10}}{1.875\,546 \times 10^{-18}}\ \text{V} = 6.552\,983 \times 10^{27}\ \text{V} \tag{4.18}$$

$$\begin{aligned} V_P &= \frac{1}{t_P}\sqrt{\pi\, Z_0\, m_e a_0 v_B} = \left(1.854\,858 \times 10^{43}\right)\sqrt{\pi\, 3.767\,303 \times 10^2\ 1.054\,572 \times 10^{-34}}\ \text{V} \\ &= 6.552\,983 \times 10^{27}\ \text{V} \end{aligned} \tag{4.19}$$

Current:

$$I_P = \frac{q_P}{t_P} = \frac{1.875\,546 \times 10^{-18}}{5.391\,247 \times 10^{-44}}\ \text{A} = 3.478\,872 \times 10^{25}\ \text{A} \tag{4.20}$$

$$\begin{aligned} I_P &= \frac{1}{t_P}\sqrt{\frac{4\pi}{Z_0}\, m_e a_0 v_B} = \left(1.854\,858 \times 10^{43}\right)\sqrt{\frac{4\pi}{3.767\,303 \times 10^2}\ 1.054\,572 \times 10^{-34}}\ \text{A} \\ &= 3.478\,872 \times 10^{25}\ \text{A} \end{aligned} \tag{4.21}$$

Power:

$$P_P^{(\text{Titus})} = \frac{E_P}{t_P} = \frac{1.229\,042 \times 10^{10}}{5.391\,247 \times 10^{-44}}\ \text{W} = 2.279\,699 \times 10^{53}\ \text{W} \tag{4.22}$$

$$P_{P,\text{std}} = \frac{\hbar}{t_P^2} = \frac{P_P^{(\text{Titus})}}{2\pi} = 3.628\,255 \times 10^{52}\ \text{W} \tag{4.23}$$

$$= \frac{c^5}{G} = 3.628\,255 \times 10^{52}\ \text{W} \tag{4.24}$$

Impedance:

$$Z_P = \frac{V_P}{I_P} = \frac{6.552\,983 \times 10^{27}}{3.478\,872 \times 10^{25}}\,\Omega = 1.883\,652 \times 10^{2}\,\Omega$$
$$= \frac{Z_0}{2} = \frac{3.767\,303 \times 10^{2}}{2}\,\Omega = 1.883\,652 \times 10^{2}\,\Omega \tag{4.25}$$

Units Checks. $[V_P] = [E/q] = \mathrm{J/C} = \mathrm{V}$, $[I_P] = [q/t] = \mathrm{A}$, $[Z_P] = [V/I] = \Omega$, $[P_P] = [E/t] = \mathrm{W}$.

5 Bohr radius (consistency and evaluation)

Canonical and Titus–consistent forms. Starting from the canonical definition,

$$a_0 = \frac{\hbar}{m_e c\,\alpha} \tag{5.1}$$

the Titus loop identity $E_P t_P = h = 2\pi\hbar$ implies

$$a_0 = \frac{E_P t_P}{2\pi\, m_e c\,\alpha} \tag{5.2}$$

and using $c = \ell_P/t_P$ (i.e. $\ell_P = c\,t_P$; Eq. (2.14) gives the length–time form

$$a_0 = \frac{E_P\, t_P^2}{2\pi\, m_e\,\alpha\,\ell_P} \tag{5.3}$$

Equations (5.1), (5.2), and (5.3) are algebraically identical under the Titus substitutions.

Bohr identity $\Rightarrow \alpha = v_B/c$ (full cancellation). Insert the Bohr action identity $\hbar = m_e a_0 v_B$ into (5.1):

$$a_0 = \frac{m_e a_0 v_B}{m_e c\,\alpha} = a_0\,\frac{v_B}{c\,\alpha} \;\Rightarrow\; 1 = \frac{v_B}{c\,\alpha} \;\Rightarrow\; \boxed{\alpha = \frac{v_B}{c}} \tag{5.4}$$

Equivalently, starting from (5.2) and using $h = 2\pi m_e a_0 v_B$ and $\ell_P = c\,t_P$ leads to the same result.

Units checks. From (5.1): $[\hbar] = \mathrm{J\,s} = \mathrm{kg\,m^2\,s^{-1}}$, and $[m_e c\alpha] = (\mathrm{kg})(\mathrm{m\,s^{-1}})(1) = \mathrm{kg\,m\,s^{-1}}$. Hence $[a_0] = \mathrm{m}$. From (5.3): $[E_P t_P^2] = (\mathrm{J})(\mathrm{s^2}) = \mathrm{kg\,m^2}$ and $[m_e\alpha\ell_P] = (\mathrm{kg})(1)(\mathrm{m}) = \mathrm{kg\,m}$, so $[a_0] = \mathrm{m}$.

Numerical validation (CODATA 2022). All three routes yield the same value.

Canonical ($\hbar$, m_e, c, α):

$$a_0 = \frac{1.054\,572 \times 10^{-34}\,\mathrm{J\,s}}{(9.109\,384 \times 10^{-31}\,\mathrm{kg})(2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}})\,(7.297\,353 \times 10^{-3})}$$
$$= 5.291\,772 \times 10^{-11}\,\mathrm{m} \tag{5.5}$$

Titus loop form (E_P, t_P, m_e, c, α):

$$a_0 = \frac{1.229\,042 \times 10^{10}\,\mathrm{J}\,(5.391\,247 \times 10^{-44}\,\mathrm{s})}{2\pi\,(9.109\,384 \times 10^{-31}\,\mathrm{kg})(2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}})\,(7.297\,353 \times 10^{-3})}$$
$$= 5.291\,772 \times 10^{-11}\,\mathrm{m} \tag{5.6}$$

Length–time form (E_P, t_P, ℓ_P, m_e, α):

$$a_0 = \frac{1.229\,042 \times 10^{10}\,\mathrm{J}\,(5.391\,247 \times 10^{-44}\,\mathrm{s})^2}{2\pi\,(9.109\,384 \times 10^{-31}\,\mathrm{kg})\,(7.297\,353 \times 10^{-3})\,1.616\,255 \times 10^{-35}\,\mathrm{m}}$$
$$= 5.291\,772 \times 10^{-11}\,\mathrm{m} \tag{5.7}$$

Velocity check (for completeness). From (5.4), $v_B = \alpha c$ gives

$$v_B = (7.297\,353 \times 10^{-3})\,(2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}) = 2.187\,691 \times 10^{6}\,\mathrm{m\,s^{-1}}, \tag{5.8}$$

consistent with earlier evaluations and confirming $\alpha = v_B/c$.

6 Elementary charge (closed loop)

Closed-loop identities (Titus and Std, step by step). Starting from the Titus/Bohr and vacuum relations Eq. (3.1),

$$q_P^2 = \frac{4\pi}{Z_0}\,m_e a_0 v_B = \frac{2E_P t_P}{Z_0} = 4\pi\,\epsilon_0\,\hbar\,c$$

and the definition $\alpha = \dfrac{e^2}{q_P^2}$, we obtain the equivalent forms for the elementary charge:

$$e^2 = \alpha\,q_P^2 \tag{6.1}$$
$$= \alpha\,\frac{4\pi}{Z_0}\,m_e a_0 v_B \tag{6.2}$$
$$= \alpha\,\frac{2E_P t_P}{Z_0} \tag{6.3}$$
$$= 4\pi\,\epsilon_0\,\hbar\,c\,\alpha \tag{6.4}$$

Equivalences: (6.2)→(6.4) uses $1/Z_0 = \epsilon_0 c$ and $\hbar = m_e a_0 v_B$; (6.3)→(6.4) uses $c = \ell_P/t_P$ and $E_P t_P = 2\pi\hbar$.

Length–time (lattice) variants. Using $\alpha = \dfrac{v_B}{c}$ and $c = \dfrac{\ell_P}{t_P}$,

$$e^2 = \alpha\,\frac{4\pi}{Z_0}\,m_e a_0 v_B = \frac{4\pi\,m_e a_0 v_B^2}{Z_0\,c} = \boxed{\frac{4\pi\,t_P\,m_e a_0 v_B^2}{Z_0\,\ell_P}} \tag{6.5}$$

$$e^2 = \alpha\,\frac{2E_P t_P}{Z_0} = \frac{2E_P t_P}{Z_0}\,\frac{v_B}{c} = \boxed{\frac{2E_P\,t_P^2\,v_B}{Z_0\,\ell_P}} \tag{6.6}$$

These show e^2 directly in terms of the Titus lattice triplet (E_P, t_P, ℓ_P) and the Bohr triplet (m_e, a_0, v_B).

Units checks. From (6.3): $[\alpha] = 1$ and $[q_P^2] = \mathrm{C}^2 \Rightarrow [e^2] = \mathrm{C}^2$. From (6.4): $[\epsilon_0] = \mathrm{C}/(\mathrm{V\,m})$, $[\hbar] = \mathrm{J\,s} = \mathrm{V\,C\,s}$, $[c] = \mathrm{m/s} \Rightarrow [\epsilon_0 \hbar c] = \mathrm{C}^2$; with $[\alpha] = 1$, $[e^2] = \mathrm{C}^2$.

Numeric validations (CODATA 2022). Two independent routes give the same value (within the small uncertainties of non-exact constants). We keep numbers compact to fit line width.

Titus route (using (6.3)*):*

$$\begin{aligned} e^2 &= \alpha \, \frac{2E_P t_P}{Z_0} \\ &= \left(7.297\,353 \times 10^{-3}\right) \frac{2\,(1.229\,042 \times 10^{10})\,(5.391\,247 \times 10^{-44})}{3.767\,303 \times 10^{2}} \ \mathrm{C}^2 \\ &= 2.566\,970 \times 10^{-38}\,\mathrm{C}^2 \end{aligned} \tag{6.7}$$

Std route (using (6.4)*):*

$$\begin{aligned} e^2 &= 4\pi \, \epsilon_0 \, \hbar \, c \, \alpha \\ &= 4\pi \left(8.854\,188 \times 10^{-12}\right) \left(1.054\,572 \times 10^{-34}\right) \left(2.997\,925 \times 10^{8}\right) \left(7.297\,353 \times 10^{-3}\right) \mathrm{C}^2 \\ &= 2.566\,970 \times 10^{-38}\,\mathrm{C}^2 \end{aligned} \tag{6.8}$$

Exact comparison from the SI definition of e:

$$e^2_{\text{exact}} = \left(1.602\,177 \times 10^{-19}\,\mathrm{C}\right)^2 = 2.566\,970 \times 10^{-38}\,\mathrm{C}^2 \tag{6.9}$$

The tiny difference between (6.7)–(6.8) and (6.9) reflects the experimental uncertainty in α and for the Std route in ϵ_0 as inferred via α, whereas e is exact in the SI.

7 Unified Planck-Scale Two-Plane Throttling, Phase–Time Reciprocity, and Applications

We model the lattice with two planes: *time* and *length*. Gravity throttles both planes equally (two-plane), while velocity dilates the time plane only (one-plane). Planck's constant h is invariant as the action per full 2π of phase; frequencies/energies shift because the lattice dilates the tick. We derive the orbit-lock condition, map energy to phase/time, and work through orbital and neutron-star examples, plus impedance/atomic checks.

7.1 Lattice throttling framework (two-plane) and redshift

In the Titus framework, gravitational potential throttles the local phase channels (time tick t_P and spatial step ℓ_P) while preserving $c = \ell_P / t_P$. Define the dimensionless potential parameter

$$\theta(r) \ \equiv \ \frac{2\,G\,M}{r\,c^2} \ = \ \frac{r_s}{r} \tag{7.1}$$

where r_s is the Schwarzschild radius. This dimensionless form is used throughout; the full angular representation $\theta = (|\phi_g|/c^2)\, 2\pi$ appears in [25].

The **throttle (phase index)** is

$$k(r) \;=\; \frac{1}{\sqrt{1-\theta(r)}} \;=\; 1 + \frac{\theta}{2} + \mathcal{O}(\theta^2) \tag{7.2}$$

At each lattice node,

$$t'_P(r) = k\, t_P \qquad \ell'_P(r) = k\, \ell_P \qquad \frac{\ell'_P}{t'_P} = c \quad \text{(invariant)} \tag{7.3}$$

For light emitted at radius r and received at infinity, the Schwarzschild redshift gives [5, 6, 7]

$$f_\infty = f_0 \sqrt{1-\theta(r)} = \frac{f_0}{k(r)} \tag{7.4}$$

and, because $c = f\lambda$ is invariant,

$$\lambda_\infty = k(r)\, \lambda_0 \tag{7.5}$$

Defining the gravitational redshift parameter,

$$z \;\equiv\; \frac{\lambda_\infty - \lambda_0}{\lambda_0} = k - 1 \tag{7.6}$$

the first-order (weak-field) relations follow directly:

$$\frac{\Delta t}{t} = \frac{\Delta \ell}{\ell} = +\frac{\theta}{2} \qquad \frac{\Delta f}{f} = -\frac{\theta}{2} \qquad \frac{\Delta \lambda}{\lambda} = +\frac{\theta}{2} \tag{7.7}$$

7.2 Action–phase mapping and cross-field comparison (one-plane vs two-plane)

Planck anchors and constants.

$$c = 2.997\,925 \times 10^{8}\,\text{m/s} \quad \text{(exact)} \qquad h = 6.626\,070 \times 10^{-34}\,\text{J s} \qquad \hbar = \frac{h}{2\pi} \tag{7.8}$$

$$t_P = 5.391\,247 \times 10^{-44}\,\text{s} \qquad f_P = \frac{1}{t_P} = 1.854\,858 \times 10^{43}\,\text{Hz} \qquad E_P = \frac{h}{t_P} \tag{7.9}$$

Planck energy identity:

$$E_P \;=\; \frac{2\pi\,\hbar}{t_P} \;=\; \hbar\omega_P \;=\; \frac{h}{t_P} \tag{7.10}$$

Energy from phase per time. Let ϑ be the phase angle accumulated over interval t':

$$E \;=\; \hbar\,\frac{\vartheta}{t'} \tag{7.11}$$

so per baseline tick t_P,

$$\frac{E}{E_P} \;=\; \frac{\vartheta}{2\pi}\,\frac{t_P}{t'} \tag{7.12}$$

Cross-field (and kinematic) comparison. Let the emitter have $(k_g, k_v)_{\text{em}}$ and the observer $(k_g, k_v)_{\text{obs}}$, where the gravitational throttle is two-plane and the (transverse) kinematic dilation is one-plane:

$$\frac{f_{\text{obs}}}{f_{\text{em}}} = \frac{k_{g,\text{obs}}}{k_{g,\text{em}}} \cdot \frac{k_{v,\text{obs}}}{k_{v,\text{em}}} \qquad \frac{E_{\text{obs}}}{E_{\text{em}}} = \frac{f_{\text{obs}}}{f_{\text{em}}} \qquad \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} = \left(\frac{f_{\text{obs}}}{f_{\text{em}}}\right)^{-1} \tag{7.13}$$

If the local tick dilates to $t' = k_{\text{tot}}\, t_P$ with

$$k_{\text{tot}} = k_g\, k_v \tag{7.14}$$

then keeping the *local* energy/action invariant requires

$$\vartheta' = k_{\text{tot}}\, \vartheta_0 \qquad E_{\text{local}} = \hbar\, \frac{\vartheta'}{t'} = \hbar\, \frac{\vartheta_0}{t_P} = E_{\text{em}} \tag{7.15}$$

and a different observer measures

$$\frac{E_{\text{obs}}}{E_P} = \frac{f_{\text{em}} t_P}{k_{\text{tot}}} = \frac{E_{\text{em}}}{E_P}\, \frac{1}{k_{\text{tot}}} \tag{7.16}$$

7.3 Orbit lock: two-plane vs one-plane balance

Let $\beta = v/c$, $\gamma_v = 1/\sqrt{1-\beta^2}$. Velocity spreads phase only in the time plane; gravity stretches both tick and step. In the weak-limit, a circular orbit corresponds to a constant phase density along the path:

$$\delta_g \simeq 2\, \delta_v \tag{7.17}$$

with $\delta_g \simeq \theta/2$ and $\delta_v \simeq \beta^2/2$, giving

$$\frac{\theta}{2} \simeq 2\, \frac{\beta^2}{2} \quad \Rightarrow \quad \boxed{\theta = 2\beta^2} \tag{7.18}$$

Using (7.1),

$$\boxed{v^2 = \frac{GM}{r}} \tag{7.19}$$

i.e. the standard circular condition; the factor of two reflects gravity acting on both planes while velocity acts on one [5, 6, 7].

7.4 Constants and Planck/Titus anchors

Unless otherwise specified, CODATA 2022 [1]:

$$G = 6.674\,300 \times 10^{-11}\, \text{m}^3\, \text{kg}^{-1}\, \text{s}^{-2} \qquad c = 2.997\,925 \times 10^{8}\, \text{m/s (exact)} \tag{7.20}$$

$$M_\oplus = 5.972\,000 \times 10^{24}\, \text{kg} \qquad M_\odot = 1.988\,470 \times 10^{30}\, \text{kg} \tag{7.21}$$

Mean orbital radii:

$$r_{\text{Moon}\leftarrow\oplus} = 3.844\,000 \times 10^{8}\,\text{m} \quad r_{\oplus\leftarrow\odot} = 1.495\,979 \times 10^{11}\,\text{m} \quad r_{\text{Mars}\leftarrow\odot} = 2.279\,392 \times 10^{11}\,\text{m} \tag{7.22}$$

Titus Planck-frequency anchor:

$$f_P = 1.854\,858 \times 10^{43}\,\text{Hz} \quad t_P = \frac{1}{f_P} = 5.391\,247 \times 10^{-44}\,\text{s} \quad \ell_P = c\,t_P = 1.616\,255 \times 10^{-35}\,\text{m} \tag{7.23}$$

7.5 Worked examples: Moon, Earth, and Mars (circular)

For each case, $v = \sqrt{GM/r}$, $\beta^2 = (v/c)^2$, $\theta = 2GM/(rc^2)$, $k_g = 1/\sqrt{1-\theta}$, $k_v = 1/\sqrt{1-\beta^2}$, $\delta_g = k_g - 1$, $\delta_v = k_v - 1$, and per-tick dilations $R_{t,g} = \delta_g t_P$, $R_{t,v} = \delta_v t_P$, $\Delta\ell_g = \delta_g \ell_P$, $\Delta\ell_v = \delta_v \ell_P$. We report $\delta_g/(2\delta_v)$.

(1) Moon about Earth

$$r = 3.844\,000 \times 10^{8}\,\text{m} \qquad v = \sqrt{\frac{GM_\oplus}{r}} = 1.018\,289 \times 10^{3}\,\text{m/s} \tag{7.24}$$

$$\beta^2 = 1.153\,721 \times 10^{-11} \qquad \theta = 2.307\,442 \times 10^{-11} \tag{7.25}$$

$$k_g = 1.000\,000 \qquad k_v = 1.000\,000 \tag{7.26}$$

$$\delta_g = 1.153\,722 \times 10^{-11} \qquad \delta_v = 5.768\,497 \times 10^{-12} \tag{7.27}$$

$$\frac{\delta_g}{2\delta_v} = 1.000\,019 \tag{7.28}$$

$$R_{t,g} = 6.219\,523 \times 10^{-55}\,\text{s} \qquad R_{t,v} = 3.109\,702 \times 10^{-55}\,\text{s} \tag{7.29}$$

$$\Delta\ell_g = 1.864\,566 \times 10^{-46}\,\text{m} \qquad \Delta\ell_v = 9.322\,651 \times 10^{-47}\,\text{m} \tag{7.30}$$

(2) Earth about the Sun (near 1 AU)

$$r = 1.495\,979 \times 10^{11}\,\text{m} \qquad v = \sqrt{\frac{GM_\odot}{r}} = 2.978\,514 \times 10^{4}\,\text{m/s} \tag{7.31}$$

$$\beta^2 = 9.870\,927 \times 10^{-9} \qquad \theta = 1.974\,185 \times 10^{-8} \tag{7.32}$$

$$k_g = 1.000\,000 \qquad k_v = 1.000\,000 \tag{7.33}$$

$$\delta_g = 9.870\,927 \times 10^{-9} \qquad \delta_v = 4.935\,464 \times 10^{-9} \tag{7.34}$$

$$\frac{\delta_g}{2\delta_v} = 1.000\,000 \tag{7.35}$$

$$R_{t,g} = 5.321\,255 \times 10^{-52}\,\text{s} \qquad R_{t,v} = 2.660\,627 \times 10^{-52}\,\text{s} \tag{7.36}$$

$$\Delta\ell_g = 1.595\,272 \times 10^{-43}\,\text{m} \qquad \Delta\ell_v = 7.976\,360 \times 10^{-44}\,\text{m} \tag{7.37}$$

(3) Mars about the Sun

$$r = 2.279\,392 \times 10^{11}\,\mathrm{m} \qquad v = \sqrt{\frac{GM_\odot}{r}} = 2.412\,975 \times 10^{4}\,\mathrm{m/s} \tag{7.38}$$

$$\beta^2 = 6.478\,349 \times 10^{-9} \qquad \theta = 1.295\,670 \times 10^{-8} \tag{7.39}$$

$$k_g = 1.000\,000 \qquad k_v = 1.000\,000 \tag{7.40}$$

$$\delta_g = 6.478\,349 \times 10^{-9} \qquad \delta_v = 3.239\,174 \times 10^{-9} \tag{7.41}$$

$$\frac{\delta_g}{2\delta_v} = 1.000\,000 \tag{7.42}$$

$$R_{t,g} = 3.492\,371 \times 10^{-52}\,\mathrm{s} \qquad R_{t,v} = 1.746\,186 \times 10^{-52}\,\mathrm{s} \tag{7.43}$$

$$\Delta\ell_g = 1.046\,987 \times 10^{-43}\,\mathrm{m} \qquad \Delta\ell_v = 5.234\,933 \times 10^{-44}\,\mathrm{m} \tag{7.44}$$

7.6 540 nm test line: Earth, Sun, and neutron stars

Constants used (CODATA 2022) [1]:

$c = 2.997\,925 \times 10^{8}\,\mathrm{m/s}$ (exact) $\quad G = 6.674\,300 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
$M_\oplus = 5.972\,000 \times 10^{24}\,\mathrm{kg}$, $R_\oplus = 6.371\,000 \times 10^{6}\,\mathrm{m}$ $\quad M_\odot = 1.988\,470 \times 10^{30}\,\mathrm{kg}$, $R_\odot = 6.963\,400 \times 10^{8}\,\mathrm{m}$

Baseline $\lambda_0 = 5.400\,000 \times 10^{2}\,\mathrm{nm} = 5.400\,000 \times 10^{-7}\,\mathrm{m}$ gives

$$f_0 = \frac{c}{\lambda_0} = \frac{2.997\,925 \times 10^{8}\,\mathrm{m/s}}{5.400\,000 \times 10^{-7}\,\mathrm{m}} = 5.551\,712 \times 10^{14}\,\mathrm{Hz} \tag{7.45}$$

Earth surface → infinity

$$\theta_\oplus = \frac{2GM_\oplus}{R_\oplus c^2} = 1.392\,216 \times 10^{-9} \tag{7.46}$$

$$k_\oplus = \frac{1}{\sqrt{1-\theta_\oplus}} = 1.000\,000 \tag{7.47}$$

Using (7.4)–(7.5):

$$f_\infty = \frac{f_0}{k_\oplus} = 5.551\,708 \times 10^{14}\,\mathrm{Hz} \qquad \Delta f = -3.864\,590 \times 10^{5}\,\mathrm{Hz} \tag{7.48}$$

$$\lambda_\infty = k_\oplus\,\lambda_0 = 5.400\,000 \times 10^{2}\,\mathrm{nm} \qquad \Delta\lambda = 3.758\,980 \times 10^{-7}\,\mathrm{nm} \tag{7.49}$$

Sun photosphere → infinity

$$\theta_\odot = \frac{2GM_\odot}{R_\odot c^2} = 4.241\,232 \times 10^{-6} \qquad k_\odot = \frac{1}{\sqrt{1-\theta_\odot}} = 1.000\,002 \tag{7.50}$$

$$f_\infty = \frac{f_0}{k_\odot} = 5.551\,700 \times 10^{14}\,\mathrm{Hz} \qquad \lambda_\infty = k_\odot\,\lambda_0 = 5.400\,011 \times 10^{2}\,\mathrm{nm} \tag{7.51}$$

Neutron stars (surface → infinity). For $M = \{1.4, 2.0\}\, M_\odot$ and $R = \{10, 12\}$ km:

Case	θ	k	$z = k - 1$	f_∞ (THz)	λ_∞ (nm)	$\Delta\lambda$ (nm)
M=1.4 $M_\odot$, R=1.200 000 × 10^1 km	0.3446	1.2352	0.2352	449.46	667.0	+127.0
M=1.4 $M_\odot$, R=1.000 000 × 10^1 km	0.4135	1.3057	0.3057	425.18	705.1	+165.1
M=2.0 $M_\odot$, R=1.200 000 × 10^1 km	0.4922	1.4033	0.4033	395.61	757.8	+217.8
M=2.0 $M_\odot$, R=1.000 000 × 10^1 km	0.5907	1.5630	0.5630	355.19	844.0	+304.0

Force from throttle gradient. Interpreting $n \equiv k$ as a phase index, the weak-field geodesic acceleration recovers Newton's law:

$$\mathbf{a} = c^2 \nabla \ln n \;\Rightarrow\; a_r = c^2 \frac{d}{dr} \ln\!\left(\frac{1}{\sqrt{1-\theta}}\right) = -\frac{GM}{r^2} \tag{7.52}$$

7.7 Neutron-star redshift with local phase compensation (540 nm)

Using the same line, $\lambda_{\rm em} = 5.400\,000 \times 10^2$ nm gives $f_{\rm em} = c/\lambda_{\rm em} = 5.551\,712 \times 10^{14}$ Hz, $E_{\rm em} = hf_{\rm em} = 3.678\,603 \times 10^{-19}$ J, and $E_{\rm em}/E_P = f_{\rm em} t_P = 2.992\,837 \times 10^{-29}$. The baseline phase per tick is

$$\vartheta_0 \;=\; 2\pi \frac{E_{\rm em}}{E_P} \;=\; 1.880\,455 \times 10^{-28} \text{ rad per } t_P \tag{7.53}$$

Gravity only: $M = 1.4\, M_\odot,\;\; R = 1.200\,000 \times 10^1$ km**.**

$$\theta = \frac{2GM}{Rc^2} = 3.445\,563 \times 10^{-1} \qquad\qquad k_g = \frac{1}{\sqrt{1-\theta}} = 1.235\,186 \tag{7.54}$$

Observer at infinity ($k_{g,\rm obs} = 1$):

$$f_\infty = \frac{f_{\rm em}}{k_g} = 4.494\,637 \times 10^{14}\,\text{Hz} \qquad \lambda_\infty = k_g\, \lambda_{\rm em} = 6.670\,000 \times 10^2\,\text{nm} \tag{7.55}$$

$$E_\infty = \frac{E_{\rm em}}{k_g} = 2.978\,178 \times 10^{-19}\,\text{J} \qquad \frac{E_\infty}{E_P} = \frac{E_{\rm em}}{E_P} \cdot \frac{1}{k_g} = 2.422\,985 \times 10^{-29} \tag{7.56}$$

Local phase compensation (source frame) keeping $E_{\rm em}$ fixed:

$$\vartheta'_{\rm NS} \;=\; k_g\, \vartheta_0 \;=\; 2.322\,711 \times 10^{-28} \text{ rad per } t_P \qquad t'_P = k_g\, t_P \tag{7.57}$$

Gravity + velocity (equatorial spin $f_{\rm spin} = 3.000\,000 \times 10^2$ Hz). Equatorial speed $v = 2\pi R f_{\rm spin} = 2.262\,000 \times 10^7$ m/s, $\beta = 7.545\,040 \times 10^{-2}$, $k_v = \gamma = 1.002\,859$.

$$k_{\rm tot} = k_g\, k_v = 1.238\,717 \tag{7.58}$$

$$f_\infty = \frac{f_{\rm em}}{k_{\rm tot}} = 4.482\,060 \times 10^{14}\,\text{Hz} \qquad \lambda_\infty = k_{\rm tot}\, \lambda_{\rm em} = 6.689\,070 \times 10^2\,\text{nm} \tag{7.59}$$

Local compensation:

$$\vartheta'_{\rm NS+spin} \;=\; k_{\rm tot}\, \vartheta_0 \;=\; 2.329\,000 \times 10^{-28} \text{ rad per } t_P \qquad t'_P \;=\; k_{\rm tot}\, t_P \tag{7.60}$$

7.8 Fine-structure constant validation via lattice throttling

The fine-structure constant couples the Bohr velocity to the Planck lattice ratio:

$$\alpha = \frac{v_B}{c} = v_B \, \frac{t_P}{\ell_P} \tag{7.61}$$

Because $c = \ell_P/t_P$, throttling t_P and ℓ_P by the same factor k leaves their ratio and therefore α invariant:

$$\alpha' = v'_B \frac{t'_P}{\ell'_P} = v'_B \frac{k_t}{k_\ell} \frac{t_P}{\ell_P} \;\Rightarrow\; v'_B = v_B \, \frac{k_\ell}{k_t} \tag{7.62}$$

If time dilates slightly more than length ($k_t > k_\ell$), the local orbital velocity v'_B decreases to preserve α, producing the same fractional frequency shift as gravitational redshift.

Numerical validation. Since $t_P/\ell_P = 1/c$, the Bohr radius a_0 tests the relation:

$$a_0 \, \frac{t_P}{\ell_P} = \frac{a_0}{c} \tag{7.63}$$

Using CODATA 2022 constants ($a_0 = 5.29177210903 \times 10^{-11}$ m, $c = 2.99792458 \times 10^{8}$ m/s)

$$\frac{1}{c} = 3.33564095198 \times 10^{-9} \text{ s/m}$$

$$\frac{a_0}{c} = (5.29177210903 \times 10^{-11})(3.33564095198 \times 10^{-9}) = 1.7651451755 \times 10^{-19} \text{ s} \tag{7.64}$$

Hence

$$\boxed{a_0 \, (t_P/\ell_P) = 1.76515 \times 10^{-19}\,\text{s}}$$

the light-travel time across one Bohr radius (0.1765 as).

7.9 Lattice impedance and coherent energy throughput (Planck application)

Classically, the impedance of free space is

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \mu_0 c = \frac{1}{\varepsilon_0 c} \tag{7.65}$$

with $c = 1/\sqrt{\mu_0 \varepsilon_0}$ ensuring $u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2} B^2/\mu_0 = u_B$, so $E/H = Z_0 \approx 3.767\,303 \times 10^2\,\Omega$. Within the Titus framework, the Planck loop $E_P t_P = h$ and the Planck charge [1]

$$q_P^2 = \frac{2h}{Z_0} \tag{7.66}$$

lead to a lattice (Planck) impedance

$$Z_P \equiv \frac{h}{q_P^2} = \frac{Z_0}{2} \tag{7.67}$$

Thus the effective vacuum impedance during complete lattice coherence is one-half the Maxwell value.

Power flow and energy partition. The time-averaged Poynting vector is

$$S = \frac{E^2}{Z_{\text{eff}}} \tag{7.68}$$

so at $Z_{\text{eff}} = Z_0/2$

$$S_P = \frac{E^2}{(Z_0/2)} = 2\,\frac{E^2}{Z_0} = 2S_0 \tag{7.69}$$

Energy-density ratio follows from

$$\frac{u_E}{u_B} = \frac{Z_{\text{eff}}^2}{Z_0^2} \quad \Rightarrow \quad \frac{u_E}{u_B}\bigg|_{Z_0/2} = \frac{1}{4} \tag{7.70}$$

Numeric check. With $Z_0 = 3.767\,303 \times 10^2\,\Omega$ [1],

$$Z_P = \frac{Z_0}{2} = 1.883\,652 \times 10^2\,\Omega \tag{7.71}$$

At $E = 1.000\,000\,\text{V/m}$:

$$S_0 = \frac{1}{376.730313} = 2.655 \times 10^{-3}\ \text{W/m}^2 \tag{7.72}$$

$$S_P = \frac{1}{188.3651565} = 5.310 \times 10^{-3}\ \text{W/m}^2 \tag{7.73}$$

so $S_P/S_0 = 2.000$, and $(u_E/u_B)_{Z_0/2} = 0.25$.

7.10 Selected dimensionless checks

- Sensitivity of magnetic permeability to α: $\dfrac{d\mu_0}{d\alpha} = 1.722 \times 10^{-4}$, from $Z_0 = 2\alpha R_K$ and $2R_K/c$ [1].
- Mass ratio $\delta = m_e/m_p = 5.446 \times 10^{-4}$, using $m_p = 1.67262192595 \times 10^{-27}\,\text{kg}$ (NIST) [23].
- Dilation parameter reminder: $\theta = 2GM/(rc^2) = r_s/r$ (dimensionless) used here; the full angular-phase form $\theta = (|\phi_g|/c^2)\,2\pi$ is discussed in [25].

7.11 Interpretation and scope

Across orbital examples, $\delta_g \approx 2\delta_v$ within rounding: the two-plane gravitational throttle offsets the one-plane velocity phase-spread so that phase density along the path is locked and the trajectory closes—the circular-orbit state. In strong fields, gravity dilates time (and length), and local phase per tick grows by the same factor to keep the local action/energy invariant; a distant observer measures multiplicative redshifts set by the k-ratio law (7.13). Classic factor-of-two effects (e.g., light bending) likewise reflect the sum of time- and length-plane contributions [5, 6, 7]. For precision timing (e.g., GNSS), gravitational and kinematic dilations combine as observed [8]. Atomic-scale checks (Sec. 7.8) and impedance scaling (Sec. 7.9) numerically align with the Planck-loop $E_P t_P = h$ and the two-plane throttling picture.

Appendix: raw numeric values (derived)

Using Table 1. Titus ("loop") values use $E_P = h/t_P$; Std ("reduced") values use $E_{P,\mathrm{std}} = \hbar/t_P$.

Energy and frequency

$$E_P = 1.229\,042 \times 10^{10}\,\mathrm{J} \qquad E_{P,\mathrm{std}} = 1.956\,081 \times 10^{9}\,\mathrm{J}$$
$$f_P = 1.854\,858 \times 10^{43}\,\mathrm{Hz} \qquad \omega_P = 1.165\,442 \times 10^{44}\,\mathrm{s}^{-1}$$

Mass and momentum

$$m_P = 1.367\,494 \times 10^{-7}\,\mathrm{kg} \qquad p_P = 4.099\,644 \times 10^{1}\,\mathrm{kg\,m\,s^{-1}}$$

Force and densities

$$F_P = 7.604\,259 \times 10^{44}\,\mathrm{N} \qquad u_P \equiv \frac{E_P}{\ell_P^3} = 2.910\,966 \times 10^{114}\,\mathrm{Pa}$$
$$\rho_P = 3.238\,887 \times 10^{97}\,\mathrm{kg\,m^{-3}}$$

Temperature and phase-per-K

$$T_P = 8.901\,917 \times 10^{32}\,\mathrm{K} \qquad T_{P,\mathrm{std}} = 1.416\,784 \times 10^{32}\,\mathrm{K}$$
$$\vartheta_{1\mathrm{K}}^{(\mathrm{Titus})} = 7.058\,239 \times 10^{-33}\,\mathrm{rad\,K^{-1}} \qquad \vartheta_{1\mathrm{K}}^{(\mathrm{std})} = 4.434\,822 \times 10^{-32}\,\mathrm{rad\,K^{-1}}$$

EM constants and quartet

$$Z_0 = 3.767\,303 \times 10^{2}\,\Omega \qquad Z_P = 1.883\,652 \times 10^{2}\,\Omega$$
$$\mu_0 = 1.256\,637 \times 10^{-6}\,\mathrm{N\,A^{-2}} \qquad \epsilon_0 = 8.854\,188 \times 10^{-12}\,\mathrm{F\,m^{-1}}$$
$$V_P = 6.552\,983 \times 10^{27}\,\mathrm{V} \qquad I_P = 3.478\,872 \times 10^{25}\,\mathrm{A}$$
$$P_P^{(\text{power, Titus})} = 2.279\,699 \times 10^{53}\,\mathrm{W} \qquad P_{P,\mathrm{std}} = 3.629\,702 \times 10^{52}\,\mathrm{W}$$

Bohr velocity and gravitational constant.

$$v_B = \alpha c = 2.187\,691 \times 10^{6}\,\mathrm{m\,s^{-1}} \qquad G = 6.674\,300 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}\ \text{(from (1.31))}$$

Charges.

$$q_P^2 = 3.517\,673 \times 10^{-36}\,\mathrm{C}^2 \qquad q_P = 1.875\,546 \times 10^{-18}\,\mathrm{C}$$

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