Wave Function of the Universe near Cosmological Singularity in three dimensions III

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Abstract

In this short report we investigate the wave function of the Universe near Cosmological Singularity in pure Einstein Gravity. The space time is considered in three dimensions for simplicity. We use the minisuperspace model and the canonical quantization . We extend the Hamiltonian Constraint , concretely , from $\mathcal{H} = 0$ to $\mathcal{H} \approx 0$. So we obtain the Schrodinger equation instead of the Wheeler-De Witt equation. The resulting wave functions and the energy levels are represented by the Harmonic Oscillator. Our models treat dS spacetime ($k = 1, \Lambda > 0$), which explains the closed expanding universe. However when we consider the neighborhood of the Big Bang Singularity, within the Planck scale, the space time has to be treated as quantum gravity. So our most interested wave function of the universe is the one near the singularity. However the identity of the Hamiltonian constraint is an open question in quantum gravity. Originally Quantum Cosmology was considered as the candidate of the quantum gravity , which is represented by the wave function instead of the metric structure..

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Contents

1 Introduction

In the past we investigated the wave function of the universe near the Big Bang singularity in three dimensions[1]. In this report we will investigate the wave functions and energy levels by quantum mechanics. In quantum cosmology where the universe is treated as the object of quantum mechanics, the universe is described by the wave function instead of the classical metric space-time structure. The wave function is the solution of the Wheeler-de Witt equation[9][10]. We are much interested in the wave function near Big Bang Singularity. No boundary wave function [3] and the Tunneling wave function [4][5]are both known as the representative quantum cosmological models. Along these directions , in lorentzian quantum cosmology where the lorentzian pathintegral is performed , some developement has been done with the Picard-Lefschetz method [7]recently. This point is not touched in this report . Originally Quantum cosmology is represented by the wave function of the universe. And the wave functions are the solutions of the Wheeler-De-Witt equation which is induced by the Hamiltonian constraint;$\mathcal{H} = 0$[8][9][10].
In this report we extend the Hamiltonian constraint, namely from $\mathcal{H} = 0$ to $\mathcal{H} \approx 0$, though we have to check more deeply. The Hamiltonis is obtained through the canonical procedure[8][11]. However in this case we could treat the wave function by Schrödinger equation , not by Schrödinger-Wheeler-De Witt equation. Through some approximation the wave function is represented by the Harmonic Oscillator wave functions.The energy levels are the ones of the harmonic oscillator. We concentrate on the wave function near the initial singularity. Regarding the extension of the hamiltonian constraint of general relativity, such a procedure would break the framework of general relativity. Lots of researchers

recognize that the Wheeler-De Witt equation is the central equation in the canonical approach to quantum gravity and is also an essential tool in quantum cosmology [12].However we are interested in the Planck scale physics where classical general relativity is no longer valid. So we may be able to extend the Hamiltonian constraint. In reality the identity of the Hamiltonian constraint is an open question in quantum gravity. We deal with the pure gravity without matter for simplicity.

Quantum Gravity is the theory we have been looking for around 100 years. However we have never seen even the small parts of it at the moment. String theory is one of the most strong candidates of it though the string theory has lots of difficulties. Quantum cosmology is the other way to quantum gravity.

The plan of this report is as follows. In Section 2 we begin by reviewing Quantum Cosmology very shortly. In Section 3 we will calculate the Ricci Scholar R and substitute it into the Einstein -Hilbert action to obtain the Lagrangian explicitly. In Section 4 we obtain the Hamiltonian by usual procedure (Legendre transformation).In Section 5 we willl extend the Hamiltonian Constraint. As the result we will obtain the Schrodinger equation instead of the Wheeler-De Witt equation. In Section 6 we will solve the Schrodinger equation precisely. We will have some discussion in Section 7.

2 Short Review of Quantum Cosmology

Quantum Cosmology is one of the quantum gravitational approaches. The most important points are the boundary conditions, namely the initial conditions. How the universe began? Nowadays two stories are known. : No-Boundary by Hartle-Hawking[3] and Tunneling by Vilenkin[4][5]. Meanwhile lots of efforts have been made by many researchers . For example the Euclidean path integral which represent the wave function of the universe was pointed out the possibility that it is not convergent.So the Lorentzian path integral for convergent was proposed with Picard -Lefschetz method[7]. However the main streets of quantum cosmology have been kept to be based on the above two approaches for many years. So we think we need the new boundary condition in order to develop the quantum cosmology. Our extension of the Hamiltonian Constraints is the trial along this line.

3 Brief Set-Up

In this section we do set-up simply. We start from the Einstein -Hilbert Action:

$$S = -\frac{1}{16\pi G}\int d^3x\sqrt{-g}(R-2\Lambda) \tag{1}$$

The metric is assumed to be the Robertson -Walker metric (homogenious and isometric space time): we set $N = 1.k = 1$

$$\begin{aligned} ds^2 &= -N^2dt^2 + a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2d\theta^2\right) \\ &= -dr^2 + a(t)^2\left(\frac{dr^2}{1-r^2} + r^2d\theta^2\right) \end{aligned}$$

We change the variables:

$$d\chi = \frac{dr}{\sqrt{1-r^2}} \tag{2}$$

$$\begin{aligned} ds^2 &= -dt^2 + a(t)^2(d\chi^2 + \sin^2\chi d\theta^2) \\ &= g_{\mu\nu}dx^\mu dx^\nu \end{aligned}$$

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & a^2 & \\ 0 & 0 & a^2\sin^2\chi \end{pmatrix} \tag{3}$$

where we set $N = 1.k = 1$, (N :lapse function) for simplicity.In addition we consider pure Gravity for simplicity
Christoffel symbols:

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2}g^{\alpha\lambda}(\partial_\mu g_{\nu\lambda} + \partial_\nu g_{\lambda\mu} - \partial_\lambda g_{\mu\nu})$$

Non vanishing terms; So we obtain the Ricci tensor;

$$R_{00} = -2\frac{\ddot{a}}{a}, R_{11} = \frac{1}{1-kr^2}(\dot{a}^2 + a\ddot{a} + k), R_{22} = r^2(\dot{a}^2 + a\ddot{a} + k) \tag{4}$$

We will obtaion the scalar curvature R.

$$R = g^{00}R_{00} + g^{11}R_{11} + g^{22}R_{22} = 2((\frac{\dot{a}}{a})^2 + \frac{k}{a^2} + \frac{2\ddot{a}}{a}) \tag{5}$$

Next we calculate the Einstein -Hilbert action after substituting (3) into (1).

$$\begin{aligned} S &= -\frac{1}{16\pi G}\int d^3x\sqrt{-g}(R - 2\Lambda) \\ &= \frac{1}{16\pi G}\int dt\int_0^\pi d\chi\int_0^\pi d\theta\sqrt{-g}\left(2\left(\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} + \frac{2\ddot{a}}{a}\right) - 2\Lambda\right) \end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{16\pi G}\int dt \int_0^\pi d\chi \int_0^\pi d\theta \sqrt{a^4 \sin^2\chi}\left(2\left(\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} + \frac{2\ddot{a}}{a}\right) - 2\Lambda\right) \\
&= -\frac{1}{16\pi G}\int dt \int_0^\pi d\chi \sin\chi \int_0^\pi d\theta a^2 \left(2\left(\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} + \frac{2\ddot{a}}{a}\right) - 2\Lambda\right) \\
&= -\frac{4\pi}{16\pi G}\int dt(\dot{a}^2 + 1 + 2a\ddot{a} - \Lambda a^2) \\
&= -\frac{4\pi}{16\pi G}\int dt(\dot{a}^2 + 1 - 2\dot{a}^2 - \Lambda a^2) \\
&= -\int dt\{\frac{1}{4G}(-\dot{a}^2 + 1 - \Lambda a^2)\} \\
&= \int dt \mathcal{L}(a, \dot{a})
\end{aligned}$$

where we used integral by parts. We have obtained the Lagrangian.

$$\mathcal{L} = -\frac{1}{4G}(-\dot{a}^2 + 1 - \Lambda a^2) \tag{6}$$

And we can get the equation of motion , namely, Euler-Lagrange Equation.

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{a}}\right) - \frac{\partial \mathcal{L}}{\partial a} = 0 \tag{7}$$

Now we substitute (5) into Euler-Laglange equation(6) to get the equation of motion.

$$\ddot{a} - \Lambda a = 0 \tag{8}$$

Next we calculate the equations of motion, namely the Friedmann Equations, from the original Einstein field equations.

$$R_{\mu\nu} - \frac{1}{2}(R - 2\Lambda)g_{\mu\nu} = 0 \tag{9}$$

$(0, 0)$ component: Freedman equation

$$\begin{aligned}
R_{00} - \frac{1}{2}(R - 2\Lambda)g_{00} &= 0 \\
\dot{a}^2 + 1 - \Lambda a^2 &= 0
\end{aligned} \tag{10}$$

(i, i) components

$$\begin{aligned}
R_{ii} - \frac{1}{2}(R - 2\Lambda)g_{ii} &= 0 \\
-\ddot{a} + \Lambda a &= 0
\end{aligned} \tag{11}$$

So we notice that Lagrangian (5) induces only (i,i)component equation of motion. The Lagrangian which induces $(0,0)$ component equation of motion is, for example, :

$$\mathcal{L} = -\frac{a}{4G}(-\dot{a}^2 + 1 - \Lambda a^2) \tag{12}$$

This Lagrangian induces equation of motion (10) in reality. In addition in pure Gravity ,we can induce E.O.M (10) from E.O.M (9) by differentiating with respect to t. So we will use the Lagrangian (12) instead of Lagrangian (6).

4 Canonical Quantization

Now we quantize the system by ordinary canonical quantization method. Our Lagrangian is

$$\mathcal{L} = -\frac{a}{4G}(-\dot{a}^2 + 1 - \Lambda a^2)$$

The conjugate momenta is

$$p = \frac{\partial \mathcal{L}}{\partial \dot{a}} = \frac{a\dot{a}}{2G} \tag{13}$$

The Hamiltonian is

$$\mathcal{H} = p\dot{a} - \mathcal{L} = p\dot{a} + \frac{a}{4G}(-\dot{a}^2 + 1 - \Lambda a^2) = \frac{G}{a}p^2 + \frac{a}{4G}(1 - \Lambda a^2) \tag{14}$$

Canonical Quantization:

$$p \to \frac{\hbar}{i}\frac{\partial}{\partial a} \tag{15}$$

So we obtain the quantum hamiltonian $\hat{\mathcal{H}}_0$

$$\hat{\mathcal{H}}_0 = -\frac{G\hbar^2}{a}\frac{\partial^2}{\partial a^2} + \frac{a}{4G}(1 - \Lambda a^2) \tag{16}$$

Here we define the new quantum Hamiltonian:

$$\hat{\mathcal{H}} = -G\hbar^2\frac{\partial^2}{\partial a^2} + \frac{a^2}{4G}(1 - \Lambda a^2) = T + U(a) \tag{17}$$

$$U(a) = \frac{a^2}{4G}(1 - \Lambda a^2) \tag{18}$$

5 Schrödinger – Wheeler – DeWitt Equation

In the cllasical level, Hamiltonian constraint has been considered as the primary constraint.

$$\mathcal{H} = 0 \tag{19}$$

In this report we will extend the interpretation of the above constraint on the quantum level , namely ,

$$\textbf{classical level} : \mathcal{H} = 0 \Longleftrightarrow \textbf{quantum level} : \hat{\mathcal{H}} \approx 0$$

$$\hat{\mathcal{H}} \approx 0 \to \mathcal{H} = 0 \, (\hbar \to 0) \tag{20}$$

So we may set the usual Schrodinger equation instead of Wheeler-De-Witt equation. Usual correspondence:

$$\mathcal{H} = 0 \Longleftrightarrow \hat{\mathcal{H}}\Psi = 0 : \textbf{Wheeler} - \textbf{DeWitt equation} \tag{21}$$

New correspondence:

$$\mathcal{H} \approx 0 \Longleftrightarrow \hat{\mathcal{H}}\Psi = E\Psi : \textbf{Schrodinger equation} \tag{22}$$

If we approve the above correspondence, we could use the Schrodinger equation instead of the Wheeler-De-Witt equation. Of course our proposal needs more deeper consideration. So $H \approx 0$ may break the framework of general relativity but we concentrate on the wave function at the Planck scale around the singularity. Around the Planck scale General relativity holds good or not , no one knows. In general, Quantum gravity seems to be needed around the Planck scale. so around the Planck scale the framework of general relativity may be broken. General relativity is a cllasical theory which represents cosmic scale physics.

6 Schrödinger Equation

Now we are going to solve the Schrodinger equation which is obtained in the previous section. That is

$$\left(-G\hbar^2 \frac{\partial^2}{\partial a^2} + U(a)\right)\Psi(a) = E\Psi(a) \tag{23}$$

We are interested in the wavefunction near the Big-Bang Singularity . So if a is assumed to be very small $(a \approx 0)$, a^4 term will be neglected.

$$U(a) = \frac{a^2}{4G}(1 - \Lambda a^2) \approx \frac{a^2}{4G} \tag{24}$$

So the Schrodinger equation becomes :

$$\left(-G\hbar^2\frac{\partial^2}{\partial a^2}+\frac{a^2}{4G}\right)\Psi(a)=E\Psi(a)$$

This equation is the same as the Harmonic osscilator in one dimension. It is convenient to rewrite it in dimensionless form.We introduce a dimensionless valuable $\xi = \alpha a$ and a dimensionless eigenvalue ε. Here we change variables as:

$$\xi=\alpha a=\frac{1}{\sqrt{2G\hbar}}a,\ \varepsilon=\frac{2E}{\hbar} \tag{25}$$

We have

$$\frac{\partial^2\Psi}{\partial\xi^2}+(\varepsilon-\xi^2)\Psi=0 \tag{26}$$

This equation is the same one as the harmonic oscillator in one dimensional quantum mechanics. So we can solve the equation by use of Hermite polynomials[2].

We follow the ordinary process;

$$\Psi(\xi)=u(\xi)e^{-\frac{1}{2}\xi^2} \tag{27}$$

We subscribe it into the equation(25) so we obtain the next equation :

$$\frac{d^2u(\xi)}{d\xi^2}-2\xi\frac{du(\xi)}{d\xi}+(\varepsilon-1)u(\xi)=0 \tag{28}$$

We do the power series expansion:

$$u(\xi)=\sum_{n=0}^{\infty}c_n\xi^n \tag{29}$$

So equation(28) is subscribed into (27) to obtaion the next recurrence formula:

$$c_{n+2}=\frac{2n-(\varepsilon-1)}{(n+1)(n+2)}c_n \tag{30}$$

In order that the power series are convergent ,

$$2n-(\varepsilon-1)=0 \tag{31}$$

We obtain

$$\frac{2E}{\hbar}=\varepsilon=2n+1 \tag{32}$$

$$E_n = \frac{\hbar}{2}(2n+1) = \hbar\left(n+\frac{1}{2}\right) \tag{33}$$

$$E_0 = \frac{\hbar}{2} \tag{34}$$

$$E_1 = \frac{3\hbar}{2} \tag{35}$$

$$E_2 = \frac{5\hbar}{2} \tag{36}$$

where $E_0 = \frac{\hbar}{2}$ is the zero-point energy. In addition we obtain the Hermite differential equation after insert (31) into (27)

$$\frac{d^2u(\xi)}{d\xi^2} - 2\xi\frac{du(\xi)}{d\xi} + 2nu(\xi) = 0 \tag{37}$$

So the solution is the Hermite polynomials

$$u(\xi) = H_n(\xi) \tag{38}$$

Finally we obtain the wavefunction:

$$\Psi(\xi) = H_n(\xi)e^{-\frac{1}{2}\xi^2} \tag{39}$$

The nth Hermite polynomial:

$$H_n(\xi) = (-1)^n e^{\xi^2}\frac{\partial^n}{\partial\xi^n}e^{-\xi^2} \tag{40}$$

The first three polynomials are

$$H_0(\xi) = 1,\ H_1(\xi) = 2\xi,\ H_2(\xi) = 4\xi^2 - 2 \tag{41}$$

We obtain the wavefunction:

$$\Psi_0(\xi) = H_0 e^{-\frac{1}{2}\xi^2} = e^{-\frac{1}{2}\xi^2},\ \Psi_1(\xi) = H_1(\xi)e^{-\frac{1}{2}\xi^2} = 2\xi e^{-\frac{1}{2}\xi^2} \tag{42}$$

$$\Psi_2(\xi) = H_2(\xi)e^{-\frac{1}{2}\xi^2} = (4\xi^2 - 2)e^{-\frac{1}{2}\xi^2} \tag{43}$$

The explict wavefunction is

$$\Psi_n(a) = C_n H_n(\alpha a)e^{-\frac{1}{2}\alpha^2 a^2} \tag{44}$$

where $C_n$is the normalizing constant. We use the generating function to normalize $\Psi_n(a)$.

$$\int_{-\infty}^{+\infty} |\Psi_n(a)|^2 da = \frac{|C_n|^2}{\alpha}\int_{-\infty}^{+\infty} \left|H_n(\xi)e^{-\frac{1}{2}\xi^2}\right|^2 d\xi = 1 \tag{45}$$

A generating function $S(\xi, s)$:

$$S(\xi, s) = e^{\xi^2-(s-\xi)} = e^{-s^2+2s\xi} = \sum_{n=0}^{\infty} \frac{H_n(\xi)}{n!} s^n \tag{46}$$

The integral on the right can be expressed as a series coefficient in the expansion of an integral containing the product of two generating functions:

$$\int_{-\infty}^{+\infty} e^{-s^2+2s\xi} e^{-t^2+2t\xi} e^{-\xi^2} d\xi = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{s^n t^m}{n!m!} \int_{-\infty}^{+\infty} H_n(\xi) H_m(\xi) e^{-\xi^2} d\xi \tag{47}$$

The left side is

$$\int_{-\infty}^{+\infty} e^{-\{(t+s)-\xi\}^2} e^{2ts} d\xi = e^{2ts} \int_{-\infty}^{+\infty} e^{-\{(t+s)-\xi\}^2} d\xi = e^{2ts}\sqrt{\pi} = \sqrt{\pi}\sum_{n=0}^{\infty} \frac{(2st)^n}{n!} \tag{48}$$

where we used the Gaussian integral;$\int_{-\infty}^{+\infty} e^{-x^2} = \sqrt{\pi}$
So we obtain

$$\sqrt{\pi}\sum_{n=0}^{\infty} \frac{(2st)^n}{n!} = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{s^n t^m}{n!m!} \int_{-\infty}^{+\infty} H_n(\xi) H_m(\xi) e^{-\xi^2} d\xi \tag{49}$$

If equal powers of s and t are equated in the series on the right sides of Eq.(48), we obtain the results:

$$\int_{-\infty}^{+\infty} H_n(\xi) H_m(\xi) e^{-\xi^2} d\xi = 2^n \sqrt{\pi} n! \delta_{mn} \tag{50}$$

Thus the normalizing constant can be chosen to be

$$C_n = \sqrt{\frac{\alpha}{2^n\sqrt{\pi}n!}} = \sqrt{\frac{\sqrt{2G\hbar}}{2^n\sqrt{\pi}n!}} \tag{51}$$

We integrate here from $-\infty$ to $+\infty$ to normalize the wavefunction . However we had better integrate from $-l_p$ to $+l_p$: l_p : the Planck length. Because we consider about the wavefunction within the Planck scale from the initial singularity point. At last we obtain the wavefunction finally;

$$\begin{aligned}
\Psi_n(a) &= C_n H_n(\alpha a) e^{-\frac{1}{2}\alpha^2 a^2} = \sqrt{\frac{\sqrt{2G\hbar}}{2^n\sqrt{\pi}n!}} H_n(\frac{1}{\sqrt{2G\hbar}}a) e^{-\frac{1}{2}\frac{1}{2G\hbar}a^2} \\
\Psi_0(a) &= C_0 H_0(\alpha a) e^{-\frac{1}{2}\frac{1}{2G\hbar}a^2} = \sqrt{\frac{\sqrt{2G\hbar}}{\sqrt{\pi}}} e^{-\frac{1}{2}\frac{1}{2G\hbar}a^2} \\
\Psi_1(a) &= C_1 H_1(\alpha a) e^{-\frac{1}{2}\frac{1}{2G\hbar}a^2} = \sqrt{\frac{\sqrt{2G\hbar}}{2\sqrt{\pi}}} 2\frac{1}{\sqrt{2G\hbar}} a e^{-\frac{1}{2}\frac{1}{2G\hbar}a^2} = \sqrt{\frac{\sqrt{2}}{\sqrt{2G\hbar\pi}}} a e^{-\frac{1}{2}\frac{1}{2G\hbar}a^2} \\
\Psi_2(a) &= C_2 H_2(\alpha a) e^{-\frac{1}{2}\frac{1}{2G\hbar}a^2} = \sqrt{\frac{\sqrt{2G\hbar}}{4\sqrt{2\pi}}} \left(4\left(\frac{1}{\sqrt{2G\hbar}}a\right)^2 - 2\right) e^{-\frac{1}{2}\frac{1}{2G\hbar}a^2}
\end{aligned}$$

7 Conclusions

We have investigated the wave function of the universe near the cosmological singularity by considering the extended Hamiltonian Constraints. The resulting wave function is the harmonic oscillator wave function . The energy spectrum are the Harmonic Oscillator ones. However this extension of the constraint has to be considered more deeply. Our result is the Quantum Mechanical result.If the zero-point energy exists , our universe would start by the use of the zero-point energy. Of course the fact we recognize such an extension of the Hamiltonian Constraint means that we break the framework of general relativity. But if our resulting wave function exists in the Planck scale, the breaking of general relativity may be permitted. Because general relativity is the classical theory and the Planck Scale must be covered by Quantum gravity. However if general relativity holds good in the Planck scale, our assumption would be not correct. In the near future we will apply this method to JT cosmology[6]. which is the two dimensional renormalizable quantum gravity theory .

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