Quantum Gravitation from Nordström's Gravitation Theory

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Abstract: This article is not a historical look at a rejected theory. It aims to recover a working scalar theory of gravitation from Nordström's old theory because the General Relativity Theory has very serious flaws that cannot be fixed. The scalar gravitational theory does not have the faults that Einstein claimed it had. I hope that the presented article shows that the scalar gravitational theory is a quite good simple classical field theory for gravitation without any serious flaws. It can be quantized easily: the scalar gravitational theory gives the free field Lagrangean of a (real) scalar field for a quantum gravitation theory and can be connected with spontaneous symmetry breaking in the Higgs mechanism, and the Higgs mechanism gives the mass to elementary particles in the Unified Field Theory. As the generating functional $W[J]$ for the free field can be exactly solved, a scalar gravitational theory allows investigating gravitation in the very early universe when all mass still formed a quantum system.

Keywords: Nordström's theory, scalar gravitation field, Einstein equations, quantum gravity.

1. Introduction

It is sad to have to start an article on a theory from explaining why the theory is not shown incorrect long ago, but is the situation with Nordström's gravitation theory. Only in the second section we can proceed to explain why this theory is of current interest.

Gunnar Nordström was the first to present a relativistic theory of gravitation [1] but unfortunately he decided to work on it with Einstein who was developing his own relativity theory [2][3]. It does seem that Einstein confused Nordström in many issues, and later he made false claims that Nordström's theory contradicts with experiments. Einstein was believed and work on the scalar theory was stopped. These false claims are still circulated and believed, see [4][5].

Einstein claimed that light does not bend in Nordström's theory and that the theory predicts the precession speed of Mercury's perihelion incorrectly, contradicting two experiments that he proposed for experimentally verifying the General Relativity Theory (GRT). These Einstein's claims have been accepted as valid reasons for rejecting a scalar gravitational theory because of empirical grounds.

Both claims are false. Paper [6] shows that light does bend in a scalar metric (i.e., a metric induced by a scalar field) under the geodesic Lagrangean for light-like world paths, which is the correct geodesic Lagrangean for light as the action integral is minimized by varying paths that light can take (i.e., those that always have $ds = 0$). The Schwarzschild metric gives a very complicated equation for this Lagrangean. Light also bends in a scalar metric under the geodesic Lagrangean for test masses, though this Lagrangean is not correct for light: the action integral is minimized over world paths that light cannot take (i.e., ds is varied and not always zero). The Schwarzschild metric can also be solved for the geodesic Lagrangean for test masses and it gives a similar result to the scalar metric. In both metrics the amount of bending of light cannot be predicted as it depends on integration constants, but the path of light is unphysical in a geodesic Lagrangean, both in the one for light and the one for a test mass.

Paper [6] also shows that the geodesic metric for a test mass is incorrect: a mass that is freely falling under this Lagrangean does not accelerate if the metric is scalar and the field is Newtonian. Paper [7] shows the same for the Schwarzshild metric: a mass that is freely falling under this Lagrangean does not accelerate if the metric is Schwarzschild's metric. Paper [8] demonstrates that in the calculation of the precession of Mercury's perihelion from the Schwarzschild metric by using the geodesic equation, the Christoffel symbols are incorrect. Paper [9] shows that the Schwarzschild metric is not a valid: the space part is not a valid Riemannian metric, the Schwarzschild metric does not have locally constant speed of light and it does not give the Minkowski metric in the tangent space of a point. For these reasons no verification of GRT from the gravitation field of the Sun or the Earth can use the Schwarzschild metric. Paper [7] shows that there is no need to calculate light bending a correction to Mercury's perihelotic precession from the geodesic metric, an improved approximation from Newtonian mechanics can explain the error with the effect of Jupiter.

2. Why a scalar gravitation theory is of current interest?

Let us look at the main reason why the scalar gravitation theory is of current interest. Taking the trace of the Einstein equations gives

$$g^{ab}\left(R_{ab}-\frac{1}{2}Rg_{ab}\right)=g^{ab}\left(k_0T_{ab}+\lambda g_{ab}\right)\qquad k_0=\frac{8\pi G}{c^4} \tag{1}$$

$$g^{ab}R_{ab}-\frac{1}{2}Rg^{ab}g_{ab}=k_0g^{ab}T_{ab}+\lambda g^{ab}g_{ab} \tag{2}$$

$$-R=R-2R=k_0T+4\lambda. \tag{3}$$

For a scalar field ϕ holds

$$R=-6\phi^{-1}\Box\phi. \tag{4}$$

Combining (3) and (4) gives the field equation

$$\phi^{-1}\Box\phi=-\frac{1}{6}k_0T=\frac{4}{3}\pi G\frac{T}{c^4}. \tag{5}$$

The interesting part in this equation is that if $T=T(x)$ is constant, it has the form of the Euler-Lagrange equations

$$\partial_\nu\partial^\nu\phi+\mu^2\phi=0 \tag{6}$$

for

$$\mu^2 = \frac{1}{6}k_0 T. \tag{7}$$

The Euler-Lagrange equation (6) comes from the free field Lagrangean

$$\mathcal{L}_0 = \partial_\nu \phi \partial^\nu \phi - \mu^2 \phi^2. \tag{8}$$

The classical field with this Lagrangean quantised by using the path integral method gives the partition functional

$$Z[J] = e^{i\hbar^{-1}\int d^4x(\mathcal{L}_0 + J\phi)}. \tag{9}$$

The generating functional of the connected Feynman diagrams is

$$W[J] = -iZ[J] \tag{10}$$

and the classical field is

$$\phi(x)_c = \frac{\delta W[J]}{\delta J(x)}. \tag{11}$$

In the case of the free field Lagrangean, $Z[J]$ and thus also $W[J]$ can be exactly calculated

$$Z[J] = Ne^{-i\hbar^{-1}\int dxdy J(x)\Delta_F(x-y)J(y)} \tag{12}$$

where $\Delta_F(x-y)$ is the Feynman propagator, i.e., a Green function for the Klein-Gordon equation, and N is a constant. The Euler-Lagrange equation that the quantised field satisfies is

$$(\partial_\nu \partial^\nu + \mu^2)\phi_c(x) = 0 \tag{13}$$

but we can add to the Langrangean the term $\mathcal{L}_1 = -(4!)^{-1}\lambda\phi^4$ and get a Lagrangean that gives spontaneous symmetry breaking

$$\mathcal{L}_0 + \mathcal{L}_1 = \partial_\nu \phi \partial^\nu \phi - \mu^2 \phi^2 - \frac{1}{4!}\lambda\phi^4. \tag{14}$$

Then the mass term μ^2 must be renormalized and we get the measured masses of particles:

$$\mu_R^2 = \mu^2 + \frac{1}{2}i\Delta_F(0) \tag{15}$$

we may find a connection between gravitation and the Higgs field.

Such quantum considerations are not the topic of this article, see [10] for some early ideas, but they hopefully motivate why a scalar gravitational field can be of interest if one can find a quantum system where gravitation is so strong that the gravitational interaction is important between particles that take part in a quantum system. Such a quantum system might be found from the very early universe. The scalar gravitation field may be a way to study the very early universe.

Where to find such a quantum system deserves a comment, especially as a Nobel Prize was given for finding a graviton. That is, a spin 2 graviton was found. There was some scepticism when the discovery of the graviton was announced, it was claimed that the measured signal could have been simply noise. There is one reason why I am still sceptical of this finding of the graviton. It is not only that the gravitational field must be a scalar field if the speed of light is locally constant, for a scalar gravitational field a gravition must be a spin 0 particle. It is mainly because there should not be gravitons for any researchers to measure.

What we can see is a gravitational field, and we can easily see for instance the gravitational field of the Earth, but we do not see that this field is composed of particles, we do not see a graviton. This is correct: fields are composed of waves, not particles. Only when a field interacts with a quantum system of materia, the interaction is in the form of a packet. Only then there exists a boson as a particle, we can see a photon or a graviton.

The idea of particle-wave dualism that causes the confusion that we could measure gravitons without finding a special quantum system that can make a transition when it receives a graviton comes from Einstein's solution to the photoelectric effect. This effect shows that light can release electrons from an atom only if the frequency of the light is sufficiently high. Several interactions of an atom with lower frequency light cannot release electrons, even if the combined energy of these interactions suffices for releasing electrons. Einstein explained this observation as showing that light is composed of photons. As light certainly also is waves, the result was the particle-wave dualism. But this is not so. Light is only waves. Only the interaction between a field and matter is in this case made by discrete exchange of a wave-packet, a quantum of energy.

However, light can interact with matter in a non-quantised manner, as it happens in Thomson and Compton scattering. In Compton scattering light can give electrons any amount of energy. This is because an electron can receive any amount of energy to its kinetic energy, but transitions in the electron cloud of an atom can only happen if the interaction transfers at one packet sufficient energy for this transition. Light interacting with matter in a non-quantised manner is not limited to small particles like electrons. There is radiation pressure, a space ship can be moved by solar wind.

Considering the situation with a gravitation field, the correspondence to Thomson and Compton scattering is the effect of a gravitational field to a test mass. Energy is not quantised in such interactions. In order to find a graviton, a quantum of gravitational energy, we need to find a quantum system that makes transitions. The gravitational force is so weak that small quantum system, like those of elementary particles, are not kept together with the gravitational force. Gravitational interaction does not cause transitions in such small quantum systems. Large systems, like planet or star systems, are kept together with gravitation, but even if such systems have stable solutions and return to them after being disturbed by an interaction that gives a small amount of energy, the time scale when they return to a stable solution is very long. Such large systems cannot be considered as quantum systems. A quantum system can make transitions and (practically) immediately emits energy that is too small for making a transition, giving it the all-or-none behavior characterizing a quantum system. This requires that there are stable solutions and that the time scale for reaching stable solutions after a disturbance is very small. Quantum systems for gravitation can appear only in very specific conditions, such as the Big Bang or inside a very dense star. Therefore I doubt that a graviton was measured.

Let us continue the classical theory of a scalar gravitational field. In addition to the field equation there is the dynamic equation for the movement of a test mass in a gravitational field. This equation is from the point of view of the test mass always an Euler-Lagrange equation derived from energy. It contains potential energy, which is given by the field potential, and kinematic energy, which always must have the same form if it is only a function of velocity v, see the proof in [11]:

$$E_k(v) = \frac{1}{2}mv^2. \tag{16}$$

Especially, [11] proves that relativistic kinetic energy (and the equivalent momentum-energy formula) is wrong: it does not conserve energy. Bertozzi's measurements refute the relativistic kinetic energy formula. They agree with apparent longitudinal mass, which means force weakening when the mass that the force of a field tries to accelerate moves close to the speed of light. The force has a finite propagation speed, in vacuum it is c, and if the mass moves away with a speed that is close to c, then the mass does not feel the full force. It feels a weakened force and this weakened force F_w is in the dynamic equation of the test mass:

$$F = \frac{\partial}{\partial x}E_p(x) \tag{17}$$

$$F_w = \gamma(\alpha)^{-1}F. \tag{18}$$

The Lagrange equation for the test mass is

$$F_w = \frac{d}{dt}\frac{\partial}{\partial \dot{x}}E_k(\dot{x}) = m\ddot{x}. \tag{19}$$

and it can be expressed as

$$\frac{\partial}{\partial x}E_p(x) - \frac{d}{dt}\frac{\partial}{\partial \dot{x}}E_{k,w}(\dot{x}) = 0 \tag{20}$$

where the apparent kinetic energy is

$$E_{k,w}(\dot{x}) = \frac{1}{2}\gamma(\alpha)m\dot{x}^2 = m_w\dot{x}^2. \tag{21}$$

where the apparent mass is

$$m_w = \gamma(\alpha)m. \tag{22}$$

For force acting orthogonally to the velocity v, the apparent mass is the transverse mass $m_{v,T} = \gamma m$, the same formula as for the relativistic mass. This was found by Kaufmann, Buchener and Lorentz. For force acting parallel to the velocity, the apparent mass is the longitudinal mass. Measurements by Bertozzi suggest that in his measurement the logitudinal mass was $m_{w,T} = \gamma^{1.6} m$, but a more natural guess is $m_{w,T} = \gamma^{1.5} m$.

Let us continue to Nordström's gravitational theory.

3. The field equation in Nordström's gravitation theory

Nordström's gravitation theory is described in two published articles [1], see also [2] and [3]. Nordström's theory essentially contains only of two equations: the field equation and the dynamic equation.

The field equation for classical Newtonian gravitation is usually derived by modifying the Gauss law in electro-magnetism

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \tag{23}$$

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to match the Newtonian gravitation potential which so is similar to the Coulomb law. Calculus of residues gives the starting point

$$\nabla \cdot \frac{\bar{r}}{|r^3|} = 4\pi\delta(r) \tag{24}$$

Therefore

$$\nabla \cdot \left(GM \frac{\bar{e}_r}{r^2} \right) = 4\pi GM\delta(r) \tag{25}$$

The term $\bar{g} = GM\frac{\bar{e}_r}{r^2}$ is the acceleration in a gravitational field, thus

$$\bar{g} = \nabla\phi$$

Inserting this result and replacing $M\delta(r)$ by a continuous mass density $\rho(r)$ we get

$$\nabla \cdot \nabla\phi = \Delta\phi = 4\pi G\rho(r) \tag{26}$$

Nordström's theory's field equation is only a slight generalization to this classical result

$$\Box\phi = -4\pi G\rho(r) \tag{27}$$

The box $\Box$ is the D'Alembertian. In Cartesian coordinates and with the signs (+,-,-,-)

$$\Box = \partial_0^2 - \partial_1^2 - \partial_2^2 - \partial_3^2 = \partial_0^2 - \Delta. \tag{28}$$

In the second version of Nordström's theory, the field equation is written as

$$\phi^{-1}\Box\phi = -4\pi T_{matter} \tag{29}$$

but it is the same equation in the classical limit: $T_{matter} = G\rho\phi^{-1}$. It has to be the same equation in the classical limit, because in the classical limit it must give (26).

We can write (27) in a geometric form by using the geometric concept of the Ricci scalar curvature R. It is useful to first scale the field so that is is a plain number, ϕ has the units m^2/s^2. The scaled field is

$$\psi = c^{-2}\phi. \tag{30}$$

For any scalar field ψ the Ricci scalar curvature is

$$R = -6\psi^{-3}\Box\psi. \tag{31}$$

Writing

$$T = \rho\psi^{-3} \tag{32}$$

$$\kappa = 12\pi G c^{-2} \tag{33}$$

$$g = \psi^2 \tag{34}$$

the field equation (27) gets the form that intentionally mimics the field equation in the General Relativity Theory (GRT)

$$-\frac{1}{2}Rg = \kappa T g \tag{35}$$

With the coordinates $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$ and signs $\eta_{00} = 1$, $\eta_{ii} = -1$, $i = 1, 2, 3$, $\eta_{ab} = 0$ if $a \neq b$ we get

$$g_{ab} = \eta_{ab} x^a x^b \tag{36}$$

and

$$g_{aa} = g \qquad \text{for} \quad \text{a} = 0, 1, 2, 3. \tag{37}$$

so, if we define the stress-energy tensor as $T_{bb} = T$, we can write (35) very much like the Eistein equations in GRT

$$-\frac{1}{2}Rg_{ab} = \kappa T_{ab}g_{ab}. \tag{38}$$

What is missing in (38) from the field equation of GRT

$$R_{ab} - \frac{1}{2}Rg_{ab} = \kappa_0 T_{ab}g_{ab} + \lambda g_{ab}. \tag{39}$$

in addition the λ-term (that Einstein initially did not want to the equation) is the Ricci tensor entry R_{ab}. Because of this entry, the Einstein equations (39) are six separate equations. When the field is a scalar field, only the diagonal entries are nonzero ($g_{ab} = 0$ it $a \neq b$ for a scalar field), but it is still four separate equations. This leads to a serious problem when trying to solve (39) for a scalar field. Let us explain the problem.

4. The General Relativity Theory really is wrong

For Cartesian coordinates the nonzero Ricci entries R_{aa} and the Ricci scalar $R = g^{aa}R_{aa}$ of a scalar field ψ are: (we write x_i instead of x^i so that indices are not confused with powers)

$$R_{00} = -\psi^{-1}\Box\psi + \psi^{-2}\sum_{i=1}^{3}\left(\frac{\partial\psi}{\partial x_i}\right)^2 + 3\psi^{-2}\left(\frac{\partial\psi}{\partial x_0}\right)^2 - 2\psi^{-1}\frac{\partial^2\psi}{\partial x_0^2}$$

$$R_{ii} = \psi^{-1}\Box\psi - \psi^{-2}\sum_{i=1}^{3}\left(\frac{\partial\psi}{\partial x_i}\right)^2 + \psi^{-2}\left(\frac{\partial\psi}{\partial x_0}\right)^2 - 2\psi^{-1}\frac{\partial^2\psi}{\partial x_i^2}$$
$$+4\psi^{-2}\left(\frac{\partial\psi}{\partial x_i}\right)^2 \qquad \text{for} \qquad \mathrm{i} = 1,2,3$$

$$R = \psi^{-2}R_{00} - \psi^{-2}R_{11} - \psi^{-2}R_{22} - \psi^{-2}R_{33} = -6\psi^{-3}\Box\psi. \tag{40}$$

Let the space be empty with one point mass at the center. The Schwarzschild solution is an exact solution of (39) in this situation. In the derivation of the Schwarzschild solution the tensor $T_{ab} = 0$ is set to zero outside the origin and is zero in the calculation. Schwarzschild understood correctly, $T_{ab} = 0$ outside the origin because in GRT

$$T_{ab} = -2\frac{\delta L_{matter}}{\delta g^{ab}} + g_{ab}L_{matter} \tag{41}$$

If matter is all concentrated to the origin, then $T_{ab} = 0$ outside the origin and is zero in the calculation of the Einstein equations. Every $g_{aa} = g$ is equal. Thus,

$$R_{aa} = \frac{1}{2}Rg + \lambda g. \tag{42}$$

for every $a = 0, 1, 2, 3$. Let us take R_{ii} and R_{jj}, $i \neq j$, $i, j \in \{1, 2, 3\}$, cancel all common terms in $R_{ii} = R_{jj}$ coming from (42), and then we get an equation that cannot be satisfied.

First we subtract

$$R_{00} - R_{ii} = (\frac{1}{2}R + \lambda)(g - g) = 0 \tag{43}$$

and then we insert R_{00} and R_{ii} from (40) and move all terms that are common to every $i = 1, 2, 3$ to the left

$$\begin{aligned} R_{00} + \psi^{-1}\Box\psi - \psi^{-2}\sum_{i=1}^{3}\left(\frac{\partial\psi}{\partial x_i}\right)^2 - \psi^{-2}\left(\frac{\partial\psi}{\partial x_0}\right)^2 \\ = 2\psi^{-1}\frac{\partial^2\psi}{\partial x_i^2} - 4\psi^{-2}\left(\frac{\partial\psi}{\partial x_i}\right)^2 \end{aligned} \tag{44}$$

The terms that are different for R_{ii} and R_{jj} give the equation

$$\frac{\partial}{\partial x_i}\left(\frac{\partial\psi}{\partial x_i}\psi^{-2}\right) = \frac{\partial}{\partial x_j}\left(\frac{\partial\psi}{\partial x_j}\psi^{-2}\right). \tag{45}$$

Equations (45) are solved by any function the form $\psi = \psi(\rho)$ where $\rho = \sum x_j$, but the solution we need is close to the radially symmetric Newtonian gravitation field in this special case of a single point mass in the origin, as that is the case in our solar system. See [12] for a short and simple proof that this is not possible. The problem can be explained easily. A solution for this situation must be close to the Newtonian gravitation potential, which is spherically symmetric. Thus, the solution is very closely spherically symmetric. Applying the left side of (45) to a function of $r = \sqrt{x_1^2 + x_2^2 + x_3^2}$ gives a function of the form $x_i^2 f(r) + h(r)$. The right side gives the function $x_j^2 f(r) + h(r)$ with the same f and h. We can see that they cannot cancel and no small addition to the Newtonian potential can give a large enough term that the equation (45) can hold. Instead, if we sum all three values of i together, as is done in calculation of R, the terms x_i^2 add to r^2 and R can be zero.

Let us locate the error in Einstein's derivation of the Einstein equations. He claimed to have obtained (39) from the Lagrangian

$$L = \frac{c^4}{16\pi G}(R - 2\lambda) - L_{matter}. \tag{46}$$

Our L_{matter} is zero outside the origin and we set $\lambda = 0$. Then the Langangian is the Ricci scalar curvature R multiplied by a constant which we can forget. The Euler-Lagrange equations are

$$-\frac{\partial}{\partial_\mu}\left(\frac{\partial L}{\partial(\partial_\mu \psi)}\right) + \frac{\partial L}{\partial \psi} = 0 \tag{47}$$

We calculate these terms for the scalar field from (40).

$$\frac{\partial R_{ii}}{\partial(\partial_i \psi)} = -\psi^{-2} 2\partial_i \psi + 4\psi^{-2} 2\partial_i \psi = 6\psi^{-2}\partial_i \psi$$

$$\frac{\partial R_{ii}}{\partial(\partial_j \psi)} = -\psi^{-2} 2\partial_j \psi$$

$$\frac{\partial R_{ii}}{\partial(\partial_0 \psi)} = \psi^{-2} 2\partial_0 \psi$$

$$\frac{\partial R_{00}}{\partial(\partial_0 \psi)} = 6\psi^{-2}\partial_0 \psi$$

$$\frac{\partial R_{ii}}{\partial(\partial_0 \psi)} = 2\psi^{-2}\partial_0 \psi$$

$$\frac{\partial R}{\partial \psi)} = 18\psi^{-4}\Box\psi. \tag{48}$$

From these results we obtain for the Ricci scalar curvature R, which is our L in this case

$$R = \psi^{-2}(R_{00} - R_{11} - R_{22} - R_{33})$$

the result

$$\frac{\partial R}{\partial(\partial_\mu \psi)} = 0$$

for all $\mu = 0, 1, 2, 3$. Consequently the Euler-Lagrange equations reduce to

$$\frac{\partial R}{\partial \psi} = 18\psi^{-4}\Box\psi = 0 \qquad (56)$$

i.e., to Nordström's field equation (27) in this empty space situation

$$\Box\psi = 0 \qquad (57).$$

Of course, we would get the same result by directly inserting R to (46), but the goal was to show that the Ricci entries in (39) really are not obtained from Euler-Lagrange equations.

The Lagrangian gives Nordström's field equation, not Einstein's equations. Einstein may have solved the Euler-Lagrange equations incorrectly, but it is very difficult to imagine how he could have ever come to (39) from (46)-(47) even while making some mistakes. Thus, Einstein's equations are not derived from anything and they are wrong.

Also the Special Relativity Theory (SRT) is seriously wrong, see [12], Einstein did not project the time values to the time axis in the moving frame in the Lorentz transform as one must do. This error and the twin paradox in the Lorentz transform invalidate the whole SRT. Before going to Nordström's dynamical equation, let us look at time dilations.

5. Time dilations from Nordström's gravitation theory

Nordström's gravitation theory gives the gravitational time dilation and the acceleration time dilation. The acceleration time dilation does not agree with Einstein's equivalence principle. Instead, it agrees with the observed time difference in GPS satellites.

The gravitational time dilation in Nordström's gravitation theory is simply that time time unit in a gravitational field ψ can be read from the metric ds^2

$$ds^2 = c^2\psi^2 dt^2 - \psi^2 dx^2 - \psi^2 dy^2 - \psi^2 + dz^2 \qquad (58)$$

The the gravitational field of a mass M makes the length of one second to the length

$$\psi^2 = \frac{GM}{c^2 r} = \frac{1}{c^2}\phi \qquad (59)$$

in the distance of r from the mass center. This equation is scaled to give seconds.

The acceleration time dilation is derived as follows. Constant acceleration a leads to the distance $s = (1/2)at^2$ in time t. Work is force times length (or integral actually) $F = ma$, $W = Fs$, thus

$$\frac{W}{m} = as = \frac{1}{2}a^2t^2 = \frac{1}{2}v^2. \tag{60}$$

What time dilation this work density could cause? A gravitational field is also energy divided by mass and it produces the gravitational time dilation. A field

$$\phi = \frac{E_p}{m} \tag{61}$$

lenghtens one second to

$$\frac{1}{c^2}\phi. \tag{62}$$

The acceleration time dilation rule that gives the correct time for GPS satellite time difference is that the ratio work to mass

$$\frac{W}{m} = \frac{1}{2}v^2 \tag{63}$$

lenghtens one second to

$$\frac{v^2}{c^2} \tag{64}$$

This rule works not only in the GPS case but also explains the muon longer lifetime in the muon-laboratory experiment. The time dilation of SRT can be removed from the theory. The calculations for the GPS satellite clock advance are in [13].

Nordström's theory does give the correct gravitational and acceleration time dilations from geometric arguments, but it does not imply that time changes because of gravitation or acceleration. The reason can simply be that the time in the heuristic Schrödinger and Dirac equations is not the time, it is the apparent time as these equations do not include the gravitational field strength unlike e.g. the period of a pendulum does.

6. The dynamic equation in Nordström's gravitation theory

This section explains how the dynamic equation in Nordström's theory is derived.

The second equation that Nordström gave in his theory is a direct modification of

$$F = ma \qquad \text{and} \qquad \mathrm{F} = \nabla\phi. \tag{65}$$

In GRT we write $\phi, b = \partial_b \phi$ where ∂_b is $\partial/\partial x_b$ (using a lower index for clarity). The notation $\dot{u_a}$ means a derivative with respect to the proper time

$$\dot{u_b} = \frac{d}{d\tau} u_b \tag{66}$$

where $\tau = c^2 \psi dt$ is the proper time scaled by c^2, to get seconds remove the c^2, ψ is a plain number and dt is seconds. The dynamic equation is very much what is should be, a denotes acceleration and u is velocity

$$F = ma = -m\nabla\phi$$

The minus sign here is because the acceleration is towards decreasing r.

$$a = -\nabla\phi$$

$$\frac{d}{dt} u_b = -\phi_{,b}$$

$$\frac{c^2\psi}{c^2\psi} \frac{d}{dt} u_b = c^2\psi \frac{d}{d\tau} u_b = -\phi_{,b}$$

$$\psi \frac{d}{d\tau} u_b = -\psi_{,b}. \tag{67}$$

The only modification Nordström made is that if the field can change in time, there should be a term containing $\dot{\psi}$. He inserted it into the equation in a natural way

$$\frac{d}{d\tau} \psi u_b = -\psi_{,b}$$

$$\psi \dot{u_b} = -\psi_{,b} - \dot{\psi} u_b. \tag{68}$$

Equation (68) is the dynamical equation and it hardly could be anything else.

7. A small generalization of Nordström's theory and conclusions

It is possible to generalize Nordström's field equation to give a different gravitational potential in the case of an empty space with a point mass in the origin. The following field equation may not be the most elegant, but it does the job.

$$\Box\phi - 6GM\beta\frac{1}{r^5} = -4\pi G\rho(r). \tag{69}$$

$$M = \int dV\rho$$

This field equation gives the solution

$$\phi = -\frac{GM}{r}\left(1 - \frac{\beta}{r^3}\right). \tag{70}$$

There is some reason for investigating a potential like in (70): such generalizations may be needed in finding a connection between the gravitational field and the Higgs field.

Nordström's gravitation theory is as close to a valid geometrized gravitation theory as can be achieved, but I doubt that any geometrized gravitation theory is correct. It is probably better to see the scalar gravitational field as a field in the Euclidean geometry rather than as geometry of the space-time.

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