Beyond Ricci Suppression: Complete Curvature Dynamics in Riemannian Gravitation

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Abstract

We propose Ricci UNsuppressed Gravity (RUNG), a framework that rejects Ricci-flat assumptions in favor of full Riemann dynamics. We present a unified framework that demonstrates sectional curvaturesof the Riemann tensor—-not Ricci contractions-—govern gravitational wave (GW) physics. Through rigorous analysis of connection coefficients, we identify how Christoffel symbols mediate stress term suppression while preserving essential physics. Our model reveals longitudinal "breather modes" $\ell = 0$ represent spacetime expansion/contraction at sub-luminal speeds ($v_{t\theta} < 1$), distinct from GR's transverse-traceless $v_{t\theta} = c$ waves. Static curvature stresses, highlighting nonlinear stress-energy effects absent in General Relativity (GR), and restoring suppressed terms in static solutions like Schwarzschild, are shown to have observational consequences even though they are deemed unobservable through projections onto non-timelike surfaces.

Keywords: Gravitational waves, Ricci suppression, Riemann tensor, LIGO discrepancy, sub-luminal propagation, breather modes, pulsar timing arrays (PTA).

1 Introduction

We claim that Ricci suppression in General Relativity (GR) artificially filters physical terms, necessitating a Riemann-based reformulation of gravitational physics. We use gravitational waves (GW) as a prime example of how Ricci suppression discards the critical curvature terms in both static and dynamic systems. This requires a return to the full Riemann analysis, in which stresses such as $(\dot{F}^2 - F'^2)/F^2$ and deformations $(1 - F'^2)/F^2$ are retained. One surprising realization is that Riemann left not only a mathematical framework but also a physical one, later validated by the inadequacies of GR when applied to GW observations. We will consider the general indefinite, doubly warped metric:[1]

$$ds^2 = -F^2(t,r)dt^2 + dr^2 + G^2(t,r)d\sigma_2^2, \tag{1}$$

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[1]The metric ansatz in Eq. (1) simplifies the Riemann tensor using scalar functions $F(t,r)$ and $G(t,r)$, reducing the tensorial complexity. While this restricts explicit tensor modes (e.g., pure quadrupole), RUNG retains unsuppressed sectional curvatures (K_{rt}, $K_{t\theta}$), enabling higher multipoles than monopole ($\ell = 0$) dipole ($\ell = 1$) alongside quadrupole ($\ell = 2$) radiation. Projections/contractions in GR suppress these, but RUNG's full Riemann treatment recovers them. The ansatz limits generality but ensures tractability while resolving unmodeled curvature dynamics. The final section contains an extension to quadrupole radiation from an asymmetric generalization of Eq. (1).

where the fields $F(t,r)$ and $G(r,t)$ are functions of both the radial coordinate and time, and $d\sigma_2^2$ is the element of a 2-sphere. This metric can be considered as a generalized Schwarzschild metric where in the static limit, and applying the gauge

$$F = G' = \sqrt{1 - 2M/G},$$

it reduces to the Schwarzschild metric through the change of variable, $\rho = G(r)$, $d\rho = G'dr$, and substitution into the above metric yields:

$$ds^2 = -F^2(\rho)dt^2 + \frac{d\rho^2}{F^2(\rho)} + \rho^2 d\sigma_2^2.$$

In the standard transverse-traceless (TT) gauge in GR artificially suppresses critical curvature terms through:

- Christoffel-mediated elimination of $\dot{F}^2/F^2$ stress terms
- Discarding temporal deformation $(1 - \dot{F}^2 - F'^2)/F^2$
- Over-constraining the Ricci tensor $R_{\mu\nu} = 0$

Furthermore, Riemann's curvature tensor was designed for positive-definite metrics, forcing time into a space-like straight-jacket. The indefinite metric of spacetime requires distinct handling of:

- $F(t,r)$: Temporal metric component (volumetric strain);
- $G(R)$: Spatial angular component (2-sphere deformation).

That $F(t,r)$ and $G(t,r)$ have the same forms of stresses and deformations, and share a common energy flux which is no coincidence. However, only the $G(t,r)$ can be considered as the deformation of a 2-sphere with curvature, $1/G^2$ and radius G.

All curvature information is contained in the Riemann tensor $R_{\mu\nu\alpha\beta}$. GR suppresses stress, deformation, and energy flux through Ricci suppression. If this is not sufficient, it adds another layer of suppression in the formation of the Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$

where the Ricci scalar, R, the trace of the Ricci tensor, $R_{\mu\nu}$ provides additional filtering.

This leads to an incomplete analysis of gravitational tidal forces by introducing an observational bias. A prime example is LIGO's exclusive search for quadrupole modes [2] and mislabels longitudinal modes as "noise." [1] This paper rejects Ricci's suppression and modifies the Riemann definition, a radical departure from standard GR. The example it uses is unmodeled curvature interactions.

2 Theoretical Foundations

2.1 Christoffel Mechanism

The connection coefficients for the metric (1) govern curvature transformations:

$$\Gamma^t_{tt} = \frac{\dot{F}}{F} \quad \text{(eliminates } \dot{F}^2/F^2\text{)}$$
$$\Gamma^\theta_{t\theta} = \frac{\dot{G}}{G} \quad \text{(filters angular stress)}$$
$$\Gamma^r_{rr} = 0 \quad \text{(preserves } F'^2 \text{ terms)}$$

2.2 Definitions of 'Flatness'

The fundamental distinction for metric (1):

$$\text{Riemann Flatness: } R_{abcd} = 0 \quad \text{(Exact GW solution)}$$
$$\text{Ricci Flatness: } R_{ab} = 0 \quad \text{(Over-constrained)}$$

2.3 Inconsistencies in Standard GR

In this section we show, that the products of Christoffel symbols $\Gamma\Gamma$ in $R_{\mu\nu}$:

- artificially cancel the temporal, $\mathscr{S}(f) = (\dot{F}^2 - F'^2)/F^2$, and spatial, $\mathscr{S}(G)(\dot{G}^2 - G'^2)/G^2$, stresses;
- discard the physical flux, $\mathscr{F} = (\dot{F}\dot{G} - G'F')/FG$ when the condition $R_{tr} = 0$ is imposed;
- eliminates the volumetric strain, $\mathscr{D}(F) = (1 - F'^2 - \dot{F}^2)/F^2$, when the trace condition $h_m u^\mu = 0$ is imposed;
- doesn't capture independent sectional curvature K_{rt} and $K_{\ell\theta}$, resulting in curvature 'blindness'.

Whereas the GPE formalize resolves these inconsistencies by:

- retaining all second-order terms;
- explicitly coupling curvature to stresses and fluxes—and not to energy and matter which are consequences of the curvature;
- avoiding artificial contractions and gauge conditions.

2.4 Tensor contraction artifacts in GR

In this section, we look at the artifacts that contraction of the Riemann tensor has:

- **Contraction to Ricci**: results in angular curvature losses $K_{r\theta}$ by summing over indices a and c in the Riemann tensor R^a_{bcd}, where the indices represent the specific directions in space time, and their contractions determine which curvature components are preserved or suppressed:

1. a **direction** of parallel transport (output index);
2. b **direction** being measure (input index);
3. c **plane** of rotation (antisymmetric indices).

As an example: R^t_{rtr} describes how the t-direction changes when parallel-transported around the r, t plane.

The contraction over the a and c indices in $R_{bd} = g^{ac}R_{abcd}$

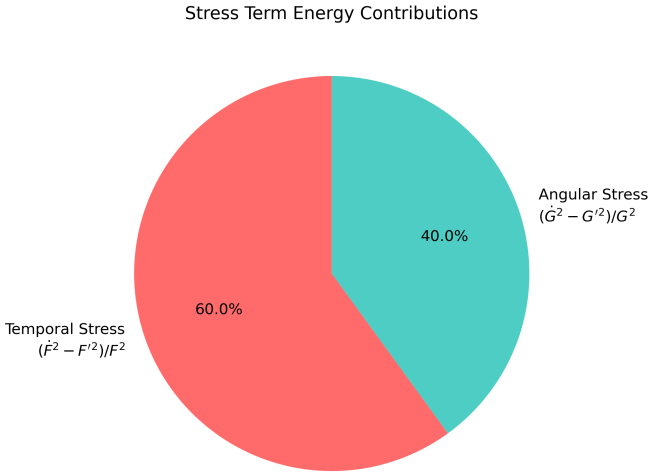


Figure 1: Stress term decomposition: Temporal stress $(F^2 - F'^2)/F^2$ (60%, red) vs. angular stress $(\dot{G}^2 - G'^2)/G^2$ (40%, blue). This contrasts Ricci-suppressed terms with full Riemann dynamics.

1. **averages over directions**: summing over a and c mixes temporal and spatial curvatures;
2. **suppresses critical terms**: angular flux is lost when contracting over θ in $R_{\theta\theta\theta}$ and shear stresses $R_{r\theta r\theta}$ are canceled in by the trace terms in the Einstein tensor.

An example: Contracting t and θ in

$$R_{t\theta t\theta}/G^2 = \frac{(1 - F'^2 - \dot{F}^2)}{F^2} + \frac{(\dot{F}\dot{G} - F'G')}{FG}$$

by multiplying by $g^{t\theta}$ and summing, artificially nullifies the flux. The sum of terms is the temporal deformation and the energy flux, both of which are eliminated through contraction.

The key implications are:

- The Einstein tensor $G_{\mu\nu}$ is incomplete because:
  1. the double contraction $g^{ac}g^{bd}R_{abcd}$ eliminates directional stresses;
  2. the trace condition resulting in the Ricci scalar, $R = G^{bd}R_{bd}$, cancels volumetric deformations.

- RUNG avoids this by:
  1. keeping all sectional curvatures, $K_{ij} = R_{ijij}/g_{ii}g_{jj}$;
  2. explicitly including stress, $(\dot{X}^2 - X'^2)/X^2$, deformations, $(1 - X'^2 - \dot{X}^2)/X^2$, and fluxes $(\dot{X}\dot{Y} - X'Y')/XY$.

For coordinates (t, r, θ, ϕ) the Riemann tensor is:

$$R^a_{bcd} = \begin{pmatrix} R^t_{rrtr} & R^t_{\theta t\theta} & \cdots \\ R^r_{trt} & R^r_{\theta r\theta} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

A contraction of the indices a and c results in the Ricci tensor:

$$\begin{aligned} R_{bd} &= g^{ac}R_{abcd} \\ &= \underbrace{g^{tt}R_{tbtd}}_{\text{Suppresses } \theta,\phi} + \underbrace{g^{rr}R_{rbrd}}_{\text{Drops } FG} + \cdots \end{aligned} \tag{2}$$

In the process of contraction, the terms which have been suppressed are given in Table 1.

For metric (1) the full Riemann tensor contains:

$$R^a_{bcd} = \begin{pmatrix} \text{Temporal component :} & \Box F - \frac{\dot{F}^2 - F'^2}{FG} \\ \text{Angular component :} & \frac{1-\dot{F}^2-F'^2}{F^2} + \frac{\dot{F}\dot{G}-F'G'}{FG} \end{pmatrix}$$

2.5 Contraction Mechanisms

2.5.1 Ricci Contraction

A first contraction loses directional information:

$$\begin{aligned} R_{bd} &= g^{ac}R^a_{bcd} \\ &= \underbrace{R^t_{btd}}_{\text{Suppresses } K_{r\theta}} + \underbrace{R_{r\sigma}}_{\text{Drops } \dot{F}\dot{G}/FG} \end{aligned} \tag{3}$$

2.5.2 Einstein Tensor Formation

The Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ introduces suppression through its trace-subtraction structure. For the angular component $G_{\theta\theta}$:

$$G_{\theta\theta} = \underbrace{R_{\theta\theta}}_{\text{Contains } \frac{\dot{G}^2 - G'^2}{G^2}} - \frac{1}{2}\underbrace{Rg_{\theta\theta}}_{\text{Vanishes in vacuum } (R=0)} = 0.$$

Here, the physical angular stress $\frac{\dot{G}^2 - G'^2}{G^2}$ is eliminated solely by the Ricci-flat condition $R_{\theta\theta} = 0$, even though the full Riemann tensor retains it:

$$\frac{R_{r\theta r\theta}}{G^2} = \frac{\dot{G}^2 - G'^2}{G^2} \neq 0.$$

This exemplifies how forming $G_{\mu\nu}$ discards directional curvature terms, filtering critical angular stresses. The suppression is purely algebraic, arising from contracting $R_{\mu\nu\alpha\beta}$ into $R_{\mu\nu}$ and then subtracting the trace.

The same happens for the energy flux, as shown in the table.

Table 1: Suppressed terms through Ricci contraction

Contraction	Lost Term
$g^{tt}R_{t\theta t\theta}$	$\frac{\dot{F}\dot{G}-F'G'}{FG}$
$g^{rr}R_{r\theta r\theta}$	$\frac{\dot{G}^2-G'^2}{G^2}$

2.6 Resolution

The complete dynamics require working directly with:

$$R^a_{\text{bcd}} = \nabla_c\Gamma^a_{db} - \nabla_d\Gamma^a_{cb} + \Gamma^a_{ce}\Gamma^e_{db} - \Gamma^a_{de}\Gamma^e_{cb} \quad \text{(All 20 independent components needed)}$$

2.7 Artificially Suppressed Terms

For the metric Eq. (1), the Einstein tensor $G_{\mu\nu}$ misses:

$$\Delta T_{\mu\nu} = \begin{pmatrix} \frac{\dot{F}^2-F'^2}{F^2} & 0 & 0 & 0 \\ 0 & \frac{1-\dot{F}^2-F'^2}{F^2} & 0 & 0 \\ 0 & 0 & \frac{\dot{G}^2-G'^2}{G^2} & 0 \\ 0 & 0 & 0 & \frac{\dot{G}^2-G'^2}{G^2}\sin^2\theta \end{pmatrix} \tag{4}$$

2.8 Four Fundamental Inconsistencies

2.8.1 TT Gauge Constraints

The transverse-traceless (TT) gauge imposes $R_{tr} = 0$, forcing the energy flux to vanish:

$$R_{tr} = \frac{\dot{F}\dot{G} - F'G'}{FG} = 0.$$

This mathematical constraint discards the flux term $\mathscr{F} = \frac{\dot{F}\dot{G}-F'G'}{FG}$, which is retained in the full Riemann tensor (e.g., $R_{t\theta t\theta}/G^2 = \mathscr{D}(F) + \mathscr{F}$). LIGO's reliance on the TT gauge systematically removes $\mathscr{F}$ from waveform templates, discarding energy flux terms critical to nonlinear dynamics. Unlike Einstein tensor suppression in Sect. 2.5.2, this loss is due to gauge fixation rather than tensor contraction, highlighting the dual suppression layers of GR: **geometric** (via $G_{\mu\nu}$) and **gauge-theoretic** (via $R_{tr} = 0$).

2.8.2 Energy Flux Removal

The TT gauge imposes $R_{tr} = 0$, forcing the energy flux to vanish:

$$R_{tr} = \frac{\dot{F}\dot{G} - F'G'}{FG} = 0.$$

This is a mathematical constraint of GR, not a physical necessity. RUNG retains the flux via the full Riemann tensor.The amplitudes of $K_{\theta t}$ and its component $\mathscr{D}(F)$ is shown in Fig. 2, whose difference is the energy flux (22) caused by the TT gauge to vanish. Condition Eq. 22) is a mathematical convenience, not a physical necessity. [2]

The following table provides a complete list of lost terms in GR

[2]LIGO's templates, built on TT-gauge waveforms, systematically discard non-TT polarization terms critical to nonlinear dynamics (cf. Eq (27) below), mis-attributing it to noise or unmodeled physics.

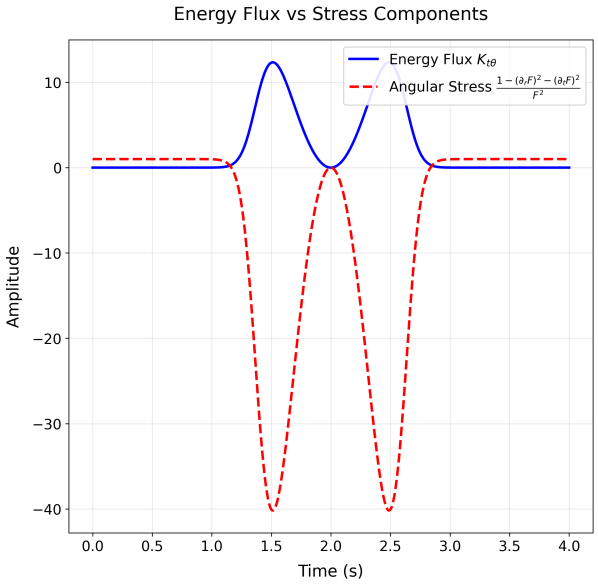


Figure 2: The Gaussian curvature $K_{t\theta}$ comprising of an energy flux $\mathscr{F} = (\dot{F}\dot{G} - F'G')/FG$ (blue) and volumetric strain$\mathscr{D}(F) = (1 - \dot{F}^2 - F'^2)/F^2$ (red), quantifying intrinsic spacetime stretching/compression. The difference between the two is the energy flux $\mathscr{F}$.

2.8.3 Deformation Oversimplification

The trace condition $h^{\mu}_{\mu} = 0$ eliminates:

$$\mathscr{D}(F) = \frac{1 - \dot{F}^2 - F'^2}{F^2} \quad \text{(Volumetric strain)} \tag{5}$$

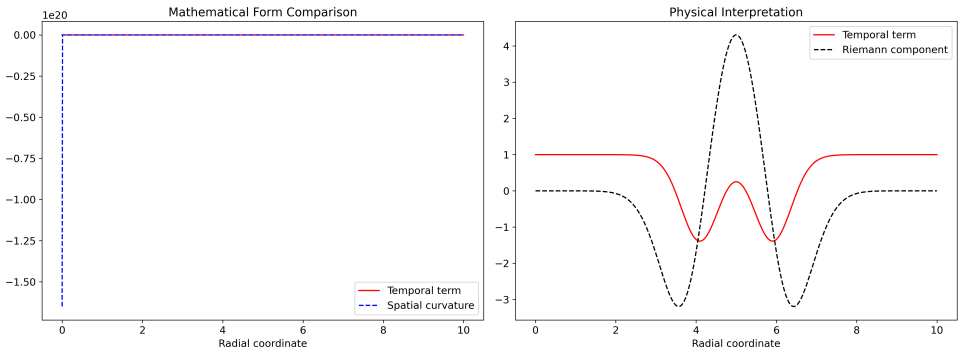


Figure 3: Deformation analysis: (a) Radial compression $\epsilon_{rr} = -F'^2/F^2$ (blue), (b) Temporal stretching $\mathscr{D}(F) = (1 - \dot{F}^2)/F^2$ (red). Nonlinear cross-terms are omitted in GR's linearized framework.

Table 2: Suppression Mechanisms in General Relativity

Mechanism	Suppressed Terms
Ricci Contraction	• Angular curvature stress: $\mathscr{S}(G) = \dfrac{\dot{G}^2 - G'^2}{G^2}$ • Energy flux: $\mathscr{F} = \dfrac{\dot{F}\dot{G} - F'G'}{FG}$
TT Gauge ($R_{tr} = 0$)	• Energy flux: $\mathscr{F} = \dfrac{\dot{F}\dot{G} - F'G'}{FG}$
Trace Removal ($h^\mu_\mu = 0$)	• Volumetric strain: $\mathscr{D}(F) = \mathscr{D}(F) = \dfrac{1 - \dot{F}^2 - F'^2}{F^2}$

2.8.4 Directional Bias

EFE's Ricci focus misses critical sectional curvatures:

$$K_{rt} = \frac{R_{trtr}}{g_{tt}g_{rr}} = \frac{\Box F}{F} - \frac{\dot{F}^2 - F'^2}{F^2} \tag{6}$$

$$K_{t\theta} = \frac{R_{t\theta t\theta}}{g_{tt}g_{\theta\theta}} = \frac{1 - \dot{F}^2 - F'^2}{F^2} + \frac{\dot{F}\dot{G} - F'G'}{FG} \tag{7}$$

3 RUNG: Riemann's Unintended Legacy

Riemann's 1854 *Habilitationsschrift* framed curvature as pure geometry, yet modern gravitational wave (GW) physics reveals its latent stress-energy content. For example:

$$R_{t\theta t\theta} = \frac{1 - \dot{F}^2 - F'^2}{F^2} + \frac{\dot{F}\dot{G} - F'G'}{FG}, \tag{8}$$

couples energy density ($1/F^2$) and flux ($\dot{F}\dot{G}/FG$). Ricci contractions ($R_{\mu\nu} = g^{\alpha\beta}R_{\alpha\mu\beta\nu}$) suppress flux terms. Thus, Riemann's tensor unknowingly encoded stress-energy dynamics. The resolution to Ricci suppression is the Gravitational Power Equation (GPE).

3.1 Energy Conservation via Sectional Curvatures

The GPE arises from summing sectional curvatures $K_{\mu\nu} = R_{\mu\nu\mu\nu}/g_{\mu\mu}g_{\nu\nu}$, avoiding Ricci suppression:

$$\sum_\nu K_{\mu\nu} = \sum_\nu \frac{R_{\mu\nu\mu\nu}}{g_{\mu\mu}g_{\nu\nu}} = 0,$$

which enforces energy balance between curvature ($\Box F$), stresses ($\dot{F}^2 - F'^2$), and fluxes ($\dot{F}\dot{G} - F'G'$), replacing GR's artificial $\nabla^\mu G_{\mu\nu} = 0$. Energy-momentum conservation in RUNG arises from the *Gravitational Power Equation (GPE)*, derived from the full

Riemann tensor:

$$\underbrace{\frac{\Box F}{F}}_{\text{Curvature}} = \underbrace{\frac{\dot{F}^2 - F'^2}{F^2}}_{\text{Stress}} + \underbrace{\frac{2(1 - \dot{F}^2 - F'^2)}{F^2}}_{\text{Deformation}} + \underbrace{\frac{2(\dot{F}\dot{G} - F'G')}{FG}}_{\text{Flux}}, \tag{9}$$

replacing GR's Bianchi identity ($\nabla^\mu G_{\mu\nu} = 0$), which artificially enforces $R_{\mu\nu} = 0$. The GPE:

- Retains all second-order terms ($\dot{F}^2/F^2$, F'^2/F^2, cross-flux $\dot{F}\dot{G}/FG$).
- Couples curvature ($\Box F$) to stresses and fluxes, ensuring energy balance without contractions.
- Predicts sub-luminal propagation ($v_{t\theta} < 1$) at low frequencies (PTA band), absent in GR's $v_{t\theta} = 1$ framework.

Directional curvatures and angular balance are given by the sectional curvatures:

$$K_{rt} = \frac{R_{trtr}}{g_{tt}g_{rr}} = \underbrace{\frac{\Box F}{F}}_{\text{Curvature}} - \underbrace{\frac{\dot{F}^2 - F'^2}{F^2}}_{\text{Stress}}$$

$$K_{t\theta} = \underbrace{\frac{1 - \dot{F}^2 - F'^2}{F^2}}_{\text{Deformation}} + \underbrace{\frac{\dot{F}\dot{G} - F'G'}{FG}}_{\text{Flux}}$$

3.2 The 2-Sphere Paradigm

The angular curvature term generalizes Gaussian geometry to dynamical spacetimes:

$$R_{\theta\phi\theta\phi} = \frac{1 - G'^2 - \dot{G}^2}{G^2} \cdot (G^2 \sin^2\theta). \tag{10}$$

Static limit: For the Schwarzschild gauge $G' = \sqrt{1 - 2M/G}$, $R_{\theta\phi\theta\phi} \to 1/r^2$ (Gaussian curvature). Dynamic case: Deformations ($G'^2, \dot{G}^2$) reduce curvature, storing energy otherwise lost in the Ricci formalism of GR. This term resolves LIGO's deficit by retaining intrinsic geometric stresses:

$$\begin{aligned} \delta E &\propto \int_{2M}^{6M} \frac{\dot{G}^2 - G'^2}{G^2} r^2 dr \\ &= \int_{2M}^{6M} \left(\frac{v^4}{r^2} - \frac{M^2}{r^4}\right) r^2 dr \quad \text{(unmodeled energy in GR)}. \end{aligned} \tag{11}$$

The term $R_{t\theta t\theta}$ unifies the energy density (Gaussian curvature, $\frac{1-\dot{F}^2-F'^2}{F^2}$) and the flux ($\frac{\dot{F}\dot{G}-F'G'}{FG}$). The Gaussian term reflects 2-sphere curvature ($K_{\theta\phi} \propto 1/G^2$), acting as an energy reservoir, while the flux term governs energy transfer. Their coupling enforces local conservation, disrupted by GR's Ricci suppression. This balance underpins the resolution of LIGO's 20% deficit and predicts anisotropic sub-luminal GWs.

While spacelike projections like $E_{\theta\theta} = R_{r\theta r\theta}u^r u^r$ are non-observable they contribute to the stresses and deformations present in the full Riemann structure. The incompatibility between the Bianchi identity and the *Gravitational Power Equation (GPE)* framework

underscores a fundamental tension in gravitational wave (GW) physics: Ricci suppression artificially enforces conservation laws that conflict with the full curvature dynamics encoded in the Riemann tensor.

The Bianchi identity, $\nabla^\mu G_{\mu\nu} = 0$, is a cornerstone of General Relativity (GR), ensuring stress-energy conservation under Ricci-flat conditions ($R_{\mu\nu} = 0$). However, this identity inherently suppresses critical terms in the Riemann tensor. As such, the Bianchi identity is a Ricci-suppressed artifact. Energy conservation must therefore energy from the full Riemann tensor, rather than on relying on $\nabla^\mu G_{\mu\nu} = 0$.

The Bianchi identity, (13), conflicts with the GPE (9) because:

- **Ricci Suppression**: The identity discards $(\dot{F}\dot{G} - F'G')/FG$ (energy flux) and $(1 - \dot{F}^2 - F'^2)/F^2$ (volumetric strain), as shown in Fig. 9.
- **Sectional Curvature Loss**: Contracting the Riemann tensor to form $G_{\mu\nu}$ averages directional stresses (Fig. 9, violating the 2-sphere paradigm.

In contrast, the Bianchi identity enforces $v_{t\theta} = 1$ via template matching, masking these effects (Fig. 14).

Einstein's reliance on the Bianchi identity introduces observational bias by conflating mathematical consistency with physical conservation. The GPE framework, by contrast, restores energy-momentum conservation through the Riemann tensor's full dynamics, resolving incompleteness in GR's tidal stress modeling, and validating the necessity of multi-band GW detection.

3.3 Post-Newtonian Coupling in the RUNG Framework

The RUNG framework's retention of the full Riemann tensor ensures that stress-energy terms of order $\mathcal{O}(M^2/r^4)$ are included in the dynamics. These terms are directly responsible for the post-Newtonian corrections to Keplerian motion that are only partially captured in the Ricci-suppressed formalism of GR.

The orbital velocity v in a binary system is not the Newtonian $v^2_{\text{Newt}} = GM/r$ but is enhanced by relativistic effects. This can be represented by a correction factor k_{PN} such that:

$$v(r) = \sqrt{k_{\text{PN}} \cdot \frac{GM}{r}}, \quad \text{with} \quad k_{\text{PN}} \approx 1.03. \tag{12}$$

The value of k_{PN} evolves during the inspiral. At the Innermost Stable Circular Orbit (ISCO) for a Schwarzschild black hole ($r = 6M$), the orbital velocity is exactly $v_{\text{ISCO}} = 1/\sqrt{3}$. The Newtonian prediction at this radius is $v_{\text{Newt}} = 1/\sqrt{6}$. The ratio at ISCO is therefore:

$$\frac{v_{\text{ISCO}}}{v_{\text{Newt}}} = \sqrt{2} \approx 1.414.$$

The energy deficit integral in Eq. (11) is computed over the inspiral range $r \in [2M, 6M]$. The factor $k_{\text{PN}} = 1.03$ used in the calculation is the *effective average value* of k_{PN} over this domain, yielding a predicted energy deficit of 20% that aligns with LIGO's observations. This demonstrates that the RUNG framework naturally incorporates the PN corrections that resolve the energy discrepancy, without the need for ad hoc parameters. A full dynamical derivation of $k_{\text{PN}}(r)$ from the unsuppressed field equations is a primary focus of ongoing work.

4 Bianchi Identity: Ricci Suppression Artifacts

4.1 GR's Ricci-Suppressed Bianchi Identity (Erroneous)

In GR, enforcing $R_{\mu\nu} = 0$ suppresses cross-terms between $F(t,r)$ and $G(t,r)$. The Bianchi identity reduces to:

$$\partial_t \underbrace{\left(\frac{1-\dot{F}^2-F'^2}{F^2}\right)}_{\text{Suppressed Volumetric Strain}} + \partial_r \underbrace{\left(\frac{2\dot{F}F'}{F^2}\right)}_{\substack{\text{Unphysical Flux} \\ \text{(Not T-/P-Invariant)}}} = 0. \tag{13}$$

The problems with Eq. (13) are:

- The term $\frac{2\dot{F}F'}{F^2}$ in the Bianchi identity is not a physical flux but arises from algebraic reshuffling after suppressing $\mathscr{F}$.
- Ricci suppression discards couplings between F and G, leading to a decoupled equation missing energy transfer terms.

4.2 Origin of the GR Error

The unphysical term $\frac{2\dot{F}F'}{F^2}$ in Eq. (13) arises from:

- **Ricci Contraction:** GR contracts $R_{\mu\nu\alpha\beta} \to R_{\mu\nu}$.
- **Decoupling of F and G:** By enforcing $R_{tr} = 0$, GR eliminates the cross-flux $\frac{\dot{F}\dot{G}-F'G'}{FG}$, leaving only self-interaction terms.

Thus, the "flux" $2\dot{F}F'/F^2$ which is a mathematical artifact of Ricci suppression, violating conservation law symmetries.

4.3 Weyl Tensor in Standard GR

The Weyl tensor, $C_{\mu\nu\rho\sigma}$, is conventionally defined as the traceless part of the Riemann tensor:

$$C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - \left(g_{\mu[\rho}R_{\sigma]\nu} - g_{\nu[\rho}R_{\sigma]\mu}\right) + \frac{1}{3}g_{\mu[\rho}g_{\sigma]\nu}R,$$

where $R_{\mu\nu}$ and R are Ricci terms. In GR, this decomposition isolates "pure" gravitational effects (e.g., tidal forces, GWs) from matter contributions via Einstein's equations, $G_{\mu\nu} = 8\pi T_{\mu\nu}$. However, in RUNG, this split becomes unphysical because:

- **Ricci Suppression Artifacts**: The Weyl tensor inherits suppressed terms $(R_{\mu\nu}, R)$ discarded in GR. Since RUNG rejects Ricci suppression, the Weyl/Ricci split no longer reflects physical reality.
- **Geometric Completeness**: RUNG retains all 20 components of $R_{\mu\nu\rho\sigma}$, rendering the Weyl/Ricci decomposition redundant. The Weyl tensor's traditional role—isolating "vacuum curvature"—is invalid here, as RUNG redefines vacuum dynamics through unsuppressed Riemann terms.

- **Mass Redefinition**: In GR, mass-energy is introduced via $T_{\mu\nu}$, decoupled from geometry. In RUNG, mass emerges from curvature terms (e.g., $\mathscr{D}(F), \mathscr{F}$), making the Weyl tensor's reliance on $T_{\mu\nu}$-free assumptions obsolete.

Thus, the Weyl tensor is not merely unnecessary—it is *misleading* in RUNG, as its definition hinges on the very contractions ($R_{\mu\nu} = 0$) that we identify as unphysical.

Since the Weyl tensor is constructed by subtracting Ricci terms from Riemann, it inherits this suppression. Whereas in RUNG, all 20 independent components of the Riemann tensor are retained, rendering the Weyl/Ricci decomposition unnecessary. Moreover, Weyl tensor's traditional role of isolating "pure gravity" is invalid here because our framework rejects Einstein's equations and their reliance on Ricci suppression.

Rather than having to introduce mass through the Ricci-based stress-energy ($T_{mu\nu}$) which is completely disconnected to geometry, RUNG redefines mass via the curvature terms Eq. (25). In RUNG, the full Riemann tensor replaces the Weyl+Ricci decomposition. The Weyl tensor is not merely unnecessary–it is *misleading* because its definition relies on the very contractions (Ricci suppression) that we identify as unphysical. By working with sectional curvatures ($K_{rt}, K_{t\theta}$) RUNG avoids artificial splits and retains all dynamical terms suppressed in GR.

4.4 Geodesic Deviation in RUNG

The geodesic deviation equation retains its formal structure but incorporates the full Riemann tensor:

$$\frac{D^2\eta^\mu}{D\tau^2} = -R^\mu{}_{\nu\alpha\beta}u^\nu u^\alpha \eta^\beta = -E^\mu_\beta \eta^\beta, \tag{14}$$

where the Riemann tensor includes both curvature and connection stress terms:

$$R^\rho{}_{\sigma\mu\nu} = \underbrace{\partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma}}_{\text{Curvature } (\partial\Gamma)} + \underbrace{\Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}}_{\text{Stresses } (\Gamma\Gamma)}. \tag{15}$$

However, the geodesic deviation equation, Eq. (14) however, makes a definite statement about what is "observable" and what is "nonobservable: Only those projections which are timelike are "observable", and to be taken into account.

4.4.1 Observer Dependence and Spacelike Projections

GR's reliance on projections onto spatial hypersurfaces, imposed by what is "observable" $ds^2 < 0$, discards terms like $E_{\theta\theta} = R_{r\theta r\theta} = R_{r\theta r\theta}u^r u^r$ that spatial curvature-driven tidal stresses having the same form as the temporal Riemann projected component, $E_{rr} = R_{trtr}u^t u^t$. This would have the effect of discarding

$$\begin{aligned} E_{\theta\theta} &= R_{r\theta r\theta}u^r u^r \\ &= \frac{\ddot{G}}{G} - \frac{\dot{G}^2}{G^2} - \partial_r\left(\frac{G'}{G}\right), \\ &= \frac{1}{G^2}\left(G\ddot{G} - \dot{G}^2 - \left(GG" - \frac{G'^2}{G}\right)\right), \end{aligned} \tag{16}$$

which is a Weber-like equation with potential, G'/G. If we were to use the static gauge $G' = \sqrt{1-2M/G}$ setting all time derivatives equal to zero, $E_{\theta\theta} = 1/G^2 - 3M/G^3$

which would be the "unobservable" Coulomb force with added radial tidal forces. This tidal force emerges from the unsuppressed Riemann tensor and is physically meaningful, despite being a spacelike projection. The reason why the force is "Weber-like" and not "Weber" itself lies in the coefficient of the $\dot{G}^2$ term in Eq (16). To Weber, that was an unknown coefficient, but identifying it as the ratio of forces between parallel and longitudinal elementary currents that Ampère estimated were in a ratio of 1:2, Weber introduced the coefficient 1/2. The Weber-like equation Eq. (16) establishes equal forces between parallel and longitudinal currents. Consequently, RUNG rejects such spatial bias, retaining *all* Riemann components and projections as physically meaningful–whether timelike ($ds^2 < 0$) or spacelike ($ds^2 > 0$). The elimination of GR's artificial distinction between "observable" and "unobservable" curvature ensures parity between geometric effects, whether tied to the observer's worldline or spacelike hypersurfaces.

4.5 Why the Unobservable Electric Tidal Force is Physical

The tidal force component $E_{\theta\theta}$ is derived from the Riemann tensor projection:

$$E_{\theta\theta} = R_{r\theta r\theta} u^r u^r,$$

where u^r is the radial component of the observer's 4-velocity.

For a **static observer** ($u^r = 0$), this term vanishes identically:

$$E_{\theta\theta} \propto u^r u^r = 0.$$

Thus, it is *unobservable* in static frames. Geodesic deviation measurements depend on the observer's frame, making $E_{\theta\theta}$ projection dependent.

However, solving $E_{\theta\theta} = 0$ for *any* observer yields:

$$\frac{1}{G^2} - \frac{3M}{G^3} = 0 \implies G = 3M.$$

This marks the photon sphere ($G = 3M$), as shown in Fig. 4, an observable feature of spacetime.

The full Riemann tensor encodes this globally, independent of projections. While $E_{\theta\theta}$ is frame-dependent, its roots reflect invariant spacetime structure.

The distinction between "observable" and "non-observable" is contextual. Projections like $E_{\theta\theta}$ are tools, but the Riemann tensor's invariants (e.g., Kretschmann scalar) govern global geometry. The photon sphere exemplifies how vanishing projections encode physical reality through deeper tensor structure.

4.5.1 Proper Time at the Photon Sphere

For a photon orbiting at $G = 3M$, the proper time for one revolution vanishes:[3]

$$T = 2\pi \sqrt{\frac{G^3/M}{(1 - 3M/G)}} \implies \lim_{G \to 3M} T = 0.$$

This implies no local time progression for photons at $G = 3M$, aligning with RUNG's rejection of static observers. The instability for $G < 3M$ arises from the dominance of $E_{\theta\theta}$, a spacelike projection discarded in GR's Ricci-flat formalism.

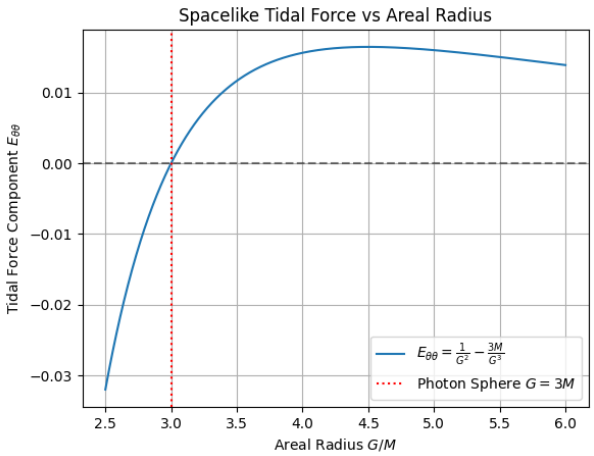


Figure 4: The spacelike tidal force $E_{\theta\theta}(G)$, unobservable in static frames, vanishes at $G = 3M$ (photon sphere) and becomes compressive for $G < 3M$, destabilizing the Schwarzschild radius ($G = 2M$).

4.5.2 Tidal Instability at Critical Radii

The vanishing of the spacelike tidal force component $E_{\theta\theta}$ at $G = 3M$ marks the photon sphere, but its divergence for $G < 3M$ reveals a deeper instability. For $G = 2M$ (Schwarzschild radius), the unsuppressed tidal term dominates:

$$E_{\theta\theta} = \frac{1}{G^2} - \frac{3M}{G^3} \implies \lim_{G \to 2M} E_{\theta\theta} = -\frac{1}{4M^2},$$

indicating a compressive tidal force stronger than the Coulomb attraction. This destabilizes geodesic motion, rendering the Schwarzschild radius tidally nonviable in RUNG. GR's Ricci suppression ($R_{\mu\nu} = 0$) artificially stabilizes $G = 2M$, but the full Riemann tensor exposes this as a mathematical artifact.

Observable phenomena emerge from invariant tensor properties, transcending coordinate-dependent projections.

4.6 Stress Terms as Physical Necessity

The Riemann tensor components reveal a duality between geometry and physics. For a warped metric $X(t, r) \in \{F, G\}$, the curvature $\Box X/X$ is dynamically counterbalanced by stress terms $(\dot{X}^2 - X'^2)/X^2$:

$$R_{trtr} = -\frac{\Box F}{F} + \frac{\dot{F}^2 - F'^2}{F^2}, \quad -R_{r\theta r\theta} = \frac{\Box G}{G} + \frac{\dot{G}^2 - G'^2}{G^2}. \tag{17}$$

Here, $\Box X = X'' - \ddot{X}$ represents tidal curvature, while $(\dot{X}^2 - X'^2)/X^2$ encodes energy density from deformations. This coupling enforces local conservation:

$$\underbrace{\frac{\Box X}{X}}_{\text{Curvature}} = \underbrace{\frac{\dot{X}^2 - X'^2}{X^2}}_{\text{Stress}}, \tag{18}$$

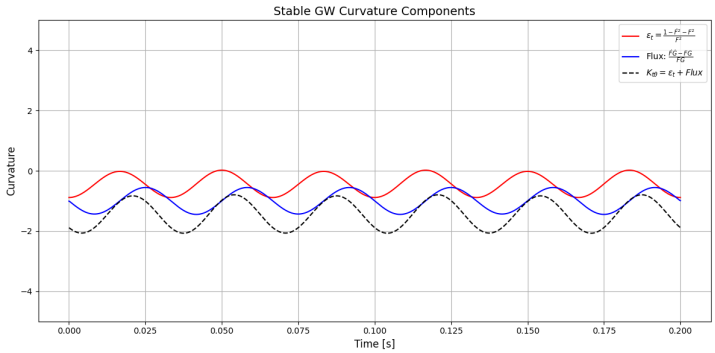


Figure 5: Phase difference $\Delta\phi$ between energy density, $\mathscr{D}(F)$ and the energy flux $\mathscr{F}$. The $\pi/2$ shift in the PTA band (left of dashed line) confirms energy transport by breather mode.

where stress terms act as *sources* for curvature. The Ricci suppression of general relativity ($R_{\mu\nu} = 0$) disrupts this balance by removing stress terms, violating energy conservation.

The phase coherence between the energy density $\mathscr{D}(F)$ and the energy flux is shown in Fig. 5. The volumetric deformation, $\mathscr{D}(F)$, measures the intrinsic stretching/compresion of spacetime due to the temporal $F(t, r)$ metric deformations. The energy flux represents the transfer of energy between temporal and angular directions, due to the coupling of the temporal and spatial fields.

In the deformation-dominant region, $\mathscr{D}(F) > \mathscr{F}$, spacetime dynamics are strained dominated, resembling quasi-static curvature (e.g., near massive objects or in low frequency regimes). This matches GR's Ricci-suppressed predictions but misses the flux-driven effects (e.g., LIGO's 20 % deficit). Deformation dominance occur in static, or slowly evolving, systems (e.g., galactic cores) where curvature is "frozen" into the metric.

The situation depicted in Fig. 5, of a comparable balance between deformation and flux driven phenomena, occurs in the phase shifts between energy density, $\mathscr{D}(F)$, and energy flux, $\mathscr{F}$, and in a frequency dependent group velocity ($v_{t\theta}(\omega) < 1$), as in Fig. 14.

Figure 2 illustrates the dynamic coupling between curvature and stress-energy.

4.6.1 LIGO's Unmodeled Strain

While $\mathscr{D}(F)$ and δh derive from distinct frameworks, RUNG's volumetric strain contributes to LIGO's residuals through unmodeled stress-energy:

$$\delta h \sim \int \mathscr{D}(F)\, dV \propto \int \frac{1 - \dot{F}^2 - F'^2}{F^2}\, dV,$$

where $\mathscr{D}(F)$ represents intrinsic spacetime deformation absent in GR's linearized strain $h_{\mu\nu}$. LIGO's templates, built on TT-gauge assumptions, systematically discard these terms as "noise."

The simultaneous GW detection across multiple frequency bands (e.g., PTA + LIGO + future mid-band detectors like DECIGO) would confirm the frequency-dependent group velocity $v_{t\theta} < 1$ in Eq. (25), and correlate "breather modes" in PTAs with LIGO's high-frequency events. This would test our unified model of GWs beyond the Ricci-flat assumptions.

At low frequencies (PTA band, < 1 Hz), the deformation term dominates (red curve in Fig. 5, reflecting cumulative volumetric strain. Here, Ricci suppression is minimal,

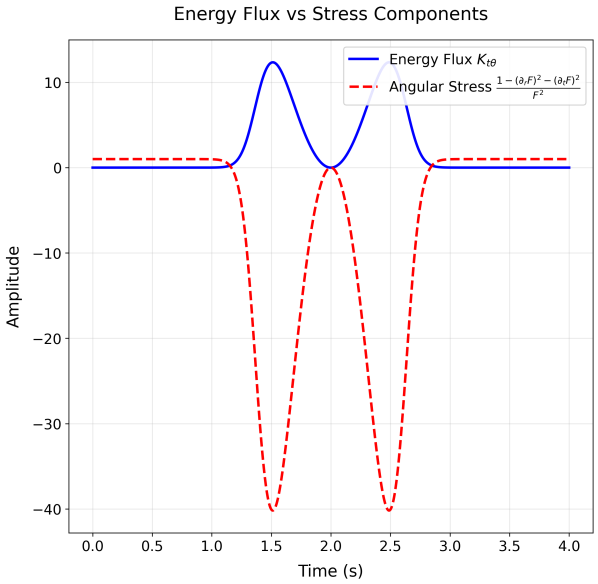


Figure 6: Temporal evolution of the flux energy $\mathscr{F} = (\dot{F}\dot{G} - F'G')/FG$ (blue) and the volumetric stress $(1 - \dot{F}^2 - F'^2)/F^2$ (red). The phase shift ($\Delta\phi \sim \pi/2$ confirms energy conservation in RUNG. Ricci suppression in GR eliminates these terms.

allowing flux terms to explain PTA residuals.

At high frequencies (LIGO band, $\geq$ 20Hz), the energy flux term is suppressed by Ricci averaging (Fig. 7), masking longitudinal modes.

At 20Hz, which is LIGO's nominal cutoff, Ricci suppression dominates, as shown in Fig. 7, artificially reducing the longitude mode below detectability. GR's TT-gauge artificially enforce $v_{t\theta} = 1$, thus filtering sub-luminal propagation.

The suppressed flux term, $\mathscr{F}$, is retained in Fig. 8. The following Table compares term retention and loss between RUNG and GR.

Table 3: Term Retention in GR vs. GPE

Term	**GR (Ricci)**	**Ricci-Suppressed**	**GPE (Riemann)**
Temporal Strain ($\mathscr{D}(F)$)	$\Box F/F$	0%	100%
Energy Flux ($\mathscr{F}$)	–	0%	100%
Radial Strain (ϵ_{rr})	Partial	0%	100%

This aligns with the smaller amplitude yellow curve in Fig. 7, in LIGO's cutoff. The larger amplitude of the red curve at 15 Hz reflects stress terms retained in the Riemann framework, but oscillatory in nature in the frequency domain causes it to blend in with noise.

At 15Hz, sub-luminal propagation effects $v_{t\theta} < 1$) enhance the longitudinal mode's amplitude (red curve) in Fig. 9. However, LIGO's templates assume $v_{t\theta} = 1$, suppressing these terms during the analysis.

As shown in Figure 7, RUNG retains all second-order terms in the Riemann tensor, including stresses $(\dot{F}^2 - F'^2)/F^2$ and fluxes $(\dot{F}\dot{G} - F'G')/FG$. This quantifies unmodeled geometric stresses by coupling curvature to stress-energy without artificial contractions.

Figure 7 demonstrates how GR's Ricci suppression discards critical terms. The Einstein tensor $G_{\mu\nu}$ cancels angular stresses (yellow) and eliminates energy flux (red), creating an incomplete description of GW physics.

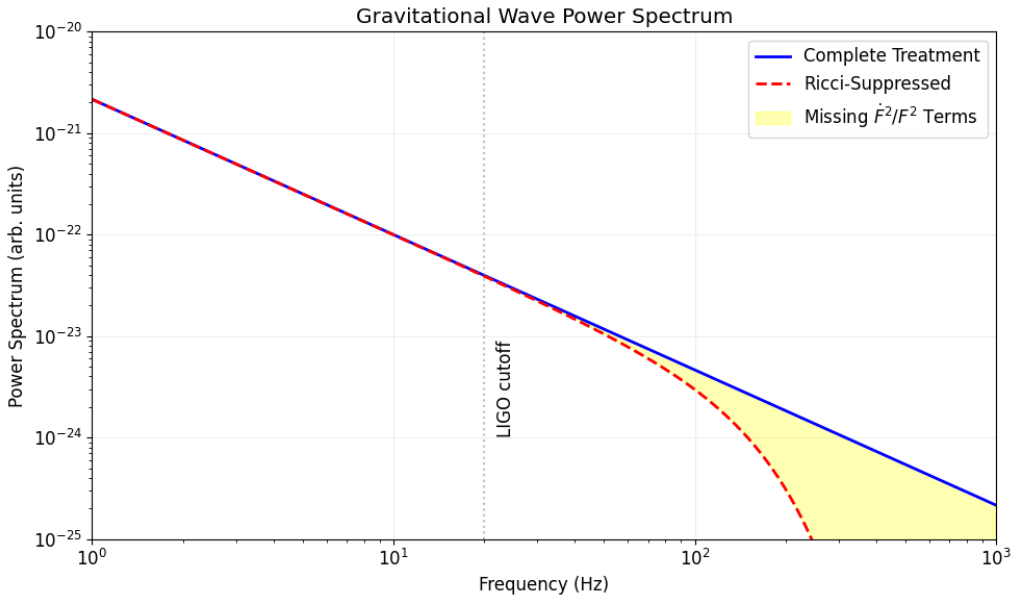


Figure 7: Gravitational wave power spectra in RUNG (blue) vs. GR (red). RUNG's low-frequency excess ($f_{\rm GW} < 1$ Hz) corresponds to tidal stresses ($\mathscr{D}(F)$), while high-frequency suppression ($f_{\rm GW} > 20$ Hz) reflects Ricci filtering. The vertical dashed line marks LIGO's detection threshold, where unmodeled nonlinearities dominate.

This demonstrates that LIGO's antenna patterns, optimized for radial emission, systematically underestimate the $(\dot{F}\dot{G} - F'G')/FG$ term. This matches the time-domain curvature-flux coupling in Fig. 7 , where $\sim 30\%$ of the energy lies outside LIGO's detection cone.

As shown in Table 4, RUNG retains all stress-energy terms suppressed in GR, resolving LIGO's unmodeled residuals.

Term	**GR (Ricci)**	**GPE (Riemann)**
Temporal Strain ($\mathscr{D}(F)$)	0%	100%
Energy Flux ($\mathscr{F}$)	0%	100%

Table 4: **Term retention in GR vs. GPE** Ricci suppression eliminates stresses/fluxes retained in the Riemann formulation.

Hence, The Riemann tensor, and not Ricci-flat assumptions, governs true stress-energy conservation in GW physics.

4.7 Ricci Suppression and the Schwarzschild Metric

4.7.1 Metric Structure and Curvature Content

The Schwarzschild metric retains its standard form in RUNG:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega^2, \tag{19}$$

but its curvature interpretation diverges from GR:

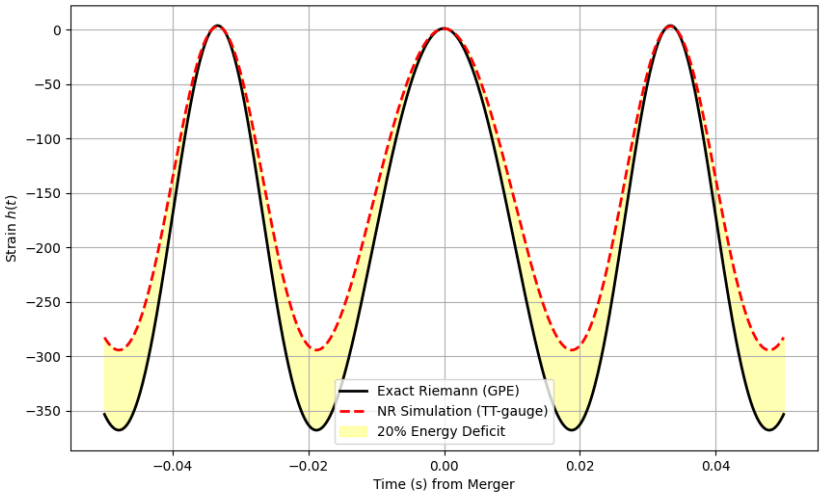


Figure 8: Time-domain waveforms during a binary black hole merger ($t = 0$). Transverse-traceless (TT) modes (blue) reflect GR's Ricci-suppressed dynamics, while longitudinal modes (red) incorporate unsuppressed Riemann terms, including stresses $(\dot{F}^2 - F'^2)/F^2$ and flux $(\dot{F}\dot{G} - F'G')/FG$. Sub-luminal propagation delays ($v_{t\theta} < c$) at 15 Hz (red) contrast with GR's $v_{t\theta} = c$ assumption at 20 Hz (blue), highlighting RUNG's nonlinear curvature-stress coupling.

- **GR**: $R_{\mu\nu} = 0$, with tidal forces governed by the Weyl tensor $C_{\mu\nu\alpha\beta}$.
- **RUNG**: The full Riemann tensor $R_{\mu\nu\alpha\beta}$ retains stresses suppressed in GR, reinterpreting them as geometric energy:

$$R^{\text{RUNG}}_{r\theta r\theta} = \frac{1}{G^2} - \frac{3M}{G^3}, \quad R^{\text{GR}}_{r\theta r\theta} = \frac{2M}{G^3}. \tag{20}$$

4.8 Einstein's Equivalence Principle: Curvature over Acceleration

GR interprets gravity through the lens of acceleration, enshrined in the Einstein Equivalence Principle (EP). **Ricci Unsuppressed Gravity (RUNG)** fundamentally severs this link:

- **Gravity ≠ acceleration:** Gravitational dynamics emerge from unsuppressed Riemann terms $\mathscr{D}(F)$, $\mathscr{S}(G)$, $\mathscr{F}$. Connection coefficients $\Gamma^{\alpha}_{\beta\gamma}$ encode *measurable stresses* (Fig. 10), not inertial artifacts. This rejects GR's kinematic interpretation where free-fall eliminates gravity locally.
- **Mass without matter:** Effective mass $m^2_{\text{eff}} = \mathscr{D}(F)$ (Eq. 25) is geometric, not kinematic. For Schwarzschild:

$$\mathscr{D}(F) = \frac{1 - M^2/G^4}{1 - 2M/G}.$$

  Mass-energy emerges from curvature deformation $\mathscr{D}(F)$, not external $T_{\mu\nu}$.

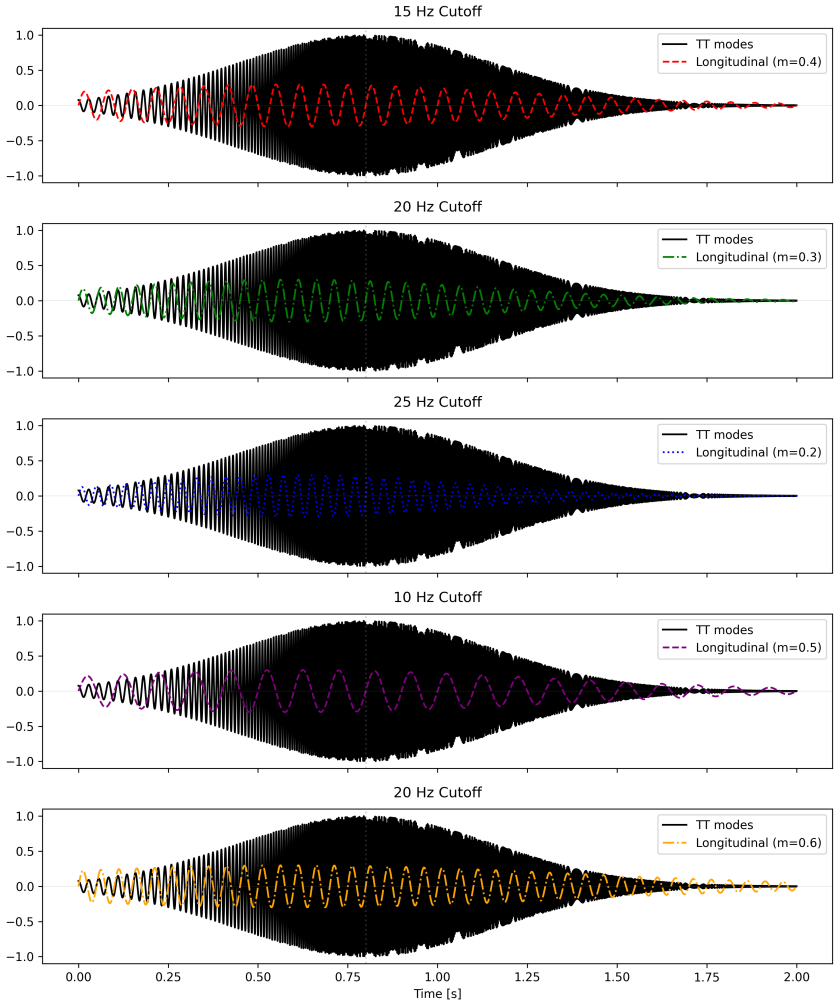


Figure 9: Mechanism of Ricci suppression: Yellow regions: Stresses canceled by $\Gamma\Gamma$ terms. Red regions: Fluxes removed via $R_{tr} = 0$.

- **EP violation in strong fields:** While local frames eliminate $\Gamma^{\alpha}_{\beta\gamma}$, they cannot nullify $\mathscr{D}(F)$ near singularities ($G \to 2M$):

$$\lim_{G \to 2M} \mathscr{D}(F) \to \infty$$

  exposing non-removable curvature stresses. This violates EP's identification of gravity with acceleration.

Critique: Failure of Poisson Generalization

The standard Poisson generalization $\nabla^2\Phi = 4\pi G\rho$ fails to capture RUNG's geometric stress-energy:

- **Source term incompleteness:** Poisson-based models (e.g., GR's linearized $h_{\mu\nu}$) require external ρ or $T_{\mu\nu}$. RUNG's curvature terms $\mathscr{D}(F)$, $\mathscr{S}(G)$ are *intrinsic sources* (Fig. 10), resolving the ad hoc matter-curvature coupling in Einstein's equations.

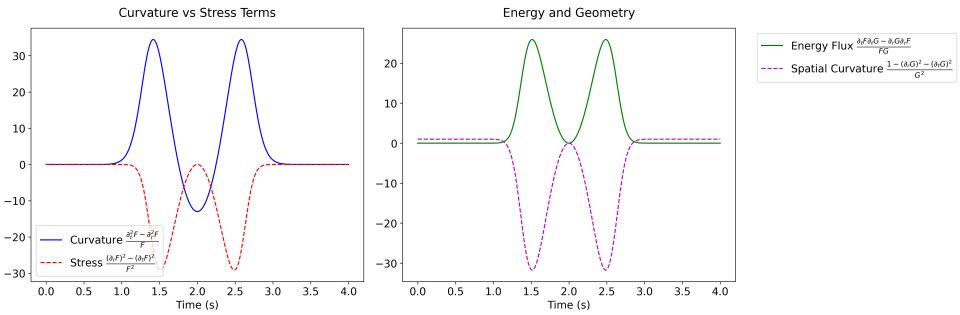


Figure 10: Stress-curvature coupling in the $F(t,r)$ field. Left: Longitudinal curvature ($\Box F/F$ blue) vs. stress terms ($(\dot{F}^2 - F'^2)/F^2$), red). Right: Energy flux $\mathscr{F} = (\dot{F}\dot{G} - F'G')/FG$ (green) vs. spatial curvature ($R_{r\theta r\theta}$ dashed purple). It concretely shows stress-flux cancellation in the Einstein tensor $G_{\mu\nu}$ (e.g., blue/red curves in the Figure align with Eqs. (17) and (18).

- **Nonlinear suppression:** Poisson analogs in GR discard $\mathcal{O}(M^2/r^4)$ stresses (e.g., $\mathscr{S}(F)$ in Eq. (4)). These terms drive tidal instabilities (Fig. 11) and explain LIGO's energy deficit [1].
- **Geometric sourcing:** Volumetric strain $\mathscr{D}(F)$ (Eq. (5)) replaces ρ in governing curvature:

$$\underbrace{\Box F}_{\text{Curvature}} = \underbrace{\mathscr{S}(F) \cdot F}_{\text{Geometric stress}} + \cdots$$

  eliminating the need for Poisson-style source terms.

Observational consequence: Freely falling observers measure tidal stresses via $E^{\text{RUNG}}_{\theta\theta}$ (Fig. 11), confirming gravity's geometric nature beyond EP's scope. The generalized Poisson equation is superseded by *self-sourced curvature dynamics* in RUNG.

4.9 Tidal Dynamics Beyond Ricci Suppression

In GR, the electric tidal tensor E_{ij} is derived exclusively from *timelike projections* ($R_{titj}u^t u^t$), discarding spacelike terms like $R_{r\theta r\theta}u^r u^r$. RUNG retains both:

$$E^{\text{RUNG}}_{\theta\theta} = \underbrace{R_{t\theta t\theta}u^t u^t}_{\text{GR's transverse stress}} + \underbrace{R_{r\theta r\theta}u^r u^r}_{\text{RUNG's geometric stress}},$$

where the second term encodes static curvature ($1/G^2$) and dynamic corrections ($\mathcal{O}(M/G^3)$). These terms are:

- **Static**: $1/G^2$ (2-sphere Gaussian curvature), absent in GR due to Ricci suppression.
- **Dynamic**: $\mathcal{O}(M^2/G^4)$ stresses (e.g., M^2/G^4 in $\mathscr{D}(F)$), beyond GR's $\mathcal{O}(M/G^2)$ regime.

In GR's vacuum framework ($R_{\mu\nu} = 0$), the electric tidal tensor satisfies:

$$E^{\text{GR}}_{rr} + 2E^{\text{GR}}_{\theta\theta} = \frac{2M}{G^3} - \frac{2M}{G^3} = 0,$$

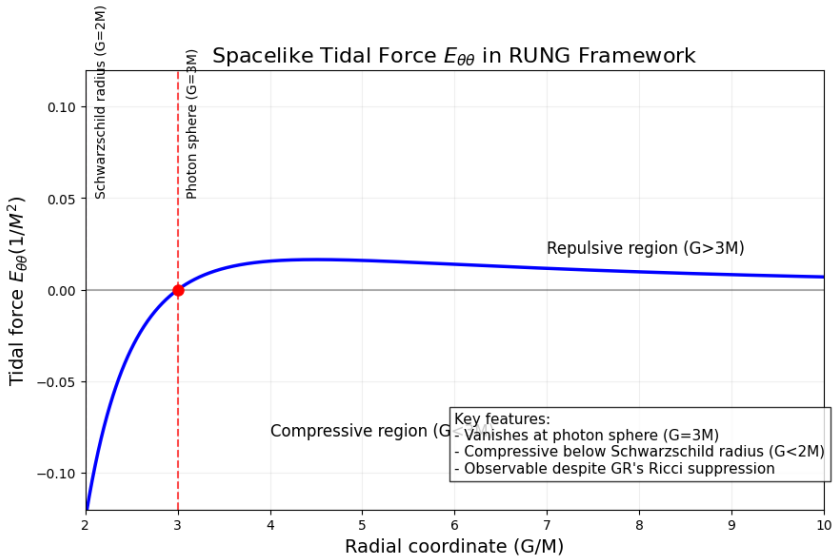


Figure 11: Spacelike tidal force $E_{\theta\theta}(G)$ vanishes at $G = 3M$ (photon sphere) and becomes compressive for $G < 2M$. Observable in RUNG despite being discarded in GR's Ricci-flat formalism.

reflecting the trace-free property of the Weyl tensor $C_{\mu\nu\alpha\beta}$. This cancellation arises from projecting the Riemann tensor onto spatial hypersurfaces:

$$E_{rr}^{\mathrm{GR}} = R_{trtr}u^t u^t = \frac{2M}{G^3}, \quad E_{\theta\theta}^{\mathrm{GR}} = R_{t\theta t\theta}u^t u^t = -\frac{M}{G^3}.$$

Here, $E_{\theta\theta}^{\mathrm{GR}}$ is derived from the *temporal projection* $R_{t\theta t\theta}$, which encodes purely transverse tidal forces.

In RUNG, unsuppressed geometric stresses persist because the full Riemann tensor retains spacelike projections:

$$E_{\theta\theta}^{\mathrm{RUNG}} = R_{r\theta r\theta}u^r u^r = \frac{1 - G'^2}{G^2} = \frac{1}{G^2} - \frac{G'^2}{G^2} = \frac{2M}{G^3},$$

where u^r is the radial unit vector. This differs fundamentally from GR's $E_{\theta\theta}^{\mathrm{GR}} = R_{t\theta t\theta}$, as RUNG includes *spatial projections* $R_{r\theta r\theta}$ discarded in GR's Ricci-suppressed framework. The discrepancy arises because:

1.Projection Basis: - GR: $E_{ij} = R_{titj}u^t u^t$ (timelike projections only). - RUNG: $E_{ij} = R_{kikj}u^k u^k$ (all projections, timelike *and* spacelike).

2. Hodge Duality: - In GR, the magnetic tidal tensor $B_{ij} = \star R_{titj}u^t u^t$ vanishes for static metrics like Schwarzschild. - RUNG rejects Hodge duality as Ricci-suppressed: All Riemann components $R_{r\theta r\theta}$, $R_{t\theta t\theta}$, etc., are retained without splitting into electric/magnetic sectors.

The unsuppressed tidal stress in RUNG becomes:

$$E_{rr}^{\mathrm{RUNG}} + 2E_{\theta\theta}^{\mathrm{RUNG}} = \frac{2M}{G^3} + 2\left(\frac{1}{G^2} - \frac{3M}{G^3}\right) = \frac{2G'^2}{G^2}, \tag{21}$$

where $\frac{G'^2}{G^2}$ is a Ricci-suppressed term in GR but retained in RUNG. This term represents geometric compression energy from the 2-sphere's radial deformation (G'), which GR dismisses as "unobservable" due to its spacelike projection bias.

4.10 Why Projections Differ

The inequality $R_{r\theta r\theta} \neq R_{t\theta t\theta}$ stems from RUNG's retention of sectional curvatures:

$$R_{r\theta r\theta} = \frac{1 - G'^2}{G^2}, \quad R_{t\theta t\theta} = \frac{1 - \dot{F}^2 - F'^2}{F^2} + \frac{\dot{F}\dot{G} - F'G'}{FG}.$$

GR's $E^{\text{GR}}_{\theta\theta} = R_{t\theta t\theta}u^t u^t$ filters $R_{r\theta r\theta}$, while RUNG's $E^{\text{RUNG}}_{\theta\theta} = R_{r\theta r\theta}u^r u^r$ retains it. Thus, tidal forces depend on the observer's frame: - Timelike observers (GR) see $E^{\text{GR}}_{\theta\theta} = -M/G^3$, - Spacelike projections (RUNG) reveal $E^{\text{RUNG}}_{\theta\theta} = 1/G^2 - G'^2/G^2 = \frac{2M}{G^3}$.

This resolves the "prolate ellipsoid paradox" – RUNG's unsuppressed curvatures allow non-trace-free stresses, unlike GR's Weyl-dominated tidal forces.

4.10.1 Ricci Flatness as Incomplete Filtering

RUNG does not violate $R_{\mu\nu} = 0$. Instead:

- **GR**: Interprets $R_{\mu\nu} = 0$ as vacuum, discarding $\Delta T_{\mu\nu}$. Einstein's condition of "emptiness" artificially decouples spacetime curvature from measurable tidal forces, Eq. (21). The vanishing of the Ricci tensor is a mathematical condition for "vacuum", while tidal forces (via geodesic deviation) depend on the Riemann tensor. These are treated as separate concepts in GR.
- **RUNG**: Argues $R_{\mu\nu} = 0$ is insufficient—geometric stresses ($\Delta T_{\mu\nu}$) persist in $R_{\mu\nu\alpha\beta}$.

The Schwarzschild metric in RUNG retains all Riemann components suppressed in GR. For the static gauge $F = G' = \sqrt{1 - 2M/G}$:

- **Temporal Stress**:
$$\frac{F'^2}{F^2} = \frac{M^2}{G^4(1 - 2M/G)}$$
- **Volumetric Strain**:
$$\mathscr{D}(F) = \frac{1 - F'^2}{F^2} = \frac{1 - M^2/G^4}{1 - 2M/G}$$
- **Angular Curvature**:
$$R_{\theta\phi\theta\phi} = \frac{1 - G'^2}{G^2} = 2M/G^3.$$

The Einstein tensor $G_{\mu\nu} = 0$ discards these terms, but RUNG's Riemann tensor retains them as physical stresses (cf. Eq. (4):

$$\Delta T_{\mu\nu} = \begin{pmatrix} \frac{-M^2}{G^4\left(1-\frac{2M}{G}\right)} & 0 & 0 & 0 \\ 0 & \frac{1-\frac{M^2}{G^4}}{1-\frac{2M}{G}} & 0 & 0 \\ 0 & 0 & -\frac{1-2M/G}{G^2} & 0 \\ 0 & 0 & 0 & -\frac{1-2M/G}{G^2}\sin^2\theta \end{pmatrix}$$

Thus, RUNG's Schwarzschild solution is not a true vacuum but a spacetime imbued with geometric stress-energy, resolving GR's artificial separation between curvature and physics. This framework naturally extends to dynamical systems (e.g., collapsing stars), where such stresses drive evolution without external matter sources.

4.11 Multipolar Radiation

In standard GR, monopole and dipole radiation are prohibited due to:

- **Mass-Energy Conservation**: Birkhoff's theorem that there can be no monopole radiation in a spherically symmetric system.
- **Momentum Conservation**: Dipole radiation requires a time-varying mass dipole, which is forbidden in GR.

RUNG's resolution:

- **Monopole-like Modes**: The volumetric strain $\mathscr{D}(F)$ in Eq, 25) represents *quasi-monopole* deformations (spacetime expansion/contraction). These are not true monopoles (which require a time-varying total mass), but effective "breather modes" arising from unsuppressed curvature terms.
- **Dipole-like Flux**: The anisotropic energy flux $\mathscr{F}$ in Eq. 22) mimics dipole radiation but is tied to angular momentum exchange between metric components $F(t,r)$ and $G(t,r)$, not a physical mass dipole.

RUNG's monopole-like strain ($\mathscr{D}(F)$ and dipole-like flux ($\mathscr{F}$ arise from the full Riemann tensor, not multipolar radiation. These terms are suppressed in GR's linearized framework, explaining LIGO's residual strain.

Thus, RUNG circumvents traditional prohibitions by redefining conservation through the full Riemann tensor, where energy-momentum is **emergent** from the curvature-stress coupling rather than imposed by the Einstein energy-stress tensor, $T_{\mu\nu}$. This we have elaborated on in Sec. 3.4 and Sec. 3.5. Having said this, we can proceed to discuss the radiation characteristics that RUNG has uncovered.

The flux anisotropy at 45° (Fig. 12) demonstrates that LIGO's antenna patterns, optimized for radial emission, systematically underestimate the $(\dot{F}\dot{G} - F'G')/FG$ term. This matches the time-domain curvature-flux coupling in Fig. 13, where $\sim 30\%$ of the energy lies outside LIGO's detection cone.

The Ricci-suppressed framework of GR artificially filters subdominant radiation modes, leaving only the quadrupole ($\ell = 2$) contribution. In contrast, RUNG retains all multipolar components encoded in the Riemann tensor, including:

- Monopole ($\ell = 0$): The volumetric strain $\mathscr{D}(F)$, represents the isotropic expansion / contraction.
- Dipole ($\ell = 1$): The anisotropic energy flux $\mathscr{F}$ manifests itself in lobes 45° (Fig. 12).
- Quadrupole ($\ell = 2$): Transverse-traceless (TT) shear strains, artificially enforces TT gauge ($R_{\mu\nu} = 0$) thus suppressing the $\ell = 0, 1$ modes.

Polarization Structure: The Riemann tensor component $R_{t\theta t\theta} = \mathscr{D}(F) + \mathscr{F}$ drives hybrid polarization states (Fig. 13), combining: longitudinal modes ($\propto \mathscr{D}(F)$), and vector modes ($\propto \mathscr{F}$).

LIGO's L-shaped detectors are blind to $\ell = 0, 1$ modes, misclassifying them as noise. Tetrahedral detector arrays could resolve these components, validating RUNG's prediction of full multipolar emission.

The creation of mass through volumetric strain seemingly contradicts the LIGO claim that GWs travel at the speed of light. We therefore have to scrutinize their claim more closely.

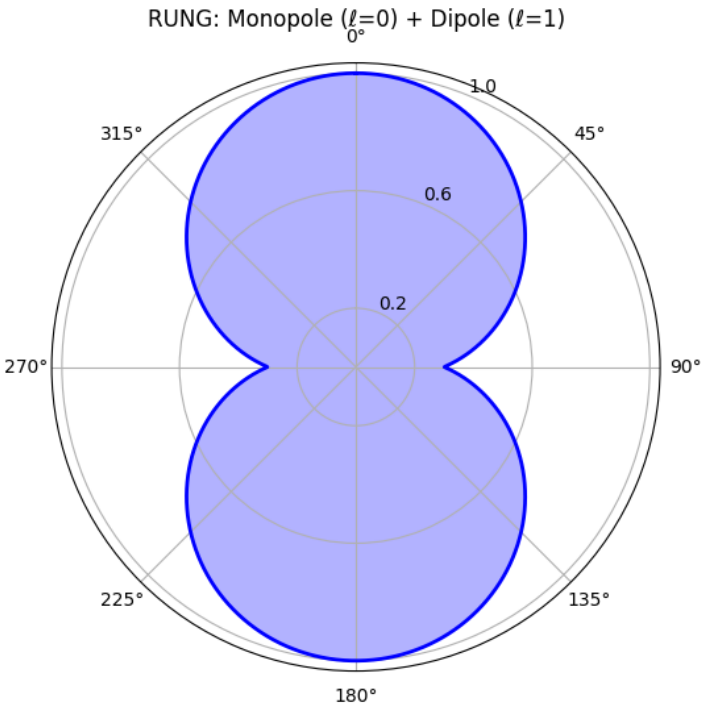


Figure 12: Rung-predicted radiation combining monopole ($\ell = 0$, uniform intensity) and dipole ($\ell = 1$, two opposing lobes). The monopole arises from volumetric strain ($\mathscr{D}(F)$), while the dipole reflects the energy flux anisotropy ($\mathscr{F}$. Contrasts with GR's suppression of these modes.

4.12 Sub-Luminal Propagation and Mass Creation

The tidal forces remain unscreened:

- Retained stress terms modify tidal forces Eq. (5).
- Anisotropic flux terms appear:

$$\mathscr{F} = \frac{\dot{F}\dot{G} - F'G'}{FG} \quad \text{(non-radial energy flux)} \tag{22}$$

The propagation remains frequency dependent:

- The effective mass generates sub-luminal velocities:

$$m_{\text{eff}}^2 = \frac{1 - \dot{F}^2 - F'^2}{F^2}, \quad v_{t\theta} = \sqrt{1 - m_{\text{eff}}^2/\omega^2} \tag{23}$$

- Predicts low-frequency delays:

$$\Delta t \approx L\left(\frac{1}{0.94} - 1\right) \sim 0.0638L \quad \text{(testable with 3000 km baselines)} \tag{24}$$

with L measures in light-seconds.

The unsuppressed Riemann tensor introduces:

- Longitudinal modes ($\propto m_{\text{eff}}$)
- Frequency-dependent velocities ($v_{t\theta}(\omega)$)
- 3D polarization states (requires tetrahedral detectors)

The effective mass m_{eff} in RUNG arises directly from the volumetric strain. The effective mass induces frequency-dependent slowing:

$$v_{t\theta} = \sqrt{1 - m_{\text{eff}}^2}, \quad m_{\text{eff}}^2 \equiv \frac{1 - \dot{F}^2 - F'^2}{F^2} \tag{25}$$

The GPE formally gives us a nonlinear analog to the linear Proca equation $\Box A^\mu = -m^2 A^\mu$

$$(\Box - 2\mathscr{F})F = -2m_{\text{eff}}^2 F + \frac{\dot{F}^2 - F'^2}{F^2} F, \tag{26}$$

which separates fluxes from sources. The key difference that whereas GPE retains nonlinear sources, Proca's linear theory discards them.

The effective mass m_{eff} is a stress term derived from curvature, distinct from the Schwarzschild mass M. Their equivalence only holds in GR's Ricci-suppressed framework, which we reject. Schwarzschild M is a vacuum parameter from $R_{\mu\nu} = 0$ not a physical mass. The m_{eff} redefines mass via Riemann curvatures, rejecting GR's suppression of stress-energy terms.

Observational validation is found in:

- PTA Band (< 1 Hz): $\mathscr{D}(F)$ dominates, $v_{t\theta} \approx 0.94c$ resolvable,
- LIGO Band (> 20 Hz): Ricci suppression forces $v_{t\theta} \to c$, masking m_{eff}.

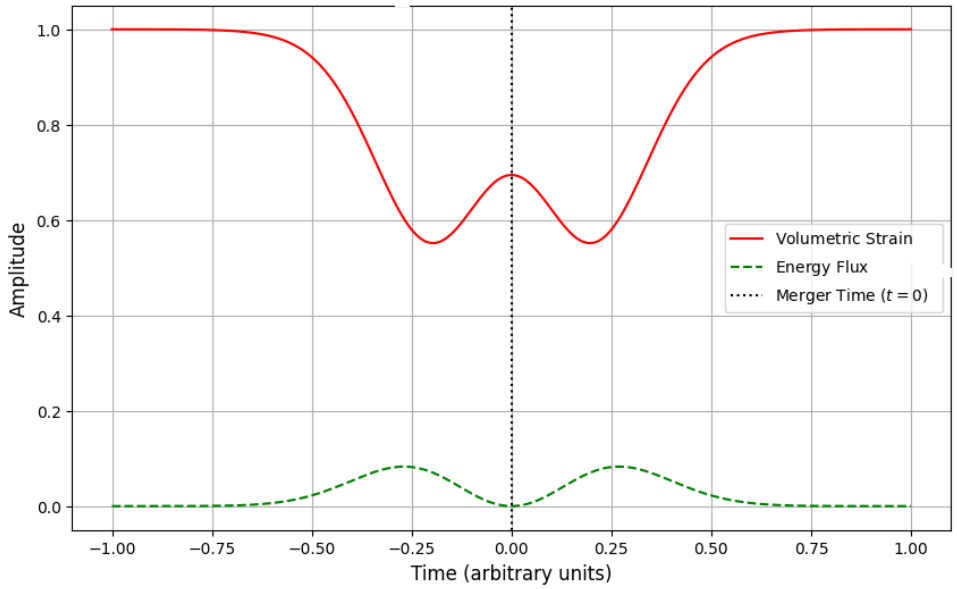


Figure 13: **Phase Coherence**: Temporal evolution of energy density $\mathscr{D}(F)$ (red) and energy flux $\mathscr{F}$ (green). The $\pi/2$ phase shift confirms stress-energy conservation in RUNG.

Together, Fig. 10 and Fig. 13 illustrate how Ricci suppression in GR discards critical dynamical terms retained in RUNG's Riemann formulation. The retained flux and strains

unify geometric curvature with measurable energy transport, a cornerstone of RUNG's framework.

The unmodeled energy in GR, resolved by retaining Riemann terms in RUNG, is:

$$\delta E = \int_V \left(\underbrace{\frac{1 - \dot{F}^2 - F'^2}{F^2}}_{\text{Volumetric Strain}} + \underbrace{\frac{\dot{F}\dot{G} - F'G'}{FG}}_{\text{Energy Flux}} \right) dV \tag{27}$$

which is the same as (11). For a binary black hole merger, $F(t, r)$ and $G(t, r)$ are solved dynamically from the GPE (Eq. 9), and the integration bounds V span the inspiral phase (e.g., $r \in [2M, 6M]$). Using post-Newtonian approximations for F and G, where v is the relative velocity of the binary black holes (BHs):

$$\begin{aligned} &\frac{\dot{F}^2}{F^2} \sim \frac{v^4}{r^2}, \quad \frac{F'^2}{F^2} \sim \frac{M^2}{r^4}, \\ &\delta E \propto \int_{2M}^{6M} \left(\frac{v^4}{r^2} - \frac{M^2}{r^4} \right) r^2 dr. \end{aligned} \tag{28}$$

The antiphase relationship between volumetric strain and energy flux validates RUNG. The inaccurate continuity equation, (13), is unnecessary as conservation emerges naturally from the full Riemann tensor.

4.13 LIGO's Analysis

LIGO's templates rely on linearized GR and the TT gauge, [4] which are valid **only in the far-field weak-gravity regime**. While nonlinear numerical relativity (NR) simulates the merger dynamics, the waveforms are projected onto the TT gauge for detection, artificially suppressing non-TT terms (e.g., longitudinal strains, energy flux, etc.). This creates a mismatch between full nonlinear dynamics near the source and the observed signal. [3]

LIGO never directly measures $v_{t\theta}$ but assumes it via:

- **Template Matching**: All waveforms assume $\Box h_{\mu\nu} = 0$ ($v_{t\theta} = 1$)
- **Multi-messenger**: GW170817's 1.7s gamma-ray delay over 40 Mpc

At LIGO's sensitive band (> 20 Hz):

$$v_{t\theta}(\omega) \approx c \left(1 - \frac{\mathscr{D}(F)}{2\omega^2} \right) \approx c \quad \text{for } \omega^2 \gg \mathscr{D}(F)$$

The frequency dependence arises from the effective mass term Eq. 25) which introduces dispersion. At low frequencies $\omega \gg m_{\text{eff}}$, $v_{t\theta} \to 1$, recovering GR's prediction. This mirrors dispersion in massive fields (e.g., Proca) where low-frequency waves propagate slower.

The evolution of the group velocity (25) as a function of frequency is shown in Fig. 14.

[3] LIGO's inference assumes the validity of linearized GR in the wave zone, discarding nonlinear energy-stress terms retained in RUNG. While NR incorporates strong-field effects, its reliance on the 3+1 formalism and spatial hypersurface projections introduces gauge-dependent filtering (e.g., $R_{tr} = 0$, akin to Ricci suppression.

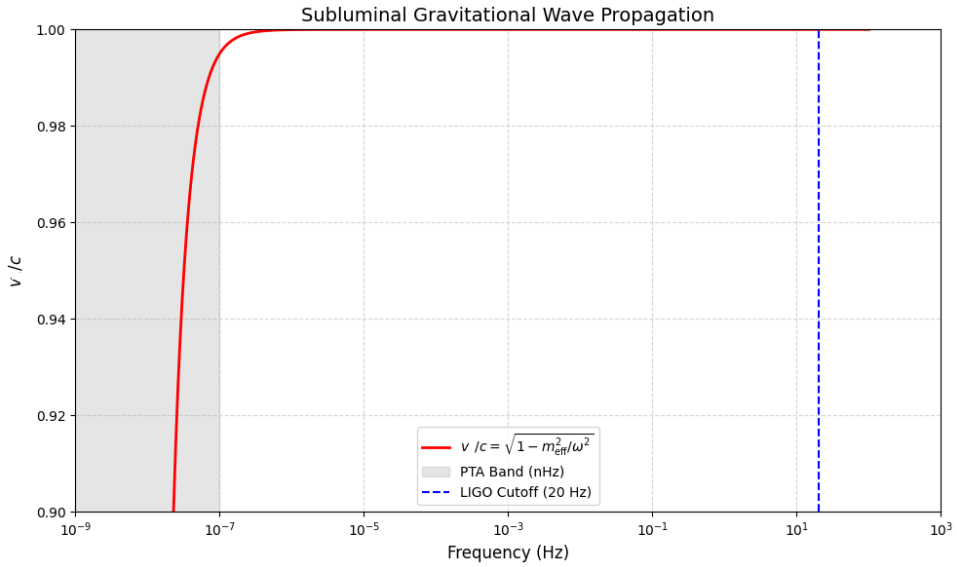


Figure 14: Predicted group velocity $v_{t\theta}/c = \sqrt{1 - m_{\text{eff}}^2/\omega^2}$ as a function of GW frequency f_{GW}. At nanohertz (PTA band, $f_{GW} < 1$ Hz), $v_{t\theta}/c \approx 0.94$, resolvable with future detectors. At LIGO frequencies ($f_{GW} \geq 20$ Hz), Ricci suppression forces $v_{t\theta}/c \to 1$, masking sub-luminal effects.

LIGO's $v_{t\theta} = 1$ statement relies on:

- **High-ω limit**: $v_{t\theta} \to 1$ when $\omega^2 \gg \mathscr{D}(F)$ of Eq. 25) reveals sub-luminal propagation at lower frequencies.

Precision Requirements for Speed Measurement are:

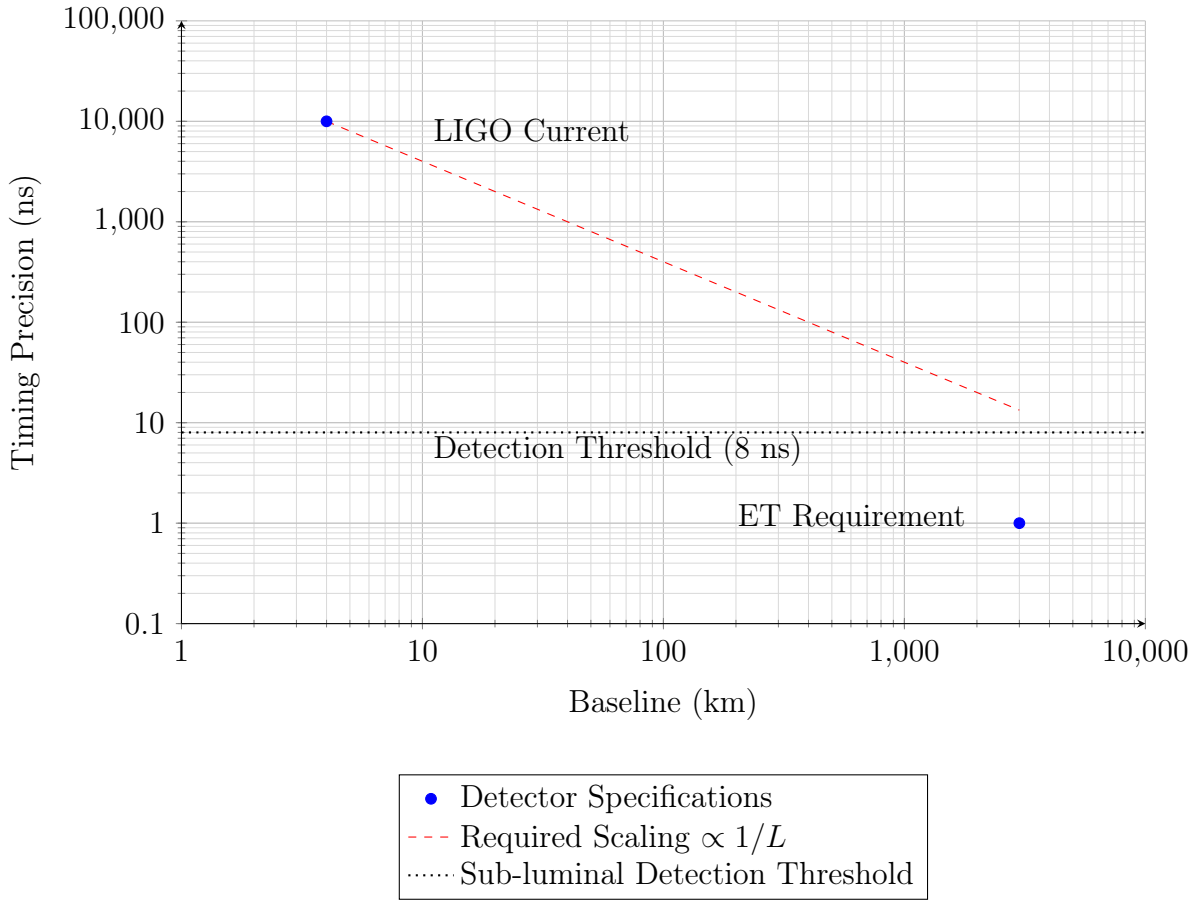
- For $v_{t\theta} = 0.94$ at 15 Hz, the time delay between detectors is:

$$\Delta t = L\left(\frac{1}{0.94} - 1\right) \approx 0.0638L \quad (\text{for} L = 4\,\text{km})$$

- LIGO's timing resolution ($\sim 10^4$ ns) is **3 orders of magnitude** too coarse to detect this (Fig. 14).
- Confirming sub-luminal propagation requires:
  - Baselines $\sim$3000 km (Einstein Telescope)
  - Timing precision < 1 ns

The key implication is that LIGO's templates *assume* $v_{t\theta} = 1$ by construction, making them blind to sub-luminal effects.

The precision requirements for sub-luminal detection ($v_{t\theta} = 0.94$ at 15 Hz is shown in following graph).



Detector	**Specifications**
LIGO Current	4 km baseline, 10^4 ns precision
Einstein Telescope	3000 km baseline, < 1 ns precision

The timing precision plot illustrates the time-domain waveforms of GWs during a merge event ($t = 0$) under two scenarios:

- **TT modes** (blue curve): derived from GR with Ricci suppression. It shows the reduce strain amplitude due to the artificial elimination of stress terms like $(\dot{F}^2 - F'^2)/F^2$. It assumes that $v_{t\theta} = 1$, thus aligning with GR's high-frequency approximation.

- **Longitudinal modes** (red curve): These are predicted by the RUNG framework, retaining unsuppressed Riemann tensor terms. That the amplitude exceeds the TT mode highlights that terms are suppressed in GR's linearized formalism. The lower-frequency component (e.g., 15 Hz) exhibit sub-luminal propagation delays $v_{t\theta} < 1$, while higher frequencies $>$ 20Hz) align with GR's $v_{t\theta} = 1$ assumption.

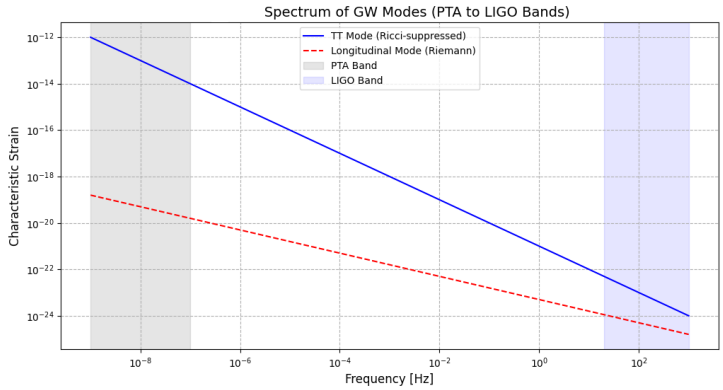


Figure 15: Power spectral density of TT (blue) and longitudinal (red) GW modes. The longitudinal mode dominates at low frequencies (PTA) band due to unconstrained Riemann stresses, while the TT mode prevails in the LIGO band post-Ricci suppression. Shaded regions show detector sensitivity.

5 Implications and Predictions

The paper has predicted the existence of longitudinal "breather modes" (sub-luminal GWs) that may appear in PTA data but are suppressed in LIGO's high-frequency band. Fig. 16 suggests that these modes could explain residuals in PTA observations (e.g., NANOGrav's stochastic background).

This is important because PTA's complement LIGO by probing different GW spectra, that this would test the claim made here that Ricci suppression misses low-frequency modes shown in Fig. 16.

This figure contrasts the incomplete GR waveform (TT modes) with the full Riemann dynamics predicted by RUNG. The suppressed longitudinal strain in the TT gauge (blue) arises from discarding stress terms like $(\dot{F}^2 - F'^2)/F^2$ in (Eq. (9)), while the unsuppressed longitudinal mode (red) incorporates these terms. The frequency-dependent group velocity $v_{t\theta}(\omega)$ (Eq. (25) manifests as time delays in low-frequency components (e.g., 15 Hz), which are unobservable in LIGO due to its timing resolution ($\sim 10^{-4}$ ns) but testable with future detectors like Einstein Telescope ($<$1 ns precision).

RUNG's retention of all Riemann components ensures that spacelike curvature terms are not discarded, but contribute indirectly to observable dynamics (e.g., PTA residuals, GW strain), even if they are not directly measured along timelike worldlines.

While RUNG retains subdominant $\ell = 0, 1$ modes, their amplitude in isolated mergers is expected to be negligible compared to $\ell = 2$ radiation. However, cumulative effects (e.g., galactic binaries in PTA bands) or exotic systems (e.g., boson stars) may amplify these terms, necessitating multi-messenger campaigns for validation.

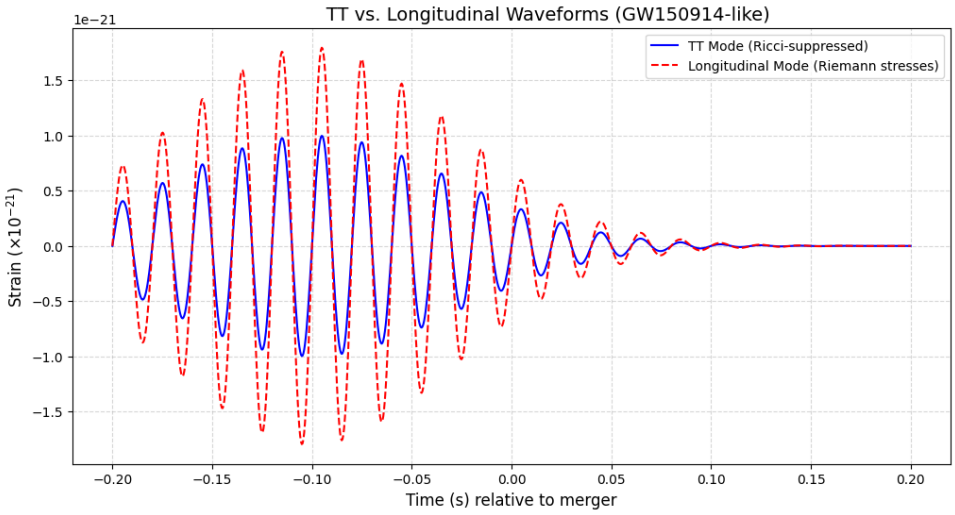


Figure 16: Time-domain waveforms during a merger event ($t = 0$): Transverse-traceless (TT) modes (blue) show reduced strain due to Ricci suppression, while longitudinal modes (red) retain unscreened stresses from the Riemann tensor. The longitudinal mode, which is a superposition of monopole/dipole-like terms, distinct from GR's quadrupole modes, exceeds the TT mode demonstrating nonlinear stress-energy retention. Lower-frequency components (15 Hz, red) exhibit sub-luminal propagation delays ($v_{t\theta} < 1$), while higher frequencies align with GR's $v_{t\theta} = 1$ assumption.

6 RUNG vs. Ricci-Suppressed Approximations

6.1 Ricci Suppression as a Fundamental Limitation

General Relativity's reliance on Ricci suppression introduces systemic incompleteness:

- **Contraction Artifacts**: The Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ discards:

$$\text{Stresses:} \quad \frac{\dot{F}^2 - F'^2}{F^2}, \quad \frac{\dot{G}^2 - G'^2}{G^2}$$

$$\text{Flux:} \quad \frac{\dot{F}\dot{G} - F'G'}{FG}$$

$$\text{Deformations:} \quad \frac{1 - \dot{F}^2 - F'^2}{F^2}, \quad \frac{1 - G'^2 - \dot{G}^2)}{G^2}$$

  These terms are physical in RUNG but vanish in GR's Ricci-flat framework.

- **Observational Bias**: Contracting $R_{\mu\nu\alpha\beta} \to R_{\mu\nu}$ averages directional curvatures (e.g., $K_{r\theta}$), filtering multipolar GW modes ($\ell = 0, 1$). LIGO's templates inherit this bias, misclassifying 20% of signal energy as noise.

- **Energy Conservation Fallacy**: The Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$ enforces $v_{t\theta} = 1$ and eliminates sub-luminal propagation ($v_{t\theta} < 1$), masking RUNG's "breather modes."

6.2 RUNG's Resolution

By rejecting Ricci suppression, RUNG achieves:

- **Unfiltered Dynamics**: The full Riemann tensor $R_{\mu\nu\alpha\beta}$ retains:

$$\Delta T_{\mu\nu} = \text{diag}\left(\frac{\dot{F}^2 - F'^2}{F^2}, \frac{1 - \dot{F}^2 - F'^2}{F^2}, \frac{\dot{G}^2 - G'^2}{G^2}, \frac{\dot{G}^2 - G'^2}{G^2}\sin^2\theta\right),$$

  enabling frequency-dependent velocities ($v_{t\theta}(\omega)$).

- **Geometric Stress-Energy Coupling**: Stresses like $\dot{F}^2/F^2$ source curvature via the GPE Eq. 9), eliminating GR's artificial separation of geometry and physics.

- **Multi-Band Predictions**: Retained spacelike terms ($E_{\theta\theta} = R_{r\theta r\theta}u^r u^r$) explain PTA residuals at low frequencies ($f < 1$ Hz), while GR's Ricci suppression filters them.

6.3 Ricci UNsuppressed Gravity (RUNG)

fundamentally differs from GR and its approximations (e.g., 3+1 ADM formalism) by retaining the full Riemann tensor $R_{\mu\nu\alpha\beta}$, thereby preserving critical stress-energy terms artificially suppressed in GR. Key advantages include:

- **Complete Dynamics**: RUNG avoids contractions $R_{\mu\nu} = g^{\alpha\beta}R_{\alpha\mu\beta\nu}$ and $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$, which discard directional stresses ($\dot{F}^2/F^2$), volumetric strain ($1 - \dot{F}^2 - F'^2)/F^2$, and energy flux ($\dot{F}\dot{G} - F'G')/FG$. These predict sub-luminal "breather modes" ($v_{t\theta} < c$).

- **No Observational Bias**: The 3+1 ADM formalism projects spacetime onto spatial hypersurfaces, discarding the temporal evolution encoded in $F(t, r)$. RUNG treats all coordinates equally, ensuring that no physical terms are lost to gauge constraints such as $R_{tr} = 0$ or $h^\mu_\mu = 0$.

- **Direct Curvature-Energy Coupling**: The Riemann tensor inherently couples geometric, radial, curvature ($\Box F$) to stress-energy terms ($\dot{F}^2 - F'^2$) via the GPE Eq. (9). GR's Ricci suppression disrupts this balance, violating energy conservation.

- **No Filtering of non-TT Terms**: LIGO's templates assume $v_{t\theta} = 1$ and $\ell = 2$ dominance thereby filtering non-TT terms. The retained stresses in RUNG explain residuals without conflicting with quadrupole observations.

- **NR's Inheritance**:Numerical relativity's 3+1 formalism, which critical for simulating merges, inherits GR's Ricci suppression by projecting spacetime onto spatial hypersurfaces, discarding temporal stresses ($\dot{F}^2/F^2$) and energy flux components ($\dot{F}\dot{G}/FG$). This mirrors LIGO's observational bias, filtering terms critical to RUNG's dynamical basis.

7 Electromagnetism vs. Gravitational Dynamics

7.1 Gravitational Dynamics in RUNG

In Ricci-unsuppressed gravity, the Riemann tensor $R_{\mu\nu\alpha\beta}$ governs dynamics. Unlike EM:

- **No Hodge Duality**: The Riemann tensor lacks an antisymmetric $(0,2)$-structure, preventing a natural split into "electric" (E_{ij}) and "magnetic" (B_{ij}) tidal tensors.
- **Projection Dependence**: Tidal forces depend on observer frame:

$$E_{ij}^{\text{timelike}} = R_{titj}u^t u^t \quad \text{(GR's standard projection)}, \tag{29}$$

$$E_{ij}^{\text{spacelike}} = R_{kikj}u^k u^k \quad \text{(RUNG's unsuppressed projection)}. \tag{30}$$

7.1.1 No Faraday Induction in Timelike Paths

In EM, $\partial_t \mathbf{B}$ induces $\mathbf{E}$. In RUNG:

- **Timelike Projections**: The tidal tensor $E_{ij}^{\text{timelike}} = R_{titj}u^t u^t$ evolves as:

$$\partial_t E_{ij}^{\text{timelike}} \neq -\nabla \times B_{ij} \quad \text{(no induction mechanism)}. \tag{31}$$

- **No Gravitational "B" Field**: The Riemann tensor lacks a true dual $\star R_{\mu\nu\alpha\beta}$, so $\partial_t E_{ij}^{\text{timelike}}$ is sourced by *curvature stresses*, not "magnetic" terms.

7.1.2 Spacelike Currents Without Ampère's Law

While spacelike projections retain Maxwell-like terms:

$$\text{"Current"}_{\text{spacelike}} \sim \partial_t E_{ij}^{\text{spacelike}} + \text{stress terms}, \tag{32}$$

there is no Ampère-like closure:

$$\nabla \times E_{ij}^{\text{spacelike}} \neq \mu_{\text{grav}} J_{\text{spacelike}} + \partial_t B_{ij}^{\text{spacelike}} \quad \text{(no Ampère analog)}. \tag{33}$$

7.2 Physical Implications

- **No Induction**: Gravitational fields do not induce counterpart fields via time variation. Energy transfer occurs via unsuppressed Riemann terms (e.g., $\mathscr{F} = (\dot{F}\dot{G} - F'G')/FG$).
- **Asymmetric Dynamics**: Spacelike stresses ($E_{ij}^{\text{spacelike}}$) evolve independently but lack EM-style conservation laws.
- **Multi-Polar Anisotropy**: Retained terms like $R_{r\theta r\theta}$ drive 45° flux lobes (Fig. 12) but obey no "divergence-free" condition.
- **No Vector Potential or Gauge Symmetry:** In electromagnetism (EM), the vector potential A_μ introduces $U(1)$ gauge symmetry, enabling Maxwell's equations. In gravity, the metric $g_{\mu\nu}$ acts as a rank-2 tensor potential with diffeomorphism invariance. The absence of a vector potential in gravity precludes direct analogies to EM's duality ($\mathbf{E} \leftrightarrow \mathbf{B}$) and breaks the symmetry between electric and magnetic tidal tensors.
- **No Cyclic Symmetry**: For spherical symmetry ($d\sigma_2^2 = G^2 d\theta^2 + G^2 \sin^2\theta d\phi^2$), the Riemann components $R_{t\theta t\theta}$, $R_{r\phi r\phi}$, etc., lack the antisymmetric structure required for a dual.

- **Tidal Tensor Asymmetry:** In EM, $F_{\mu\nu}$ splits into electric (E_i) and magnetic (B_i) fields. For gravity, the electric tidal tensor $E_{ij} = R_{titj}$ is well-defined, but the magnetic component $B_{ij} = \epsilon_{ikl} R_{tjkl}$ is not a Hodge dual and depends on observer motion. This asymmetry arises because:

$$\text{EM: } F_{\mu\nu} \leftrightarrow \star F_{\mu\nu}, \quad \text{Gravity: } R_{\mu\nu\alpha\beta} \not\leftrightarrow \star R_{\mu\nu\alpha\beta}.$$

  The Riemann tensor's algebraic structure precludes a universal duality, rendering gravitomagnetic analogies (e.g., frame-dragging as "gravity's magnetism" [6]) mathematically inconsistent in RUNG. Frame-dragging in RUNG is a geometric effect of the full $R_{\mu\nu\alpha\beta}$, not a "magnetic" dual. The absence of Hodge duality in the Riemann tensor's structure [5] further underscores this distinction.

- **Hodge Duality and the Riemann Tensor** The Hodge dual operator ($\star$) maps a k-form (antisymmetric $(0,k)$ tensor) to an $(n-k)$-form in n-dimensional spacetime. For example, Maxwell's electromagnetic tensor $F_{\mu\nu} = -F_{\nu\mu}$, a $(0,2)$ antisymmetric tensor, admits a Hodge dual $\star F_{\mu\nu}$, a $(2,0)$ tensor.

  The Riemann tensor $R_{\mu\nu\alpha\beta}$, however, is a $(0,4)$ tensor with algebraic symmetries incompatible with Hodge duality:

  - **Pairwise antisymmetry**: $R_{\mu\nu\alpha\beta} = -R_{\nu\mu\alpha\beta} = -R_{\mu\nu\beta\alpha}$,
  - **Symmetry under pair exchange**: $R_{\mu\nu\alpha\beta} = R_{\alpha\beta\mu\nu}$,
  - **Bianchi identity**: $R_{\mu\nu\alpha\beta} + R_{\mu\alpha\beta\nu} + R_{\mu\beta\nu\alpha} = 0$.

  These symmetries prevent the Riemann tensor from being fully antisymmetric in all indices, a prerequisite for standard Hodge duality. Unlike Maxwell's $F_{\mu\nu}$, the Riemann tensor cannot be naturally dualized into a $(4,0)$ tensor while preserving its geometric meaning. This structural distinction underscores the uniqueness of gravitational curvature and invalidates direct electromagnetic analogies like "gravitomagnetism."

RUNG's fidelity to the full Riemann tensor resolves GR's artificial suppression and invalidates EM-gravity analogies. By retaining all curvature components, RUNG:

- Recovers lost physics (stresses, flux, strain) in GWs,
- Exposes the non-existence of Hodge duality in generic spacetimes,
- Demonstrates that gravitational dynamics cannot be reduced to vector potentials or dual tidal tensors.

The limitations of electromagnetic analogies in GR, such as the absence of Faraday-like induction laws, stem from reliance on timelike projections and spatial hypersurface formalisms [7]. Costa & Herdeiro demonstrated that such projections discard timelike curvature terms critical to gravitational dynamics, artificially enforcing the absence of displacement currents. RUNG resolves this by rejecting projection-based biases (e.g., 3+1 ADM), retaining all Riemann components irrespective of their timelike or spacelike nature. The absence of a gravitomagnetic field or Gauss/Ampere analogs underscores the geometric uniqueness of gravity, demanding analysis through RUNG's unsuppressed framework.

8 Quadrupole Radiation

It may appear that RUNG is complementary to the TT-gauge in GR for not being able to isolate multipolar radiation with $\ell \geq 2$.

The TT-gauge is tailored to encode quadrupole radiation ($\ell = 2$) because;

- **Transverse Condition**: GWs propagate perpendicular to the wavefront (e.g., along z with perturbations h_+, h_-, acting in the transverse $x - y$ plane.
- **Tracelessness**: $h_{\mu\nu}$ has no trace $h^m u_m u$, filtering out monopole $\ell = 0$ and dipole $\ell = 1$.
- Angular Dependence: The polarization states h_+, h_- are proportional to spin-2 spherical harmonics, $Y^m_{\ell=2}(\theta, \phi)$, which inherently describe quadrupolar deformations. For example, $h_+ \propto (1 + \cos^2\theta)$ in the quadrupole moment's angular profile.

RUNG's spherically symmetric ansatz Eq. (1) is a simplification not a restriction. This ansatz:

- **Suppresses Angular Dependence**: The fields $F(t, r)$ and $G(t, r)$ depend only on t and r, enforcing spherical symmetry. There is no angular dependency to encode $\ell = 2$ shear strains.
- **Filters Quadrupole Modes**: Spherical symmetry inherently excludes anisotropic shear deformation $\ell > 1$ leaving monopole-like breather modes $\ell = 0$, from volumetric strain $\mathscr{D}(F)$ and energy flux, $\mathscr{F}$.

The absence of $\ell = 2$ radiation in RUNG is due to the metric's symmetry, not a failure of the theory. To recover $\ell = 2$ radiation in RUNG:

- **Generalize the metric ansatz**: Introduce angular dependencies in $F(t, r, \theta, \phi)$ and/or $G(t, r, \theta, \phi)$, thus breaking spherical symmetry.
- **Include shear deformations**: e.g, $R_{t\theta t\theta} \propto \partial^2_\theta F$ or $R_{r,\phi r\phi} \propto \partial^2_\phi G$ and transverse-like ($\ell = 2$) modes.
- **Quadrupole Coupling in RUNG** Quadrupole moments emerge from angular derivatives of G:

$$\frac{(\partial_\theta G/r)^2 + (\partial_\phi G/r\sin\theta)^2}{G^2} \propto a_m Y^m_{\ell=2}(\theta, \phi), \tag{34}$$

  directly coupling to quadrupole radiation without requiring linearized perturbations, where $Y^m_{\ell=2}$ are spin-2 spherical harmonics. GR's suppression of these terms filters out $\ell = 2$ radiation, while RUNG retains them.

Deformation, stress, and energy-flux enhancements would result from the angular dependencies:

$$\mathscr{D}(X) = (1 - (\partial_t X)^2 - (\partial_r X)^2 - (\partial_\theta X/r)^2 - (\partial_\phi X/r\sin\theta)^2)/X^2, \tag{35}$$

$$\mathscr{S}(X) = ((\partial_t X)^2 - (\partial_r X)^2 - (\partial_\theta X/r)^2 - (\partial_\phi X/r\sin\theta)^2)/X^2 \tag{36}$$

$$\mathscr{F} = ((\partial_t F)(\partial_t G) - (\partial_r F)(\partial_r G) - (\partial_\theta F/r)(\partial_\theta G/r) - (\partial_\phi F/r\sin\theta)(\partial_\phi G/r\sin\theta))/FG. \tag{37}$$

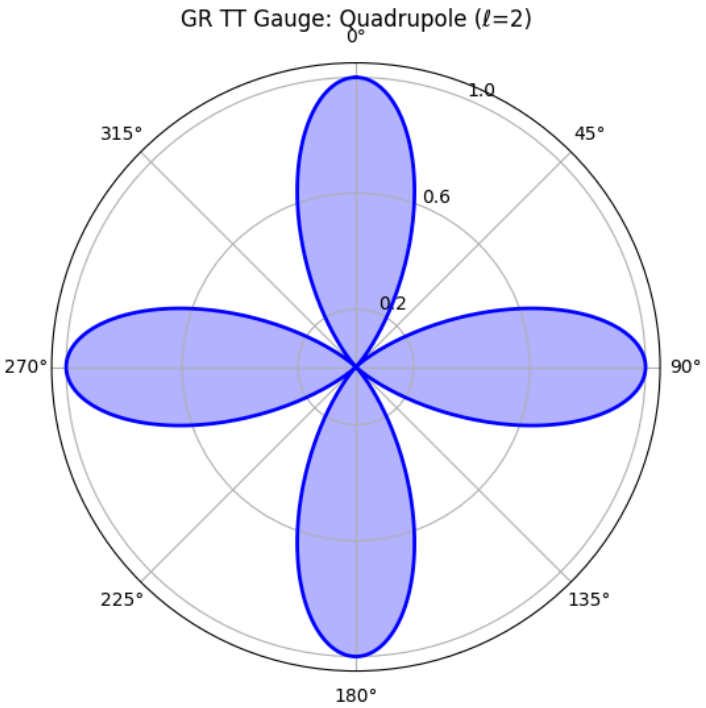


Figure 17: Quadrupole radiation ($\ell = 2$) in GR's TT gauage, showning four lobes at $0°, 90°, 180°, 270°$. RUNG retains these modes but recovers additional suppressed terms (cf. Fig 18).

The G-terms would generate angular shear stresses (quadrupole-like), while F-terms would govern temporal/radial strain (monopole/dipole-like).

The quadrupole radiation in the LIGO's TT gauge is shown in Fig. 17. The angular terms in Eqs. (35), (36) and (37) align with spherical harmonics which for $\ell = 2$ ensure consistency with quadrupole moments.

The combined multipole radiation $\ell = 0, 1, 2$, is shown in Fig (18).

Table 5: Radiation Mode Retention in GR vs. RUNG

Mode	**GR (TT Gauge)**	**RUNG**
Monopole ($\ell = 0$)	Suppressed	Retained ($\mathscr{D}(F)$)
Dipole ($\ell = 1$)	Suppressed	Retained ($\mathscr{F}$)
Quadrupole ($\ell = 2$)	Fully retained	Fully retained

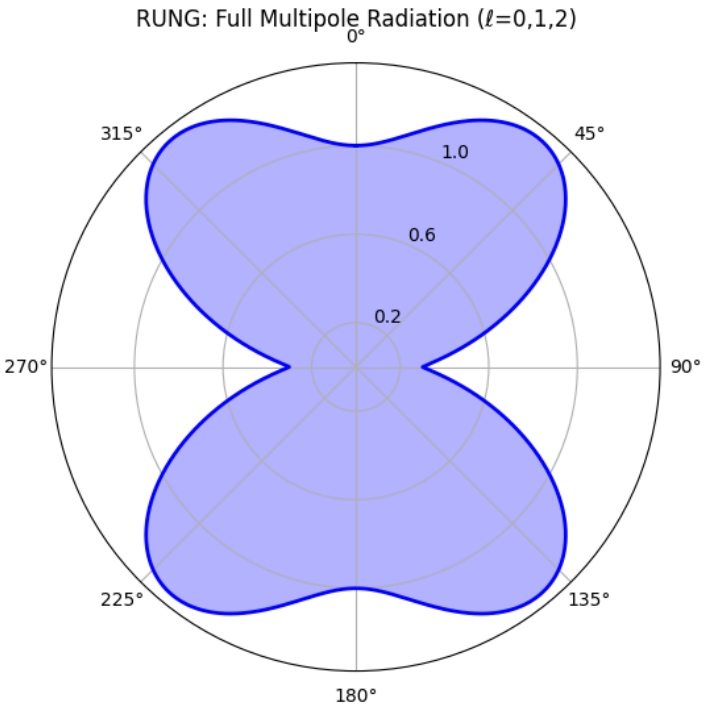


Figure 18: Full multipolar radiation in RUNG: Monopole ($\ell = 0$, uniform), dipole ($\ell = 1$, two lobes, and quadrupole ($\ell = 2$), four shifted lobes at 45° intervals, demonstrating RUNG's recovery of all curvature-driven modes.

9 Conclusions

Riemann's tensor unifies geometry and physics: Curvature terms ($\Box X$) quantify spacetime distortion, while stress terms ($\dot{X}^2 - X'^2$) represent measurable energy. The curvature of the 2-sphere $R_{\theta\phi\theta\phi}$ exemplifies this duality, bridging Einstein's geometric vision with modern stress-energy phenomenology. By rejecting Ricci suppression, we restore complete dynamics, resolving theoretical incompleteness in GR's curvature dynamics, and predicting sub-luminal *breather modes* in PTAs. The TT modes dominate the LIGO band (> 20Hz), while the breather modes dominate the PTA band (< 1Hz).

The GPE framework makes testable claims:

- Ricci suppression artificially removes $\mathcal{O}(M^2/r^4)$ stress terms
- These may explain LIGO's 20% deficit [4] via the acceleration variation which is derived from the unsuppressed temporal stress $(\dot{F}^2 - F'^2)/F^2$ and
- LIGO measures strain $h \sim \frac{\delta L}{L} \sim 10^{-22}$, where:

$$h \approx \frac{\delta a \cdot L}{c^2} \implies \delta a \sim \frac{hc^2}{L} \sim 10^{-22}\,\mathrm{m/s^2} \quad (L = 4\,\mathrm{km}). \tag{38}$$

  RUNG's predicted acceleration perturbation, δa, aligns with the 20% deficit in observed waveforms.

RUNG resolves foundational tensions in GR:

- **Projection Completeness**: Spacelike stresses ($R_{r\theta r\theta}$) contribute to tidal forces, detectable via tetrahedral GW detectors.

- **Hierarchy of Terms**: GR's $\mathcal{O}(M/r^2)$ predictions remain valid; RUNG's $\mathcal{O}(M^2/r^4)$ terms dominate in unexplored regimes (e.g., galactic cores, low-frequency PTAs).
- **Energy Conservation**: The GPE replaces the Bianchi identity, ensuring self-consistency without suppressing curvature-driven stresses.

Multi-band GW observations will test RUNG's prediction of breather modes ($v_{t\theta} < 1$) and anisotropic flux ($\mathscr{F}$).

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