Maxwell's equations are not Lorentz invariant

Jorma Jormakka
Vantaa, Finland
jorma.o.jormakka@gmail.com

Abstract: Section 1 of the article shows that the first Maxwell's equation is not Lorentz invariant when the charge is nonzero and not stationary. The reasons for the failure of Lorentz invariance is that in order for the transformed equation to allow a solution of the untransformed equation, the charge must transform because of velocity of the inertial frame. This is not possible for reasons explained in Section 3. Section 2 looks at the Lorentz covariant formulation of Maxwell's equations that is used in quantum gauge field theories. The equations this formulation use a Lorentz invariant operator but a Lorentz invariant operator does not imply that the equation is Lorentz invariant. Maxwell's equations are not Lorentz invariant.

Keywords: Maxwell's equations, Lorentz invariance, wave equation, covariant.

1. Maxwell's equations are not Lorentz invariant

Maxwell's equations are the four equations of classical electromagnetism

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \mathbf{B} = 0 \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right). \tag{2}$$

From these equations follows two equation having the form of a wave equation

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \mathbf{E} = 0 \tag{3}$$

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \mathbf{B} = 0. \tag{4}$$

The differential operator

$$D = \alpha \frac{\partial^2}{\partial x^2} + \beta \frac{\partial^2}{\partial t^2} \tag{5}$$

is invariant under the coordinate transform

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = M \begin{pmatrix} x \\ t \end{pmatrix} \qquad M = \begin{pmatrix} a & b \\ (a^2-1)/b & a \end{pmatrix} \qquad \beta = -\alpha \frac{a^2-1}{b^2} \tag{6}$$

$$D = \alpha \frac{\partial^2}{\partial x^2} + \beta \frac{\partial^2}{\partial t^2} = \alpha \frac{\partial^2}{\partial x'^2} + \beta \frac{\partial^2}{\partial t'^2} = D'. \tag{7}$$

Especially, if $\alpha = -1$, $\beta = c^{-2}$ and require that $b = -va$ in order to interpret the transform of $x' = a(x - vt)$ as the frame (x', t') moving with the speed v, then the coordinate transform M under which the operator D is invariant is the Lorentz transform

$$M = \begin{pmatrix} \gamma & -v\gamma \\ -\frac{v}{c^2}\gamma & \gamma \end{pmatrix}. \tag{8}$$

There is another fact connected with a wave equation. The wave equation

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) f = 0 \tag{9}$$

has a solution $f = Ce^{k(x-ct)}$. The term $x - ct$ is invariant under the coordinate transform T in the functional form, but gets multiplied by a constant that depends on v

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = T \begin{pmatrix} x \\ t \end{pmatrix} \qquad T = \begin{pmatrix} a & b \\ b/c^2 & a \end{pmatrix} \tag{10}$$

$$x - ct = (ax' - bt') - c\left(at' - \frac{b}{c^2}x' \right) = \left(a + \frac{b}{c} \right)(x' - ct'). \tag{11}$$

The Lorentz transform has the form of T and of M. Especially if $T = M$, then $b/c^2 = (a^2 - 1)/b$ and we get the same equation $b^2 = c^2(a^2 - 1)$ that we get by requiring that $\alpha = -1$, $\beta = 1/c^2$ in (6). With the additional requirement $b = va$, the transform can only be the Lorentz transform. For the Lorentz transform (11) is

$$x - ct = \gamma(1 - v/c)(x' - ct'). \tag{12}$$

These facts probably are the reason for claiming that Maxwell's equations are Lorentz invariant.

However, the classical four Maxwell's equations (1) are not Lorentz invariant. Let us take the first Maxwell's equation and a simple case where $\mathbf{E}$ from the equation (3) is a basic wave solution

$$E_x = Ce^{k(x-ct)} \quad E_y = C_1 e^{k(x-ct)} \quad E_z = C_2 e^{k(x-ct)}. \tag{13}$$

The first of Maxwell's equations states that

$$\nabla \cdot \mathbf{E} = \frac{\partial}{\partial x} E_x = \frac{\rho}{\epsilon_0}. \tag{14}$$

Let the coordinate transform be the Lorentz transform (8) and let us define

$$D = \frac{\partial}{\partial x} \qquad D' = \frac{\partial}{\partial x'} \tag{15}$$

where D' is obtained from D by renaming the coordinates x, t as x', t', and we call D that is written in the coordinates x', t' with the name

$$D'' = D_{x(x',t')} = \frac{\partial}{\partial x(x', t')} \tag{16}$$

$$= \frac{\partial x'}{\partial x}\frac{\partial}{\partial x'} + \frac{\partial t'}{\partial x}\frac{\partial}{\partial t'} = \gamma\frac{\partial}{\partial x'} - \frac{v}{c^2}\gamma\frac{\partial}{\partial t'}. \tag{17}$$

We also define

$$E_x = Ce^{k(x-ct)} \qquad E_{x'} = Ce^{k(x'-ct')}. \tag{18}$$

where $E_{x'}$ is obtained form E_x by renaming x, t to x', t', and we call E_x written in coordinates x', t' by using (12) with the name

$$E''_{x'} = E_{x(x',t')} = Ce^{k\gamma(1-(v/c))(x'-ct')}. \tag{19}$$

The first Maxwell equation in this simple special case is

$$DE_x = \frac{\rho}{\epsilon_0}. \tag{20}$$

Renaming x, t' as x', t' we get an equation that necessarily also holds:

$$D'E_{x'} = \frac{\rho}{\epsilon_0} \tag{21}$$

as we only renamed the variables. Also, a valid manipulation to an equation by changing the variables from x, t to x', t' by a coordinate transform gives an equation that necessarily holds:

$$D''E''_{x'} = \frac{\rho}{\epsilon_0}. \tag{22}$$

Let us confirm that equation (22) does hold, it must, (22) is (20) in a different form

$$D''E''_{x'} = \gamma\left(\frac{\partial}{\partial x} - \frac{v}{c^2}\frac{\partial}{\partial t}\right)Ce^{k\gamma(1-(v/c))(x'-ct')} \tag{23}$$

$$= \gamma\left(1+\frac{v}{c}\right)\gamma\left(1-\frac{v}{c}\right)kCe^{k\gamma(1-(v/c))(x'-ct')} \tag{24}$$

$$= kCe^{k\gamma(1-(v/c))(x'-ct')} = DE_x = \frac{\rho}{\epsilon_0}. \tag{25}$$

The equations (21) and (22) follow from (20) without any condition on the invariance of (20) under the coordinate transform. What is required from invariance under a coordinate transform is

$$D''E_{x'} = \frac{\rho}{\epsilon_0} \tag{26}$$

or

$$D'E''_{x'} = \frac{\rho}{\epsilon_0}. \tag{27}$$

We will show that (26) is not true:

$$D''E_{x'} = \gamma\left(\frac{\partial}{\partial x'} - \frac{v}{c^2}\frac{\partial}{\partial t'}\right)Ce^{k(x'-ct')} \tag{28}$$

$$= \gamma \left(1 - \frac{v}{c}\right) kCe^{k(x'-ct')} = \gamma(1 - \frac{v}{c})D'E_{x'} \tag{29}$$

Thus

$$D''E_{x'} = \gamma(1 - \frac{v}{c})\frac{\rho}{\epsilon_0}. \tag{30}$$

In order to give (30) the form of (20), we should define a transformed charge $\rho' = \gamma(1 - \frac{v}{c})\rho$ but a charge does not depend on velocity for reasons given in Section 3. This means that Maxwell's equations are not Lorentz invariant.

2. Covariant Maxwell's equations do not prove Lorent invariance

The covariant formulation for Maxwell's equations expresses Maxwell's equations in the Lorentz gauge as

$$\partial_\mu \partial^\mu A^\nu = \mu_0 J^\nu. \tag{31}$$

The Lorentz gauge is

$$\partial_\mu A^\mu = \partial^\mu A_\mu = 0. \tag{32}$$

The four-vector A^μ is defined as

$$A^\mu = \left(\frac{\Phi}{c}, \mathbf{A}\right) \tag{33}$$

where Φ is the electric potential and $\mathbf{A}$ is the magnetic vector potential

$$\mathbf{B} = \nabla \times \mathbf{A}. \tag{34}$$

The operator

$$\partial_\mu \partial^\mu = \partial^2 = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \tag{35}$$

is Lorentz invariant as was seen in Section 1. This does not mean that the equation (31) is Lorentz invariant as the right side is not zero. A Lorentz invariant operator can, and in this case does, multiply the function that it acts on with a constant. If the right side, being the charge for $\nu = 0$, does not change correspondingly because of velocity, the equation is not Lorentz invariant.

Let us look at a standard textbook [20], it was used as a textbook on a Ph.D. course on theoretical physics that I attended 1986. The authors of [20] claim that A^μ is Lorentz invariant. If this were the case, then (31) would be a Lorentz invariant equation. On page 35 the authors explain why the vector potential A^μ in the covariant formulation of Maxwell's equations is Lorentz invariant. The text is this: "Under Poincare transformation

$$\mathbf{x} \to \mathbf{x}' = \lambda \mathbf{x} + \mathbf{a} \tag{36}$$

and the vector potential transforms as

$$A^\mu(x) \to A'^\mu(x') = \lambda^\mu_\nu A^\nu(x). \tag{37}$$

Thus A^μ is invariant under an infinitesimal translation..."

It is not quite so. The transform that A^μ(x) does is that the variables x, t in it are changed to x', t' by the Lorentz transform. If **E** and **B** are waves and have the term $x - ct$, then A^μ also contains the term $x - ct$. This term transforms as in (12), $(x - ct)$ transforms to $\gamma(1 - v/c)(x' - ct')$. The term is at the exponent of A^μ (i.e., as a variable in a sinusoidal wave). The term has a multiplicative part $\gamma(1 - v/c)$ and because of it, A^μ is not invariant under an infinitesimal translation.

It is not so obvious that (31) gives Maxwell's equations. Let us calculate them because it shows a small error in the literature. According to [20] page 17 equation (31) is derived from a Langangean

$$\mathcal{L} = -\frac{1}{2} \left(\partial_\mu A_\nu - \partial_\nu A_\mu\right) \left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) - \mu_0 J_\mu A^\mu \tag{38}$$

which gives the Euler-Lagrangean equations

$$-\partial_\mu \left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) = -\mu_0 J^\nu \tag{39}$$

Inserting the Lorentz gauge into (39) gives (31), but it is easier to derive Maxwell's equations form (39)

Only two Maxwell's equations need to be solved because **A** solves two equations: setting

$$\mathbf{E} = -\frac{\partial}{\partial t}\mathbf{A} - \nabla\Phi \tag{40}$$

satisfies equations two and three

$$\nabla \cdot B = \nabla \cdot \nabla \times \mathbf{A} = 0 \tag{41}$$

$$\nabla \times \mathbf{E} = -\nabla \times \frac{\partial}{\partial t}\mathbf{A} - \nabla \times \nabla\Phi = -\frac{\partial}{\partial t}\nabla \times \mathbf{A} = -\frac{\partial}{\partial t}\mathbf{B}. \tag{42}$$

Let us set $\nu = 0$ to (39), write separately $\mu = 0$ and use summation over $1, 2, 3$

$$-\partial_0\partial^0 A^0 - \sum_{\mu=1}^{3} \partial_\mu \partial^\mu A^0 + \partial_0\partial^0 A^0 + \partial_0 \sum_{\mu=1}^{3} \partial^\mu A^\mu = -\mu_0 J^0 \tag{43}$$

Inserting

$$\partial^0 = \frac{1}{c}\frac{\partial}{\partial t} \quad , \quad \partial^i = -\frac{\partial}{\partial x_i} \quad i = 1, 2, 3 \tag{44}$$

$$\partial_\mu = \frac{1}{g_{\mu\mu}}\partial^\mu \tag{45}$$

equation (40) can be written as

$$\nabla \cdot \nabla A^0 - \frac{1}{c}\frac{\partial}{\partial t}\nabla \cdot \mathbf{A} = -\mu_0 J^0. \tag{46}$$

Inserting A^0

$$\nabla \cdot \left(-\frac{1}{c}\nabla\Phi - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{A} \right) = \mu_0 J^0. \tag{47}$$

Inserting $\mathbf{E}$ and $J^0 = \rho c$ gives

$$\nabla \cdot \mathbf{E} = c\mu_0 J^0 = c^2\mu_0\rho = \frac{\rho}{\epsilon_0}. \tag{48}$$

Literature gives $J^0 = \rho$, this is the promised error.

Next we calculate the $\nu = 1, 2, 3$ part. We start from

$$\nabla\times\mathbf{A} = (\frac{\partial}{\partial x_2}A_3 - \frac{\partial}{\partial x_3}A_2)\vec{e_1} + (\frac{\partial}{\partial x_3}A_1 - \frac{\partial}{\partial x_1}A_3)\vec{e_2} + (\frac{\partial}{\partial x_1}A_2 - \frac{\partial}{\partial x_2}A_1)\vec{e_3} \tag{49}$$

where A_i are components of $\mathbf{A}$, but by (34) they are also the space components of the four-vector A^μ. Then

$$\nabla \times \nabla \times \mathbf{A} = \left(\frac{\partial}{\partial x_2}(\frac{\partial}{\partial x_1}A_2 - \frac{\partial}{\partial x_2}A_1) - \frac{\partial}{\partial x_3}(\frac{\partial}{\partial x_3}A_1 - \frac{\partial}{\partial x_1}A_3) \right)\vec{e_1} \tag{50}$$

$$+ \left(\frac{\partial}{\partial x_3}(\frac{\partial}{\partial x_2}A_3 - \frac{\partial}{\partial x_3}A_2) - \frac{\partial}{\partial x_1}(\frac{\partial}{\partial x_1}A_2 - \frac{\partial}{\partial x_2}A_1) \right)\vec{e_2} \tag{51}$$

$$+ \left(\frac{\partial}{\partial x_1}(\frac{\partial}{\partial x_3}A_1 - \frac{\partial}{\partial x_1}A_3) - \frac{\partial}{\partial x_2}(\frac{\partial}{\partial x_2}A_3 - \frac{\partial}{\partial x_3}A_2) \right)\vec{e_3} \tag{52}$$

$$= -\left(\frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} \right) A_1\vec{e_1} + \frac{\partial}{\partial x_1}\left(\frac{\partial}{\partial x_2}A_2 + \frac{\partial}{\partial x_3}A_3 \right)\vec{e_1} \tag{53}$$

$$-\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2} \right) A_2\vec{e_2} + \frac{\partial}{\partial x_2}\left(\frac{\partial}{\partial x_1}A_1 + \frac{\partial}{\partial x_3}A_3 \right)\vec{e_2} \tag{54}$$

$$-\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} \right) A_3\vec{e_3} + \frac{\partial}{\partial x_3}\left(\frac{\partial}{\partial x_1}A_1 + \frac{\partial}{\partial x_2}A_2 \right)\vec{e_3}. \tag{55}$$

We can write (53)-(55) as

$$= -\left({\partial_2}^2 + {\partial_3}^2\right) A_1\vec{e_1} + \partial_1\left(\partial_2 A_2 + \partial_3 A_3\right)\vec{e_1} \tag{56}$$

$$-\left({\partial_1}^2 + {\partial_3}^2\right) A_2\vec{e_2} + \partial_2\left(\partial_1 A_1 + \partial_3 A_3\right)\vec{e_2} \tag{57}$$

$$-\left({\partial_1}^2 + {\partial_2}^2\right) A_3\vec{e_3} + \partial_3\left(\partial_1 A_1 + \partial_2 A_2\right)\vec{e_3}. \tag{58}$$

At this point it is good to raise the indices and change to the four-vector A^μ:

$$= \left(\partial_2\partial^2 + \partial_3\partial^3\right) A^1\vec{e_1} - \partial_1\left(\partial^2 A^2 + \partial^3 A^3\right)\vec{e_1} \tag{59}$$

$$+ \left(\partial_1\partial^1 + \partial_3\partial^3\right) A^2\vec{e_2} - \partial_2\left(\partial^1 A^1 + \partial^3 A^3\right)\vec{e_2} \tag{60}$$

$$+\left(\partial_1\partial^1+\partial_2\partial^2\right)A^3\vec{e_3}-\partial_3\left(\partial^1A^1+\partial^2A^2\right)\vec{e_3}. \tag{61}$$

Adding and subtracting terms

$$=\left(\partial_0\partial^0+\partial_1\partial^1+\partial_2\partial^2+\partial_3\partial^3\right)A^1\vec{e_1}-\partial_1\left(\partial^0A^0+\partial^1A^1+\partial^2A^2+\partial^3A^3\right)\vec{e_1} \tag{62}$$

$$+\left(\partial_0\partial^0+\partial_1\partial^1+\partial_2\partial^2+\partial_3\partial^3\right)A^2\vec{e_2}-\partial_2\left(\partial^0A^0+\partial^1A_1+\partial^2A^2+\partial^3A^3\right)\vec{e_2} \tag{63}$$

$$+\left(\partial_0\partial^0+\partial_1\partial^1+\partial_2\partial^2+\partial_3\partial^3\right)A^3\vec{e_3}-\partial_3\left(\partial^0A^0+\partial^1A^1+\partial^2A^2+\partial^3A^3\right)\vec{e_3} \tag{64}$$

$$-\partial_0\partial^0A^1\vec{e_1}-\partial_1\partial^1A^1\vec{e_1}+\partial_1\partial^0A^0\vec{e_1}+\partial_1\partial^1A^1\vec{e_1} \tag{65}$$

$$-\partial_0\partial^0A^2\vec{e_2}-\partial_2\partial^2A^2\vec{e_2}+\partial_2\partial^0A^0\vec{e_2}+\partial_2\partial^2A^2\vec{e_2} \tag{66}$$

$$-\partial_0\partial^0A^3\vec{e_3}-\partial_3\partial^3A^3\vec{e_3}+\partial_3\partial^0A^0\vec{e_3}+\partial_3\partial^3A^3\vec{e_3}. \tag{67}$$

Some terms cancel in (65)-(67), we can write the equations with μ and ν

$$\sum_{\nu=1}^{3}\partial_\mu(\partial^\mu A^\nu-\partial_\nu A^\mu)\vec{e_\nu}-\sum_{\nu=1}^{3}(\partial_0^2A^\nu-\partial_0\partial^\nu A^0)\vec{e_\nu}. \tag{68}$$

The result gives Maxwell's fourth equation

$$\nabla\times\mathbf{B}=\sum_{\nu=1}^{3}\partial_\mu(\partial^\mu A^\nu-\partial_\nu A^\mu)\vec{e_\nu}-\sum_{\nu=1}^{3}(\partial_0^2A^\nu-\partial_0\partial^\nu A^0)\vec{e_\nu} \tag{69}$$

$$=\mu\mathbf{J}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{A}-\frac{1}{c}\frac{\partial}{\partial t}\nabla A^0 \tag{70}$$

$$\nabla\times\mathbf{B}=\mu\mathbf{J}+\frac{1}{c^2}\frac{\partial}{\partial t}\left(-\frac{\partial}{\partial t}\mathbf{A}-\nabla\Phi\right) \tag{71}$$

$$\nabla\times\mathbf{B}=\mu\mathbf{J}+\frac{1}{c^2}\frac{\partial}{\partial t}\mathbf{E}. \tag{72}$$

There is no reason why (31) or (39) should refute the conclusion in Section 1 that the first of Maxwell's equations is not Lorentz invariant because Lorentz invariance of the first of Maxwell's equations implies that the charge changes with velocity which is shown impossible in Section 3. It is not enough to have a Lorentz invariant operator for the equation to be Lorentz invariant.

3. Observable properties cannot depend on velocity

We saw that (20) is not a Lorentz invariant equation because for (20) to be invariant, charge ρ should depend on velocity. Let us explain why this is not possible.

Observable properties, such as charge, cannot depend on the velocity of the inertial frame because of the following simple argument. The observable property can change either in the fixed inertial frame or in the moving inertial frame.

Assume that an observable property changes because of the velocity in the fixed inertial frame, then physical experiments in the fixed frame of a laboratory can measure the changed property. No physical experiments show that the charge of fast moving electrons when measured in the fixed inertial frame of the laboratory is any different from their charge when they are moving slowly.

Assume next that an observable property changes because of the velocity in the moving inertial frame. Then we can imagine an inertial frame that moves very fast with respect to the laboratory where we are, choose this other frame as the fixed inertial frame and the laboratory as the moving inertial frame. Then we can measure the observable property in the laboratory. We can easily make the physical test: simply imagine the other frame and see if anything changes, like the charge of an electron, can we set the charge of an electron to another figure simply by imagining another inertial frame? Imagining a fast moving inertial frame does not change the charge of an electron in any measurements made in the laboratory.

The same argument goes for all other observable properties, such as mass, time and length. In the beginning of the 20th century physicists did measure increasing mass of fast electrons, but they interpreted it as apparent mass. They defined two different apparent masses: transverse mass and longitudinal mass. Only after Einstein's Special Relativity Theory the view that the mass growth is apparent changed to the view that there is relativistic mass, but [4] shows that the original idea of apparent longitudinal mass fits better to Bertozzi's measurements. Paper [4] also gives a mathematical refutation of the relativistic mass concept: relativistic kinetic energy formula violates conservation of energy. Mass does not change because of velocity.

Concerning time, the Lorentz transform gives the coordinate time in the moving frame as

$$t'_c = \gamma^{-1}t. \tag{73}$$

This is derived by solving $x = \gamma^{-1} + vt$ from $x' = \gamma(x - vt)$ and inserting it to $t' = \gamma(t - (v/c^2)x$. This gives $t' = \gamma^{-1}t - (v/c^2)x'$ showing that t' is a local time. The coordinate time, i.e., the time on the t'-coordinate axis is the projection of (x', t') on the t'-coordinate axis. This projection is done by varying x' in $t' = \gamma^{-1}t - (v/c^2)x'$ into a line and seeing where the line intersects the t'-coordinate axis. The intersection point is $(0, t'_c)$. The mapping of the time axes from (x, t) to (x', t') takes t to t'_c. The time t'_c is the real time in the moving inertial frame R' while t' is a local time. The speed of e.g. light is calculated in R' by taking the projections on the t' and x' axes and dividing the projections.

The twin paradox is to consider two inertial frames R_1 and R_2 that have a relative velocity v. If R_1 is chosen as (x, t) and R_2 is chosen as (x', t'), then the scales of the t_1 and t_2 coordinate axes of R_1 and R_2 relate as $t_2 = \gamma^{-1}t_1$. If R_2 is chosen as (x, t) and R_1 is chosen as (x', t'), then the scales of the t_1 and t_2 coordinate axes of R_1 and R_2 relate as $t_1 = \gamma^{-1}t_2$. This means that $t_1 = \gamma^2 t_1$ and $\gamma^2 = 1$. Einstein claimed to have solved the original twin paradox with two astronauts by requiring that the astronauts meet twice. There is no such way out of the above formulation of the twin paradox. The twin paradox is not a paradox, it is a correct refutation of the Special Relativity Theory and it cannot be explained away.

The speed of light not being c in the moving inertial frame and the twin paradox refute the claim by the Special Relativity Theory that time depends on velocity. Time and also length do not depend on velocity of the frame.

Every transform M in (6) that does not have $a = 1$ gets a contradiction from the twin paradox. Therefore the fact that the equation (3) is invariant under coordinate transforms of the type (6) does not have any physical meaning: those transformed coordinates have the twin paradox and local time, and they do not have the speed of light as c. Let us notice that just like (3), every wave equation for some speed $w \neq c$ has transforms of the type (6) under which the wave function is invariant, but this is not considered to be of any physical importance. It is not concluded that w is the maximal speed in the universe, or that every equation of motion should be invariant under transform of the type M where the speed is w and not c. Equations do have invariances under some transforms. Such transforms do not need to have any physical meaning.

4. Conclusions

Maxwell's equations in the classical (1) and covariant (31) formulations include inhomogeneous equations. It should be obvious that if a homogeneous equation is Lorentz invariant, then the same equation as inhomogeneous is not Lorentz invariant if the part that makes the equation inhomogeneous, like the charge, does not change correctly in the Lorentz transform. Even Einstein did not claim that charge changes because of velocity though he made this claim of time, length and mass, which in itself is a strange claim because velocity is always relative velocity and depends on what inertial frames we choose.

However, a standard textbook [20] claims that the four-vector A^{μ} is Lorentz invariant and even the Google AI, drawing from many sources, claims that Maxwell's equations are Lorentz invariant. The article shows that this is not true.

Invariance of an inhomogenous equation $Df = g$ can be shown as follows. Let us denote a linear differential operator D transformed by Lorentz transfor with D'' and D written by changing x, t with x', t' by D'. Similarly, f'', g'' are Lorentz transformed f, g, while f', g' are f, g written with x, t changed to x', t'. Lorentz invariance of $DF = g$ can be demonstrated by showing that D'' has the same solutions as $D'f' = g'$. Let D be Lorentz invariant. This means $D'' = D'$. In Maxwell's first equation $f'' = Cf'$ for a constant C but g, being the charge, does not change by velocity: $g'' = g' = g$. The equations $D''f'' = g'' = g$ and $D'f' = g' = g$ are both true. However, $D'f'' = g''$ is $CD'f' = g'$. It is not the same equation as $D'f' = g'$ and it is not true. Likewise $D''f' = g'$ is the same as $D''f'' = Cg' = Cg''$. It is not the same as $D''f'' = g''$ and it is not true. If g

transforms as $g'' = Cg'$, then the equation $Df = g$ is Lorentz invariant.

Invariance of an equation can be a weaker claim to invariance of an operator in it because the function on which the operator acts may ignore some differences in two operators, this happens in $F = ma$ under the Galileo transform, or it can be a stronger requirement: an inhomogenous equation can have a Lorentz invariant operator and not be a Lorentz invariant equation, and example being (31) where the right hand side has charge in J^0 and charge does not change by velocity but A^0 does.

Lorentz covariance does not imply Lorentz invariance. Lorentz covariance only means that the equation transforms in a covariant way.

This article points to serious errors in the General Relativity Theory. These are not the only errors in the Relativity Theory, both Special and General. The references [1]-[19] give a selected list of the author's findings. Almost all papers show a serious error in the Relativity Theory.

Statement of conflicting interests and funding

There are no conflicting interests and this work has not been funded.

5. References

[1] Jormakka J., The Schwarzschild metric is not a valid metric and the Einstein equations are not Lorentz invariant, 2025. Available at the ResearchGate.

[2] Jormakka J., Three fatal errors in the Relativity Theory, 2025. Available at the ResearchGate.

[3] Jormakka J., Refutation that experiments verify the relativity theory and a more reasonable proof of $E = mc^2$, 2025. Available at the ResearchGate.

[4] Jormakka J., Calculation of the longitudinal mass from Bertozzi's experiment, 2025. Available at the ResearchGate.

[5] Jormakka J., The Dirac equation is not Lorentz invariant, 2025. Available at the ResearchGate.

[6] Jormakka J., A Better Solution to the Precession of Mercury's Perihelion, 2024. Available at the ResearchGate.

[7] Jormakka J., Irrefutable proof that the relativistic mass formula is wrong and Einstein did not prove $E = mc^2$, 2024. Available at the ResearchGate.

[8] Jormakka J., Error in Einstein's geodesic equation and the failure of his geometrization idea, 2024. Available at the ResearchGate.

[9] Jormakka J., On light-like geodesics in the Schwarzschild metric, 2024. Available at the ResearchGate.

[10] Jormakka J., A proof that light does not travel along geodesics of a gravitational field, 2024. Available at the ResearchGate.

[11] Jormakka J., On the field equation in gravitation, 2024. Available at the ResearchGate.

[12] Jormakka J., On the field equation in gravitation, part 2, 2024. Available at the ResearchGate.

[13] Jormakka J., The error in the General Relativity Theory and its cosmological considerations, 2024. Available at the ResearchGate.

[14] Jormakka J., Failure of the geometrization principle and some cosmological considerations, 2023. Available at the ResearchGate.

[15] Jormakka J., A New Look on Nordström's Gravitation Theory, 2023. Available at the ResearchGate.

[16] Jormakka J., The Essential Questions in Relativity Theory, 2023. Available at the ResearchGate.

[17] Jormakka J., "Relativity Theory needs some fixing", in *Redefining Standard Model Physics* (ed. Prof. Brian Robson), Intechopen, 2023, ISBN: 978-1-83768-017-7, London. (Later the whole book was cancelled by the publisher, not because of this paper.) Available at the ResearchGate.

[18] Jormakka J. Quantization of gravitation, 2020. Available at the ResearchGate.

[19] Jormakka J. Einstein's field theory is wrong and Nordström's is correct, 2020. Available at the ResearchGate.

[20] Bailin D., Love A., Introduction to Gauge Field Theory, 1986. Adam Hilger, Bristol and Boston.