The Schwarzschild metric is not a valid metric and the Einstein equations are not Lorentz invariant

Jorma Jormakka
Vantaa, Finland
jorma.o.jormakka@gmail.com

Abstract: The Schwarzschild metric is not a valid metric. Additionally, it does not make the local speed of light equal to c to all directions. It follows that all physical experiments which claim to verify the General Relativity theory by using the Schwarzschild solution as a substantial part of the argument are invalid as verifications of the theory. These include the Pound-Rebka experiment, precession of the perihelion of Mercury, clock dilatation in GPS satellites, Shapiro delay, black holes and bending of light in a gravitational field. Alternative explanations that do not use the the Schwarzschild metric exist [3][6]19]. By selecting a valid metric that agrees with the Schwarzschild metric on the x-axis the article proves that the Einstein equations are not Lorentz invariant. This fact makes the requirement that equations of motion should be Lorentz invariant irrelevant.

Keywords: Schwarzschild metric, valid metric, Einstein equations, Lorentz invariance.

1. The Schwarzschild metric is not a valid metric

The Schwarzschild metric is defined as

$$ds^2 = c^2 d\tau^2 = A(r)c^2 dt^2 - B(r)dr^2 - r^2 d\theta^2 - r^2 \sin^2(\theta) d\phi^2 \tag{1}$$

where

$$A(r) = \left(1 - \frac{r_s}{r}\right) \quad B(r) = \left(1 - \frac{r_s}{r}\right)^{-1} \tag{2}$$

and r_s is a constant, so called Schwarzschild radius. Changing the differential form (1) to Cartesian coordinates gives

$$ds^2 = A(r)c^2 dt^2 - (B(r) - 1)dr^2 - \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2\right) \tag{3}$$

$$= A(r)c^2 dt^2 - (B(r) - 1)dr^2 - \left(dx^2 + dy^2 + dz^2\right) \tag{4}$$

$$= A(r)c^2 dt^2 - ((B(r) - 1)\frac{x^2}{r^2} + 1)dx^2 - ((B(r) - 1)\frac{y^2}{r^2} + 1)dy^2 \tag{5}$$

$$- ((B(r) - 1)\frac{z^2}{r^2} + 1)dz^2 - (B(r) - 1)\frac{xy}{r^2}dxdy \tag{6}$$

$$- (B(r) - 1)\frac{xz}{r^2}dxdz - (B(r) - 1)\frac{yz}{r^2}dydz. \tag{7}$$

The space differential part in (1)

$$d\rho^2 = B(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2 \tag{8}$$

should be a Riemannian metric, but it is invalid.

In order to explain what is wrong with the metric (3-7), let us start from very simple concepts because the relativity theory community considers the Schwarzschild metric as valid, which it is not, and the reason why not needs to be explained in a very easy way. In the two-dimensional Euclidean space

$$r^2 = x^2 + y^2. \tag{9}$$

Dividing by a very large number N that approaches infinity

$$dr^2 = \left(\frac{r}{N}\right)^2 = \left(\frac{x}{N}\right)^2 + \left(\frac{y}{N}\right)^2 = dx^2 + dy^2. \tag{10}$$

Here

$$\frac{dy}{dx} = \frac{y}{N}\frac{N}{x} = \frac{y}{x} \qquad dy = \frac{y}{x}dx. \tag{11}$$

The relation between dy and dx does not mean that they are not independent. The variables x, y are independent and dx, dy are independent. The relation only means that the scaling number N must be the same in the whole equation, else we do not get

$$dr^2 = dx^2 + dy^2. \tag{12}$$

Derivating (9) gives

$$dr = \frac{\partial r}{\partial x}dx + \frac{\partial r}{\partial y}dy = \frac{x}{r}dx + \frac{y}{r}dy. \tag{13}$$

Squaring the equation gives

$$dr^2 = \left(\frac{x}{r}\right)^2 dx^2 + \left(\frac{y}{r}\right)^2 dy^2 + 2\frac{xy}{r^2}dxdy. \tag{14}$$

This expression must equal (12), and it does when we insert $dy = (y/x)dx$. Both equations give

$$dr = \frac{r}{x}dx \qquad \rightarrow \qquad \frac{dr}{dx} = \frac{r}{x}. \tag{15}$$

Notice that $\partial r/\partial x = x/r$ in (13) and that (15) does not mean that y is a function of x, the variables are free. The reason for (15) is the scaling condition $dy = (y/x)dx$.

Comparing (5-7) with (14), we see that both have cross terms like $dxdy$. Such terms can appear if a metric is given in an uncommon form, but every point in a Riemannian manifold has an infinitesimally small open environment where the metric is Euclidean, i.e., when going to an infinitesimally small environment, curvature disappears and the space becomes flat. The small open environment has local Cartesian coordinates.

In the general case a n-dimensional manifold cannot be smoothly embedded into the Euclidean space $\mathbb{R}^n$, it can only be smoothly embedded in $\mathbb{R}^{2n+2}$. There are only local coordinates in a small environment of a point, but the spherical coordinates in the Schwarzschild metric (1) are global (i.e., for the whole $\mathbb{R}^4$).

They provide the embedding into $\mathbb{R}^4$ and for the space dimensions into $\mathbb{R}^3$. Therefore, in an infinitesimally small environment of a point (r, ϕ, θ), expressed as (x, y, z) in global Cartesian coordinates, the metric (1) should be a valid Riemannian metric. We will try to construct this valid metric in Cartesian coordinates.

From (14) we can see how to find the Cartesian coordinate representation. If in (14) we set $y = dy = 0$, then

$$dr^2 = dx^2. \tag{16}$$

Likewise, if we set $x = dx = 0$, then

$$dr^2 = dy^2. \tag{17}$$

The Cartesian coordinate representation can only be of the form

$$dr^2 = A(x, y)dx^2 + B(x, y)dy^2. \tag{18}$$

because a Riemannian metric is induced by an inner products and $(\bar{e}_i, \bar{e}_j) = \delta_{ij}$. We notice that (12) matches (16) and(17) for $A(x, y) = B(x, y) = 1$ and calculating as in (15) does prove that (12) is the Cartesian form of (14).

In the same way we set $y = dy = z = dz$ in (5-7) to get the metric on the x-axis

$$ds^2 = A(r)c^2dt^2 - B(r)dx^2 \tag{19}$$

and similarly on the y and z-axes respectively

$$ds^2 = A(r)c^2dt^2 - B(r)dy^2 \qquad ds^2 = A(r)c^2dt^2 - B(r)dz^2. \tag{20}$$

If (1) is a valid metric, then in Cartesian coordinates it must be

$$ds^2 = A(r)c^2dt^2 - B(r)dx^2 - B(r)dy^2 - B(r)dz^2. \tag{21}$$

If (1) is a valid metric, then converting (21) to spherical coordinates must give (1), but

$$ds^2 = c^2d\tau^2 = A(r)c^2dt^2 - B(r)dx^2 - B(r)dy^2 + B(r)dz^2 \tag{22}$$

$$= A(r)c^2dt^2 - B(r)dr^2 - r^2B(r)d\theta^2 - r^2\sin^2(\theta)B(r)d\phi^2 \tag{23}$$

is not (1). This shows that (1) is not a valid metric.

Metric (1) is not (21), but we can look at it at a close vicinity to the x-axis. In an infinitesimally small environment of a point, (1) should give the Minkowski metric. This is not true for (1). In Cartesian coordinates on the x-axis the metric (1) is

$$ds^2 = A(r)c^2dt^2 - B(r)dx^2 - dy^2 - dz^2 \tag{24}$$

while the Minkowski metric is

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 \tag{25}$$

We can work out a valid metric out of the invalid metric (1), though it will not give the Minkowski metric as the limit. On the x-coordinate axis, where $y = z = 0$, the space part of the metric is

$$d\rho^2 = B(x)dx^2 + dy^2 + dz^2. \tag{26}$$

This is an acceptable Riemannian metric. However, this acceptable Riemannian metric is valid in (1) only on the line $y = z = 0$, not in a small open environment of a point on the x-axis. In Section 2 we will extend this metric into an open environment of a point.

The speed of light along the x-axis is solved for the Minkowski metric by setting $ds = 0$ for a light-like world path in (25), $dy = dz = 0$ for a world path along the x-axis, and solving

$$0 = c^2dt^2 - dx^2 \tag{27}$$

$$c^2 = \frac{dx^2}{dt^2}. \tag{28}$$

We get similar equations for a light-like world path on the y and z-axis directions.

The time infinitesimal dt can be taken the same for measurements of the speed of light to all space directions. This is shown by measuring the speed of light to two space dimensions at the same time

$$0 = c^2dt^2 - dx^2 - dy^2. \tag{29}$$

Clearly, dt is the same for x and y directions. We can do like this to all combinations of x, y, x, z and y, z and conclude that the time differential can be the same and $cdt = dx = dy = dz$ holds in measurement of the speed of light. This does not mean that dt, dx, dy, dz are not independent. They are independent. It only means that in order to measure the speed of light, it is good to use the same length infinitesimals dx, dy and dz.

In a similar way we get the speed of light along the x-axis ($dy = dz = 0$) for the metric (24)

$$0 = A(r)c^2dt^2 - B(r)dx^2 \tag{30}$$

$$c'^2 = \frac{B(r)}{A(r)}\frac{dx^2}{dt^2} = \frac{B(r)}{A(r)}c^2 \tag{31}$$

$$c'^2 = c^2\frac{\left(1 - \frac{r_s}{r}\right)^{-1}}{\left(1 - \frac{r_s}{r}\right)} = c^2\left(1 - \frac{r_s}{r}\right)^{-2} \tag{32}$$

Similarly, in the (y, z)-plane the speed of light in (11) is

$$c'^2 = c^2\frac{1}{\left(1 - \frac{r_s}{r}\right)} = c^2\left(1 - \frac{r_s}{r}\right)^{-1}. \tag{33}$$

The local speed of light in the Schwarzschild metric is not the same to all directions and it is not c to any direction. Physical experiments show that the speed of light must be locally constant c to every direction.

A third problem with the Schwarzschild metric (1) is that in the tangent space the metric must be flat. This means that when the open environment of a point decreases, the metric must approach the Minkowski metric multiplied by the squared value of the field on that point. This does not happen with (1).

The Schwarzschild metric (1) is not a valid metric, light does not have the speed c to each direction at each point in the metric (1) and the metric (1) does not give the Minkowski metric at the tangent space of a point as it should. Because of these reasons the Schwarzschild metric is invalid and does not give a valid approximation to the Newtonian gravitation field in any of the tests of the General Relativity Theory (GRT) that Einstein proposed. Consequently, GRT has not been verified by any of these experiments.

The most general valid metric that is radially symmetric, does not depend on time, and has the speed of light constant at c is induced by a scalar field

$$ds^2 = c^2 d\tau^2 = B(r)c^2dt^2 - B(r)dx^2 - B(r)dy^2 + B(r)dz^2 \tag{34}$$

$$= B(r)c^2dt^2 - B(r)dr^2 - r^2B(r)d\theta^2 - r^2\sin^2(\theta)B(r)d\phi^2. \tag{35}$$

We continue to proving that the Einstein equations are not Lorentz invariant by extending the metric (24).

2. The Einstein equations are not Lorentz invariant

It is assumed to be proven that the Einstein equations are Lorentz covariant. We well show that this is not the case. For the proof we need one valid solution to the Einstein equations. Such a solution is not the Schwarzschild metric (1), but for the same situation of a point mass in an otherwise empty space we can find a valid solution to the Einstein equations. In this situation the Einstein equations reduce to all Ricci tensor entries being zero:

$$R_{ab} = 0. \tag{36}$$

Though the Schwarzschild metric (1) is not a valid metric, we can extend (26) to hold in an open environment of a point, then (26) is a valid Riemanninan metric for the space coordinates in the open environment and

$$ds^2 = A(x,r)c^2dt^2 - B(x,r)dx^2 - dy^2 - dz^2 \tag{37}$$

is a valid pseudo-Riemannian metric for the Einstein equations. It can be considered as a pseudo-metric in an open environment of a point on the x-axis. The metric (37) is not spherically symmetric and approximates (1) only at a close vicinity to the x-axis.

In spherical coordinates this metric is

$$ds^2 = Ac^2dt^2 - \left((B\cos^2\phi + \sin^2\phi)\sin^2\theta + \cos^2\theta\right)\, dr^2 \tag{38}$$

$$- \left((B\cos^2\phi + \sin^2\phi)\cos^2\theta + \sin^2\theta\right)\, r^2 d\,\theta^2 \tag{39}$$

$$- \left((B \sin^2 \phi + \cos^2 \phi \right) r^2 \sin^2 \theta \, d\phi^2 \tag{40}$$

$$- (1 - B) 2r \sin^2 \theta \sin \phi \cos \phi \, dr \, d\phi \tag{41}$$

$$- (\sin^2 \phi + B \cos^2 \phi - 1) 2r \sin \theta \cos \theta \, dr \, d\theta \tag{42}$$

$$- (1 - B) 2r^2 \sin \phi \cos \phi \sin \theta \cos \theta \, d\theta \, d\phi \tag{43}$$

showing that this metric is not (1). It does not have the same solution as the Schwarzschild metric, but this metric is a valid metric, shown by the local representation in Cartesian coordinates, and we can find a valid solution to the Einstein equations in an open environment of a point on the x-axis.

Calculating the Ricci tensor entries gives two equations only. First, the Christoffel symbols for $A(x, r)$, $B(x, r)$ have only six nonzero terms

$$\Gamma^0_{00} = \frac{1}{2} A^{-1} \frac{\partial A}{\partial t} \quad \Gamma^1_{01} = \frac{1}{2} B^{-1} \frac{\partial A}{\partial t} \quad \Gamma^0_{11} = -\frac{1}{2} A^{-1} \frac{\partial B}{\partial t} \tag{44}$$

$$\Gamma^1_{11} = \frac{1}{2} B^{-1} \frac{\partial B}{\partial x} \quad \Gamma^0_{10} = \frac{1}{2} A^{-1} \frac{\partial B}{\partial x} \quad \Gamma^1_{00} = -\frac{1}{2} B^{-1} \frac{\partial A}{\partial x} \tag{45}$$

The Ricci tensor entries R_{ab} where $a \neq b$ are zero because the Cartesian coordinates are orthogonal. $R_{22} = 0$ and $R_{33} = 0$ because all terms contributing to these Ricci tensor entries have partial derivation by y and z respectively. This leaves only two nonzero Ricci tensor entries:

$$R_{00} = \Gamma^0_{00} \left(\Gamma^1_{01} + \Gamma^2_{02} + \Gamma^3_{03} \right) + \Gamma^1_{00} \left(\Gamma^1_{11} + \Gamma^2_{12} + \Gamma^3_{13} - \Gamma^0_{10} \right) \tag{46}$$

$$+ \Gamma^2_{00} \left(\Gamma^1_{21} + \Gamma^2_{22} + \Gamma^3_{23} - \Gamma^0_{20} \right) + \Gamma^3_{00} \left(\Gamma^1_{31} + \Gamma^2_{32} + \Gamma^3_{33} - \Gamma^0_{30} \right) \tag{47}$$

$$- \left(\Gamma^1_{01} \right)^2 - \left(\Gamma^2_{02} \right)^2 - \left(\Gamma^3_{03} \right)^2 \tag{48}$$

$$+ \Gamma^1_{00,0} + \Gamma^2_{00,2} + \Gamma^3_{00,3} - \Gamma^1_{01,0} - \Gamma^2_{02,0} - \Gamma^3_{03,0} \tag{49}$$

$$R_{11} = \Gamma^0_{11} \left(\Gamma^0_{00} + \Gamma^2_{02} + \Gamma^3_{03} - \Gamma^1_{01} \right) + \Gamma^1_{11} \left(\Gamma^0_{10} + \Gamma^2_{12} + \Gamma^3_{13} \right) \tag{50}$$

$$+ \Gamma^2_{11} \left(\Gamma^0_{20} + \Gamma^2_{22} + \Gamma^3_{23} - \Gamma^1_{21} \right) + \Gamma^3_{11} \left(\Gamma^0_{30} + \Gamma^2_{32} + \Gamma^3_{33} - \Gamma^1_{31} \right) \tag{51}$$

$$- \left(\Gamma^0_{10} \right)^2 - \left(\Gamma^2_{12} \right)^2 - \left(\Gamma^3_{13} \right)^2 \tag{52}$$

$$+ \Gamma^0_{11,0} + \Gamma^2_{11,2} + \Gamma^3_{11,3} - \Gamma^0_{10,1} - \Gamma^2_{12,1} - \Gamma^3_{13,1}. \tag{53}$$

Inserting the Cristoffel symbols from (44,45) gives

$$R_{00} = \frac{1}{4} A^{-1} B^{-1} \frac{\partial A}{\partial t} \frac{\partial B}{\partial t} - \frac{1}{4} B^{-2} \left(\frac{\partial B}{\partial t} \right)^2 - \frac{1}{2} \frac{\partial}{\partial t} \left(B^{-1} \frac{\partial B}{\partial t} \right) \tag{54}$$

$$+ \frac{1}{4} A^{-1} B^{-1} \left(\frac{\partial A}{\partial x} \right)^2 - \frac{1}{4} B^{-2} \frac{\partial A}{\partial x} \frac{\partial B}{\partial x} - \frac{1}{2} \frac{\partial}{\partial x} \left(B^{-1} \frac{\partial A}{\partial x} \right) \tag{55}$$

$$R_{11} = \frac{1}{4}B^{-1}A^{-1}\frac{\partial B}{\partial x}\frac{\partial A}{\partial x} - \frac{1}{4}A^{-2}\left(\frac{\partial A}{\partial x}\right)^2 - \frac{1}{2}\frac{\partial}{\partial x}\left(A^{-1}\frac{\partial A}{\partial x}\right) \tag{56}$$

$$+ \frac{1}{4}B^{-1}A^{-1}\left(\frac{\partial B}{\partial t}\right)^2 - \frac{1}{4}A^{-2}\frac{\partial B}{\partial t}\frac{\partial A}{\partial t} - \frac{1}{2}\frac{\partial}{\partial t}\left(A^{-1}\frac{\partial B}{\partial t}\right). \tag{57}$$

Let us solve $R_{00} = R_{11} = 0$ in the case that A and B depend only on x and not on t. The Ricci scalar is

$$R = g^{aa}R_{aa} = A^{-1}R_{00} + B^{-1}R_{11} = \left(\frac{\partial A}{\partial x}\right)^2\left(\frac{1}{4}A^{-2}B^{-1} - \frac{1}{4}A^{-2}B^{-1}\right) \tag{58}$$

$$+ \left(\frac{\partial A}{\partial x}\frac{\partial B}{\partial x}\right)\left(\frac{1}{4}A^{-1}B^{-2} - \frac{1}{4}A^{-1}B^{-2}\right) \tag{59}$$

$$- \frac{1}{2}A^{-1}\frac{\partial}{\partial x}\left(B^{-1}\frac{\partial A}{\partial x}\right) - \frac{1}{2}B^{-1}\frac{\partial}{\partial x}\left(A^{-1}\frac{\partial A}{\partial x}\right) \tag{60}$$

If $R_{aa} = 0$, then $R = 0$. This gives the following equation:

$$B\frac{\partial}{\partial x}\left(B^{-1}\frac{\partial A}{\partial x}\right) = A\frac{\partial}{\partial x}\left(A^{-1}\frac{\partial A}{\partial x}\right). \tag{61}$$

Writing $A' = \frac{\partial}{\partial x}A$ and $A'' = \frac{\partial}{\partial x}A'$

$$B(-B^{-2})B'A' + A'' = A^{-1}(A')^2 - A'' \tag{62}$$

$$-2\frac{A''}{A'} + \frac{A'}{A} = -\frac{B'}{B} \tag{63}$$

$$\ln A' - \frac{1}{2}\ln A = \frac{1}{2}\ln B + C \tag{64}$$

where C is an integration constant. The solution is

$$B = C_1\left(\frac{\partial A}{\partial x}\right)^2 A^{-1} \tag{65}$$

where C_1 is constant.

Inserting (65) to R_{00} when $A = A(x)$, $B = B(x)$ gives

$$R_{00} = \frac{1}{4}A^{-1}B^{-1}\left(\frac{\partial A}{\partial x}\right)^2 - \frac{1}{4}B^{-2}\frac{\partial A}{\partial x}\frac{\partial B}{\partial x} - \frac{1}{2}\frac{\partial}{\partial x}\left(B^{-1}\frac{\partial A}{\partial x}\right). \tag{66}$$

Denoting $A' = \frac{\partial A}{\partial x}$ as there are no partial time derivatives

$$4C_1R_{00} = A^{-1}(A'^{-2}A)A'^2 - (A')^{-2}A^2A'\left(A''2A'A + A'^2A'\right) \tag{67}$$

$$-2\frac{\partial}{\partial x}\left(A'^{-2}AA'\right) \tag{68}$$

$$4C_1R_{00} = 1 - 2A''A'^{-2}A + 1 - 2A''(-A'^{-2})A - A'^{-1}A' \tag{69}$$

$$4C_1R_{00} = 1 - 2A''A'^{-2}A + 1 + 2A''A'^{-2}A - 2 = 0. \tag{70}$$

This shows that $R_{00} = 0$. As $R = 0$ and $R_{00} = 0$, it follows that $R_{11} = 0$ and for any a and b $R_{ab} = 0$, the Einstein equations are fulfilled. We notice that any $A(x)$ and $B(x)$ satisfying (65) for any C_1 is a solution to the Einstein equations in this case.

Let us select one such solution, there are many. We do not need to specify the solution. Then the Ricci tensor entries calculated from g_{ab} having these $A(x)$ and $B(x)$ and being of the form (24) satisfy the Einstein equations

$$R_{ab}(g_{ab}) = 0. \tag{71}$$

We can write the equation (71) with coordinate names t', x', y', z'. In these coordinates

$$R_{a'b'}(g_{a'b'}) = 0 \tag{72}$$

where $g_{a'b'}$ has the same functions A and B and is of the form (24), but the functions appear as $A(x')$ and $B(x')$ in the metric in coordinates t', x', y', z'.

Next we connect t', x', y', z' and t, x, y, z with the Lorentz transform $x' = \gamma(x - vt)$, $t' = \gamma(t - (v/c^2)x)$, $y' = y$, $z' = z$. The Ricci tensor entries change when we insert the partial derivatives in t', x', y', z' expressed in t, x, y, z. Notice, if the Ricci tensor entries in coordinates t', x', y', z' operate on $g_{a'b'}$ it is not any invariance under a coordinate transform, it is simply changing the names of the coordinates, like the change of (71) to (72). The Lorentz transformed Ricci entries must operate on g_{ab} for showing Lorentz invariance.

Let us calculate $R_{00} = 0$. The partial differential operators must be changed to t, x, y, z with the Lorentz transform:

$$\frac{\partial}{\partial t'} = \frac{\partial t}{\partial t'}\frac{\partial}{\partial t} + \frac{\partial x}{\partial t'}\frac{\partial}{\partial x} = \gamma\frac{\partial}{\partial t} + v\gamma\frac{\partial}{\partial x} \tag{73}$$

$$\frac{\partial}{\partial x'} = \frac{\partial t}{\partial x'}\frac{\partial}{\partial t} + \frac{\partial x}{\partial x'}\frac{\partial}{\partial x} = \frac{v}{c^2}\gamma\frac{\partial}{\partial t} + \gamma\frac{\partial}{\partial x}. \tag{74}$$

In the metric g_{ab} the functions in (24) are $A = A(x)$ and $B = B(x)$. We can forget the partial derivative $\frac{\partial}{\partial t}$, it can only give zero. Thus we can replace

$$\frac{\partial}{\partial t'} = v\gamma\frac{\partial}{\partial x} \qquad \frac{\partial}{\partial x'} = \gamma\frac{\partial}{\partial x}. \tag{75}$$

Inserting to R_{00}

$$4R_{00} = A(x)^{-1}B(x)^{-1}v^2\gamma^2A'(x)B'(x) - B(x)^{-2}v^2\gamma^2B'(x)^2 \tag{76}$$

$$- 2v^2\gamma^2\frac{\partial}{\partial x}\left(B(x)^{-1}B'(x)\right) \tag{77}$$

$$+ A^{-1}B(x)^{-1}\gamma^2 A'(x)^2 - B^{-2}\gamma^2 A'(x)B'(x) - 2\gamma^2 \frac{\partial}{\partial x}\left(B(x)^{-1}A'(x)\right). \quad (78)$$

Because of v, we get two equations

$$0 = A^{-1}B^{-1}A'B' - B^{-2}B'^2 + 2B'^2B^{-2} - 2B^{-1}B'' \quad (79)$$

$$0 = -B^{-2}A'B' + A^{-1}B^{-1}A'^2 + 2B'B^{-2}A' - 2B^{-1}A''. \quad (80)$$

Simplifying

$$\frac{A'}{A} = 2\frac{B''}{B'} - \frac{B'}{B} \quad (81)$$

$$\frac{B'}{B} = 2\frac{A''}{A'} - \frac{A'}{A}. \quad (82)$$

Equation (82) follows from (65). Equation (81) has the solution

$$A = C_2 B'^2 B^{-1} \quad (83)$$

The two equations (83) and (65) can be solved by multiplying them

$$C_2 B'^2 B^{-1} B = C_1 A'^2 A^{-1} A \quad (84)$$

$$B' = \pm \left(\frac{C_1}{c_2}\right)^{\frac{1}{2}} A' \quad (85)$$

$$B = \pm \left(\frac{C_1}{C_2}\right)^{\frac{1}{2}} A + C_3 \quad (86)$$

where C_3 is an integration constant. Setting $C_3 = 0$ for an easy example solution

$$\pm \left(\frac{C_1}{C_2}\right)^{\frac{1}{2}} = C_1 A'^2 A^2 \quad (87)$$

$$A'A = C_4 \quad \rightarrow \quad A^2 = 2C_4 x - C_5 \quad (88)$$

where C_5 is an integration constant and $C_4 = |\left(\frac{C_1}{C_2}\right)^{\frac{1}{4}} / \sqrt{C_1}|$. It is not the A from the Schwarzschild solution, but metric (24) is not the false metric (1).

There are functions $A(x)$ and $B(x)$ that satisfy (65) and fail (83). We select such functions and form from them metric g_{ab} as in (24). Metric g_{ab} solves the Einstein equations (71) for this case. Then metric g_{ab} is written in coordinates t, x', y', z and called $g_{a'b'}$. The equation (72) is fulfilled. But when (72) is transformed by the Lorentz transform, the Einstein equations are not fulfilled for g_{ab}. We showed this in the case of R_{00}, it is zero only if (65) and (83) hold. This means that the Einstein equations are not Lorentz invariant.

3. When stationary the Ricci scalar is Lorentz invariant?

Notice that this discussion is only for metric of the form (37). A stationary metric of the type (37) has $A = A(x)$, $B = B(x)$. The Ricci scalar R is defined in the left side of (58). We sum it from (54) and (56) as (58) does not have the time derivatives, thus

$$R = -\frac{1}{2}A^{-1}\frac{\partial}{\partial t'}\left(B^{-1}\frac{\partial B}{\partial t'}\right) - \frac{1}{2}A^{-1}\frac{\partial}{\partial x'}\left(B^{-1}\frac{\partial A}{\partial x'}\right) \tag{89}$$

$$-\frac{1}{2}B^{-1}\frac{\partial}{\partial x'}\left(A^{-1}\frac{\partial A}{\partial x'}\right) - \frac{1}{2}B^{-1}\frac{\partial}{\partial t'}\left(B^{-1}\frac{\partial B}{\partial t'}\right). \tag{90}$$

The equation can be written as

$$R = -A^{-1}B^{-1}\frac{\partial^2 B}{\partial t'^2} - A^{-1}B^{-1}\frac{\partial^2 A}{\partial x'^2} \tag{91}$$

$$+\frac{1}{2}A^{-1}B^{-2}\frac{\partial A}{\partial x'}\frac{\partial B}{\partial x'} + \frac{1}{2}A^{-2}B^{-1}\frac{\partial A}{\partial t'}\frac{\partial B}{\partial t'} \tag{92}$$

$$+\frac{1}{2}A^{-2}B^{-1}\left(\frac{\partial A}{\partial x'}\right)^2 + \frac{1}{2}A^{-1}B^{-2}\left(\frac{\partial B}{\partial t'}\right)^2 \tag{93}$$

Making the Lorentz transform (60) turns the equation for the Ricci scalar into the form

$$R = \gamma^2 v^2\left(-A^{-1}B^{-1}B'' + \frac{1}{2}A^{-2}B^{-1}A'B' + \frac{1}{2}A^{-1}B^{-2}(B')^2\right) \tag{94}$$

$$+\gamma^2\left(-A^{-1}B^{-1}A'' + \frac{1}{2}A^{-1}B^{-2}A'B' + \frac{1}{2}A^{-2}B^{-1}(A')^2\right) \tag{95}$$

The equation has the velocity v^2 that can be chosen freely. The parts with partial derivatives of time and space must separately give zero:

$$-A^{-1}B^{-1}B'' + \frac{1}{2}A^{-2}B^{-1}A'B' + \frac{1}{2}A^{-1}B^{-2}(B')^2 = 0 \tag{96}$$

$$-A^{-1}B^{-1}A'' + \frac{1}{2}A^{-1}B^{-2}A'B' + \frac{1}{2}A^{-2}B^{-1}(A')^2 = 0 \tag{97}$$

Multiplying (97) by $B^{-1}2A/A'$ changes it to

$$\frac{B'}{B} = 2\frac{A''}{A'} - \frac{A'}{A} \tag{98}$$

This equation is (65). Multiplying (96) by $A^{-1}2B/B'$ changes it to

$$\frac{A'}{A} = 2\frac{B''}{B'} - \frac{B'}{B} \tag{99}$$

The equation is (66) and the solution is (68).

In order for the Ricci scalar to be zero (65) and (83) must hold. The Einstein equations are not Lorentz invariant, but the equations have some solutions that are Lorentz invariant.

4. Conclusions

The Schwarzschild metric is not a valid metric. Additionally, it does not make the local speed of light equal to c to all directions. It follows that all physical experiments which claim to verify the General Relativity theory by using the Schwarzschild solution as a substantial part of the proof are invalid. These include the Pound-Rebka experiment, precession of the perihelion of Mercury, clock dilatation in GPS satellites, Shapiro delay, black holes and bending of light in a gravitational field. Alternative explanations that do not use the the Schwarzschild solutions exist, e.g. in [3][6][19].

A metric that is the same as the Schwarzschild metric on the x-axis, yet a valid metric, was used to prove that the Einstein equations are not Lorentz invariant. This fact makes the requirement that equations of motion should be Lorentz invariant irrelevant.

The article points to serious errors in the General Relativity Theory. These are not the only errors in the Relativity Theory, both Special and General. The references [1]-[19] give a selected list of the author's findings. All but one of the papers show one or more serious errors in the Relativity Theory.

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