

A Wavelike Gravitational Potential as the Origin of Dark Matter and Energy

V. B. VERMA ¹

¹*Independent Researcher, Boulder, CO, 80305, USA*

ABSTRACT

Motivated by the observation made by R. H. Dicke in 1957 that the speed of light seems to be correlated with the gravitational potential of the entire universe, we suggest a modification of general relativity to incorporate a wavelike gravitational potential. We show that the gravitational force derived from such a potential is an emergent property of matter and critically dependent on the distribution of mass on both local and global scales throughout the universe. The theory is free of gravitational singularities. It provides a path to numerically computing the critical acceleration in modified Newtonian dynamics theory (MOND), providing an alternative explanation for dark matter. Finally, we develop a cosmological model and fit the model to high-redshift supernova data from the Supernova Cosmology Project, showing that we obtain an excellent fit with only baryonic matter.

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1. THE GRAVITATIONAL WAVEFUNCTION

There is a serious problem with general relativity which is rarely discussed in textbooks or the scientific literature. Namely, the theory predicts that the entire universe is the event horizon of a black hole. In 1957 R. H. Dicke published “Gravitation Without a Principle of Equivalence” in which he made the observation that twice the gravitational potential of the observable universe is approximately equal to the speed of light squared (R. H. Dicke 1957). This is precisely the event horizon condition. The reason this is a serious problem is that according to general relativity, despite measuring a speed of light of $2.99 \times 10^8 m/s$ in a local inertial frame, when we look out into the surrounding universe the speed of light should appear to be nearly zero. Clearly this is not consistent with observations of the universe. In general relativity the gravitational potential of the universe is typically neglected and only the massive bodies of interest are taken into account, but there is no logical reason for doing this. The gravitational potential in the Einstein equation should represent the potential generated by all massive bodies in the universe. Dicke did not come to this conclusion in his paper. This is because he was considering the fact that the deflection of light around a gravitating body can be modeled as a local change in the refractive index of the form

$$n = \frac{c_0}{c} = 1 + \frac{2GM}{rc_0^2}. \quad (1)$$

He suggested that the number 1 on the right-hand side of this equation has its origin in the gravitational potential of all matter in the universe. However, this conclusion is also not logical because if it is the gravitational potential that determines the square of the speed of light then the speed of light ought to increase in the vicinity of a gravitating body instead of decrease. The preceding arguments suggest that what is needed is a means by which the gravitational potential can be either positive or negative in sign, depending on distance.

We are interested in determining the gravitational potential in the vicinity of mass M , and assume that the rest of the universe is very far away from this mass. How we define “in the vicinity” and “very far away” will be discussed later. The crux of the problem is to figure out why matter which is “very far away” should have a gravitational potential of opposite sign when compared to the gravitational potential of mass which is “nearby”. The Minkowski or flat-spacetime metric can then be considered to be produced by matter which is far away, while the metric perturbations

are due to matter which is nearby. This provides a more unified view of spacetime and matter when compared to general relativity, since spacetime is no longer simply a background which exists independently of matter, but is fundamentally co-dependent upon matter for its existence. We begin with the relativistic energy-momentum relation

$$E^2 = p^2 c^2 + m_0^2 c^4. \quad (2)$$

If we rearrange terms we obtain the following quadratic equation for c^2

$$m_0^2 (c(\mathbf{r})^2)^2 + p^2 (c(\mathbf{r})^2) - E^2 = 0. \quad (3)$$

Solving for $c(\mathbf{r})^2$ we obtain

$$c(\mathbf{r})^2 = \sqrt{\left(\frac{p^2}{2m_0^2}\right)^2 + \left(\frac{E}{m_0}\right)^2} - \frac{p^2}{2m_0^2}. \quad (4)$$

We can see immediately that we may be on the correct path, as c^2 is determined by two terms with opposite sign.

Now we come back to Dicke's observation. According to expression (4) we require the gravitational potential of the entire universe to be associated with the square root term. The kinetic term $p^2/(2m_0^2)$ represents the gravitational potential only of matter which is nearby. Next, we must define, and if possible quantify, what we mean by nearby or far away. We may gain some insight about how to accomplish this from the Einstein equation of general relativity

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}. \quad (5)$$

The cosmological constant Λ is generally moved to the right-hand-side of (5) and treated as a source of gravitation with positive energy density and negative pressure. This has become known as "dark energy" which could be considered the energy of the vacuum. We will consider a different interpretation here. Given that $G_{\mu\nu}$ contains second derivatives of the metric $g_{\mu\nu}$, it is interesting to note the similarities between (5) and the inhomogeneous Helmholtz equation having the general form

$$\nabla^2 \phi(\mathbf{r}) + k^2 \phi(\mathbf{r}) = 8\pi G \rho(\mathbf{r}). \quad (6)$$

In our interpretation, $k = \Lambda^{1/2}$ serves as the magnitude of a wavevector for the gravitational potential. While Λ is considered to be a scalar in general relativity, in its most general form the wavevector will be a complex number

$$\mathbf{k} = \boldsymbol{\beta} + i\boldsymbol{\alpha}. \quad (7)$$

Equation (6) has a solution of the form

$$\begin{aligned} \phi(\mathbf{r}) &= -\frac{2GM}{r} e^{i\mathbf{k}\cdot\mathbf{r}} \\ &= -\frac{2GM}{r} e^{-\alpha r} [\cos(\beta r) + i \sin(\beta r)]. \end{aligned} \quad (8)$$

In keeping with general relativity in the following we will assume $\alpha = 0$ so that the cosmological constant remains a scalar and equation (8) simply becomes

$$\phi(\mathbf{r}) = -\frac{2GM}{r} [\cos(\beta r) + i \sin(\beta r)]. \quad (9)$$

We will refer to equation (9) as the gravitational wavefunction. While this wavefunction does resemble a wavefunction in quantum mechanics in the sense that it is periodic and has both real and imaginary parts, it should be emphasized that we are not implying that this wavefunction has properties such as collapse upon measurement or the interpretation as a probability density.

Equation (9) represents the gravitational wavefunction of only one mass. What we will be interested in is the net sum of all gravitational wavefunctions from all masses in the universe which we can express as follows

$$\begin{aligned} \phi(\mathbf{r}) &= \sum_i \phi_i(\mathbf{r}) \\ &= \sum_i -\frac{2GM_i}{r_i} [\cos(\beta r_i) + i \sin(\beta r_i)] \cdot \\ &= -(\phi_R + i\phi_I) \end{aligned} \quad (10)$$

where we have defined

$$\phi_R = \sum_i \frac{2GM_i}{r_i} \cos(\beta r_i) \quad (11)$$

$$\phi_I = \sum_i \frac{2GM_i}{r_i} \sin(\beta r_i). \quad (12)$$

Returning to equation (4), we propose the following relationships

$$\frac{p^2}{2m_0^2} = \phi_R \quad (13)$$

$$\frac{E}{m_0} = \phi_I. \quad (14)$$

Using these expressions in equation (4) yields

$$\begin{aligned} c^2(\mathbf{r}) &= \sqrt{\phi_R^2 + \phi_I^2} - \phi_R \\ &= |\phi| - \phi_R \\ &= |\phi| \left(1 - \frac{\phi_R}{|\phi|}\right) \end{aligned} \quad (15)$$

Before moving on, we should be careful about specifying whether equation (15) refers to the frame-invariant speed of light, the coordinate speed of light, or the g_{00} component of the metric tensor in general relativity. The Schwarzschild metric in general relativity is given by

$$ds^2 = -c_0^2 \left(1 - \frac{2GM}{rc_0^2}\right) dt^2 + \left(1 - \frac{2GM}{rc_0^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (16)$$

Note that equation (15) resembles the g_{00} component of the metric (the coefficient of the dt^2 term)(S. M. Carroll 2019). By comparing our expression in equation (15) with g_{00} we find that $|\phi| = c_0^2$ represents the squared frame-invariant speed of light. The coordinate speed of light measured by an observer far from the mass M will measure a radial speed of light given by setting $ds^2 = 0$ and solving for dr^2/dt^2 to obtain

$$\frac{dr^2}{dt^2} = c_0^2 \left(1 - \frac{2GM}{rc_0^2}\right)^2 \quad (17)$$

which we may express in our notation as

$$\frac{dr^2}{dt^2} = |\phi| \left(1 - \frac{\phi_R}{|\phi|}\right)^2. \quad (18)$$

We will show below that in a homogeneous and isotropic universe, $\phi_R = 0$, so that the frame-invariant speed of light, the radial coordinate speed of light, and g_{00} , are all equal and determined by $c_0^2 = |\phi_I|$. This will be important in Section 2 where we develop a cosmological model. It is also consistent with the notion that the frame-invariant speed of light should be determined by matter which is “far away” since ϕ_I is proportional to $\sin(\beta r)$.

Next, we would like to determine the magnitude of the wavevector β in equation (9). We can do this by writing the sums in equation (11) and equation (12) as integrals and assuming an average mass density $\rho = 4.08 \times 10^{-28} \text{ kg/m}^3$. This is 4.6% of the critical density in the Λ CDM model which is an estimate of the percentage of ordinary matter in the universe based on measurements of the inhomogeneity of the cosmic microwave background performed by the WMAP experiment(G. Hinshaw et al. 2013). Therefore, we are neglecting dark matter and dark energy in our model.

If we choose to integrate over a full wavelength the integrals take the following form

$$\begin{aligned}
\phi_R &= \sum_i \frac{2GM_i}{r_i} \cos(\beta r_i) \\
&= \int_0^{2\pi/\beta} \frac{2G(4\pi r^2 \rho)}{r} \cos(\beta r) dr \\
&= 8\pi G\rho \int_0^{2\pi/\beta} r \cos(\beta r) dr \\
&= 0
\end{aligned} \tag{19}$$

$$\begin{aligned}
\phi_I &= \sum_i \frac{2GM_i}{r_i} \sin(\beta r_i) \\
&= \int_0^{2\pi/\beta} \frac{2G(4\pi r^2 \rho)}{r} \sin(\beta r) dr \\
&= 8\pi G\rho \int_0^{2\pi/\beta} r \sin(\beta r) dr \\
&= -\frac{16\pi^2 G\rho}{\beta^2}.
\end{aligned} \tag{20}$$

By integrating over one wavelength we are implying that this wavelength physically represents the distance of the furthest mass in the universe to which we are gravitationally connected. Just as we have a particle horizon in cosmology representing the maximum distance to which we are causally connected, the wavelength of the gravitational wavefunction could be considered to be a gravitational horizon. This wavelength is connected to the cosmological constant by $\lambda = 2\pi/\beta = 2\pi/\sqrt{\Lambda}$. Using equations (19) and (20) in equation (15) yields

$$\beta = \sqrt{\frac{16\pi^2 G\rho}{c_0^2}} = 6.92 \times 10^{-27} m^{-1}. \tag{21}$$

The gravitational horizon is then given by

$$\lambda = \frac{2\pi}{\beta} = 9.08 \times 10^{26} m. \tag{22}$$

Note that the diameter of the observable universe is $8.8 \times 10^{26} m$. Thus, we have a remarkable coincidence which is very likely not a coincidence at all - the gravitational horizon corresponds very closely with the diameter of the observable universe.

To gain further insight into the physics implied by equation (15) we can plot the gravitational potential and acceleration for the mass of the Earth in an otherwise empty universe. We will define the potential and acceleration using our existing notation as $2\Phi(\mathbf{r}) = c^2(\mathbf{r}) = -g_{00}$ with $c^2(\mathbf{r})$ given by (15). The potential is given by

$$\Phi(\mathbf{r}) = \frac{1}{2}c^2(\mathbf{r}) = \frac{GM}{r} [1 - \cos(\beta r)] \tag{23}$$

Figure 1(a) shows a plot of equation (23). There are two important points to be learned from this plot. The first is that the potential is everywhere positive, which it must be since the potential represents $c^2(\mathbf{r})$. This is the first departure from Newtonian gravity. The second point is that there is no singularity at the origin. Instead, the origin is the location of the event horizon of general relativity where $g_{00} = 0$ and the coordinate speed of light goes to zero.

Figure 1(b) shows the gravitational acceleration as derived from equation (23)

$$a(\mathbf{r}) = -\nabla\Phi(\mathbf{r}) = -\frac{1}{2}\nabla c^2(\mathbf{r}) = \frac{GM}{r^2} [1 - \cos(\beta r)] - \beta \frac{GM}{r} \sin(\beta r) \tag{24}$$

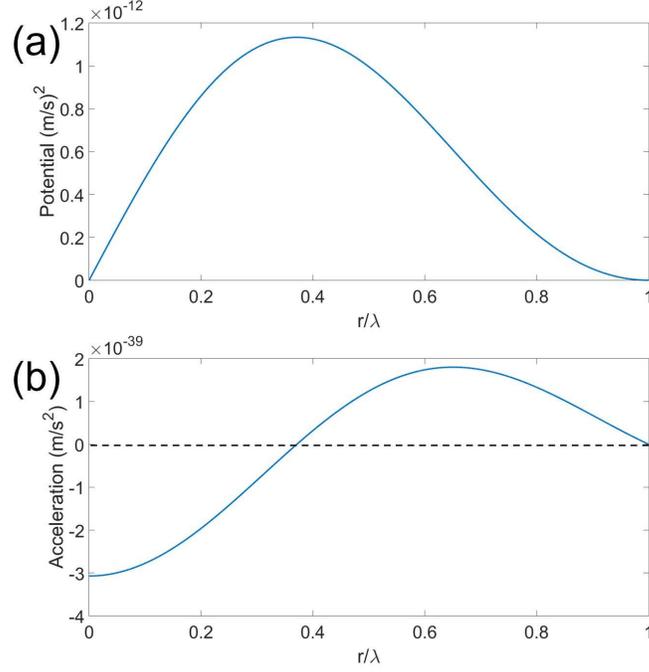


Figure 1. (a) Gravitational potential and (b) gravitational acceleration near an isolated mass equal to that of the Earth.

The most important point from this plot is that the acceleration transitions from being attractive to repulsive at 0.37λ . Again, this is a drastic departure from both Newtonian gravity and general relativity where gravity is only attractive. In addition, the acceleration approaches a finite value at the origin instead of tending to infinity. In general, the acceleration approaches $-\beta^2 GM/2$ at the origin which can be obtained by applying L'Hôpital's rule to equation (24). Therefore, there is nothing particularly special about a black hole. The gravitational acceleration approaches a finite limit. Physics does not break down because there is nothing beyond the event horizon which lies at the origin.

It is notable that the gravitational acceleration near the surface of the Earth in an otherwise empty universe is not the familiar $-9.8m/s^2$ but $-3 \times 10^{-39}m/s^2$ according to Fig. 1(b). This highlights an important aspect of our theory, which is that gravitational forces and accelerations are not fundamentally created by mass. Rather, gravitational forces emerge from the cooperative interactions between the gravitational wavefunctions of all masses in the universe. When we take into account the matter in the rest of the universe, equation (15) is simply

$$c^2(\mathbf{r}) \approx c_0^2 - \frac{2GM}{r} \quad (25)$$

and the acceleration reduces to the familiar Newtonian result

$$a(\mathbf{r}) = -\frac{1}{2}c^2(\mathbf{r}) = -\frac{GM}{r^2}. \quad (26)$$

It is possible to calculate the net radial acceleration our galaxy experiences due to the rest of the matter in the universe using $a(\mathbf{r}) = -\nabla\Phi(\mathbf{r}) = -\frac{1}{2}c^2(\mathbf{r})$ and the general form of $c^2(\mathbf{r})$ given in equation (15). Evaluating the cosine term in front of the square root we obtain

$$\begin{aligned} & -\frac{1}{2}\nabla\left[-\sum_i\frac{2GM_i}{r_i}\cos(\beta r_i)\right] = \sum_i\nabla\left[\frac{GM_i}{r_i}\cos(\beta r_i)\right] \\ & = \sum_i-\frac{GM_i}{r_i^2}\cos(\beta r_i) + \sum_i-\beta\frac{GM_i}{r_i}\sin(\beta r_i) \\ & = -4\pi G\rho\int_0^{2\pi/\beta}\cos(\beta r_i) - 4\pi G\rho\beta\int_0^{2\pi/\beta}r\sin(\beta r_i) \\ & = -4\pi G\rho\beta\left[-\frac{2\pi}{\beta^2}\right] = \frac{8\pi^2 G\rho}{\beta}. \end{aligned} \quad (27)$$

Evaluating the square root term we obtain

$$\begin{aligned}
& -\frac{1}{2}\nabla\sqrt{\left[\sum_i\frac{2GM_i}{r_i}\cos(\beta r_i)\right]^2+\left[\sum_i\frac{2GM_i}{r_i}\sin(\beta r_i)\right]^2} \\
&= -\frac{1}{4}\left[\left[\sum_i\frac{2GM_i}{r_i}\cos(\beta r_i)\right]^2+\left[\sum_i\frac{2GM_i}{r_i}\sin(\beta r_i)\right]^2\right]^{-1/2} \\
&\times\left\{\begin{aligned} & \left[\sum_i\frac{4GM_i}{r_i}\cos(\beta r_i)\right]\times\left[\sum_i-\frac{2GM_i}{r_i^2}\cos(\beta r_i)-\sum_i\beta\frac{2GM_i}{r_i}\sin(\beta r_i)\right] \\ & +\left[\sum_i\frac{4GM_i}{r_i}\sin(\beta r_i)\right]\times\left[\sum_i-\frac{2GM_i}{r_i^2}\sin(\beta r_i)+\sum_i\beta\frac{2GM_i}{r_i}\cos(\beta r_i)\right] \end{aligned}\right\} \quad (28) \\
&= -\frac{1}{4}\left[\left[8\pi G\rho\int_0^{2\pi/\beta}r_i\cos(\beta r_i)\right]^2+\left[8\pi G\rho\int_0^{2\pi/\beta}r_i\sin(\beta r_i)\right]^2\right]^{-1/2} \\
&\times\left\{\begin{aligned} & \left[16\pi G\rho\int_0^{2\pi/\beta}r_i\cos(\beta r_i)\right]\times\left[-8\pi G\rho\int_0^{2\pi/\beta}\cos(\beta r_i)-8\pi G\rho\beta\int_0^{2\pi/\beta}r_i\sin(\beta r_i)\right] \\ & +\left[16\pi G\rho\int_0^{2\pi/\beta}r_i\sin(\beta r_i)\right]\times\left[-8\pi G\rho\int_0^{2\pi/\beta}\sin(\beta r_i)+8\pi G\rho\beta\int_0^{2\pi/\beta}r_i\cos(\beta r_i)\right] \end{aligned}\right\}.
\end{aligned}$$

From the following identities

$$\begin{aligned}
& \int_0^{2\pi/\beta}r_i\cos(\beta r_i) = 0 \\
& \int_0^{2\pi/\beta}\sin(\beta r_i) = 0
\end{aligned} \quad (29)$$

we find that (28) is identically zero, leaving only the contribution from (27). Using $\rho = 4.08 \times 10^{-28} \text{ kg/m}^3$ and $\beta = 6.92 \times 10^{-27} \text{ m}^{-1}$, we obtain an inward (positive) acceleration acting toward the center of the galaxy of $a = 3.1 \times 10^{-10} \text{ m/s}^2$. The order of magnitude of this acceleration should be familiar to astrophysicists who study dark matter, as it is approximately the critical acceleration in Modified Newtonian dynamics (MOND)(M. Milgrom 1983, 2008, 2015). For accelerations smaller than approximately $1.2 \times 10^{-10} \text{ m/s}^2$, neither Newtonian mechanics nor general relativity can account for the flat velocity versus distance curves for the outer regions of the Milky Way galaxy and many other galaxies(K. G. Begeman et al. 1991; G. Gentile et al. 2011; F. Lelli et al. 2016; P. Li et al. 2018). Equation (15) tells us that the flat velocity curves are due to the fact that the acceleration becomes dominated by the rest of the universe pushing inward on the outer regions of the galaxy, instead of the inward pull from the galaxy's center.

2. COSMOLOGY

We will now outline the predictions of this theory on cosmology. Modern cosmology is currently based on the Friedmann equations which govern the expansion of the universe in general relativity

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{\kappa c^2}{a^2} + \frac{\Lambda c^2}{3} \quad (30)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} \quad (31)$$

where $\kappa = k/R_0^2$ is the spatial curvature, R_0 is the radius of curvature, Λ is the cosmological constant, and \dot{a} and \ddot{a} represent first and second time derivatives of the scale factor(S. M. Carroll 2019). The parameter k in the spatial curvature may take the values $+1$, -1 , or 0 corresponding to closed, open, and flat universes.

We return now to equation (21) which was derived from our expression for $c^2(\mathbf{r})$ in equation (15)

$$\beta = \sqrt{\frac{16\pi^2 G\rho}{c^2}}. \quad (32)$$

We may rewrite this equation as

$$c^2 = \frac{16\pi^2 G\rho}{\beta^2} = 4G\rho \left(\frac{2\pi}{\beta}\right)^2 = 4G\rho\lambda^2. \quad (33)$$

where $\lambda = 2\pi/\beta$ again is the wavelength of the gravitational wavefunction or the distance of the gravitational horizon. Taking the square root of both sides yields

$$c = \sqrt{4G\rho\lambda}. \quad (34)$$

Since we have already argued that the cosmological constant is simply a wavevector for the gravitational potential and not an energy density, we will now consider a cosmological model with only mass and spatial curvature but without dark energy. The Robertson-Walker metric for such a universe takes the following form

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[dr^2 + S_k(r)^2 d\Omega^2 \right] \quad (35)$$

where

$$S_k(r) = \begin{cases} R_0 \sin(r/R_0) & (k = +1) \\ r & (k = 0) \\ R_0 \sinh(r/R_0) & (k = -1) \end{cases} \quad (36)$$

and k is the curvature and R_0 is the radius of curvature (B. Ryden 2017). The Friedmann equation for a universe with only matter and curvature is

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{\kappa c^2}{a^2}. \quad (37)$$

In our model we will consider a universe with negative curvature and therefore take $k = -1$ so that $\kappa = -1/R_0^2$. Equation (37) then becomes

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{c^2}{R_0^2 a^2}. \quad (38)$$

Referring back to equation (33) we can express c^2 as

$$c^2 = 4G\rho\lambda^2 = 4G\rho\lambda_0^2 a^2 \quad (39)$$

where λ_0 is the size of the gravitational horizon at the present time. Substituting this expression for c^2 into equation (38) yields

$$\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G\rho}{3} + \frac{4G\rho\lambda_0^2 a^2}{R_0^2 a^2} \\ &= \frac{8\pi G\rho}{3} + \frac{4G\rho\lambda_0^2}{R_0^2} \\ &= \frac{8\pi G\rho}{3} \left[1 + \frac{3}{2\pi(R_0/\lambda_0)^2} \right] \\ &= \frac{8\pi G\rho}{3} \left[1 + \frac{3}{2\pi\alpha} \right] \end{aligned} \quad (40)$$

where we have defined $\alpha = (R_0/\lambda_0)^2$ as the squared ratio of the radius of negative curvature of the universe to the size of the gravitational horizon. It is interesting that although equation (40) was derived from a universe with baryonic matter and negative curvature, it appears effectively to be a flat universe with a baryonic mass density that is amplified by a factor of $1 + 3/(2\pi\alpha)$. Since $\dot{a}/a = H$ is the Hubble constant, we may solve equation (40) in terms of the other known parameters to obtain

$$\alpha = \frac{4G\rho}{H^2 - 8\pi G\rho/3} = \frac{(c/\lambda)^2}{H^2 - 8\pi G\rho/3} \quad (41)$$

where we have used equation (39) to replace $4G\rho$ with c^2/λ^2 . If we use a value for the cosmological constant of $70 \text{ km/s/Mpc} = 2.27 \times 10^{-18} \text{ s}^{-1}$ and set the average density of the universe to $\rho = 4.08 \times 10^{-28}$ we obtain $\alpha = 0.0221$. From equation (39) we can calculate the wavelength of the gravitational wavefunction as $\lambda_0 = c_0/\sqrt{4G\rho_0} =$

$9.08 \times 10^{26} m$ and then use the relation $\alpha = (R_0/\lambda_0)^2$ to solve for the radius of curvature from which we obtain $R_0 = 1.35 \times 10^{26} m$. Note that the radius of the Hubble sphere is $c_0/H_0 = 1.32 \times 10^{26} m$, so the radius of negative curvature lies just beyond the radius of the Hubble sphere.

Note that the value we obtained for $\alpha = 0.0221$ is not very different from the fine structure constant. To suggest that α should somehow be connected to the fine structure constant because it is the correct order of magnitude would certainly be numerology. However, it is worth considering what the average density of baryonic matter would need to be in order to arrive at a value of $\alpha = 0.0073 = 1/137$. We may use equation (41) to compute this density from which we obtain $\rho = 1.39 \times 10^{-28} kg/m^3$. This is not so far from our current estimates of the average density to be completely unreasonable. Current estimates of this density are based on the Λ CDM model which differs significantly from ours. It would be interesting to re-evaluate observations of the cosmic microwave background, baryon acoustic oscillations, and big bang nucleosynthesis data within the framework of our proposed model to better understand if there is any support for a lower average density which is closer to $\rho = 1.39 \times 10^{-28} kg/m^3$. If we use the value $\rho = 1.39 \times 10^{-28} kg/m^3$ for the average density of the universe in computing the value of the critical acceleration from MOND theory using equation (27) we obtain $1.8 \times 10^{-10} m/s^2$ which is closer to the experimentally measured value of $1.2 \times 10^{-10} m/s^2$ than our previous estimate of $3.1 \times 10^{-10} m/s^2$ where we assumed an average density of $4.08 \times 10^{-28} kg/m^3$. This provides some additional support for the idea that the average density may be lower than what is currently predicted in the Λ CDM framework. If true, it could prove to be yet another link between electromagnetism and gravitation.

We will now develop a model for this universe so that we can fit the model to high-redshift supernova data from the Supernova Cosmology Project. As we mentioned previously, although our model has contributions from both matter and curvature, we may consider our model to consist of only matter but with a correction factor for the mass density of $1 + 3/(2\pi\alpha)$. In a matter-only universe, the scale factor evolves in time according to

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}. \quad (42)$$

where t_0 is the age of the universe which is related to H_0 by $t_0 = 2/(3H_0)$. The proper distance to an astronomical object is given by

$$d_p(t_0) = \int_{t_e}^{t_0} \frac{c}{a(t)} dt. \quad (43)$$

where t_e is the time light from the source was emitted (B. Ryden 2017). It is important to note that in our model the speed of light varies in cosmological time. From (34) we have $c = \sqrt{4G\rho\lambda}$. Since $\lambda \propto a(t)$ and $\rho \propto a(t)^{-3}$, we have $c = c_0 a(t)^{-1/2}$ where c_0 is the present value for the speed of light. Therefore equation (43) becomes

$$\begin{aligned} d_p(t_0) &= c_0 \int_{t_e}^{t_0} \frac{1}{a(t)^{3/2}} dt = c_0 \int_{t_e}^{t_0} \frac{1}{t/t_0} dt = c_0 t_0 \ln \left[\frac{t}{t_0} \right]_{t_e}^{t_0} \\ &= c_0 t_0 \left\{ \ln[1] - \ln \left[\frac{t_e}{t_0} \right] \right\} \\ &= c_0 t_0 \ln \left[\frac{t_0}{t_e} \right]. \end{aligned} \quad (44)$$

The redshift is related to the scale factor by $1 + z = 1/a(t)$

$$1 + z = \frac{1}{a(t)} = \left(\frac{t_0}{t}\right)^{2/3} \quad (45)$$

which we can solve for t_0/t to obtain

$$\left(\frac{t_0}{t}\right) = (1 + z)^{3/2}. \quad (46)$$

Our expression (44) for the proper distance then becomes

$$\begin{aligned} d_p(t_0) &= c_0 t_0 \ln(1 + z)^{3/2} = \frac{2c_0}{3H_0} \ln(1 + z)^{3/2} \\ &= \frac{c_0}{H_0} \ln(1 + z). \end{aligned} \quad (47)$$

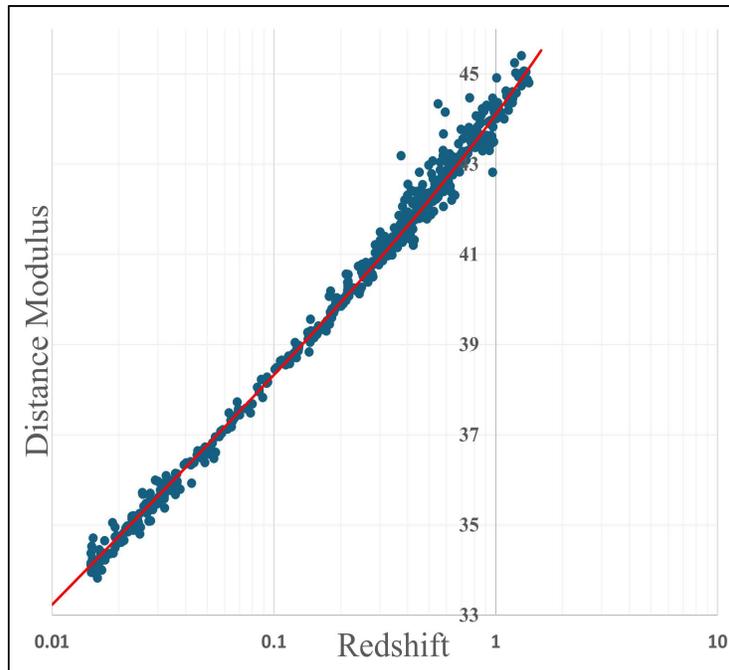


Figure 2. Plot of distance modulus vs. redshift using data from the Supernova Cosmology Project. The solid red line shows the model prediction.

where we have used the fact that in a matter-only universe $t_0 = 2/(3H_0)$.

Although we have so far neglected curvature in regard to all equations related to the expansion of the universe, with regard to luminosity distance we must take into account the negative curvature of the universe. In this case luminosity distance is related to the proper distance by

$$\begin{aligned}
 d_L &= S_k(r)(1+z) \\
 &= R_0(1+z)\sinh(r/R_0) \\
 &= R_0(1+z)\sinh(d_p(t_0)/R_0).
 \end{aligned} \tag{48}$$

Using expression (47) for the proper distance in equation (48) and taking $R_0 \approx c_0/H_0$ equal to the radius of the Hubble sphere we then obtain

$$\begin{aligned}
 d_L &= \frac{c_0}{H_0}(1+z)\sinh(\ln(1+z)) \\
 &= \frac{c_0}{2H_0} \left[(1+z)^2 - 1 \right].
 \end{aligned} \tag{49}$$

The luminosity distance is in turn related to the distance modulus defined as the difference between the apparent magnitude and absolute magnitude of a light source by

$$m - M = 5 \log_{10} \left(\frac{d_L}{1 \text{ Mpc}} \right) + 25. \tag{50}$$

Figure 2 shows a plot of distance modulus as a function of redshift based on publicly available data from the Supernova Cosmology Project Union 2.1 Compilation (N. Suzuki et al. 2012; V. Ruhlmann-Kleider et al. 2017). The red line shows our model fit based on a luminosity distance given by equation (49). Note that the model provides an excellent fit to the data even at high redshift where the Λ CDM model fails without dark matter and dark energy. We are only considering ordinary baryonic matter in our model.

This supernova data was previously interpreted as implying that the rate of expansion of the universe is accelerating (A. G. Riess et al. 1998; S. Perlmutter 2003). This conclusion was drawn based on the use of the Λ CDM model. In our model the scale factor increases as if the universe is geometrically flat and contains only baryonic matter but with an amplification factor of $1 + 3/(2\pi\alpha)$ for the baryonic mass density. In a flat universe with only baryonic matter

the deceleration parameter is $q = +0.5$, meaning the expansion of the universe is decelerating, not accelerating. The universe only appears to be accelerating due to the fact that the speed of light is decreasing in cosmological time according to $c = c_0 a(t)^{-1/2}$ and the negative geometric curvature affects the calculation of luminosity distance.

There is another fine point that must be made about our model before concluding this section. The fact that the speed of light varies in cosmological time naturally implies other physical constants must also vary in time. For example, the speed of light is related to the permittivity and permeability of free space, meaning that either one or both of those constants must also vary in time. We must check that the variability of other constants which are connected to the speed of light is consistent with experiment (J.-P. Uzan 2003).

The fine structure constant is given by

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}. \quad (51)$$

Experiments utilizing electronic transitions in atomic clocks have constrained the time variation of α to $\dot{\alpha}/\alpha \sim 10^{-17} \text{yr}^{-1}$, which strongly suggests that it is in fact a constant (N. Leefer et al. 2013). We have already shown that in our model we have $c \propto a(t)^{-1/2}$. Since $c = 1/\sqrt{\mu_0\epsilon_0}$ we could take $\mu_0 \propto a(t)^{1/2}$ and $\epsilon_0 \propto a(t)^{1/2}$ which would give the correct dependence of the speed of light on $a(t)$. Note from the expression for α in equation (51) if $c \propto a(t)^{-1/2}$ and $\epsilon_0 \propto a(t)^{1/2}$ then α does not depend on $a(t)$ as long as \hbar and e are true constants.

In our cosmological model we also assumed that the wavelength of atomic transitions does not depend on $a(t)$. In other words, we assumed that a hydrogen atom emitted light at the same wavelengths in the past as it does now. The redshift we observe is due solely to the expansion of the universe so that $1 + z = 1/a(t)$, and not due to any intrinsic variation in the properties of the hydrogen atom that would cause it to emit at different wavelengths in the past. The wavelength at which hydrogen emits is given by the expression

$$\lambda_{n,m} = \frac{4\pi\hbar}{m_e c \alpha^2} \left(\frac{1}{m^2} - \frac{1}{n^2} \right)^{-1}. \quad (52)$$

If α and \hbar are true constants, and $c \propto a(t)^{-1/2}$, then in order for hydrogen to emit at the same wavelength independent of the scale factor we must also have $m_e \propto a(t)^{1/2}$. It would seem reasonable to assume that all particle masses are varying with the scale factor, not only the electron mass. Note that the Bohr radius (the effective size of an atom) which is given by

$$a_0 = \frac{\hbar}{m_e c \alpha} \quad (53)$$

also remains constant in this framework.

Lunar laser ranging experiments have also constrained time variation in the gravitational constant to $\dot{G}/G \sim 10^{-13} \text{yr}^{-1}$ (J. G. Williams et al. 2004). However, this constraint is based on the assumption that mass is not varying which is not correct in our model. These laser ranging experiments are not actually measuring variations in G directly but variations in GM . Since masses are varying like $M \propto a(t)^{1/2}$, we must have $G \propto a(t)^{-1/2}$ in order for the product GM to be conserved. The dependence of various physical constants and their dependence on scale factor $a(t)$ is summarized in Table 1.

Finally, we note that in a cosmology with $c \propto a(t)^{-1/2}$ the speed of light would have been significantly higher in the early universe. Variable speed of light theories have been investigated in the past as an alternative to inflation to explain the isotropy of the CMB (A. Albrecht & J. Magueijo 1999). This could be considered another potential advantage of the cosmological framework we have presented.

3. CONSISTENCY WITH GENERAL RELATIVITY

General relativity has been very successful in making predictions which have been confirmed experimentally. Therefore, any theory of gravitation must reduce to general relativity in some limit. This evidence includes explaining the precession of the perihelion of Mercury (R. S. Park et al. 2017), gravitational redshift (R. V. Pound & G. A. Rebka 1959), deflection of light by the sun (F. W. Dyson et al. 1920), the detection of gravitational waves (B. P. Abbott et al. 2016), and the measurement of frame dragging effects in Earth orbit by Gravity Probe B (C. W. F. Everitt et al. 2011). We will now outline how our theory can be made compatible with the mathematics of general relativity.

The first important issue which must be addressed is causality. In general relativity, the speed of light is a universal constant which is completely disconnected from matter. In a universe devoid of matter the speed of light would still be

Table 1. Dependence of various natural constants on scale factor

Parameter	Symbol	Units	Variation
Gravitational Constant	G	$m^3/kg \cdot s^2$	$a(t)^{-1/2}$
Particle Mass	m	kg	$a(t)^{1/2}$
Fine Structure Constant	α		<i>constant</i>
Planck Constant	h	$kg \cdot m^2/s$	<i>constant</i>
Speed of Light	c	m/s	$a(t)^{-1/2}$
Gravitational Horizon	λ	m	$a(t)$
Permittivity of Free Space	ϵ_0	F/m	$a(t)^{1/2}$
Permeability of Free Space	μ_0	H/m	$a(t)^{1/2}$
Electric Charge	e	C	<i>constant</i>

$c = 2.99 \times 10^8 m/s$. By contrast, in our Machian approach, the wavelike Newtonian gravitational potential determines the speed of light. However, in a causal universe where the gravitational potential cannot propagate faster than light, it must be the time-retarded potentials which determine the speed of light. Here we run into a logical dilemma, which is that the time-retarded potentials would not know what speed to propagate at, since it is those same potentials that determine the speed of causality. In other words, the origin of causality cannot itself be causal.

There is only one solution to this problem, which is to assume that the Newtonian gravitational potentials responsible for determining the speed of light propagate instantaneously. The mass and distance used in calculating the potential GM/r must be calculated in the instantaneous rest frame of each mass. What we are proposing is that there are both nonlocal and local aspects to gravitation. The nonlocal aspect is that the instantaneous Newtonian gravitational potentials determine the speed of light in the flat-space Minkowski metric. The perturbations of the Minkowski metric derived from the Einstein equation obey causality and propagate at c .

If one is willing to accept that there is a nonlocal aspect to gravitation, then it is possible to combine our Machian framework with general relativity. Motivated by our discussion in Section 1 and equation (15), we will write the frame-invariant speed of light in the Einstein equation as

$$c_0^2 = |\phi| = \sqrt{\phi_R^2 + \phi_I^2}. \quad (54)$$

It is this frame-invariant speed of light which is determined by the instantaneous gravitational potentials. As we showed in Section 1, this expression will primarily be dominated by ϕ_I given by equation (20) which is determined by matter that is far away from the location of the observer. The contribution from ϕ_R given by equation (19) would be zero in a homogeneous and isotropic universe, but local inhomogeneity will result in small corrections to the frame-invariant speed of light in equation (54).

The second change we must make to the Einstein equation is in the cosmological constant term. The Einstein equation is usually written as

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \quad (55)$$

If we write the metric as the Minkowski metric plus a perturbation $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, we replace $g_{\mu\nu}$ with minus the metric perturbation $-h_{\mu\nu}$ in the cosmological constant term

$$G_{\mu\nu} - \Lambda h_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \quad (56)$$

We emphasize that c^2 is given by equation (54). The energy-momentum tensor contains factors of c , and the Minkowski metric is given by $\eta_{\mu\nu} = \text{diag}(-c^2, 1, 1, 1)$ with c^2 given by equation (54).

In a homogeneous and isotropic universe, coordinates may be chosen such that $h_{\mu\nu} = 0$ at any point in spacetime so that the cosmological constant term is eliminated in the Friedmann equations. This justifies our neglect of the cosmological constant term in the previous section on cosmology.

We will represent the components of $h_{\mu\nu}$ as

$$\begin{aligned} h_{00} &= -2\Phi \\ h_{0i} &= w_i \\ h_{ij} &= 2s_{ij} - 2\Psi\delta_{ij} \end{aligned} \tag{57}$$

where Ψ encodes the trace of h_{ij} and s_{ij} is traceless so that we have

$$\begin{aligned} \Psi &= -\frac{1}{6}\delta^{ij}h_{ij} \\ s_{ij} &= \frac{1}{2}\left(h_{ij} - \frac{1}{3}\delta^{kl}h_{kl}\delta_{ij}\right). \end{aligned} \tag{58}$$

We will assume $|h_{\mu\nu}| \ll 1$. In the transverse gauge the components of the Einstein equation in units where $c = 1$ can be written as follows (S. M. Carroll 2019)

$$G_{00} = 2\nabla^2\Psi + 2\Lambda\Phi = 8\pi GT_{00} \tag{59}$$

$$G_{0j} = -\frac{1}{2}\nabla^2 w_j + 2\partial_0\partial_j\Psi - \Lambda w_j = 8\pi GT_{0j} \tag{60}$$

$$\begin{aligned} G_{ij} &= (\delta_{ij}\nabla^2 - \partial_i\partial_j)(\Phi - \Psi) - \partial_0\partial_{(i}w_{j)} + 2\delta_{ij}\partial_0^2\Psi \\ &\quad - \square s_{ij} - \Lambda(2s_{ij} - 2\Psi\delta_{ij}) = 8\pi GT_{ij}. \end{aligned} \tag{61}$$

The (0,0) equation in equation (59) reduces to

$$\nabla^2\Psi + \Lambda\Phi = 4\pi G\rho. \tag{62}$$

In the case of dust where $T_{\mu\nu} = \rho u_\mu u_\nu$ from equation (61) we have $\Psi = \Phi$ so that equation (59) reduces to an inhomogeneous Helmholtz equation

$$\nabla^2\Phi + \Lambda\Phi = 4\pi G\rho. \tag{63}$$

The solution will have both real and imaginary parts as we showed in Section 1. Since we are only interested in the real part, the solution for the metric will be

$$g_{\mu\nu} = \eta_{\mu\nu} + \text{Re}\{h_{\mu\nu}\}. \tag{64}$$

For example the solution to equation (63) will be

$$\Phi(r) = -\frac{GM}{rc^2} [\cos(\beta r) + i \sin(\beta r)] \tag{65}$$

where $\beta = \sqrt{\Lambda}$. Since $g_{00} = \eta_{00} + h_{00}$ and $h_{00} = -2\Phi$ we have

$$g_{00} = -c^2 \left(1 - \frac{2GM}{rc^2} \cos(\beta r)\right) \tag{66}$$

which is consistent with our discussion from Section 1. The remaining diagonal elements of the metric are given by

$$g_{ii} = \left(1 + \frac{2GM}{rc^2} \cos(\beta r)\right) \tag{67}$$

which is the Schwarzschild solution in the weak-field limit, with the addition of the factor $\cos(\beta r)$ in the potential.

4. CONCLUSION

Motivated by the observation made by R. H. Dicke in 1957 that the speed of light has its origin in the gravitational potential of the rest of the universe, we developed a cosmological model based on a wavelike gravitational potential. This model is consistent with cosmological observations such as the flat rotation curves of galaxies and the prediction of the critical acceleration in MOND. The model also provides an excellent fit to high-redshift supernova data without the need for dark matter or dark energy. However, further work is still needed to better understand if this model in which various fundamental constants are varying in cosmological time is self-consistent with observations of the CMB, baryon acoustic oscillations, and constraints imposed by big bang nucleosynthesis. While the idea that the speed of light is determined through a nonlocal and noncausal mechanism may be difficult to accept, we should keep in mind that nonlocality is an inherent part of physics and remain open-minded to the possibility. Experiment has proven time and again that quantum mechanics is inherently nonlocal, despite Einstein's personal aversion to the concept. Accepting this idea may be a key in determining the connection between quantum mechanics and gravitation. In equation (4), the foundation of our theory, we expressed $c(\mathbf{r})^2$ in terms of kinetic and potential energy components. This suggests that there may be a link between the potential and kinetic energy carried by the ground state of quantum fields and gravitation, an idea we will explore in future work.

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