Black holes' edges are not like you were taught: Radial null geodesics in the Oppenheimer–Snyder metric

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Abstract

It is widely assumed that collapsing stars become black holes. It is also commonly accepted that the edge of a black hole, its so-called "event horizon", is a surface that can be crossed inward but not outward. However, a collapsing star is properly described by the Oppenheimer–Snyder metric, which corresponds to an eternal asymptotic process in Schwarzschild coordinates, and, as we show, its edge can be crossed outward, but not inward.

1 Motivation and scope

The aim of this note is modest and intentionally playful: to highlight a counterintuitive feature of the Oppenheimer–Snyder (OS) collapse when it is described using Schwarzschild time t. In that foliation, the process is *eternally asymptotic*: the stellar boundary $r_b(t)$ approaches the gravitational radius $r_0 = 2GM/c^2$ but never reaches it at finite t. Within this setting, we exhibit two late-time behaviors that run contrary to the textbook slogan for black holes:

- Late *inward* null signals emitted from outside fail to catch the moving boundary and cannot enter the collapsing body (Eq. (9)).
- Meanwhile, *outward* radial null geodesics emitted within a thin shell just inside the boundary *can* reach r_b and then propagate to larger radii, crossing r_0 without difficulty (Eqs. (12)–(16)).

Here the term "edge" refers, operationally, to the time-dependent boundary $r_b(t)$ of the OS dust cloud rather than to a global event horizon. The discussion is confined to the OS model (pressureless, homogeneous interior matched to an exterior Schwarzschild region) and to its late-time asymptotics in Schwarzschild t. For a broader critique of the idea that apparent non-formation in t is merely a "signal delay" masking a completed process in proper time, see [2, 3].

2 Summary of Oppenheimer–Snyder's model

In their original 1939 paper [1], Oppenheimer and Snyder start from a spherically symmetric metric of the form

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2\big(d\theta^2 + \sin^2\theta\, d\phi^2\big). \tag{1}$$

Outside the boundary r_b,

$$e^{\nu} = 1 - \frac{r_0}{r}, \qquad e^{\lambda} = \Big(1 - \frac{r_0}{r}\Big)^{-1}, \tag{2}$$

where "r_0 is the gravitational radius, connected with the gravitational mass m of the star by $r_0 = \dfrac{2mG}{c^2}$, and a constant". That is, in the surrounding vacuum the metric must coincide with Schwarzschild's solution.

Concerning r_b: "Near the surface of the star, where the pressure must be in any case low, we should expect to have a local observer see falling inward with a velocity very close to that of light; to a distant observer this motion will be slowed up by a factor $\left(1-\dfrac{r_0}{r_b}\right)^{1/2}$."

In order to otain an expression for the inner metric, Oppenheimer and Snyder "solve the field equations with the limiting form of the energy–momentum tensor in which the pressure is zero"; that is, a dust energy–momentum tensor.

At late times, they provide the following asymptotic equations:

$$\begin{aligned} e^{\lambda} &\sim 1-\left(\frac{r}{r_b}\right)^2\left\{e^{t/r_0}+\frac{1}{2}\left[3-\left(\frac{r}{r_b}\right)^2\right]\right\}^{-1}, \\ e^{\nu} &\sim e^{\lambda-2t/r_0}\left\{e^{t/r_0}+\frac{1}{2}\left[3-\left(\frac{r}{r_b}\right)^2\right]\right\}. \end{aligned} \tag{3}$$

3 Boundary and outer trajectories

In the asymptotic limit, radially inward (centripetal) trajectories in the Schwarzschild metric, both null and non-null, are of the form

$$r(t) \sim r_0 + (r_A - r_0)\, e^{K-\frac{1}{r_0}(t-t_A)}, \tag{4}$$

where r_A and t_A are the initial radius and time (i.e., the light signal or the freely falling particle were emitted from $r = r_A$ at $t = t_A$), and K is a trajectory constant that also depends on the initial conditions.

Remark. Equation (4) is proven in detail in our previous work [2]. The notation has been slightly modified for simplicity and to avoid symbol overlap with Oppenheimer–Snyder's paper.

For light:

$$K_{\text{light}} = \frac{r_A - r_0}{r_0}. \tag{5}$$

For massive particles:

$$K_p = K_{\text{light}} + \Delta K, \qquad \Delta K > 0, \tag{6}$$

where ΔK depends on the initial conditions, including the energy per unit mass of the freely falling particle.

Assume that the boundary of the collapsing star began to "fall" at $r_A = r_0 + \Delta r$ at $t_A = 0$. At late times,

$$r_b(t) \sim r_0 + \Delta r\, e^{\Delta K + \frac{\Delta r}{r_0} - \frac{t}{r_0}}. \tag{7}$$

At $t_A = \Delta t$, we emit a radial light signal from $r_A = r_0 + \Delta r$ toward the collapsing star. For the signal to intersect the boundary at some finite time, the emission must be early enough that the intersection occurs *before* both trajectories settle into their late-time parallel regime. Indeed, from Eqs. (7)–(8),

$$r_b(t) - r_0 \;\sim\; \Delta r\, e^{\Delta r/r_0}\, e^{\Delta K}\, e^{-t/r_0}, \qquad r_{\text{signal}}(t) - r_0 \;\sim\; \Delta r\, e^{\Delta r/r_0}\, e^{\Delta t/r_0}\, e^{-t/r_0}.$$

Thus, at late times they are parallel exponentials with the *same* decay rate and different constant prefactors; an intersection in that regime can only happen in the limiting case $e^{\Delta t/r_0} = e^{\Delta K}$, i.e., when $\Delta t/r_0 = \Delta K$. Consequently, an actual crossing must occur earlier, which requires

$$\Delta t \;<\; r_0\, \Delta K. \tag{9}$$

The parameter $\Delta K > 0$ precisely encodes that, during the initial transient (from $t = 0$ up to times of order r_0), the boundary falls more slowly than the asymptotic law, making it catchable by early light rays; once the late-time regime is reached, it becomes unattainable.

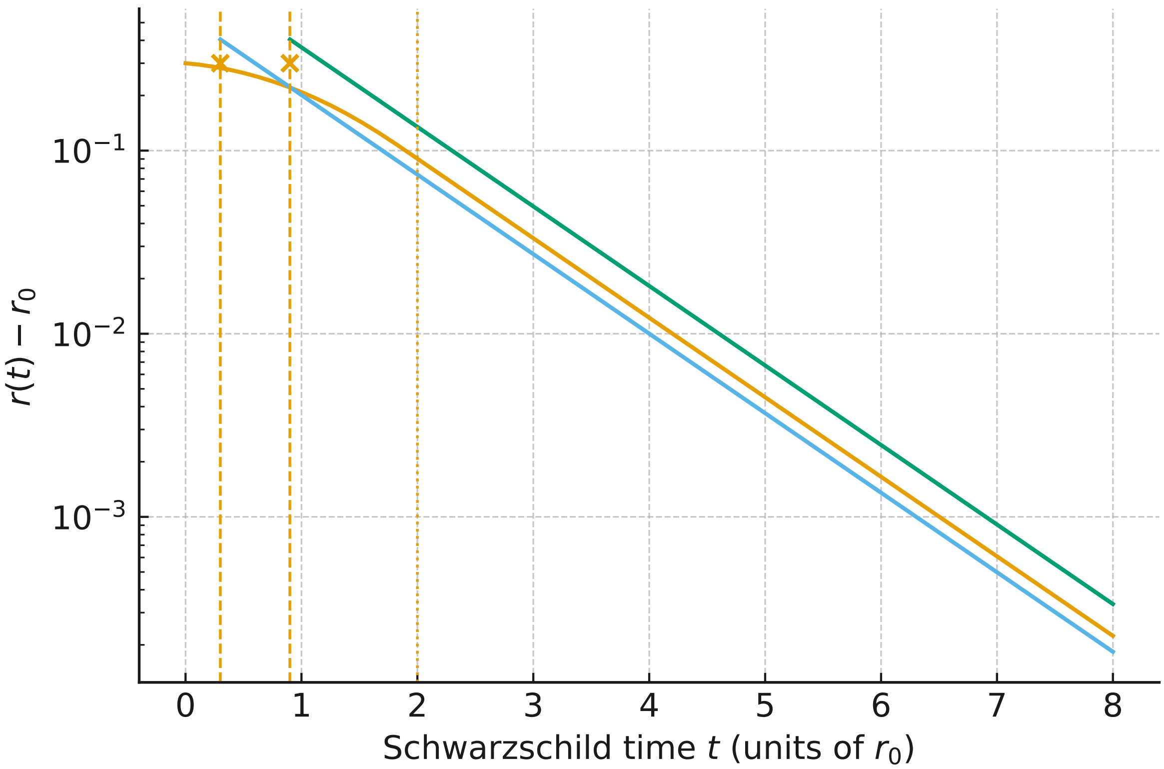

Figure 1: Illustrative semilog plot of $(r - r_0)$ versus Schwarzschild time t (units of r_0). The two null rays depict early crossing (left) and late non-crossing (right), as discussed around Eq. (9).

4 Inner trajectories

Radial null geodesics satisfy $ds^2 = 0$ and $d\Omega^2 = 0$, so that

$$\left(\frac{dr}{dt}\right)^2 = e^{\nu-\lambda} = e^{-2t/r_0}\left\{e^{-t/r_0} + \frac{1}{2}\left[3 - \left(\frac{r}{r_b}\right)^2\right]\right\}. \tag{10}$$

We are interested in outgoing (centrifugal) rays, i.e., those trying to escape from the collapsing star:

$$\frac{dr}{dt} = +\,e^{-t/r_0}\sqrt{e^{-t/r_0} + \frac{1}{2}\left[3 - \left(\frac{r}{r_b}\right)^2\right]}. \tag{11}$$

Since we restrict our study to late times, $e^{-t/r_0} \ll \frac{1}{2}\left[3 - (r/r_b)^2\right]$ and it can be neglected in the sum. Similarly, as those rays able to escape are located in a thin shell close to r_b, we may take $r/r_b \approx 1$, and then Eq. (11) becomes

$$\frac{dr}{dt} \sim e^{-t/r_0}. \tag{12}$$

If a ray departs from $r_1 = r_0 + \Delta r_1$ at time t_1, integrating yields

$$r(t) \sim r_0 + \Delta r_1 + r_0\left(e^{-t_1/r_0} - e^{-t/r_0}\right). \tag{13}$$

Imposing that it reaches r_b at time t_2 gives

$$\Delta r\, e^{\Delta K + \frac{\Delta r}{r_0}}\, e^{-t_2/r_0} = \Delta r_1 + r_0 e^{-t_1/r_0} - r_0 e^{-t_2/r_0}. \tag{14}$$

from which we solve for t_2:

$$t_2 = -\, r_0 \,\ln\left\{ \frac{\Delta r_1 + r_0 e^{-t_1/r_0}}{r_0 + \Delta r\, e^{\,\Delta K + \Delta r/r_0}} \right\}. \tag{15}$$

For t_2 to be real and finite, we need

$$\Delta r_1 + r_0 e^{-t_1/r_0} > 0, \quad \text{and thus:}$$

$$\Delta r_1 > -\, r_0 e^{-t_1/r_0}. \tag{16}$$

Therefore, all light rays emitted from $r_0\big(1 - e^{-t_1/r_0}\big) < r < r_b$ at time t_1 will be able to reach the boundary of the collapsing star and escape from it. Note that they also cross the gravitational radius r_0 without difficulty.

5 Discussion and limitations

This note deliberately adopts the coordinate-dependent, operational viewpoint in Schwarzschild time t for the OS model. As such, several caveats naturally arise:

- **Model idealization.** The OS interior is homogeneous pressureless dust. Realistic collapse includes pressure, viscosity, radiation, and deviations from spherical symmetry; these effects could modify late-time behavior near r_b.
- **Global vs. operational horizons.** The event horizon is a global null surface defined by the entire future of spacetime. Our "edge" is the time-dependent matter boundary $r_b(t)$; all claims here are explicitly about signal chasing and escaping relative to that boundary in Schwarzschild t.
- **Late-time asymptotics.** Eqs. (3), (11)–(16) are used in the $t \to \infty$ regime. Finite-time corrections may alter thresholds but not the qualitative dichotomy emphasized here.
- **On the "signal delay" narrative.** The common argument that completion in proper time with only a redshifted appearance in t explains horizon formation is contested in [2, 3], where additional kinematical and causal considerations are discussed.

In summary: in the OS setting with Schwarzschild t, late-time inward signals cannot catch the boundary, whereas suitably placed outward signals from just inside *can* reach the boundary and escape, crossing r_0 along the way.

References

[1] J. R. Oppenheimer and H. Snyder, On Continued Gravitational Contraction, *Physical Review* **56** (1939) 455–459. doi:10.1103/PhysRev.56.455

[2] M. Piñol Ribas, *Signal Transmission in the Schwarzschild Metric: an Analogy with Special Relativity*, viXra:2109.0158 (2021). https://vixra.org/abs/2109.0158

[3] M. Piñol Ribas, *Signal Transmission in the Schwarzschild Metric*, viXra:2502.0032 (2025). https://vixra.org/abs/2502.0032