# On the model of neutrinos oscillating at the speed of light, described by the Klein-Gordon equation of infinite degree.

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**Abstract**
This paper discusses equations containing high powers of the Klein-Gordon operator. In the limit of large powers, these equations are shown to have solutions that propagate at light speed and oscillate. These solutions exhibit a definite helicity. Such equations could be relevant for developing models of massless oscillating neutrinos.

**Keywords**:non-locality, neuitrinos.

Prior to the experimental discovery of neutrino oscillations, neutrinos were considered massless particles, much like photons. Oscillations are the primary experimental evidence that neutrinos have non-zero mass and cannot travel exactly at the speed of light.

This conclusion follows from experiment and the well-established and repeatedly verified mathematical model, in which solutions moving at the speed of light have a stationary form. The fact that neutrinos have mass has necessitated a revision of the Standard Model. The existence of mass and the inability to reach the speed of light imply either the obligatory existence of both left-handed and right-handed sterile neutrinos, or the introduction of new fields. However, only left-handed neutrinos and right-handed antineutrinos are observed experimentally. The existence of sterile neutrinos remains an open question. There is also no experimental confirmation yet that the neutrino is a Majorana particle.

Thus, the conclusion about neutrino mass generates certain theoretical problems.

If we consider the equations of motion from which it follows that plane-wave solutions traveling at the speed of light are necessarily stationary, we can note that these equations use only a single application of a mathematical differential operator. For example, a single d'Alembert operator for the massless Klein-Gordon equation. For example, a single d'Alembert operator $\Box = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial t^2}$ in the massless Klein-Gordon equation:

$$\Box u = 0 \tag{1}$$

Or a single power of the Weyl differential operator in the equation for a left-handed particle:

$$i\sigma^\mu \partial_\mu \psi_L = 0 \tag{2}$$

The most straightforward formal method to remain within left-handed spinors and obtain an equation for an oscillating solution propagating at the speed of light of the form

$$\psi = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\alpha(t-z)} \cos(\beta t) = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

where $u = e^{i\alpha(t-z)} \cos(\beta t)$, is a technique of compensating for the non-zero action of one degree of the d'Alembert operator $I\Box$ by the sum of the non-zero contributions from other degrees of the d'Alembert operator. Indeed, let us first consider an expression of the form

$$u - \Box\frac{u}{\beta^2} = e^{i\alpha(z-t)}\cos(\beta t) - \Box\frac{e^{i\alpha(z-t)}\cos(\beta t)}{\beta^2} = \frac{2i\alpha}{\beta}e^{i\alpha(z-t)}\sin(\beta t) \quad (3)$$

The identity matrix $I$ is omitted for convenience. Applying the same operator to the previous expression yields

$$u - \Box\frac{u}{\beta^2} - \frac{\Box}{\beta^2}\left(u - \Box\frac{u}{\beta^2}\right) = -\frac{2^2 i^2 \alpha^2}{\beta^2}e^{i\alpha(z-t)}\cos(\beta t). \quad (4)$$

If $|\frac{2i\alpha}{\beta}| < 1$, then with each successive application of these operations, the expression will decrease in magnitude. Proceeding further,

$$u - \Box\frac{u}{\beta^2} - \frac{\Box}{\beta^2}\left(u - \Box\frac{u}{\beta^2}\right) - \frac{\Box}{\beta^2}\left(u - \Box\frac{u}{\beta^2} - \frac{\Box}{\beta^2}(u - \Box\frac{u}{\beta^2})\right) = -\frac{2^3 i^3 \alpha^3}{\beta^3}e^{i\alpha(z-t)}\sin(\beta t). \quad (5)$$

By letting the number of such transformations tend to infinity, we obtain 0. These transformations can be simplified and written as (for even $N$)

$$\left(1 - \frac{\Box}{\beta^2}\right)^N u = (-1)^{N/2}\frac{2^N i^N \alpha^N}{\beta^N}e^{i\alpha(z-t)}\cos(\beta t). \quad (6)$$

Here, $\left(1 - \frac{\Box}{\beta^2}\right)$ is the operator of the standard Klein-Gordon equation with mass $\beta^2$ for a scalar function $v$:

$$\left(1 - \frac{\Box}{\beta^2}\right) v = 0. \quad (7)$$

Thus, for an infinite number of such transformations $N \to \infty$, we can write a peculiar equation by equating the result of their action to zero:

$$\left(1 - \frac{\Box}{\beta^2}\right)^\infty u = 0. \quad (8)$$

A solution to this kind of equation is a function of the form $e^{i\alpha(t-z)}\cos(\beta t)$, and hence also a function of the form

$$\psi = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\alpha(t-z)}\cos(\beta t) = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

Therefore, expression (8) can be written as

$$\left(I - I\frac{\Box}{\beta^2}\right)^\infty \psi = 0 \quad (9)$$

when replacing the d'Alembertians in equation (8) with the operator $I\Box$, where $I$ is the identity matrix.

Consequently, such equations significantly expand the class of their solutions compared to conventional equations with finite-order derivatives, for instance, second order for the standard wave equation.

Another reason for interest in such exotic relations is the possibility of a deterministic description of nonlocality. The violation of Bell's inequalities, which has been experimentally observed, is one of the main proofs of the existence of quantum nonlocality [1-3]. There are several interpretations of quantum nonlocality, the most widely accepted of which is the Copenhagen interpretation [4], which employs the concepts of measurement and instantaneous wave function collapse. One way to provide an alternative deterministic description of Lorentz-invariant nonlocality is through direct description using an equation with derivatives of infinite order.

## References


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